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THE PRINCIPLES OF THE DOCTRINE OF LIFE-ANNUITIES.

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THE

PRINCIPLES

OF THE

DOCTRINE

OF

LIFE-ANNUITIES;

EXPLAINED IN A FAMILIAR MANNER,

SO AS TO BE

Intelligible to Perſons not acquainted with the Doctrine of CHANCES;

AND ACCOMPANIED

With a VARIETY of NEW TABLES

Of the Values of ſuch ANNUITIES at ſeveral different Rates of Intereſt, both for Single Lives and for Two Joint Lives,

ACCURATELY COMPUTED FROM OBSERVATIONS.

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PREFACE.

THE following work is grown to ſo much greater a ſize than was at firſt intended, that it ſeems to be neceſſary to give my readers ſome previous information of its contents, that they may be able to diſtinguiſh between the different parts of it, (which are by no means all equally intereſting) and to ſelect thoſe which ſhall moſt excite their curioſity, and be thought moſt deſerving of their peruſal.

The principles of the whole doctrine are contained in the firſt 90 pages, which I would therefore recommend to the attentive peruſal of every reader. Of theſe the two firſt pages contain an explanation of the *data*, or grounds, upon which the computations of the values of annuities for lives are built. Theſe are, firſt, the decreaſe of the preſent value of a future ſum of money ariſing from the mere diſtance of the time at which it is to be paid, and the conſequent diſcount that is to be allowed to the purchaſer of it for prompt payment, (the quantity of which diſcount, it is evident, will depend on the rate of the intereſt of money;) and, ſecondly, the chance which, when the payment of ſuch future ſum is not made certain, but is to depend on the continuance of the life of a perſon of a given age, the grantor of it has of eſcaping the neceſſity of paying it at all by means of the death of the ſaid perſon before it becomes due; in order to determine which chance, it is neceſſary to have recourſe to certain tables of the ſeveral probabilities of the duration of human life at every different year of age, which have been formed from obſervations of the numbers of perſons who have died every year, in the courſe of a long ſeries of years, at different ages, in divers cities and pariſhes, and other numerous bodies of men.

In pages 3, 4, 5, 6, an account is given of two tables of theſe probabilities of life that appear to me to be better grounded, and conſequently fitter to be adopted, than any others; to wit, thoſe of Monſieur Kerſſeboom and Monſieur de Parcieux: and the tables themſelves are exhibited. And in pages 7, 8, 9, &c. —— 15, a compariſon is made between theſe two tables, in order to diſcover which of them repreſents human life, at ſeveral different ages, as the more durable, or makes the probabilities of living greater than the other: and it is found, upon the ſaid compariſon, that till the age of 70 years, or for all perſons under the age of 70 years, the probabilities of living are rather greater according to Monſieur de Parcieux's table than according to Monſieur Kerſſeboom's; but that after the age of 70 years, or for perſons above the age of 70 years, the probabilities of living are greater according to Monſieur Kerſſeboom's table than according to Monſieur de Parcieux's.

In page 15 a preference is given to Monſieur de Parcieux's table above that of Monſieur Kerſſeboom; and the reaſon of the ſaid preference is ſtated.

In page 16 mention is made of the *Breſlaw* table of the probabilities of the duration of human life, which was formed and publiſhed by the celebrated Dr. Edmund Halley towards the end of the laſt century, (and which, I believe, was the firſt table of the kind ever publiſhed) and of the *London* table of thoſe probabilities publiſhed by Mr. Smart and adopted by the learned Mr. Thomas Simpſon, of Woolwich, and likewiſe of two other tables of thoſe probabilities publiſhed by the Rev. Dr. Richard Price, of Newington-Green, which were derived from obſervations made at Norwich and Northampton. And in page 17 the reader is referred to the moſt celebrated writers on this ſubject for an explanation of the manner in which theſe tables of probabilities are formed from pariſh-regiſters, or other memorials, of the births and burials of mankind. And in pages 18 and 19, I have inſerted ſome very uſeful remarks of the late very learned Mr. de Moivre on the ſeveral different merits of the four tables beforementioned, of Dr. Halley, Monſieur Kerſſeboom, Monſieur de Parcieux, and Meſſieurs Smart and Simpſon. And with theſe remarks I conclude the account of the *data*, or grounds, upon which the computations of the values of life-annuities are to be founded.

I then

I then proceed to lay down the fundamental maxim of the doctrine of life-annuities, or to give a definition of what is meant by the *fair price* or *value* of a life-annuity; which I have done with as much care and exactneſs as I was able, in pages 20 and 21, articles XXI and XXII.

And, having thus ſettled the grounds of the enſuing methods of computing the values of annuities either for terms of years, or for lives, I then proceed to deliver thoſe methods themſelves by the ſolution of four problems that contain them.

The firſt problem is, "To find the preſent value of a future ſum of money, which is certainly to be paid at the end of one or more years, according to any given rate of intereſt." The ſolution of this problem, with an illuſtration of it by an example, and a corollary to it, is contained in pages 21 and 22. And in page 23 an account is given of Mr. Smart's moſt valuable tables of intereſt, which are computed upon the principles explained in this problem.

The ſecond problem begins in page 24, and ſhews "how we may find the preſent value of a future ſum of money that is to be received at the end of a given number of years in caſe a perſon of a given age ſhall then be living, but not otherwiſe, according to any given rate of intereſt." Of this problem two ſolutions are given; the firſt a particular one, in which the given number of years, at the end of which the money is to be paid, and the number of years in the age of the perſon on whoſe life it depends, are both ſpecified; the other general, in which the ſaid numbers are repreſented, (as is uſual in Algebra,) by two letters of the alphabet. The former of theſe ſolutions, it is hoped, will make the ſubject familiar to the reader, and ſerve to facilitate to him the peruſal of the latter ſolution, which will be neceſſary for his more perfect ſatisfaction. Theſe two ſolutions take up pages 24, 25, 26, and 27. And in corollary 2 of this problem the ſolution is extended to the determination of the preſent value of a number of ſuch future payments of a given ſum of money, to be made at the ends of ſeveral ſucceſſive years, in caſe of the continuance of the life of a perſon of a given age, or, in other words, to the determination of the value

of a life-annuity. After which, in pages 28, 29, 30, 31, and 32, the ſubject is further illuſtrated by examples of the actual computation of the values of life-annuities by the method deſcribed in the ſaid ſecond corollary; and then, in pages 33 and 34, there is a 3d corollary to this problem, relating to the value of a remote life-annuity that is not to take place till the end of a given number of years, together with an example of the calculation of the value of ſuch an annuity.

In this part of the work I have diſcovered an error in one of the arithmetical operations in page 30, which I muſt deſire the reader to correct. In dividing £31.8212 by 118, I have made the quotient £.2612, whereas it ought to be £.2696. This error is attended with ſome others derived from it, which will all be particularly mentioned in the liſt of *Errata*.

In pages 34, 35, 36, 37, 38, 39, and 40, I have given a ſcholium containing an account of the ſubſtance of a bill that was patronized by Sir George Savile and the late Mr. Dowdeſwell, (the member for Worceſterſhire) and other gentlemen of eminence and abilities, and which paſſed the Houſe of Commons in the ſpring of the year 1773, but was thrown out of the Houſe of Lords in conſequence of a ſpeech of Lord Camden. It was intended to operate as an encouragement to journeymen manufacturers, handicrafts-men, houſhold ſervants, and others, to induſtry and frugality, by offering them a ſafe and convenient method of employing the money they could ſave out of their earnings, in the purchaſe of remote life-annuities that were to take place in the latter periods of their lives, when they ſhould become leſs able to ſupport themſelves by their labour; which annuities were to have been ſecured upon the poor's rates of their reſpective pariſhes. As I ſtill think ſuch an eſtabliſhment is very practicable, and might be attended with very uſeful conſequences, I was willing to take this opportunity of again recommending it to the notice of the publick, and of removing, in the beſt manner I was able, the objections that had been made to it, and particularly that upon which the noble and learned Lord who oppoſed the bill, ſeemed to lay the greateſt ſtreſs, which was the danger occaſioned by it of bringing a new and heavy burthen upon the poor's rate.

After

After the ſcholium, in which this project for eſtabliſhing life-annuities in pariſhes is ſet forth and defended, there follow a 4th and a 5th corollary to this 2nd problem, ſhewing how the value of an immediate, but imperfect, life-annuity, not reaching to the utmoſt extremity of life, and likewiſe that of a diſtant and imperfect life-annuity, may be determined upon the principles explained in the ſolution of the problem. Theſe corollaries, with an example to each of them, are contained in pages 40 and 41, and conclude the whole doctrine of the computation of the values of life-annuities depending upon only one life.

Problem 3d relates to the computation of the preſent value of a future ſum of one pound, ſterling, that is to take place at the end of a certain number of years, provided two perſons of given ages ſhall then be living, and upon the ſuppoſition of a given rate of the intereſt of money. This problem is ſolved in a double manner, as well as the former, to wit, 1ſt. in the caſe of a particular example, and, 2ndly, in general terms. Theſe ſolutions are contained in pages 42, 43, 44, 45, 46, and 47. And from them are deduced, in pages 47 and 48, two corollaries which extend them to the determination of the preſent values of any future ſums (that are greater or leſs than one pound, ſterling,) and of a number of ſuch ſums to be paid at the end of every year during the continuance of the lives of both the perſons of given ages, or, in other words, of an annuity for their joint lives. And in pages 48, 49, 50, 51, and 52, the method of computing the value of an annuity for two joint lives, preſcribed in corollary 2, is illuſtrated by an example.

After the example to corollary 2, follow three more corollaries relating to annuities for two joint lives, to wit, coroll. 3, which relates to remote annuities for two joint lives; and coroll. 4, which relates to immediate, but imperfect, annuities for two joint lives; and coroll. 5, which relates to remote and imperfect annuities for two joint lives. Theſe three corollaries are contained in pages 52 and 53.

After theſe five corollaries the principles of the ſolution of this problem are extended, in coroll. 6, to the caſe of a future payment depending on the joint continuance of three lives. This, as well

as

as the former ſolutions, is done in a two-fold manner, to wit, firſt, in the caſe of a particular example, and afterwards in general terms. It takes up part of page 53, and all the 54th, 55th, and 56th pages, and part of page 57; it being rather a complex and difficult buſineſs, and much pains having been beſtowed upon it to make it as plain as poſſible. And in coroll. 7 the concluſions obtained in coroll. 6 are applied to the determination of the value of a number of ſucceſſive future payments of one pound each that are to be made at the end of every ſucceſſive year during the joint lives of three perſons of given ages, or, to the determination of the value of an annuity of one pound *per annum* for three joint lives. And this concludes the whole doctrine of the computation of the values of annuities for two, or more, *joint* lives.

The fourth problem relates to the value of a future ſum of money, the payment of which depends not upon the *joint* continuance of the lives of two perſons of given ages, but upon the continuance of the life of *either* of them; which value, it is evident, will be very different from the other, and will greatly exceed it. This problem is rather more difficult than the foregoing one, which relates to the joint continuance of the two lives. It is ſolved (as all the former ones,) in a two-fold manner, to wit, firſt, in the caſe of a particular example; and, ſecondly, in general terms. The former of theſe ſolutions is contained in pages 58 and 59; and the latter in pages 59, 60, and 61. From theſe ſolutions ſeveral corollaries are deduced.

In the 1ſt corollary (which is in page 62) it is ſhewn that the value of ſuch future payment of a given ſum of money, depending upon the longeſt of two given lives, has a remarkable relation to the value of the future payment of the ſame ſum of money depending upon the joint continuance of the ſame lives, and may be eaſily deduced from the ſaid latter value together with the values of the future payment of the ſame ſum of money depending upon the ſame two lives ſeparately; for that the value of ſuch future payment of a given ſum of money depending on the longeſt of two given lives, is equal to the exceſs of the ſum of the two values of the future payments of the ſame ſum of money depending upon the ſeparate continuance of the ſame two lives, above the value of the future payment

ment of the ſame ſum of money depending upon their joint continuance.

The 2nd corollary extends the ſolution of the problem, which related only to the value of a future payment of *one pound*, to the value of a future of *any other* ſum, greater, or leſs, than one pound.

The 3d corollary extends the ſolution of the problem to the determination of the value of a number of equal future payments of one pound each, to be received at the end of every year during the life of either of two perſons of given ages; or, in other words, to the determination of the value of an annuity of one pound a year during the longeſt of two given lives.

The 4th corollary ſhews that there is the ſame relation between the value of a life-annuity for the longeſt of two given lives, and that of the ſame annuity for the joint continuance of the ſame lives, together with thoſe of the ſame annuity for the two ſeparate lives, as there is between the value of a ſingle future payment depending on the continuance of either of the two lives, and the value of the ſame future payment in caſe of the continuance of both lives, together with the values of it in caſe of the continuance of each of the two lives ſeparately; or, that the value of an annuity of one pound a year for the longeſt of two given lives is equal to the exceſs of the ſum of the values of two ſeparate annuities of one pound a year for the ſame ſingle lives, above the value of an annuity of one pound a year for the joint continuance of both lives. This corollary is contained in pages 63, 64, 65, and 66, and is evidently of very great importance, inaſmuch as it enables us, when we have tables of the values of annuities for ſingle lives and for two joint lives ready computed to our hands, to derive from them the values of annuities for the longeſt of two given lives by the eaſy operations of addition and ſubtraction.

In pages 67 and 68 are three more corollaries, to wit, corollaries 5th, 6th, and 7th, which relate to the values of remote life-annuities for the longeſt of two given lives, and of immediate, but imperfect, life-annuities depending on the continuance of either of two given lives, and of remote and imperfect life-annuities depending likewiſe on the continuance of either of two given lives.

After

After theſe three corollaries (which are ſhort and eaſy) comes the 8th corollary to this fourth problem, which is much longer and more difficult to underſtand than any preceeding part of the book. It contains an extenſion of the ſolution of this problem to the caſe of a future ſum of money depending on the continuance *of any one of three* lives of given ages; which, though it is determined by the ſame principles as the former caſe of a future ſum of money depending on the continuance of *either of two given lives*, is greatly more complicated than that caſe, and conſequently requires a much longer inveſtigation. This inveſtigation is carried on (like all the former ſolutions,) in two different manners, namely, firſt, in the caſe of a particular example, and afterwards in general terms. The particular inveſtigation is contained in pages 68, 69, 70, 71, 72, 73, 74, and 75; and the general inveſtigation is contained in pages 76, 77, 78, and part of page 79. And, after theſe two inveſtigations, certain difficulties, that will probably ariſe in the attentive reader's mind after he has peruſed them, are examined and removed in pages 79 and 80, and the beginning of page 81.

Theſe inveſtigations are followed by three corollaries, namely, corollaries 9th, 10th, and 11th. The firſt of theſe corollaries is contained in pages 81 and 82, and is employed in ſhewing the relation of the value of a future payment of one pound depending on the longeſt of three lives of given ages to the value of a future payment of the ſame ſum of money depending on the joint continuance of all the three lives, and to the three different values of a future payment of the ſame ſum of money depending on the joint continuance of every two of the ſame three lives, and to the three different values of a future payment of the ſame ſum of money depending on the continuance of each of the ſame three lives taken ſeparately; the ſaid firſt value being equal to the exceſs of the ſum of the ſaid ſecond value (which relates to the joint continuance of all the three lives) and the ſaid three laſt values (which relate to the three ſingle lives) above the ſum of the ſaid third, fourth, and fifth values, which relate to the joint continuance of every two of the ſaid lives. The 10th corollary extends the foregoing inveſtigations to the caſe of a number of future payments of a ſum of one pound that are to be made at the end of every year during the continuance of any one of three lives of given ages, or, in other words, to the caſe of a life-annuity depending

depending on the longeſt of three lives of given ages. This corollary is very ſhort, and is contained in the firſt part of page 83. But the 11th corollary is a very long one, and takes up the remainder of page 83, and all the 84th, 85th, 86th, 87th, 88th, and 89th, pages, together with a part of page 90. But it is alſo very important and uſeful, and for that reaſon is ſo much enlarged upon. For it ſhews, that the value of an annuity of one pound a year for the longeſt of three lives of given ages, is equal to the exceſs of the ſum of the value of a like annuity for the joint continuance of the ſame three lives and the three values of the like annuity for the ſame three lives taken ſeparately, above the ſum of the three values of the like annuity for the joint continuance of every two of the ſaid lives; and conſequently that, whenever we have tables of the values of annuities for ſingle lives and for two and three joint lives ready calculated, we may eaſily deduce from them the values of annuities for the longeſt of three lives by mere addition and ſubtraction.

And here ends the fundamental part of the whole work, or the explanation of the principles of the doctrine of life-annuities. The remainder of the book, long as it is, is taken up in applications of theſe principles, and illuſtrations of them by numerous examples, in order to render theſe computations familiar to the reader, and in contrivances to abridge the labour of them, and in other ſuch matters, which are much leſs curious and important than the explanation of the principles themſelves of this uſeful ſpecies of computation, which are contained in the foregoing 90 pages. And therefore I expect that many of my readers will wholly paſs over many large parts of this remainder, which they will eſteem, and perhaps juſtly, not worth the trouble of peruſing them; though others of my readers, (who may have more leiſure and a greater liking to the ſubject,) will, I imagine, be inclined to go through every page of it. I ſhall therefore here continue the account of the contents of the book throughout this long and leſs intereſting remainder of it, in the ſame manner as I have done already with reſpect to the firſt and more important 90 pages of it; to the end that the former ſet of readers may eaſily determine before-hand which parts of it they will paſs over, and that even the latter claſs of readers may be the better able to judge which parts of it they will chuſe to read firſt, and which they will read with the moſt attention.

Pages 90, 91, 92, 93, 94, and 95 relate to the computation of the values of annuities for terms of years certain, and contain ſhort and convenient, algebräic, expreſſions of the ſaid values. And here I have inſerted a geometrical demonſtration of the rule for finding the ſum of the terms of a decreaſing geometrical progreſſion, conſiſting of a finite number of terms, as A, B, C, D, E, to wit, that it is equal to $\frac{AA - BE}{A - B}$, or to the quotient that ariſes by dividing the exceſs of the ſquare of the firſt, or greateſt, term above the rectangle, or product, contained under the ſecond term and the laſt, or leaſt, term, by the exceſs of the firſt, or greateſt, term above the ſecond term.

Page 96 contains a like ſhort and convenient, algebräic, expreſſion of the value of an immediate annuity of one pound a year for a given number of years depending on a ſingle life of a given age; and page 97 contains a like expreſſion for the value of ſuch an immediate annuity during the whole life of a perſon of a given age, which is only a particular caſe of the former expreſſion. And in the remainder of page 97, and in page 98 and part of page 99, are contained examples of the computation of the values of ſuch life-annuities by means of the ſaid algebräic expreſſions.

Then follows in pages 99 and 100 a like algebräic expreſſion for the value of *a remote* annuity for a given number of years, depending on a life of a given age; and in pages 100, 101, and 102 is contained an example of the computation of the value of ſuch a remote life-annuity by means of ſuch algebräic expreſſion.

All theſe examples of the computations of the values of life-annuities are the very ſame which were given in the former part of the work after the ſcholium of Prob. 2nd; only they are here performed by the help of the algebräic expreſſions here given of thoſe values, in order to make the uſe of thoſe expreſſions familiar to the reader. And it was in performing the computation of the laſt of theſe examples that I diſcovered the ſlip in an arithmetical operation which has been already mentioned in this preface to have been made in page 30, by making the quotient of the diviſion of 31.8212 by 118. be .2612 inſtead of .2696. See the note in page 102, Art. XCII.

After

After theſe firſt examples, which relate to the values of annuities for the lives of very old perſons, I have, in pages 103, 104, 105, 106, and 107, inſerted the calculation of the value of an annuity of one pound a year for the life of a perſon only 10 years old; which exhibits a clear view of the labour that is neceſſary to be employed in making theſe computations exactly, when the lives are young, and, conſequently, of the reaſon which, probably, induced Mr. De Moivre, and ſome other calculators of the values of life-annuities, to decline the foregoing accurate method of computing them, and reſort to hypotheſes concerning the probabilities of the duration of human life, which they knew to be not perfectly true, in order to facilitate the computation of them. The principal hypotheſis uſed by Mr. De Moivre for this purpoſe is deſcribed in a ſubſequent part of the work.

After the above-mentioned long example of the computation of the value of a life-annuity of one pound a year for the whole life of a child of 10 years of age, there is in Art. XCVII, pages 107, 108, an example of the computation of the value of an annuity of one pound a year for the firſt 30 years of the ſaid life; which is involved in, or makes only a part of, the foregoing computation.

Art. XCVIII, pages 108, 109, contains ſome remarks on the fluctuation of the price of the public funds, and of the intereſt of money in general; which makes it expedient to have the values of life-annuities calculated according to ſeveral different rates of intereſt.

In pages 109, 110, 111, and 112, an account is given of a very eaſy and convenient method of deducing the value of a life-annuity of one pound a year for a life of any given age from the value of the ſame annuity for a life that is older than the former by one year: by the help of which method a whole table of the values of a life-annuity of one pound a year for every age of human life, proceeding from the older ages to the younger by the conſtant difference of a year, may be computed with nearly the ſame labour as is neceſſary to obtain the value of the ſame annuity for the firſt, or youngeſt, life in the table. This method was firſt communicated to me by Dr. Price: but it was publiſhed in the year 1779 by Mr. William Morgan, the actuary to the Society for Equitable Aſſurances near

Black-friars Bridge, in his Treatiſe on the Doctrine of Annuities and Aſſurances on Lives, pages 56, 57; and it had been publiſhed before by Dr. Price himſelf in his Treatiſe on Reverſionary Payments, Note O of the Appendix, and likewiſe by Mr. Thomas Simpſon in his book on Life-annuities, Prob. 1, Coroll. 7; which laſt book was publiſhed ſo long ago as the year 1742. But I ſhould ſuſpect that it was not known to Mr. De Moivre, when he calculated his tables of the values of life-annuities. For, if it had, I ſhould imagine he would hardly have thought it neceſſary to have recourſe to a certain inaccurate hypotheſis concerning the probabilities of life, in order to diminiſh the labour of his computations, which would have been almoſt equally facilitated by the uſe of this excellent method. This method I have not only deſcribed, but explained and demonſtrated in the fulleſt and cleareſt manner I was able, in the ſaid pages 109, 110, 111, and 112.

In pages 112, 113, and 114 I have applied this method to the computation of the values of a life-annuity of one pound a year for lives of the ages of 8 years, 7 years, 6 years, 5 years, 4 years, and 3 years, from the value of a like annuity for a life of the age of 9 years (which had been computed by it in pages 110 111, from the value of a like annuity for a life of the age of 10 years,) and from each other. And afterwards, in pages 115, 116, 117, 118, 119, 120, &c. —— 130, in order to make this method ſtill more familiar to the curious and induſtrious reader (for all others, I ſuppoſe, will paſs over this part of the book,) I have applied it to the computation of a compleat table of the values of a life-annuity of one pound a year for every age of life, from the age of 93 years up to the age of 3 years, according to Monſieur de Parcieux's table of the probabilities of the duration of human life, and upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. And here I have diſcovered another error in one of the arithmetical operations, which pervades all the ſubſequent computations, and renders them in a ſmall degree erroneous. For at the bottom of page 123, in deducting the value of an annuity of one pound a year for a life of the age of 41 years from £15.580,939, or the value of the ſame annuity for a life of the age of 42 years, I have made the product of the multiplication of 16.580,939 by the fraction $\frac{643}{650}$ amount to £16.448,528,

£16.448,528, whereas in truth it amounts to only £16.402,375; in conſequence of which miſtake the quantity $\frac{1}{1.035} \times \frac{643}{650} \times$ £16.580,939, (which is equal to the value of an annuity of one pound a year for a life of the age of 41 years,) appears to be equal to $\frac{1}{1.035} \times$ £16.448,528, or £15.892,297, or 15*l.* 17*s.* 10*d.* inſtead of $\frac{1}{1.035} \times$ £16.402,375, or £15.847,705, or 15*l.* 16*s.* 11½*d.* which is leſs than the former ſum, 15*l.* 17*s.* 10*d.* by a difference of 10½*d.* This error neceſſarily affects all the following values of a life-annuity of one pound a year for lives of younger ages than 41 years, which are ſet down in pages 124, 125, 126, 127, 128, 129, and 130. But it is not of much importance with reſpect to the value of the annuity for a life of the age of 41 years, in which it occurs; the difference of 10½*d.* being but a trifling ſum of money in compariſon of 15*l.* 16*s.* 11½*d.* which is the true value of that annuity. And in all the following values of a life-annuity of one pound a year, for lives of younger ages than 41 years, the error, or difference of the falſe and true values, grows continually leſs and leſs. For the erroneous value of this annuity for a life of the age of 10 years, obtained in page 128, is only £20.752,981, or 20*l.* 15*s.* 0¾*d.* which exceeds the true value of it obtained before in Art. xciv, page 106, to wit, £20.739,25, or 20*l.* 14*s.* 9¾*d.* by only 3½*d.* and the erroneous value of the ſame annuity for a life of the age of 3 years, obtained in page 130, is only £19.987,654, or 19*l.* 19*s.* 9*d.* which exceeds the true value of it, obtained before in Art. cv, page 114, to wit, £19.978,21, or 19*l.* 19*s.* 6¾*d.* by only 2¼. I therefore hope the reader will be of opinion that it was hardly worth while, on account of this miſtake, to be at the trouble and expence of computing all theſe numbers over again, and of cauſing ſo many pages to be reprinted; more eſpecially as the true numbers, which expreſs the values of all theſe annuities, are all ſet down in two different places in the ſubſequent part of this work, namely, in Table XI, pages 214, 215, 216, 217, and in Table XV, page 224. See more concerning this miſtake in Art. cc, pages 218, 219.

After

After this computation of a compleat ſet of the values of a life-annuity of one pound a year for every year of human life in pages 115, 116, &c. ——— 130, the ſaid values are all ſet down in regular order in Table III, pages 131, 132, 133, 134, 135, and are expreſſed there in two different manners, to wit, in decimal parts of a pound in the ſecond column, and in pounds, ſhillings, and pence in the fourth column. And the differences of the ſaid values are likewiſe ſet down in the ſame table, and are expreſſed alſo in two different manners, namely, in decimal parts of a pound in the third column, and in pounds, ſhillings, and pence in the fifth column.

In this table, it is evident, the numbers relating to ages younger than 42 years are in a ſmall degree greater than they ſhould be, being the ſame with thoſe obtained in the foregoing computation. But theſe numbers the reader may correct, if he thinks fit, by means of Table XI, pages 214, 215, &c. or Table XV. page 224.

In pages 136, 137, 138, 139, 140, 141, are ſome obſervations which will be of uſe to a calculator of ſuch a table in diſcovering any arithmetical errors he may happen to have fallen into in the courſe of his computations; though it does not anſwer that purpoſe ſo compleatly as another method given us for that purpoſe by the above-mentioned ingenious Mr. Morgan, in his *Doctrine of Annuities and Aſſurances on Lives*, and which I have afterwards inſerted in this work and explained in as ample a manner as I could, in pages 208, 209, 210, 211, 212, and 213.

Having in theſe firſt 142 pages of the book treated only of the true and exact methods of computing the values of life-annuities from tables of the probabilities of the duration of human life, I next proceed to give ſome account of what Mr. De Moivre and other writers on this ſubject have called *the decrements of human life*, and of the obſervations which they have made on the courſe, or variations, of the ſaid decrements, as the ages of life increaſe; by which obſervations Mr. De Moivre was induced to adopt the hypotheſis concerning the probabilities of human life which has been already alluded to, (though not perfectly agreeable to the truth,) for the

ſake

ſake of facilitating his computations. Theſe *decrements of life* are the numbers of perſons who, out of a given original number of perſons, all of the ſame age, ſuppoſed to be living at a very early age, are repreſented, in tables of the probabilities of the duration of human life, as dying in the ſeveral ſucceeding years. In order to make the courſe of theſe decrements the more apparent, I have again inſerted the above-mentioned tables of the probabilities of life of Mr. Kerſſeboom and Monſieur de Parcieux, under the titles of Table IV and Table V, with an additional column in each, containing the decrements of life correſponding to every year of life, or the number of perſons who have died in the courſe of it. Theſe tables are contained in pages 144, 145, 146, 147, 148, and 149. And from them it appears that in ſeveral parts of human life, the decrements of life, or the numbers of perſons dying every year, are equal for ſeven or eight, and, in ſome parts, for twelve, or more, years together. Now, whenever this happens, the terms that expreſs the preſent values of the future payments of one pound each, which are to be received at the ends of thoſe years, will be a ſet of fractions, of which the numerators will form a decreaſing arithmetical progreſſion, and the denominators will form a decreaſing geometrical progreſſion; and therefore the ſum of all thoſe terms may be found at once by a ſingle ſhort expreſſion, without taking the pains to compute each of the terms ſeparately and then add them up into one ſum. The method of finding a fit expreſſion for this purpoſe is explained in the following 18 pages, to wit, pages 151, 152, 153, &c. —— 168. And then in pages 169, 170, an account is given of Mr. De Moivre's hypotheſis concerning the decrements of human life, and of the grounds upon which he conceived that the values of life-annuities computed from it would differ but in a ſmall degree from their true values.

Page 171 contains Table VI, which is a ſmall table of the values of a life-annuity of one pound a year for every fifth year of human life, computed by Dr. Halley from the Breſlaw table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 6 per cent. This was, probably, the firſt table of the values of life-annuities that ever was publiſhed. And with one of theſe values, but he does not ſay which, Mr. De Moivre tells us that he compared the value of the ſame annuity for a life of the

the ſame age computed from his own hypotheſis, and found the two values to be ſo very little different from each other that, for all uſeful and practical purpoſes, they might well be conſidered as the ſame. And from hence he concluded (but, I think, too haſtily,) that, in the buſineſs of computing the values of life annuities, he might ſafely neglect the tables of probabilities deduced from obſervations, and proceed upon the ground of his own hypotheſis.

Pages 172, 173, are employed in ſhewing how the conjectural probabilities of the duration of human life, reſulting from Mr. De Moivre's hypotheſis, may be compared with the real probabilities of the ſame, as exhibited in Monſieur de Parcieux's table of them. And then in pages 174, 175, a compariſon is made between the ſaid probabilities, by ſetting down in Table VII, in two columns adjoining to each other, the numbers that expreſs the probabilities of life according to Monſieur de Parcieux's table, and likewiſe thoſe which expreſs the ſame probabilities according to Mr. De Moivre's hypotheſis: and at the end of this table of compariſon between the ſaid probabilities, ſome obſervations are made, in pages 176, 177, and 178, concerning the differences between them and the manner in which the values of life-annuities, computed from them, would be affected by them.

Afterwards in page 178, Art. CLIX, it is ſhewn that the numbers exhibiting the probabilities of human life according to Mr. De Moivre's hypotheſis may be reduced to much ſmaller numbers, and even to ſuch as are only equal to thoſe which are neceſſary to compleat the numbers of years in the ſeveral ages of life to the number contained in the utmoſt ſuppoſed extent of life, and which, from this circumſtance, Mr. De Moivre calls the *complements of life*. And then in Table VIII, page 179, thoſe loweſt numbers expreſſing the probabilities of human life according to Mr. De Moivre's hypotheſis, and which are the complements of life to its greateſt poſſible extent, are ſet down in order, ſo as to enable the reader to compute the value of any life-annuity according to Mr. De Moivre's hypotheſis with the greateſt eaſe and readineſs.

After Table VIII, I have added the computations of the values of a life-annuity of one pound a year for the ſeveral ages of 1 year, 3 years,

3 years, 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35 years, 40 years, 45 years, 50 years, 55 years, 60 years, 65 years, and 70 years, from Table VIII, or Mr. De Moivre's hypotheſis, upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. in order to compare them with the true values of the ſame annuity for the ſame lives obtained before from Monſieur de Parcieux's table of probabilities. Theſe computations are contained in pages 180, 181, 182, &c. —— 193. And the reſults of them, or the values of an annuity of one pound a year for lives of the ſaid ages obtained by them, are ſet down in Table IX, page 194, together with the correſponding true values of the ſame annuity for the ſame lives according to Monſieur de Parcieux's table of probabilities, and with the differences between the ſaid true values and the ſaid conjectural values ſet down in an adjoining column.

By means of this table the reader is fully enabled to judge of the degree of exactneſs with which Mr. De Moivre's hypotheſis enables us to find the values of life-annuities. And it appears that theſe conjectural values of life-annuities differ from their true values in many ages of life by more than a whole year's purchaſe, and after the age of 65 years by more than two years purchaſe, though about the age of 45 years they are very nearly equal to each other. Theſe obſervations, with a few more of the ſame kind, are made in page 195, and a general concluſion is drawn from them againſt the expediency of ever making uſe of Mr. De Moivre's hypotheſis in computing the values of life-annuities.

In pages 196, 197, 198, an account is given of another method of computing the values of life-annuities, which is different both from that above-explained in Prob. II, and its corollaries, and likewiſe from Mr. De Moivre's method by means of his hypotheſis. This method was given by Mr. Weyman Lee, a Barriſter at Law and Bencher of the Inner Temple, in a book he publiſhed on life-annuities in the year 1738. It is exceedingly erroneous, and gives the values of life-annuities, throughout the greateſt part of human life, much greater than they ſhould be. In the younger ages of life the difference of the erroneous value from the true one amounts to about 3 years purchaſe. Yet the principle, upon which Mr. Lee grounds this method, has ſomething in it that is plauſible at firſt ſight, and

is apt to miſlead the underſtanding with an appearance of truth and ſimplicity, unleſs it be examined with a great degree of attention. And for this reaſon I have thought it worth mentioning and examining at conſiderable length. The method itſelf is deſcribed in page 197; and ſome remarks on the difference of the values of life-annuities, reſulting from it, from the true values of the ſame annuities, are made in page 198. And in pages 199, 200, 201, I have inſerted a whole table of the values of a life-annuity of one pound a year for every age of life, computed by this method, upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. This table is called Table X; and in it are ſet down, beſides theſe erroneous values of a life-annuity of one pound a year for the different ages of life, computed according to Mr. Lee's method, the true values of the ſame annuity for the ſame ages computed in the manner above-explained in Prob. II, and its corollaries, and likewiſe the differences between the ſaid erroneous values and true values, in an adjoining column.

In page 202 ſome remarks are made on this table; and in the five following pages, to wit, pages 203, 204, 205, 206, and 207, the reaſons alledged by Mr. Lee in ſupport of his method of determining the values of life-annuities are ſtated in his own words, and examined, and the fallacy, or inſufficiency, of them is pointed out.

From this account of what has been delivered concerning Mr. De Moivre's hypotheſis and Mr. Weyman Lee's erroneous method of valuing life-annuities, the reader muſt ſee that all the laſt 66 pages, from page 142 to page 207, incluſively, are not neceſſary to the right underſtanding of the true method of computing the values of life-annuities, but only to the knowledge of the moſt remarkable methods and opinions that have been adopted for this purpoſe by writers of note. And therefore ſuch of my readers as have no curioſity for this hiſtorical kind of information, will do well to paſs over all theſe pages, and go at once from page 142 to page 208. And ſuch of them as are only deſirous of underſtanding the right method of computing the values of theſe annuities, without wiſhing to make theſe computations familiar to them, will do well to paſs over in like manner the foregoing 28 pages from the end of page 114 to the end of Art. CXXIV in page 142.

In

In page 208 I return to the more uſeful part of the ſubject, to wit, the true method of computing the values of life-annuities according to the principles of Prob. II, and its corollaries, and have employed that and the five following pages, to wit, pages 209, 210, 211, 212, and 213, in explaining a method invented by the ingenious Mr. Morgan above-mentioned for proving the truth of the computations of the values of life-annuities, (when they are deduced one from the other in a regular ſucceſſion proceeding from the older lives to the next younger,) as faſt as they are made. This method is ſo ſatisfactory, and anſwers the purpoſe, for which it was invented, ſo compleatly, that nothing further need be wiſhed for on the ſubject. And therefore I thought it would be proper, not only to explain it and demonſtrate the truth of it, but alſo to illuſtrate it by a number of examples. This is done in Table XI, pages 214, 215, 216, and 217; in which table all the values of a life-annuity of one pound a year for every different age of life, are regularly computed one from the other by means of the expreſſion given by Mr. Morgan for that purpoſe, from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. and ſet down in the ſecond column of the table over againſt the correſponding ages in the firſt column, and three other ſets of numbers, which (according to this laſt-mentioned method of Mr. Morgan, that is explained in pages 208, 209, 210, 211, and 212,) are neceſſary to prove the truth of the former numbers, (or of the values of the annuities in the ſecond column,) are ſet down in regular order in a third, a fourth, and a fifth column. Theſe values may be depended upon, as exact: and it was by means of this table, (which carries its own proofs with it) that I found out the miſtake I had made in page 123, where the value of an annuity of one pound a year for a life of the age of 41 years is made to be equal to £15.892,297, inſtead of £15.847,705; of which miſtake and its conſequences I have already ſaid enough.

Pages 221, 222, 223, &c. —— —— 232, contain twelve tables of the values of a life-annuity of one pound a year for all the different ages of human life, from the age of 3 years to the age of 93 years, accurately computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, according to the twelve following rates of intereſt; to wit, 2 per cent. $2\frac{1}{2}$ per cent.

3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. This is a greater variety of tables of the values of life-annuities than has ever yet been publiſhed. And I will add, for the ſatisfaction of my readers, and of all ſuch perſons as ſhall be inclined to make uſe of theſe tables, that theſe values have, all of them, been confirmed by the computation of the correſponding proof numbers, in the manner juſt now mentioned; tho' it has not been thought neceſſary to inſert thoſe numbers in the tables.

Theſe tables are all computed upon a ſuppoſition that the annuities are to be paid only once in every year, namely, at the end of every new compleat year after the purchaſe of them. But it more uſually happens that life-annuities are made payable every half-year, and ſometimes even every quarter of a year. In theſe caſes the values of them will be ſomewhat greater than when they are payable only at the end of every year. But to determine exactly *how much* greater the values of them will be in thoſe caſes than when they are paid at the end of every year, (as they are ſuppoſed to be in the foregoing tables,) is a difficult and intricate queſtion, and hardly worth the trouble of the ſolution. Yet, for the ſake of my more curious and induſtrious readers, I have taken the firſt of theſe caſes, to wit, that of annuities paid every half-year, into conſideration, and have given a ſolution of it by approximation, which I find to agree, in the reſult of it, with that given by Mr. Thomas Simpſon, in his doctrine of annuities; the reſult, or concluſion, of both inveſtigations being, "that a life-annuity payable half-yearly is worth more than the ſame annuity, when payable only at the end of every year, by ſomewhat leſs than a quarter of a year's purchaſe." The inveſtigation of this queſtion in the following work takes up no leſs than 27 pages, to wit, pages 233, 234, 235, &c.——259, and is extremely perplexing and unentertaining. Yet I do not know how to make it leſs ſo. I therefore adviſe the generality of my readers to paſs it over, and take mine and Mr. Simpſon's word for the truth of the concluſion juſt now mentioned, to wit, "that the difference of the values of a yearly and half-yearly life-annuity is leſs than a quarter of a year's purchaſe." I have followed this advice myſelf with reſpect to the values of life-annuities that are payable quarterly; not having taken the trouble of inveſtigating the ſaid values, but believing, upon the authority of the ſame Mr. Thomas Simpſon, that thoſe annuities are worth about three eighth parts of a year's purchaſe more than thoſe which are paid only once a year, or one eighth part of a year's purchaſe more than thoſe that are payable every half-year.

From

From the ſubject of the above-mentioned tedious and perplexing inveſtigation, I was led to another inquiry, that bears ſome analogy to it, but is much eaſier and more entertaining, namely, concerning the limit of the ſum of money to which the intereſt made of a given ſum of money in a year, or in any other given time, may be made to increaſe, by increaſing the number, and diminiſhing the lengths, of the terms for which the money is lent, ſo as to improve the money during the ſaid given time, by means of the ſaid repeated loans, at compound intereſt. This problem and its corollaries are contained in pages 260, 261, 262, &c.——271. And I imagine, the reader will be entertained by them. The concluſion from the whole is, that the advantages that may be made by lending a ſum of money for very ſmall portions of a year, (as, for example, for 52nd parts, or weeks,) and then receiving the intereſt due upon it, and immediately lending both principal and intereſt at the ſame rate of intereſt; I ſay, the advantages that may be made by this method of proceeding above the intereſt that will ariſe from it by lending it at once for the whole year, are ſo very ſmall as not to be worth attending to. Thus, for example, the intereſt of £ 100 in a whole year, if thus lent for only a week at a time, at the intereſt of 5 per cent. *per annum*, will be leſs than £ 5. 2*s*. 6*d*. or will exceed the intereſt of it, when lent at once for a whole year, by leſs than half a crown; and the utmoſt poſſible increaſe of this intereſt, or, to ſpeak more properly, the *limit* of the ſaid increaſe, or the quantity to which the intereſt of the ſaid ſum of money approaches continually, and to which it may be made to come as near as we pleaſe, as the number of the ſhort terms, for which the money is lent, is increaſed *ad infinitum*, is only £ 5. 2*s*. 6½*d*. together with the 158th part of a penny.

In pages 271, 272, a remarkable analogy is pointed out between the infinite ſeries that had been found in pages 268, 269, 270, for the limit of the increaſe of a ſum of money at compound intereſt in the courſe of a year, and a certain ordinate of a logarithmick curve. And in pages 272, 273, 274, &c.——278, the ſaid infinite ſeries, expreſſing the ſaid limit of the increaſe of a ſum of money at compound intereſt, is further conſidered and explained.

Pages 278, 279, contain a ſuggeſtion that it might be an uſeful publick meaſure for the government, when it eſtabliſhes annuities for a term of 30 years, to enlarge the grant of ſuch annuities to the extent of the lives of the purchaſers of them, in conſequence of ſuch

ſuch an additional ſum of money, to be paid by the ſaid purchaſers for ſuch enlargement of their intereſt in the ſaid annuities, as ſuch enlargement may be fairly worth. Now the value of ſuch an enlargement of a purchaſer's intereſt in an annuity for 30 years certain is evidently the value of a remote life-annuity of the ſame annual amount for the life of the ſaid purchaſer, to commence at the expiration of the ſaid 30 years. And therefore in pages 280, 281, 282, I have ſhewn how (without having recourſe to new computations upon the principles delivered in Prob. II. and itscorollaries) the value of ſuch a remote life-annuity may eaſily be derived from that of an immediate life-annuity for the life of the ſame perſon, which, if the intereſt of money is ſuppoſed to be either 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, 8, 9, or 10 per cent. muſt be contained in one of the preceeding twelve tables of the values of life-annuities in pages 221, 222, 223, &c. ——— 232. And then, in pages 283, 284, 285, 286, and 287, I have inſerted four tables of the values of ſuch remote life-annuities, computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, at the four following rates of intereſt, to wit, $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent. which are the rates that are moſt likely to be adopted in the eſtabliſhment of any government ſecurities.

In this part of the work I have digreſſed from the doctrine of life-annuities, (which is the proper ſubject of it,) to make ſome reflections on the National Debt, and the moſt likely methods of paying off a part of it. Theſe reflections, together with various applications of the foregoing doctrine of life-annuities to different methods which may be taken for ſhortening the duration of this enormous debt, by converting a part of the perpetual annuities of which it conſiſts, into annuities of a finite duration, ſuch as life-annuities, or annuities for terms of years and likewiſe for life, and the like,) extend through no leſs than 102 pages, namely, from page 287 to page 389. And after this very long, but, I hope, not uſeleſs, digreſſion, I have reprinted a very valuable pamphlet on the ſame ſubject of the National Debt, that was written by Sir Nathaniel Gould, a merchant of eminence, and a Director of the Bank, in the reign of King George the 1ſt. It was firſt publiſhed in the year 1726, and went through at leaſt four editions: and Dr. Price has told us in his *Appeal to the Publick on the Subject of the National Debt*, that he ſets ſo high a value

value upon it that he wiſhes he could put it into every hand in the kingdom. This declaration of ſo eminent a writer induced me firſt to peruſe it, and, afterwards, finding it anſwer the high character that he had given of it, to cauſe it to be reprinted. And, to the end that gentlemen may be able to procure it without purchaſing this large volume upon a ſubject that may not happen to intereſt them, I have cauſed an additional number of copies of this pamphlet to be printed off, which may be bought ſeparately. This pamphlet ends at page 447. And in the following pages I have added a few reflections connected with the ſubject of the ſaid pamphlet, concerning the expediency of an equal aſſeſſment of the land-tax. Theſe reflections are contained in pages 448, 449, 450, 451, 452, 453, and 454. And with them ends every thing in the book that relates to the values of life-annuities for *ſingle lives*.

In pages 455, 456, a ſhort and convenient, algebraïc, expreſſion is given of the value of an annuity of one pound a year for a certain number of years, depending on the *joint continuance of two lives* of given ages; and a like expreſſion is given for the value of ſuch an annuity for the *whole* of the joint continuance of the ſaid lives; which is but a particular caſe of the former expreſſion. And in pages 457, 458, 459, and 460, examples are given of the computation of the values of ſuch annuities, depending on two joint lives, by means of the ſaid expreſſions.

In pages 460, 461, a like ſhort and general, algebraïc, expreſſion is given for the value of a remote annuity of one pound a year depending on the joint continuance of two lives of given ages; and in pages 462, 463, 464, an example is given of the computation of the value of ſuch a remote annuity by means of the ſaid expreſſion.

Then, in pages 465, 466, 467, and 468, I have ſet forth and explained another very uſeful method, with which we have been favoured by the before-mentioned Mr. Morgan, in his *Doctrine of Annuities and Aſſurances on Lives*; by which the computations of the values of an annuity for two joint lives are as much facilitated as the computations of the values of annuities for ſingle lives are by his former method, which is mentioned and explained in the foregoing

going part of this work, in pages 109, 110, 111, and 112. For by it we are enabled to deduce with great eaſe (by an expreſſion ſimilar to that obtained in Art. 109, 110, 111, and 112,) the value of an annuity of one pound *per annum* for the joint continuance of any two lives whatever, from the value of the ſame annuity for the joint continuance of two lives that are reſpectively older than the two former lives by one year; ſo that a whole table of the values of ſuch an annuity for two lives, that differ from each other by any given number of years, may be computed with almoſt as little labour as the two youngeſt lives poſſible that differ by the ſame number of years. Thus, for example, if the difference of the ages is 30 years, it will be almoſt as eaſy, by means of this expreſſion of Mr. Morgan, to compute a whole table of the values of an annuity of one pound a year for the joint continuance of two lives of the ages of 93 years and 63 years, 92 years and 62 years, 91 years and 61 years, 90 years and 60 years, and ſo on up to the ages of 33 years and 3 years, as to compute the value of a ſingle annuity of 1 *l.* a year for the joint continuance of the ſaid laſt two lives, which are of the ages of 33 years and 3 years. The great uſefulneſs of ſuch a method in facilitating the buſineſs of computing compleat tables of the values of annuities for two joint lives, need not be pointed out. I have therefore not only ſtated it, and demonſtrated the truth of it, in the fulleſt and cleareſt manner I was able, in pages 464, 465, 466, and 467, but have afterwards illuſtrated it by a great number of examples in pages 468, 469, 470, 471, 472, 473, 474, and 475; ſo that I doubt not the attentive reader will have no difficulty in making himſelf compleatly maſter of it.

This method of Mr. Morgan for computing the values of annuities for two joint lives one from another, proceeding from the older lives to the younger, is evidently liable to the ſame inconvenience as his method of computing the values of annuities for ſingle lives one from another, proceeding likewiſe from the older lives to the younger; namely, that when an error has once crept into the calculation of the value of any one of the annuities, it will affect thoſe of all the remaining ones, that relate to younger lives. But Mr. Morgan has provided a ſimilar and a very compleat remedy for this inconvenience in both caſes, by ſhewing us how, by a certain counter-calculation accompanying that of the values of the annuities themſelves, we

we may conſtantly verify thoſe values as faſt as we obtain them. This remedy in the caſe of annuities for ſingle lives I have deſcribed and demonſtrated in pages 209, 210, 211, 212, and 213, as has been already mentioned. And here, in pages 476, 477, 478, 479, 480, &c. —— 487, I have explained and illuſtrated by examples the method preſcribed by Mr. Morgan for effecting the ſame purpoſe in the caſe of annuities for two joint lives. And in pages 488, 489, I have inſerted a table, to wit, Table XXVIII, containing the values of an annuity of one pound a year for the joint lives of two perſons of the ages of 94 years and 84 years, 93 years and 83 years, 92 years and and 82 years, 91 years and 81 years, 90 years and 80 years, and the ten next younger ages, up to thoſe of 80 years and 70 years, computed one from the other by the above-mentioned expreſſion of Mr. Morgan, which is explained and illuſtrated in pages 464, 465, &c. —— 475; and containing likewiſe, in three adjoining columns, to wit, in columns 3^{d}, 4th, and 5th, the proof-numbers which the ſaid laſt-mentioned method of Mr. Morgan for confirming his calculations of the ſaid values, and which is explained and illuſtrated in pages 476, 477, &c. —— 487, directs us to compute: which proof-numbers manifeſtly confirm the values of the annuities ſet down in the foregoing, or third, column; ſo that no doubt can be entertained of the truth of the ſaid values.

And thus, by the help of Mr. Morgan's excellent methods, the computation of a table of the values of annuities for two joint lives, whoſe ages differ by the ſame number of years, is rendered as eaſily practicable, and as little liable to error, as that of a table of the values of annuities for ſingle lives.

But it is evident that the number of the differences that may be ſuppoſed to ſubſiſt between the ages of two perſons is ſo great that it is almoſt impoſſible to compute tables enough to exhibit the values of an annuity of one pound a year for two joint lives of all the different ages that may be taken. For this would require, at one rate of intereſt only, no fewer than 94 different tables, to wit, a table for two perſons whoſe ages differ by 93 years, another for two perſons whoſe ages differ by 92 years, a third for two perſons whoſe ages differ by 91 years, a fourth for two perſons whoſe ages differ by 90 years, and ſo on till we come to two perſons of the ſame age. And

conſequently, for the twelve different rates of intereſt above-mentioned, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. it would require twelve times 94, or near 1200, ſuch tables: which it will, probably, never be thought worth while to cauſe to be computed. With reſpect therefore to the values of annuities for two joint lives, and ſtill more with reſpect to thoſe of annuities for three joint lives, (the varieties of which are ſtill greater, and in a prodigious degree, than thoſe of annuities for two joint lives,) we muſt be contented with a few tables accurately computed by the methods above-mentioned, for lives whoſe ages differ by only a moderate number of differences, far ſhort of all the differences poſſible; and we muſt have recourſe to methods of approximation for determining the values of ſuch of theſe annuities as the tables we are poſſeſſed of do not exhibit. It is, however, deſireable to have as many tables of the values of annuities for two joint lives accurately computed, as can be conveniently procured. And I think we have ſome reaſon to be ſurprized that ſo few tables of this kind have hitherto been publiſhed. To ſupply this defect in ſome degree I have procured ten different tables of the values of a life-annuity of one pound a year for two joint lives to be accurately computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. to wit, one table for two lives of the ſame age, another for two lives whoſe ages differ by five years, a third for two lives whoſe ages differ by 10 years, and a fourth, fifth, ſixth, ſeventh, eighth, ninth, and tenth, for two lives whoſe ages differ by 20 years, 30 years, 40 years, 50 years, 60 years, 70 years, and 80 years. Theſe tables are contained in pages 492, 493, 494, &c.——501; and in page 502 is given the value of an annuity of one pound a year for two joint lives of the ages of 93 years and 3 years; which is the only annuity that is poſſible, according to Monſieur de Parcieux's table of the probabilities of life, with a difference of 90 years between the ages.

After theſe tables, and a few remarks upon the difficulty of procuring compleat ſets of the values of all poſſible annuities for two joint lives, which make the ſubject of Art. ccccx in pages 502, 503, I have ſhewn how, by a certain method of *interpolation* between the values

values of ſuch annuities for two joint lives as are contained in the foregoing tables, we may derive very uſeful approximations to the values of other annuities for two joint lives that are not contained in them, at the ſame intereſt of $3\frac{1}{2}$ per cent. This method of interpolation is deſcribed and illuſtrated by examples in pages 503, 504, 505, 506, &c.——512. And the degree of exactneſs with which the numbers obtained by this method of interpolation, approach to, or exhibit, the true values of the annuities ſought by them, is inquired into and ſhewn in pages 512, 513, 514, &c.——518. And by thoſe means the reader is tolerably well enabled to find the value of one pound a year for the joint continuance of two lives of any ages, when the intereſt of money is $3\frac{1}{2}$ per cent.

I have then added the like number of tables of the values of an annuity of one pound a year for two joint lives, computed likewiſe from Monſieur de Parcieux's table of the probabilities of life, when the intereſt of money is $4\frac{1}{2}$ per cent. Theſe tables are contained in pages 519, 520, 521, 522, &c.——528; and they exhibit the values of the ſaid annuity of one pound a year for two joint lives, when the ages of the lives are equal, and when the differences of the ſaid ages are (as before) 5 years, 10 years, 20 years, 30 years, 40 years, 50 years, 60 years, 70 years, and 80 years. And in Art. CCCCXXXIII, page 529, is added the value of the ſaid annuity of one pound a year for two joint lives of the ages of 93 years and 3 years, which is the only annuity that can be ſuppoſed, according to Monſieur de Parcieux's table of the probabilities of life, for two lives whoſe ages differ from each other by 90 years.

By the help of this ſecond ſet of tables, and the method of interpolation before explained, the reader is tolerably well enabled to find the value of an annuity of one pound a year for the joint continuance of two lives of any ages whatſoever, when the intereſt of money is $4\frac{1}{2}$ per cent.

After this ſecond ſet of tables of the values of annuities for two joint lives, which relates to the rate of $4\frac{1}{2}$ per cent. I have endeavoured to extend the uſe of theſe two ſets of tables, which relate to the rates of $3\frac{1}{2}$ and $4\frac{1}{2}$ per cent. to the determination of the values of annuities for the ſame lives at the contiguous rates of intereſt of

3 per cent. 4 per cent. and 5 per cent. This is done by a method of proceeding not very different from that of the foregoing method of interpolation, and which may be called *The method of interpolation and continuation*. This method is deſcribed and explained in pages 530, 531, 532. And in pages 532, 533, 534, &c. —— 539, an enquiry is made into the degree of exactneſs to which this method exhibits the values ſought by it.

Theſe methods of *interpolation* and *interpolation and continuation*, will only enable us to find near values of life-annuities for two joint lives at the ſame rates of intereſt as thoſe which are computed in the two foregoing tables, to wit, the rates of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. and the three neareſt rates of intereſt to them, and which differ from them by only $\frac{1}{2}$ per cent. to wit, 3 per cent. 4 per cent. and 5 per cent. In other rates of intereſt, that differ more conſiderably from thoſe at which the tables are computed, it ſeems neceſſary to have recourſe to ſome other method of approximating to the values of theſe annuities, that has no relation to thoſe tables. This I have endeavoured to do in pages 540, 541, &c. —— 546; in which I have given an expreſſion of the value of an annuity of one pound a year for two joint lives, which I conceive to be, in moſt caſes, a tolerable approximation to the true value of ſuch annuity, and which may be applied with an equal probability of ſucceſs in all the different rates of intereſt. This expreſſion of the value of ſuch an annuity for two joint lives is derived, by conjectural, but probable, reaſonings, from the value of the like annuity for the older of the two ſingle lives; which latter value will, if the intereſt of money is either 2 per cent. or $2\frac{1}{2}$ per cent. or 3 per cent. or $3\frac{1}{2}$ per cent. or 4 per cent. or $4\frac{1}{2}$ per cent. or 5 per cent. or 6 per cent. 7 per cent. 8 per cent. 9 per cent. or 10 per cent. be contained in one of the twelve tables of the values of annuities for ſingle lives in pages 221, 222, 223, 224, &c. —— 232. And by this means the ſaid twelve tables of the values of annuities for ſingle lives are rendered ſubſervient to the purpoſe of finding the values of the like annuities for two joint lives: which ſeems to be a very convenient and uſeful principle to proceed upon in the valuation of theſe latter annuities, and of other annuities for more than two lives, which are ſtill more numerous and complicated; provided the values of annuities for joint lives found by this method approach tolerably near to their

their true values. But, whether they do, or not, approach thus nearly to the ſaid true values, can only be determined by trying them in a few caſes, in which the true values have been actually computed, and comparing them with the ſaid true values. This inquiry is made, with reſpect to the above-mentioned expreſſion of the near value of an annuity of one pound a year for two joint lives (given in pages 540, 541, &c. —— 546,) in pages 546, 547, 548, &c. —— 556, by applying the ſaid expreſſion in a variety of examples to the inveſtigation of near values of ſome of the annuities which are contained in the two foregoing ſets of tables, and comparing the near values thereby obtained with the true values of the ſame annuities ſet down in the ſaid tables: and the reſults of this compariſon are ſet down in a table in pages 557 and 558. Theſe reſults ſuggeſt a correction of the aforeſaid expreſſion (given in pages 540, 541, &c. —— 546,) by multiplying it into the fraction $\frac{104}{100}$, which makes the ſaid expreſſion approach (in moſt caſes) conſiderably nearer than before to the true values of the annuities ſought by it. The reſults of the compariſon of the near values, ariſing from this corrected expreſſion, with the true values of the ſame annuities, are ſet down in another table in pages 558, 559, and 560. And in pages 560, 561, a remark is made on the ſaid laſt table, in which the ſaid corrected expreſſion, to wit, $\overset{\text{£}}{A} \times \frac{104}{100} \times \frac{P^{1}}{P} \times \frac{g \times P^{1} - h \times P^{1}}{g \times P^{1} - h \times P^{11}}$, is finally recommended as a tolerably exact method of obtaining a near value of an annuity of one pound a year for two joint lives, when the intereſt of money is either leſs than 3 per cent. or greater than 5 per cent. and the before-mentioned methods of *interpolation*, and *interpolation and continuation* conſequently cannot be applied.

The remaining part of the book relates to the values of annuities depending on three lives. In Art. CCCCLXXI, pages 561, 562, 563, a ſhort and convenient, algebraick, expreſſion is given of the value of an annuity of one pound a year for a given number of years, provided three lives of given ages ſhall all continue ſo long in being; which expreſſion, whenever the ſaid given number of years is equal to the greateſt poſſible number of years during which the oldeſt of the three lives can be extended, will become the expreſſion of the value of ſuch an annuity during the whole joint continuance of the three lives.

lives. And in Art. CCCCLXXIII, pages 564, 565, 566, an example is given of the computation of the value of ſuch an annuity for the joint continuance of three lives of the ages of 75 years, 80 years, and 85 years, by means of the ſaid expreſſion. But, as the computation of theſe values of annuities for three joint lives occaſionally, or as we want them, by means of the aforeſaid expreſſion, in the manner exemplified in Art. CCCCLXXIII, is in moſt caſes, and eſpecially when the lives are all young, extremely tedious; — and the immenſe variety of combinations with each other, which the ages of three different lives will admit of, makes it almoſt impoſſible to compute any competent numbers of theſe values; —— it is ſtill more neceſſary than in the caſe of annuities for two joint lives, to have recourſe to a ſhorter, though leſs accurate, method of computing them. With this view I have endeavoured to find a method of deriving the value of an annuity of one pound a year for three joint lives of given ages from the value of the ſame annuity for the joint continuance of the two older of the ſaid lives, which I have ſuppoſed to be either contained in ſome of the tables that have been computed for two joint lives, or to be derivable from thoſe tables by the above-mentioned methods of *Interpolation* and *Interpolation and Continuation*, or to be obtainable by means of the corrected expreſſion recommended for this purpoſe in pages 560, 561. And I have ſucceeded in ſome degree, (though not ſo well as I could wiſh,) in this endeavour, having found an expreſſion for the value of ſuch an annuity for three joint lives, which exhibits the ſaid value, for the moſt part, within leſs than a 20th part of the truth. The method in which this expreſſion is obtained, is explained in pages 568, 569, and 570. And the following twenty pages, to page 590, are employed in making a variety of trials of the degree of exactneſs with which this expreſſion exhibits the values of annuities for three joint lives, by comparing the values of ſuch annuities, obtained by means of the ſaid expreſſion, with the values of the ſame annuities accurately computed, as they are contained in certain tables of Mr. Morgan and Dr. Price. From all theſe trials, taken together, I have ventured to conclude in Art. CCCCXCV, page 590, that it ſeems reaſonable to conjecture that the ſaid expreſſion given in pages 568, 569, and 570, to wit, $\overset{\pounds}{B} \times \frac{P^{\prime}}{P} \times \frac{g \times P^{\prime} - b \times P^{\prime}}{g \times P^{\prime} - b \times P^{\prime\prime}}$, will always give

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us a tolerable approximation to the true value of an annuity of one pound a year for three joint lives of any ages whatſoever.

In pages 590, 591, 592, another method of approximating to the value of an annuity of one pound a year for the joint continuance of three lives of given ages, is delivered; which was publiſhed, and, I preſume, invented, by the late very learned mathematician, Mr. Thomas Simpſon, of Woolwich. And in pages 592, 593, an example is given of the computation of the value of ſuch an annuity by this method in the caſe of three equal joint lives, all of the age of 60 years: and the near value of the ſaid annuity, obtained by this computation, is found to be ſomewhat nearer than the near value of the ſame annuity reſulting from the former expreſſion, $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, to the true value of that annuity. And in the following five pages, to wit, 594, 595, 596, 597, and 598, further trials are made of the exactneſs of this method of Mr. Simpſon; from all which it is concluded in page 599, that the ſaid method of Mr. Simpſon is, for the moſt part, more exact, as well as always much ſhorter and eaſier to practice, than the other approximation, by means of the expreſſion $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, and upon the whole, deſerves to be preferred to it. Yet, as there are now and then ſome inſtances in which the other method, by means of the expreſſion $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, comes nearer to the truth than this method of Mr. Simpſon, I think it is convenient to be poſſeſſed of both methods, to the end that in doubtful caſes we may reſort to one of them as a kind of confirmation of the reſult obtained by the other, to a certain degree of exactneſs.

In Art. ccccccx, page 599, it is obſerved, that, in all the trials of Mr. Simpſon's approximation to the values of annuities for three joint lives, which have been made in the preceeding pages, the differences of the near values thereby obtained, from the true values of the ſame annuities, reſpectively, ſeldom exceed an eighth part of a year's

year's annuity: which is a ſufficient degree of exactneſs for all practical purpoſes, and conſequently is a great recommendation of the ſaid method.

The five remaining pages of the book, to wit, pages 600, 601, 602, 603, 604, relate to the values of annuities for the longeſt of two, or three, lives of given ages. As it has been ſhewn in the former part of this work, in the 4th and 11th corollaries to Prob. IV, that the values of all annuities of this kind may be derived from thoſe of the ſame annuities for the ſame lives taken ſingly and jointly, it is ſhewn that, if there be three lives of different ages, and A be put for the value of an annuity of one pound a year for the youngeſt life, and B for the value of a like annuity for the middle life, and C for the value of a like annuity for the oldeſt life; and AB be put for the value of a like annuity for the joint continuance of the firſt, or youngeſt life, and the middle life, and AC for the value of the like annuity for the joint continuance of the firſt, or youngeſt life, and the oldeſt life, and BC for the value of a like annuity for the joint continuance of the middle life and the youngeſt life; and ABC be put for the value of a like annuity for the joint continuance of all the three lives; I ſay, it is ſhewn that, if this notation be uſed, the value of a like annuity of one pound a year for the longeſt of the firſt and ſecond lives will be equal to $A + B - AB$, and the value of a like annuity for the longeſt of all the three lives will be equal to $A + B + C + ABC - AB - AC - BC$, or $A + B + C - AB - AC - BC + ABC$. Then, in Art. CCCCCXII, pages 601, 602, an example is given of the computation of the value of an annuity of one pound a year for the longeſt of two lives of the ages of 20 years and 30 years, by means of the expreſſion $A + B - AB$; and in Art. DXIII, pages 602, 603, 604, an example is given of the computation of a like annuity of one pound a year for the longeſt of three lives of the ages of 20 years, 30 years, and 40 years, by means of the expreſſion $A + B + C - AB - AC - BC + ABC$. After which example this treatiſe on life-annuities is concluded in page 604.

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The Appendix to this treatiſe on Life-annuities conſiſts of the abovementioned *Bill for eſtabliſhing Life-annuities in Pariſhes*, which the late Mr. Dowdeſwell brought into the Houſe of Commons in the year 1773, together with the two tables of the values of life-annuities, at the intereſt of 3 per cent. for the uſe of pariſhes in *London* and in the country, which Sir *George Savile* procured to be computed, under the inſpection of Dr. *Price*, for the purpoſes of the ſaid bill, and which were conſidered as a part of it. The reaſons for reprinting theſe tables and the bill to which they refer, are ſtated in page 605.

A LIST of ERRATA, to be corrected in the following Work, and of ſome AMENDMENTS to be added to it.

In page 30, line 5th, inſtead of "£.2612" read "£.2696".

In page 31, line 5th, inſtead of "£3.7212, or £3. 14*s*. 5*d*." read "£3.7296, or £3. 14*s*. 7*d*."

In the ſame page, line 8th, inſtead of "£.2612" read "£.2696";
and in line 9th, inſtead of "£.7814" read "£.7898";
and in line 11th, inſtead of "£3.7212" read "£3.7296";
and in line 13th, inſtead of "£.7814" read "£.7898".

In page 32, dele the note at the end of Art. XXXIII, beginning thus, *N. B. This reſult is greater*, and containing ſix lines.

In page 34, line 4th, inſtead of "£.2612" read "£.2696";
and line 14th, inſtead of "£.8650" read "£.8734".

In page 48, Art. XLIX, line 3d, after the word "annuity" inſert the words "of one pound a year."

In page 96, line 4th, dele the comma after the word "as".

In page 107, Art. xcvi, lines 8th and 9th, instead of the words "almost one half a year's payment, and consequently would be worth more than 21 years purchase," read these words, "about one half of one of those half-yearly payments, or a quarter of a year's annuity, and consequently would be worth very nearly 21 years purchase."

In page 123, the last line, instead of "$\frac{1}{1.035} \times$ £16.448,528 =) £15.892, 297, or £15. 17s. 10d." read "$\frac{1}{1.035} \times$ £16.402,375 =) £15.847, 705, or £15. 16s. 11½d.

In page 136, line 2nd, after the words "one pound" insert the words "a year."

In page 137, line 7th, after the words "one pound" insert the words "a year."

——— line 18th, after the word "pence" place a semicolon; and add these words, "which does not differ greatly from 2 pence, 3 pence, farthing, and 3 pence, half-penny, which are the differences of the four next preceeding differences."

In page 159, line 5th, instead of "$\frac{1}{r-1} \times \frac{1}{r^n \times r-1}$" read "$\frac{1}{r-1} - \frac{1}{r^n \times r-1}$."

In page 161, Art. cxli, line 1st, instead of "n" read "a"; and in line 6th, or the line at the bottom of the page, instead of "equal to a" read "equal to o".

In page 168 the operation of the algebraic division of the series $\frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3}$ — &c. — $\frac{1}{nr^n - 1}$ by $r - 1$ might have been set forth more fully by inserting the last term of it, to wit, — $\frac{1}{nr^n - 1}$, after the &c. and at the same time inserting the corresponding terms in the quotient and in the series of successive dividends. If this had been done, (which, perhaps, it would have been better to do) the whole operation would have been as follows.

$r - 1)$

$$r-1\Big)\frac{n-1}{n}-\frac{1}{nr}-\frac{1}{nr^2}-\frac{1}{nr^3}-\&c.-\frac{1}{nr^{n-1}}\Big(\frac{n-1}{nr}+\frac{n-2}{nr^2}+\frac{n-3}{nr^3}+\frac{n-4}{nr^4}+\&c.+\frac{n-\overline{n-1}}{nr^{n-1}},$$

$$\frac{n-1}{n}-\overline{\frac{n-1}{nr}}$$

$$*+\frac{n-2}{nr}-\frac{1}{nr^2}$$

$$+\frac{n-2}{nr}-\overline{\frac{n-2}{nr^2}}$$

$$*+\frac{n-3}{nr^3}-\frac{1}{nr^3}$$

$$+\frac{n-3}{nr^2}-\overline{\frac{n-3}{nr^3}}$$

$$*+\frac{n-4}{nr^3}-\&c.$$

or $+\frac{1}{nr^{n-1}}$.

$$+\frac{n-\overline{n-1}}{nr^{n-2}}-\frac{1}{nr^{n-1}}.$$

$$+\frac{n-\overline{n-1}}{nr^{n-2}}-\frac{1}{nr^{n-1}}$$

$$*\qquad\qquad *$$

In page 216, in the fifth column of numbers, the 22nd number, 4,697,003, inſtead of a comma place a full ſtop " . " after the higheſt figure, 4.

In page 225, Table XVI, in the fourth column of numbers, in the firſt, the eighth, and the fifteenth numbers, which anſwer to the ages of 34 years, 41 years, and 48 years, place full ſtops, " . " " . " and " . " after the two higheſt figures, 16, 14, and 13, of thoſe numbers reſpectively.

In page 240, line 4th, after the &c. inſert theſe words, " and with $r\frac{1}{2}\times\frac{\mathcal{L}1}{2P}\times$. "

And in line 5th, inſtead of " $\frac{\mathcal{L}}{1}{2P}$ " read " $\frac{\mathcal{L}}{1}{P}$. "

And in line 7th, inſtead of " $\frac{\Pi E-N+1}{rE-N+1}$ " read " $r\frac{1}{2}\times\frac{\mathcal{L}}{1}{2P}\times\frac{\Pi E-N+1}{rE-N+1}$. "

And

And in lines 8th and 9th, after "$+\frac{P^{IV}}{r^5}+$ &c." read as follows;

"$+\frac{r^{\frac{1}{2}}}{4}\times\frac{\pounds}{V}+r^{\frac{1}{2}}+\frac{\pounds}{2P}\times$ the term $\frac{\Pi^{E-N+1}}{r^{E-N+1}}$ = (if we neglect this laſt quantity $r^{\frac{1}{2}}\times\frac{\pounds}{2P}\times\frac{\Pi^{E-N+1}}{r^{E-N+1}}$ on account of its extreme ſmallneſs) &c." as in the book.

In page 300, line 5th from the bottom, inſtead of "this third method" read "this fourth method".

In page 302, line 20th, inſtead of "$\frac{1000,0000}{554,338}$" read "$\frac{1000,000}{554,338}$".

In page 368, the laſt line, in the number £4,711,195,1609, place a full ſtop before the four laſt figures, thus, 4,711,195.1609.

In page 502, line 6th, after the words, "by 90 years," inſert theſe words, "for the joint continuance of which an annuity will be of any value,"

——— In line 9th, at the end of Art. ccccix, add theſe words, "See below, page 529, Art. ccccxxxiii."

In page 560, line 2nd from the bottom, inſtead of "to the inſtances" read "in the inſtances".

In page 581, Art. ccccLxxxv, line 11th, inſtead of "livesin" read "lives in".

In page 592, Art. ccccxcix, line 7th, after the word "intereſt" inſert the word "of".

In page 597, in the marginal abſtract of Art. ccccc v, line 10th of the ſaid abſtract, after the words "than the near values" inſert the words, "of the ſame annuities".

THE

THE

PRINCIPLES, &c.

ARTICLE I.

THE doctrine of life-annuities is by no means of so abstruse and difficult a nature as many people are apt to imagine. A moderate share of common sense, or capacity to reason justly, and a knowledge of common arithmetick, are all the qualities that are necessary to a right understanding of the principles on which it is founded; even so far as to be able to compute the value of any proposed annuity for any given life or number of lives, if a person is disposed to undergo the labour of performing all the necessary arithmetical operations that arise in such a computation. To explain these principles in an easy and familiar manner, so as to make them intelligible to as many readers as possible, without having recourse to Algebra or the books written on the doctrine of chances, is the design of the following pages: which, as the subject of life-annuities is a matter of very general concern, will, I flatter myself, be considered by the publick as an useful and commendable undertaking. Design of this tract.

II. A life-annuity is a set of equal sums of money to be paid at certain future times, at the distance of a year one after the other, during the whole course of the life of the person for whose life the annuity is granted. Thus, if a man grants an annuity of 100*l.* a year to a young man of twenty years of age, he thereby undertakes to pay him one sum of 100*l.* at the end of one year from the time of making him the grant, and another sum of 100*l.* at the end of two years from the same time, and a third sum of 100*l.* at the end of three years from the same time, and a fourth sum of 100*l.* at the end of four years from the same time; and so on, paying a new sum of 100*l.* at the end of every following year throughout the whole life of the grantee. If therefore we can find the values of these several single payments of 100*l.* each, which are to be made to the grantee of the life-annuity at the end of the several future years of his life, but of which no part is to be paid to his executors, or other representatives, when he is dead, we may, by adding all these values together, find the value of the sum of all these payments, or of the whole life-annuity. We must there- Definition of a life-annuity.

fore endeavour to find ſome method of determining the preſent value of a ſingle future payment of any given ſum of money to be made to a perſon of any given age at the diſtance of any given number of years, or when he ſhall have attained to any greater given age. Now this value will depend upon two circumſtances; which muſt therefore be previouſly agreed upon, before it can be known: namely, the rate of intereſt at which money may be improved, and the probability of the duration of human life. For, the higher the intereſt of money is, the leſs ought the granter to take for his grant of a given ſum of money, to be paid at a future time: becauſe, when the intereſt of money is high, he may increaſe ſuch leſſer ſum of money, in the interval of time between the grant and the future payment, ſo as to make it amount to as great a ſum at that time as would have been produced by a greater ſum of money received at the time of making the grant, and improved likewiſe at compound intereſt during the ſame interval, if the intereſt of money had been lower. And the greater is the frailty of human life, or the greater the probability that the grantee of the propoſed future payment will die before it becomes due, the leſs alſo ought to be the ſum of money paid to the granter for his grant of it, becauſe he will run a ſmaller riſque of being obliged to pay it. We muſt therefore in every queſtion of this ſort determine beforehand at what rate of intereſt the money paid to the granters for ſuch future payments may be improved, and with what degree of probability it may be expected that the perſons to whom they are granted will, or will not, live till they become due.

Of the preſent value of a future contingent payment of a given ſum of money.

III. Now, as to this latter circumſtance of the degree of probability that a perſon of a given age will, or will not, live to any other given age, or till the ſum of money granted to him becomes due, it is obviouſly in many caſes a matter of very great uncertainty, and will be often very different in different perſons of the ſame age. The chance which a man of 30 years of age, who is in good health and leads a temperate and quiet life in the country, has to live 20 years, or till he is 50 years of age, is evidently much greater than that of another man of the ſame age of 30 years, and of the ſame degree of health and vigour of body, who is going into a hot and unhealthy climate, to which he has never been accuſtomed, as, for example, to Senegal in Africa: and it is likewiſe greater than that of another man of 30 years of age and of the ſame degree of health and vigour, but who lives in a capital city and in ſcenes of pleaſure and debauchery; and ſtill more evidently it is greater than that of another man of 30 who is of a weakly and unhealthy conſtitution of body, or who by his daily occupation is expoſed to many dangers of his life, from which the generality of mankind are exempt, as is the caſe with ſoldiers and ſailors in time of war. But theſe are circumſtances out of the reach of calculation, and muſt be left to be conſidered by the perſons who grant and purchaſe life-annuities according to their own judgement and diſcretion in

Of the probability of the duration of human life at a given age.

in the particular caſes in which they occur. All that can be done by any general rules upon this ſubject, is to eſtimate the degree of probability with which it may reaſonably be expected that a perſon of any given age will live to any other given age upon a ſuppoſition that he has neither a better nor a worſe chance of doing ſo than the majority of other perſons of the ſame age. And this medium, or average, chance of living is determined by tables that exhibit the numbers of perſons which, out of a certain pretty large number of children of one, or two, or three years of age, (which is uſually not leſs than 1000,) all living at the ſame time, are found (by methods of reaſoning that are grounded on long ſerieſes of obſervations,) to be living at the end of every ſubſequent year of human life to its extreme period, which ſome of the tables carry to 86, and others to more than 90, years. The inſtances of the prolongation of human life to more than 100 years are ſo unfrequent, that they are not thought to be worth attending to in forming any general rules upon this ſubject.

Of tables of the ſaid probability.

IV. The moſt exact tables of this kind that have hitherto been publiſhed ſeem to be thoſe of Mr. Kerſſeboom, and Monſieur de Parcieux; which are inſerted in the Appendix to Mr. De Moivre's Treatiſe on the Valuation of Annuities. The former was publiſhed in an eſſay of the aforeſaid Mr. Kerſſeboom on the number of people in the provinces of Holland and Weſt-Frieſland, written in the Dutch language, about the year 1738, (of which an account is given in the 9th volume of the Abridgement of the Philoſophical Tranſactions, page 326,) and is ſaid to have been formed from certain tables of aſſignable annuities for lives in Holland, which had been kept there for 125 years, and in which the ages of the ſeveral perſons dying in that period had been truly entered. And Mr. de Parcieux's table was made, as Mr. De Moivre informs us, by a like uſe of the liſts of the *French Tontines*, or *Long annuities*; and the numbers of it were verified by the *necrologies*, or mortuary regiſters, of ſeveral religious houſes of both ſexes. Theſe ſeem to be the moſt ſolid and authentick grounds upon which it is poſſible to form any tables of this kind: whereas there are ſome circumſtances of doubt and uncertainty in the methods of forming all the other tables of the probable duration of human life which prevent them from being intirely ſatisfactory. And therefore I conceive theſe two tables to be more exact and fit to be adopted in computing the values of life-annuities than any other tables I have ſeen; and particularly in computing the values of any annuities for lives which the government of this kingdom may at any time think fit to grant, if that method of raiſing money ſhould hereafter be adopted, (as is the caſe at this time in Ireland,) or it ſhould be thought expedient to diſcharge a part of the national debt in that way, by converting a part of the perpetual three per cent. annuities, payable at the Bank, into annuities for the lives of their reſpective proprietors, or for a term certain of 20 or 30 years and further for their lives. Theſe two tables are as follows.

Of the tables of probability of Mr. Kerſſeboom and Monſieur de Parcieux.

V. TABLE I.

Repreſenting the probabilities of the duration of human life at the ſeveral ages therein mentioned from the time of birth to the age of an hundred years: grounded on the regiſters of certain aſſignable annuities for lives granted by the government of Holland, which had been kept there for 125 *years, and in which the ages of the ſeveral annuitants dying during that period had been truly entered.*

By Mr. KERSSEBOOM.

Age.	*Perſons living.*	*Age.*	*Perſons living.*	*Age.*	*Perſons living.*	*Age.*	*Perſons living.*
0	1400	26	760	52	482	78	130
1	1125	27	747	53	470	79	115
2	1075	28	735	54	458	80	100
3	1030	29	723	55	446	81	87
4	993	30	711	56	434	82	75
5	964	31	699	57	421	83	64
6	947	32	687	58	408	84	55
7	930	33	675	59	395	85	45
8	913	34	665	60	382	86	36
9	904	35	655	61	369	87	28
10	895	36	645	62	356	88	21
11	886	37	635	63	343	89	15
12	878	38	625	64	329	90	10
13	870	39	615	65	315	91	7
14	863	40	605	66	301	92	5
15	856	41	596	67	287	93	3
16	849	42	587	68	273	94	2
17	842	43	578	69	259	95	1
18	835	44	569	70	245	96	0.6
19	826	45	560	71	231	97	0.5
20	817	46	550	72	217	98	0.4
21	808	47	540	73	203	99	0.2
22	800	48	530	74	189	100	0.0
23	792	49	518	75	175		
24	783	50	507	76	160		
25	772	51	495	77	145		

VI. The

VI. The meaning of the foregoing table is this; That from the obſervations on which it is grounded there is reaſon to conclude that out of 1400 new-born children 1125 have lived to the end of one year, and 1075 to the end of two years, and 1030 to the end of three years; and ſo on throughout the table; the figures that are parallel and contiguous to every age expreſſing the number of perſons out of the aforeſaid original number of 1400 infants that are living at that age. As to the figures 0.6, 0.5, 0.4, 0.2, adjoining to the 96th, 97th, 98th, and 99th year of the hundred, which denote the decimal fractions $\frac{6}{10}$, $\frac{5}{10}$, $\frac{4}{10}$, and $\frac{2}{10}$, it is obvious that in a ſtrict ſenſe they are inapplicable to any human, or other living, creatures: and therefore they muſt be underſtood in the following manner. We muſt ſuppoſe the original number of new-born infants, who were the root, or baſis, of the table, to have been increaſed tenfold, ſo as to become 14000 inſtead of 1400: in which caſe it is evident that all the following numbers in the table, repreſenting the perſons living at the ſeveral ages therein ſet down, will likewiſe be increaſed to ten times their former quantity, and therefore the number of perſons living at the age of 95 years will be 10 inſtead of 1. Now the meaning of the figures 0.6, 0.5, 0.4, 0.2, and of the cyphers 0.0 adjoining to the 100th year, is that, out of the ſaid 10 perſons who would in ſuch caſe have been found to be living at the age of 95 years, only 6 would have been found to be living at the age of 96, and 5 at the age of 97, and 4 at the age of 98, and 2 at the age of 99, and none at all at the age of an hundred years.

VII. The ſecond table above-mentioned, given us by Mr. De Parcieux, and inſerted by Mr. De Moivre in the appendix to his treatiſe on the valuation of annuities, is as follows.

TABLE

TABLE II.

Repreſenting the probabilities of the duration of human life at the ſeveral ages therein mentioned, from the age of 3 *years to the age of* 95: *grounded on liſts of the French* Tontines, *or* Long Annuities, *and verified by a compariſon thereof with the necrologies, or mortuary regiſters, of ſeveral religious houſes of both ſexes.*

By Monſieur DE PARCIEUX.

Age.	*Perſons living.*	*Age.*	*Perſons living.*	*Age.*	*Perſons living.*	*Age.*	*Perſons living.*	*Age.*	*Perſons living.*
3 yrs.	1000	23	790	43	636	63	423	83	71
4	970	24	782	44	629	64	409	84	59
5	948	25	774	45	622	65	395	85	48
6	930	26	766	46	615	66	380	86	38
7	915	27	758	47	607	67	364	87	29
8	902	28	750	48	599	68	347	88	22
9	890	29	742	49	590	69	329	89	16
10	880	30	734	50	581	70	310	90	11
11	872	31	726	51	571	71	291	91	7
12	866	32	718	52	560	72	271	92	4
13	860	33	710	53	549	73	251	93	2
14	854	34	702	54	538	74	231	94	1
15	848	35	694	55	526	75	211	95	0
16	842	36	686	56	514	76	192		
17	835	37	678	57	502	77	173		
18	828	38	671	58	489	78	154		
19	821	39	664	59	476	79	136		
20	814	40	657	60	463	80	118		
21	806	41	650	61	450	81	101		
22	798	42	643	62	437	82	85		

VIII. Both the foregoing tables repreſent human life as being more permanent, or the number of perſons dying every year, out of a given number of perſons alive at any given younger age, as being ſmaller, throughout both youth and middle age, and even till about the age of 70, than they appear to be by moſt other tables. And this is what we ought naturally to have expected, on account of the difference of the *data*, or grounds, on which theſe and thoſe other tables have been conſtructed. For both theſe tables of Mr. Kerſſeboom and Mr. de Parcieux were formed, as we before obſerved, from the regiſters of the deaths of perſons who had purchaſed life-annuities from the governments of Holland and France: whereas thoſe other tables have been formed, for the moſt part, from the regiſters of the births and burials of all the inhabitants in general of the places for which they are computed. Now it is reaſonable to believe that thoſe perſons who purchaſe annuities of a government are, for the moſt part, in eaſier circumſtances, and leſs expoſed to the dangers and hardſhips that deſtroy, or ſhorten, human life, than the reſt of mankind, and, conſequently, than the whole body of mankind (themſelves included) upon an average: and therefore it is not likely that they ſhould die off in the ſame average, or general, proportion, but at a ſlower rate.

Of the difference between the two foregoing tables of probabilities.

IX. But though both theſe tables of Mr. Kerſſeboom and Monſieur de Parcieux are formed from the regiſters of the deaths of government life-annuitants, and both of them repreſent the probability of the duration of human life as greater than it appears to be by any other tables, until towards the age of 70 years, yet they do not intirely agree with each other; but the table of Mr. de Parcieux repreſents that probability as ſtill greater than that of Mr. Kerſſeboom, till towards the ſaid advanced age of 70 years, and from that time ſomewhat leſs. The degree of difference between them in this reſpect may be collected from a compariſon of them with each other at the ſeveral ages of 20, 30, 40, 50, 60, 70, and 80 years, which may be made in the following manner.

A compariſon of them with each other with reſpect to perſons of the age of 20 *years.*

X. Let us firſt take the perſons living together at the age of 20 years in each of theſe tables, and ſee how many of thoſe perſons are repreſented in each of them to be living at the following ages of 30, 40, 50, 60, 70, 80, and 90, years, and then inquire in which of theſe two tables the proportions by which theſe numbers continually decreaſe are greateſt.

Now

Now it appears by Mr. Kerſſeboom's table that out of 817 perſons of 20 years of age, all living at the ſame time,

	711	will have lived to the age of 30 years,
and	605	to the age of 40,
and	567	to the age of 50,
and	382	to the age of 60,
and	245	to the age of 70,
and	100	to the age of 80,
and	10	to the age of 90 years.

And by the table of Monſieur de Parcieux it appears that out of 814 perſons of the ſame age of 20 years, all living at the ſame time,

	734	will have lived to the age of 30 years,
and	657	to the age of 40,
and	581	to the age of 50,
and	463	to the age of 60,
and	310	to the age of 70,
and	118	to the age of 80,
and	11	to the age of 90 years.

It appears therefore that according to Monſieur de Parcieux's table there will be more perſons found to have been living at the ſeveral ages of 30, 40, 50, 60, 70, 80, and 90, years, out of an original number of only 814 perſons of the age of 20 years, all living at the ſame time, than are found to have been living at the ſame ages reſpectively, according to Mr. Kerſſeboom's table, out of the greater original number of 817 perſons of the ſame age of 20 years all living at the ſame time; and that in the very conſiderable proportions of 734 to 711, 657 to 605, 581 to 507, 463 to 382, 310 to 245, 118 to 100, and 11 to 10. It is evident therefore that a life-annuity for the life of a perſon of 20 years of age is worth conſiderably more money according to Mr. de Parcieux's table than according to that of Mr. Kerſſeboom.

The like compariſon of them with reſpect to the age of 30 *years.*

XI. In the next place we will compare theſe two tables together with reſpect to the age of 30 years.

Now

Now by Mr. Kerſſeboom's table it appears that out of 711 perſons of the age of 30 years, all living at the ſame time,

	605	will have lived to the age of 40 years,
and	507	to the age of 50,
and	382	to the age of 60,
and	245	to the age of 70,
and	100	to the age of 80,
and	10	to the age of 90 years.

And by the table of Monſieur de Parcieux it appears that out of 734 perſons of 30 years of age, all living at the ſame time,

	657	will have lived to the age of 40 years,
and	581	to the age of 50,
and	463	to the age of 60,
and	310	to the age of 70,
and	118	to the age of 80,
and	11	to the age of 90 years:

and conſequently out of 711 perſons of 30 years of age, all living at the ſame time, (which is the number of perſons living at that age in Mr. Kerſſeboom's table,)

	$\left(657 \times \frac{711}{734}\right.$, or) 636	will have lived to the age of 40 years,
and	$\left(581 \times \frac{711}{734}\right.$, or) 563	to the age of 50,
and	$\left(463 \times \frac{711}{734}\right.$, or) 448	to the age of 60,
and	$\left(310 \times \frac{711}{734}\right.$, or) 300	to the age of 70,
and	$\left(118 \times \frac{711}{734}\right.$, or) 114	to the age of 80,
and	$\left(11 \times \frac{711}{734}\right.$, or) 10	to the age of 90 years.

It appears therefore that out of the ſame original number of 711 perſons of 30 years of age, all living at the ſame time, there will be more perſons found to be living at the ſeveral ſubſequent ages of 40, 50, 60, 70, and 80, years, according to Monſieur de Parcieux's table than according to

 that

that of Mr. Kerſſeboom, in the ſeveral very conſiderable proportions of 636 to 605, 563 to 507, 448 to 382, 300 to 245, and 114 to 100, and the ſame number, to wit, 10 perſons, at the age of 90 years. It is evident therefore that a life-annuity for the life of a perſon of 30 years of age would be worth a conſiderably greater ſum of money according to Monſieur de Parcieux's table than according to that of Mr. Kerſſeboom.

The like compariſon of them with reſpect to the age of 40 *years.*

XII. We will next compare theſe two tables together with reſpect to the age of 40 years.

Now by Mr. Kerſſeboom's table it appears that out of 605 perſons of 40 years of age, all living at the ſame time,

	507	will have lived to the age of 50,
and	382	to the age of 60,
and	245	to the age of 70,
and	100	to the age of 80,
and	10	to the age of 90 years.

And by the table of Mr. de Parcieux it appears that out of 657 perſons of 40 years of age, all living at the ſame time,

	581	will have lived to the age of 50 years,
and	463	to the age of 60,
and	310	to the age of 70,
and	118	to the age of 80,
and	11	to the age of 90 years:

and conſequently out of 605 perſons of 40 years of age, all living at the ſame time,

	$\left(581 \times \frac{605}{657}\right.$, or) 540	will have lived to the age of 50 years,
and	$\left(463 \times \frac{605}{657}\right.$, or) 426	to the age of 60,
and	$\left(310 \times \frac{605}{657}\right.$, or) 285	to the age of 70,
and	$\left(118 \times \frac{605}{657}\right.$, or) 108	to the age of 80,
and	$\left(11 \times \frac{605}{657}\right.$, or) 10	to the age of 90 years.

It

It appears therefore that out of the ſame original number of 605 perſons of 40 years of age, all living at the ſame time, there will be more perſons found to have been living at the ſeveral ſubſequent ages of 50, 60, 70, and 80 years according to Monſieur de Parcieux's table than according to that of Mr. Kerſſeboom, in the ſeveral conſiderable proportions of 540 to 507, 426 to 382, 285 to 245, and 108 to 100; and the ſame number, to wit, 10 perſons, at the age of 90 years. It is evident therefore that a life-annuity for the life of a perſon of 40 years of age would be worth conſiderably more by Mr. de Parcieux's table than by that of Mr. Kerſſeboom.

The like compariſon of them with reſpect to the age of 50 years.

XIII. We will next compare theſe two tables with each other with reſpect to the age of 50 years.

Now by Mr. Kerſſeboom's table it appears that out of 507 perſons of 50 years of age, all living at the ſame time,

	382 will have lived	to the age of 60 years,
and	245	to the age of 70,
and	100	to the age of 80,
and	10	to the age of 90 years.

And by the table of Monſieur de Parcieux it appears that out of 581 perſons of the age of 50 years, all living at the ſame time,

	463 will have lived	to the age of 60 years,
and	310	to the age of 70,
and	118	to the age of 80,
and	11	to the age of 90 years:

and conſequently out of 507 perſons of 50 years of age, all living at the ſame time, (which is the number of perſons living at that age in Mr. Kerſſeboom's table)

$\left(463 \times \frac{507}{581}, \text{ or}\right)$	404 will have lived	to the age of 60 years,
and $\left(310 \times \frac{507}{581}, \text{ or}\right)$	270	to the age of 70,
and $\left(118 \times \frac{507}{581}, \text{ or}\right)$	103	to the age of 80,
and $\left(11 \times \frac{507}{581}, \text{ or}\right)$	10	to the age of 90 years.

It appears therefore that out of the ſame original number of 507 perſons of 50 years of age, all living at the ſame time, there will be more perſons found to have been living at the ſeveral ſubſequent ages of 60, 70, and 80 years, according to Mr. de Parcieux's table than according to that of Mr. Kerſſeboom, in the proportions of 404 to 382, 270 to 245, and 103 to 100; and the ſame number, to wit, 10 perſons, at the age of 90 years. It is evident therefore that a life-annuity for the life of a perſon of 50 years of age, as well as a life-annuity for the life of a perſon of 20, or 30, or 40, years of age, will be worth more according to the table of Monſieur de Parcieux than according to that of Mr. Kerſſeboom. But the difference of the values of ſuch an annuity according to theſe two different tables will not be ſo great at the age of 50 as at thoſe younger ages.

The like compariſon of them with reſpect to the age of 60 years.

XIV. We will next examine the two tables with reſpect to the age of 60 years.

Now by Mr. Kerſſeboom's table it appears that out of 382 perſons of 60 years of age, all living at the ſame time,

245 will have lived to the age of 70 years,
and 100 to the age of 80,
and 10 to the age of 90 years.

And by the table of Monſieur de Parcieux it appears that out of 463 perſons of the age of 60 years, all living at the ſame time,

310 will have lived to the age of 70 years,
and 118 to the age of 80,
and 11 to the age of 90 years:

and conſequently that out of 382 perſons of 60 years of age, all living at the ſame time, (which is the number of perſons living at that age in Mr. Kerſſeboom's table,)

$\left(310 \times \frac{382}{463},\text{ or}\right)$ 256 will have lived to the age of 70 years,
and $\left(118 \times \frac{382}{463},\text{ or}\right)$ 97 to the age of 80,
and $\left(11 \times \frac{382}{463},\text{ or}\right)$ 9 to the age of 90 years.

It

It appears therefore that out of the ſame original number of 382 perſons of 60 years of age, all living at the ſame time, more perſons will be found to have been living at the age of 70 years by Mr. de Parcieux's table than by Mr. Kerſſeboom's in the proportion of 256 to 245; but that at the ſubſequent ages of 80 and 90 years fewer will be found to have been living by Mr. de Parcieux's table than by that of Mr. Kerſſeboom, in the proportions of 97 to 100, and 9 to 10. Now it is evident that the value of a life-annuity (which is, as we before obſerved, a ſet of equal future payments to be made at the end of every year during the continuance of the life for which it is granted,) depends more on the value of the payments that are to be made during the firſt years of its continuance than on thoſe which are to be made at more remote periods, both becauſe thoſe firſt payments are nearer at hand, and becauſe the probability of their becoming due by the continuance of the life for which they are granted is greater than the probability of the ſame event with reſpect to the latter payments. It ſeems reaſonable therefore to conclude, (though this cannot be affirmed with certainty without actually making the calculation,) that a life-annuity for the life of a perſon of 60 years of age will be worth ſomething more according to Mr. de Parcieux's table than according to that of Mr. Kerſſeboom, but not in ſo great a degree as at the younger ages before-mentioned.

The like compariſon with reſpect to the age of 70 years.

XV. We will next examine the two tables with reſpect to the age of 70 years.

Now by Mr. Kerſſeboom's table it appears that out of 245 perſons of 70 years of age, all living at the ſame time,

	231	will have lived	to the age of 71 years,
and	175		to the age of 75,
and	100		to the age of 80,
and	45		to the age of 85,
and	10		to the age of 90 years.

And by the table of Monſieur de Parcieux it appears that out of 310 perſons of 70 years of age, all living at the ſame time,

	291	will have lived	to the age of 71 years,
and	211		to the age of 75,
and	118		to the age of 80,
and	48		to the age of 85,
and	11		to the age of 90 years:

and

and conſequently that out of 245 perſons of 70 years of age, all living at the ſame time (which is the number of perſons living at that age in Mr. Kerſſeboom's table,)

$\left(291 \times \frac{245}{310}, \text{ or}\right)$ 230 will have lived to the age of 71 years,

and $\left(211 \times \frac{245}{310}, \text{ or}\right)$ 167 to the age of 75,

and $\left(118 \times \frac{245}{310}, \text{ or}\right)$ 93 to the age of 80,

and $\left(48 \times \frac{245}{310}, \text{ or}\right)$ 38 to the age of 85,

and $\left(11 \times \frac{245}{310}, \text{ or}\right)$ 9 to the age of 90 years.

It appears therefore that out of an original number of 245 perſons of 70 years of age, all living at the ſame time, there will be fewer perſons found to have been living at the ſeveral ſubſequent ages of 71, 75, 80, 85, and 90, years by Monſieur de Parcieux's table than by that of Mr. Kerſſeboom, in the ſeveral proportions of 230 to 231, 167 to 175, 93 to 100, 38 to 45, and 9 to 10. And conſequently the value of a life-annuity for the life of a perſon of 70 years of age will be leſs according to Mr. de Parcieux's table than according to that of Mr. Kerſſeboom, though it has been ſeen that in all younger ages from 20 years upwards the value of a life-annuity is greater by the former table than by the latter.

The like compariſon with reſpect to the age of 80 *years.*

XVI. In the laſt place we will examine the two tables with reſpect to the age of 80 years.

Now it appears by Mr. Kerſſeboom's tables that out of 100 perſons of the age of 80 years, all living at the ſame time,

87 will have lived to the age of 81 years,
and 45 to the age of 85,
and 10 to the age of 90 years.

And

And by the table of Monſieur de Parcieux it appears that out of 118 perſons of the age of 80 years, all living at the ſame time,

101 will have lived to the age of 81 years,
and 48 to the age of 85,
and 11 to the age of 90 years:

and conſequently that out of 100 perſons of the age of 80 years, all living at the ſame time, (which is the number of perſons living at that age in Mr. Kerſſeboom's table,)

$\left(101 \times \frac{100}{118}, \text{ or}\right)$ 85 will have lived to the age of 81 years,

and $\left(48 \times \frac{100}{118}, \text{ or}\right)$ 40 to the age of 85,

and $\left(11 \times \frac{100}{118}, \text{ or}\right)$ 9 to the age of 90 years.

It appears therefore that out of an original number of 100 perſons of the age of 80 years, all living at the ſame time, there will be fewer perſons found to have been living at the ſeveral ſubſequent ages of 81, 85, and 90 years according to Mr. de Parcieux's table than according to that of Mr. Kerſſeboom, in the ſeveral proportions of 85 to 87, 40 to 45, and 9 to 10. And conſequently the value of a life-annuity for the life of a perſon of 80 years of age, as well as that of an annuity for the life of a perſon of 70 years of age, will be leſs according to Mr. de Parcieux's table than according to that of Mr. Kerſſeboom, though in the younger ages of 20, 30, and 40 years, the value of a life-annuity was conſiderably greater by the former table than by the latter.

Of the preference due to Monſieur de Parcieux's table of probabilities above that of Mr. Kerſſeboom, with reſpect to tables of life-annuities to be calculated for the uſe of Engliſhmen.

XVII. Thus much may be ſufficient to ſhew the difference between theſe two tables of Mr. Kerſſeboom and Mr. de Parcieux. But which of them upon the whole deſerves to be conſidered as the more exact, I will not pretend to determine. Only thus much I will venture to obſerve concerning them; That, as the ſoil and temperature of the air in England bear a greater reſemblance, as I conceive, to the ſoil and temperature of the air in the northern parts of France than to thoſe of Holland, which is ſo full of moiſt vapours ariſing from the waters amongſt which it is ſituated; and the Dutch are in general reckoned to be ſhorter-lived than either

either the French or the Engliſh; it ſeems reaſonable to ſuppoſe that Mr. de Parcieux's table, which is formed from obſervations made in France, is more likely to afford a juſt meaſure of the duration of the lives of Engliſhmen in the like ſituation and circumſtances of life, that is, proprietors of government life-annuities, than the table of Mr. Kerſſeboom, which is formed from the like obſervations made in Holland. And therefore I conceive that, with reſpect to the valuation of annuities on the lives of perſons living in England, and more eſpecially of annuities to be granted at any time by the government, Mr. de Parcieux's table deſerves to be preferred to the other. And accordingly I ſhall have recourſe to it in the enſuing pages for the ſolution of the few queſtions, or examples, upon this ſubject which I ſhall have occaſion to conſider.

Of the Breſlaw and London tables of the probabilities of the duration of human life.

XVIII. Mr. De Moivre has given us, in his appendix above-mentioned to his treatiſe on the valuation of annuities, two other tables of the probable duration of human life, which he conſiders as very uſeful for the purpoſe of computing the values of life-annuities. Theſe are, 1ſt, Dr. Halley's table, which he formed from obſervations on the births and burials of the inhabitants of the city of Breſlaw, (which is the capital of the dutchy of Sileſia in Germany,) during a ſeries of five years in the latter part of the laſt century, to wit, the years 1687, 1688, 1689, 1690, and 1691; and 2dly, that of Mr. Smart, which he formed from ſimilar obſervations on the births and burials in London. This laſt table has been adopted by the late acute and very learned Mr. Thomas Simpſon, of Woolwich, in his treatiſe called "*The doctrine of annuities and reverſions*," and made the foundation of all his calculations of the values of annuities for lives, which he deſigned principally for the uſe of the inhabitants of London. And Dr. Price has likewiſe given us theſe two tables of Dr. Halley and Mr. Smart, together with two other tables of the ſame kind founded on obſervations of the births and burials at Norwich and Northampton, in his very uſeful treatiſe called "*Obſervations on reverſionary payments*," which contains the greateſt variety of tables for the valuation of annuities, both for lives and terms of years, of any book that I have met with. And to this laſt book, (which is deſervedly in every body's hands,) I muſt refer my readers for ſuch of theſe tables as they may have occaſion to conſult, as I think it would be needleſs to inſert them in this tract in which I propoſe to make uſe only of Mr. de Parcieux's table for the ſolution of the few queſtions and examples concerning future payments depending upon lives, that will occur in it.

Of

Of the manner of forming a table of the probabilities of the duration of human life.

XIX. The curious reader will probably be glad to be informed in what manner theſe tables of the probable duration of human life are formed from obſervations of the births and burials of mankind at the ſeveral different ages of human life in any given diſtrict. For ſatisfaction in this particular I muſt refer him to the two following diſcourſes; to wit, firſt, to Dr. Halley's celebrated tract in the Philoſophical Tranſactions, intitled, "*An Eſtimate of the degrees of the mortality of mankind, drawn from curious tables of the births and funerals at the city of Breſlaw; with an attempt to aſcertain the price of annuities upon lives. By Edmund Halley, R. S. S.*;" which is likewiſe publiſhed in the firſt volume of the *Miſcellanea Curioſa*, page 280; and, ſecondly, to the before-mentioned treatiſe of Dr. Price upon reverſionary payments, in which he will find, from page 235 to page 276, an ample and curious diſcourſe upon this ſubject, intitled, "*Obſervations on the proper method of conſtructing tables for determining the rate of human mortality, the number of inhabitants, and the values of lives, in any town or diſtrict, from bills of mortality in which are given the numbers dying annually at all ages.*" The part of Dr. Halley's diſcourſe above-mentioned which relates to this ſubject is contained in the firſt five pages of it, as it is printed in the *Miſcellanea Curioſa*, which is the part preceding the table. The ſubſequent part of the diſcourſe ſets forth the ſeveral uſes to which the table may be applied; of which the four firſt uſes are explained in a very clear and eaſy manner, ſo as to be intelligible to a reader not acquainted with the mathematicks, but the following parts are much more difficult, and require much more knowledge and attention to underſtand them rightly. This I mention in order to ſave the unlearned reader, who ſhall look into that diſcourſe, unneceſſary trouble.

XX. As we have mentioned in the 18th article ſeveral different tables of the probability of the duration of human life, which different authors have formed from different ſets of obſervations, and particularly thoſe of Dr. Halley and Meſſieurs Smart and Simpſon, it will, I doubt not, be a ſatisfaction to the reader to be informed of what ſo great a judge of theſe matters as Mr. De Moivre has delivered as his opinion concerning their reſpective merits and uſes. I ſhall therefore now inſert in this place the remarks of that able writer upon the four tables of Dr. Halley, Mr. Kerſſeboom, Mr. de Parcieux, and Meſſieurs Smart and Simpſon, which he has inſerted in the appendix to his learned treatiſe on the valuation of annuities. See his Doctrine of Chances, third edition, pages 345, 346, 347, 348. Theſe remarks are as follows.

Mr. De Moivre's remarks on the four tables of the probability of the duration of human life publiſhed by Dr. Halley, Mr. Kerſſeboom, Mr. de Parcieux, and Meſſieurs Smart and Simpſon, inſerted in his treatiſe on the valuation of annuities.

" The firſt table is that of Dr. Halley, compoſed from the bills of " mortality of the city of Breſlaw; the beſt, perhaps, as well as the firſt " of its kind; and which will always do honour to the judgement and " ſagacity of its excellent author.

" Next follows a table of the ingenious Mr. Kerſſeboom, founded " chiefly upon regiſters of the Dutch annuitants, carefully examined and " compared for more than a century backward. And Monſieur de Parcieux " by a like uſe of the liſts of the *French Tontines*, or *Long Annuities*, has " furniſhed us with Table III; whoſe numbers were likewiſe verified upon " the *necrologies*, or mortuary regiſters of ſeveral religious houſes of both " ſexes.

" To theſe is added the table of Meſſieurs Smart and Simpſon, adapted " particularly to the city of London; whoſe inhabitants, for reaſons too " well known, are ſhorter-lived than the reſt of mankind.

" Each of theſe tables may have its particular uſe: The *Second* or " *Third* in valuing the *better* ſort of lives, upon which one would chuſe to " hold an annuity: The *Fourth* may ſerve for London, or for lives ſuch " as thoſe of its inhabitants are ſuppoſed to be: while Dr. Halley's num- " bers, falling between the two extremes, ſeem to approach nearer to the " general courſe of nature. And in caſes of combined lives, two or more " of the tables may perhaps be uſefully employed.

" Beſides theſe, the celebrated Monſieur *de Buffon* has lately given us " a new table, from the actual obſervations of Monſieur *du Pré de Saint* " *Maur* of the French Academy. This gentleman, in order to ſtrike a " juſt *mean*, takes three populous pariſhes in the city of Paris, and ſo many " country pariſhes as furniſh him nearly an equal number of lives: and " his care and accuracy in that performance have been ſuch as to merit the " high approbation of the learned Editor. It was therefore propoſed to " add this table to the reſt; after having purged its numbers of the " inequalities that neceſſarily happen in fortuitous things, as well as of " thoſe ariſing from the careleſs manner in which ages are given in to the " pariſh-clerks; by which the years that are multiples of 10 are generally " over-loaded.

" But

" But this having been done with all due care, and the whole reduced " to Dr. Halley's denomination of 1000 infants of a year old; there " refulted only a mutual confirmation of the two tables; Mr. Du Pré's " table making the lives fomewhat better as far as 39 years, and thence " a fmall matter worfe than they are by Dr. Halley's.

" We may therefore retain this laft as no bad ftandard for mankind in " general; till a better police, in this and other nations, fhall furnifh the " proper *data* for correcting it, and for expreffing the decrements of life " more accurately and in larger numbers.

" For which purpofe, the parifh regifters ought to be kept in a better " manner, according to one or other of the forms that have been propofed " by authors. Or, if we fuppofe the numbers annually born to have " been nearly the fame for an age paft, the thing may be done at once, " by taking the numbers of the living, with their ages, throughout every " parifh in the kingdom: as was in part ordered fome time ago by the " right reverend the bifhops: but their order was not univerfally obeyed; " for what reafon we pretend not to guefs. Certain it is that a *cenfus* of " this kind once eftablifhed, and repeated at proper intervals, would fur- " nifh to our governours and to ourfelves much important inftruction, of " which we are now in a great meafure deftitute: Efpecially if the whole " was diftributed into the proper claffes of *married* and *unmarried*, *in-* " *duftrious* and *chargeable* poor, *Artificers* of every kind, *Manufacturers*, &c. " and if this was done in each county, city, and borough, feparately; " that particular ufeful conclufions might thence be readily deduced, as " well as the general ftate of the nation difcovered, and the rate according " to which human life is wafting from year to year. See, on this fubject, " the judicious Obfervations of Mr. Corbyn Morris, addreffed to Thomas " Potter, Efq; in the year 1751."

Thus far Mr. De Moivre. Since that time Dr. Price has publifhed two more tables of the probability of the duration of human life formed from obfervations on the births and burials at Northampton and Norwich, which may be feen in his book above-mentioned, intitled, " *Obfervations on reverfionary payments*," Tables IV and V, pages 317 and 318. And the grounds upon which they are conftructed, as well as the method of forming them from thofe grounds, are fet forth in the fame book in the difcourfe intitled, " *Obfervations on the proper method of conftructing tables, &c.*" from page 240 to page 281, third edition.

XXI. I now proceed to the main defign of this tract, which is to fhew how, by the help of any one fuch table of the probabilities of the duration of human life as thofe above-mentioned, the value of any future contin-

gent payment, depending upon the continuance of one or more lives, may be computed. Now the great fundamental maxim upon which all computations of this kind are grounded, may be explained as follows.

The fundamental maxim of the doctrine of life-annuities.

In every bargain between two perſons concerning a grant of a ſum of money to be paid by the one to the other at a given future time, in caſe the grantee, or purchaſer, ſhall be then alive, or in caſe the grantee and one or more other perſons of given ages ſhall be then alive, the fair price of ſuch future ſum of money, according to a given rate of the intereſt of money and a given table of the probabilities of the duration of human life, is to be aſcertained in the following manner. We muſt ſuppoſe, in the firſt place, that the grantor of the future ſum of money makes ſeveral hundred grants of the ſame kind, and upon exactly the ſame conditions, to as many different grantees, or purchaſers, all of the ſame age with the firſt grantee; and, in the ſecond place, that theſe ſeveral purchaſers and their companions, (or the perſons upon the continuance of whoſe lives, as well as their own, their right to the ſaid future ſums depends,) die off, in the interval between the time of making the grants and the time of payment, in the ſame proportions as perſons of the ſame ages reſpectively are repreſented to do in the table of the probabilities of the duration of human life by which the calculation is to be governed: and, in the third place, we muſt ſuppoſe that the ſeveral ſums of money paid by the ſeveral grantees of theſe future payments to the grantor of them as the price thereof, are improved by the ſaid grantor at compound intereſt, at the rate ſuppoſed in the queſtion, during the whole interval of time between the time of making the grants and the time at which the payments become due. And then we muſt inquire what ſum each of the ſaid grantees ought to pay to the grantor to the end that, upon theſe three ſuppoſitions, he may, at the end of the ſaid interval, or when the ſaid payments become due, be neither a gainer nor a loſer by the ſum total of all his bargains, but be poſſeſſed of juſt enough money, ariſing from the ſums formerly paid him by the ſaid grantees, to ſatisfy all the demands which will then be made upon him. And the ſum which ought thus to be paid him by each of the ſaid grantees, when he makes a great number of the ſaid grants to different perſons, is the fair price which a ſingle grantee ought to pay him for a grant of the ſame future ſum of money, ſubject to the ſame conditions and contingencies, when he makes only one ſuch grant.

This is a maxim which, I preſume, will be admitted as ſelf-evident; it being hardly poſſible to doubt of its truth. But if the reader ſhould not admit it upon its own evidence, I confeſs I am unable to demonſtrate it by means of any other propoſition more evident than itſelf. And therefore in

in this caſe I muſt deſire him to conſider it as a definition of what is meant in the following pages by the expreſſions of the *fair price*, or *true value* of ſuch a future contingent payment; ſince it is only in that ſenſe that the fair price, or true value, of ſuch a future contingent payment can be collected from the tables of the probabilities of the duration of human life above-deſcribed.

Of the preſent values of future certain payments of money.

XXII. It appears from the foregoing article that, in order to find the true value of a future contingent payment, depending upon the continuance of one or more given lives, it is neceſſary that we ſhould be able to determine, what ſum of money, paid down at preſent to the grantor of the ſeveral future payments above-mentioned, will, if improved by him at compound intereſt at the rate of intereſt ſuppoſed in the queſtion, during the whole interval of time before the ſaid future payments become due, amount at that time to the whole ſum of money which he will then be obliged to pay to his ſeveral grantees. This preſent ſum of money which, being ſo improved at compound intereſt at a certain given rate of intereſt, will in a certain number of years amount to ſuch larger given ſum, is what the writers upon this ſubject uſually call the *preſent value* of ſuch larger ſum at the ſaid rate of intereſt. And, as this is a ſhort and convenient expreſſion to denote the ſaid leſſer ſum, I ſhall frequently make uſe of it in the courſe of the following pages.

XXIII. Now the method of finding the preſent value of any given ſum of money that is to be received at the end of a given number of years, according to any given rate of intereſt, is as follows.

PROBLEM I.

To find the preſent value of any given ſum of money which is payable at the end of any given number of years, according to any given rate of intereſt.

SOLUTION.

Let S be put for the given ſum of money whoſe preſent value is to be determined; and n for the number of years at the end of which it is to be received. And let r be to 1 as the ſum that ariſes by adding together any ſum of money and its intereſt for one year according to the rate ſuppoſed in the queſtion, is to the ſaid principal ſum of money alone, or without its intereſt. And let x be put for the preſent value of the given ſum S, which is to be found. Then it is evident that x will be to x with its intereſt for one

one year as 1 to r, and conſequently that x with its intereſt for one year will be equal to $\frac{rx}{1}$, or rx; that is, the amount of x at the end of the firſt year will be rx. In like manner the amount of x at the end of the firſt year will be to the ſame amount together with the intereſt of it for one year, that is, to the amount of x at the end of the ſecond year, as 1 to r; and conſequently the amount of x at the end of the ſecond year will be equal to r multiplied into the amount of x at the end of the firſt year, or to r multiplied into rx, or to rrx. And in like manner it will appear that the amount of x at the end of the third year will be r^3x, and at the end of the fourth year r^4x, and at the end of the fifth year r^5x, and at the end of the nth, or laſt, year r^nx. Therefore r^nx is equal to the given ſum S whoſe preſent value x is to be found. Therefore x is equal to $\frac{S}{r^n}$; that is, the preſent value of the given ſum S is equal to the quotient that ariſes by dividing that given ſum by that power of r whoſe index is the number of years at the end of which the ſaid ſum becomes payable. QEI.

An example of the foregoing ſolution.

Thus, for example, if the ſum of 265*l.* is to be received at the end of three years, and the rate of intereſt is three per cent. we muſt proceed as follows.

Since the intereſt of money is 3 per cent. we ſhall have 100 : 103 : : 1 : r; and conſequently r will be $= \frac{103\times1}{100} = \frac{103}{100} = 1.03$. Therefore rr is $= 1.03\times1.03 = 1.0609$, and $r^3 = rr\times r = 1.0609\times1.03 = 1.092{,}727$. Therefore x, or the preſent value of S, or 265*l.* is $= \frac{£.265}{1.092{,}727} = £.242.51$, or 242*l.* 10*s.* 2*d.* $\frac{1}{2}$. QEI.

COROLL. It follows from the foregoing ſolution that the preſent value of one pound at the end of n years is $\frac{1}{r^n}$ of a pound. For in this caſe S is equal to £ 1; and conſequently x, or the preſent value of S, which is in all caſes equal to $\frac{S}{r^n}$, is in this caſe equal to $\frac{1l.}{r^n}$, or $\frac{1}{r^n}$ of a pound. QED.

XXIV. If

XXIV. If the intereſt of money is

2 per cent.	r will be	$= 1.02$,
$2\frac{1}{2}$ per cent.		$= 1.025$,
3 per cent.		$= 1.03$,
$3\frac{1}{2}$ per cent.		$= 1.035$,
4 per cent.		$= 1.04$,
$4\frac{1}{2}$ per cent.		$= 1.045$,
5 per cent		$= 1.05$,
6 per cent.		$= 1.06$,
7 per cent.		$= 1.07$,
8 per cent.		$= 1.08$,
9 per cent.		$= 1.09$,
10 per cent.		$= 1.10$.

Of Mr. Smart's tables of intereſt.

XXV. If one pound ſterling be denoted by 1, it is evident that r will be the amount of one pound ſterling together with its intereſt for one year, or the amount of one pound ſterling at the end of a year; and r^2 will be its amount at the end of two years; and r^3 its amount at the end of three years; and r^4 its amount at the end of four years; and, in general, r^n its amount at the end of n years. Now all theſe powers of r, from the firſt power to the hundredth, according to the twelve different values of it above-mentioned, to wit, 1.02, 1.025, 1.03, 1.035, 1.04, 1.045, 1.05, 1.06, 1.07, 1.08, 1.09, and 1.10, (which correſpond to the ſeveral rates of intereſt above-mentioned, of 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent.) or all theſe amounts of one pound ſterling, at the end of one, two, three, and four, years, and of every following year as far as the hundredth, incluſively, according to thoſe ſeveral rates of intereſt; together with the ſeveral amounts of it at the end of all the intermediate half years from the beginning of the firſt year to the end of the hundredth year; are ready calculated to our hands, to no leſs than eight places of decimal fractions, by Mr. John Smart in his moſt uſeful tables of intereſt publiſhed in quarto in the year 1726. Theſe amounts, or powers of the different values of r, conſtitute what he calls his *firſt table of compound intereſt*, which takes up eight pages of his book, to wit, the 52d, 53d, 54th, 55th, 56th, 57th, 58th, and 59th pages. And in his *ſecond table of compound intereſt*, contained in pages 60, 61, 62, 63, 64, 65, 66, 67, he has given us the reciprocals of theſe amounts, or powers of r, or the *preſent values* of one pound payable at the end of any number of years or half years, in the ſpace of an hundred years, according to the ſaid rates of intereſt, computed to the ſame degree of exactneſs. By theſe moſt uſeful tables the greater part of the labour of computing

computing the value of a future contingent payment depending upon the continuance of a life or lives, or that of a life-annuity, (which is only the ſum total of ſeveral ſuch payments,) is taken away; as will be evident from the method of computing the values of ſuch payments, which we now proceed to explain.

PROBLEM II.

XXVI. To find the ſum of money which the purchaſer of a future payment of one pound ſterling, to be received at the end of any given number of years, provided the ſaid purchaſer ſhall be then living, ought to pay for it: the age of the ſaid purchaſer, and the rate of intereſt of money, and the probabilities of the duration of human life, being all given.

A ſolution of this problem in the caſe of a particular example.

Let the rate of intereſt of money be ſuppoſed to be 3 per cent. and the probabilities of the duration of human life ſuch as they are repreſented to be in Monſieur de Parcieux's table above-mentioned. And let the number of years at the end of which the ſaid ſum of one pound is to be paid to the grantee, or purchaſer of it, if he be then alive, be 30: and the age of the ſaid grantee, or purchaſer, 25 years.

Then, in the firſt place, we muſt look into Mr. de Parcieux's table to ſee how many perſons of 25 years of age are there ſuppoſed to be all living at the ſame time. This number we ſhall find to be 774. We muſt therefore ſuppoſe that the grantor of the one pound to the purchaſer propoſed in the queſtion does not confine himſelf to that ſingle grant, but makes 773 more ſuch grants, of one pound each, to as many different perſons of the ſame age of 25 years, to be paid to them at the end of 30 years, or when they ſhall be 55 years old, if they ſhall then be living, but not to be paid to their executors, or other repreſentatives, if they ſhall then be dead; that is, we muſt ſuppoſe that he makes 774 ſuch grants in all, including that to the purchaſer propoſed in the queſtion. And we muſt likewiſe ſuppoſe that all theſe 774 purchaſers have the ſame chance, one with the other, of living any given number of years, or that there is no apparent reaſon for ſuppoſing that any one of them is more likely to live to any given future age than any other. This done, we muſt inquire how many of theſe 774 purchaſers of one pound each will be alive at the end of 30 years, ſuppoſing them to die off in the proportion mentioned in Mr. de Parcieux's table. Now it appears by Mr. de Parcieux's table that out of 774 perſons of the age of 25 years, all living at the ſame time, 526 will be alive at the age of 55 years, or at the diſtance of 30 years. Therefore

out

out of the ſaid 774 purchaſers of theſe future payments of one pound, to be received at the end of 30 years, 526 will live to be intitled to them. Therefore at the end of the ſaid 30 years the grantor of theſe future payments will have 526 ſums of one pound each to pay to the ſaid ſurviving purchaſers. And conſequently, to the end that the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive at the time of making the ſaid grants, 526 times the preſent value of one pound payable at the end of 30 years, when the intereſt of money is 3 per cent. or 526 times the ſum which, being improved continually at compound intereſt during the ſaid term of 30 years at the ſaid rate of intereſt, will at the end of that time amount to one pound; becauſe in that caſe, if he improves the ſaid ſum (of 526 times the preſent value of one pound) ſo received at compound intereſt at the ſaid rate of 3 per cent. during the whole 30 years, it will in that time increaſe to juſt 526 pounds, which is the ſum he will then be obliged to pay to the ſurviving purchaſers. Now it appears by Mr. Smart's ſecond table of compound intereſt, page 61, that the preſent value of one pound payable at the end of 30 years, without being liable to any contingency, when the intereſt of money is 3 per cent. is .411,986,76 of a pound. Therefore 526 times .411,986,76 of a pound, or £216.705,035,76, is the ſum which the ſaid grantor ought to receive at the time of making the ſaid grants from all the 774 purchaſers of them. Therefore the ſum which each of them ought then to pay him is the 774th part of £216.705,035,76, or .279,980,66 of a pound, or, nearly, .28 of a pound, or 5*s*. 7*d*.$\frac{1}{4}$. And conſequently, by Art. 21, when he makes only one ſuch grant to a purchaſer of 25 years of age, he ought to receive for it the ſame ſum of .279,980,66 of a pound, or .28 of a pound, or 5*s*. 7*d*.$\frac{1}{4}$. QEI.

XXVII. I have ſolved the foregoing problem in the caſe of a particular example for the ſake of making the method of ſolution as clear and familiar as poſſible. But it is eaſy to ſee that the reaſonings uſed in it extend to all other caſes whatſoever, and conſequently that the ſolution is really general. But now, that no doubt may be left on this head in the reader's mind, I ſhall repeat the ſolution, at the ſame length as before, in general terms and with a notation ſuited to them; by which means we ſhall obtain a ſhort and general expreſſion of the value of any ſuch future and contingent payment as is hereby ſought, which it will be eaſy to apply to any other inſtances.

A general ſolution of the foregoing problem.

XXVIII. Let 1 be put for one pound ſterling, and r for the ſum of one pound and its intereſt for one year according to the rate of intereſt given in the queſtion, or the amount of one pound, improved at that rate of intereſt, at the end of a year. And let n denote the number of years

at the end of which the ſum of one pound is to be paid to the purchaſer, if he is then alive, but not to his executor, or other repreſentative, if he ſhall then be dead. And let N denote the number of years in the age of the purchaſer at the time of his making the purchaſe of this one pound; and conſequently $N+n$ the number of years contained in his age at the time he becomes intitled to receive it, if he lives ſo long.

Then, in the firſt place, we muſt look into the table of the probabilities of the duration of human life by which the calculation is to be governed, and which I ſhall ſuppoſe to be that of Mr. de Parcieux, to find how many perſons of N years of age are there repreſented as living at the ſame time. This number we will call P. We muſt then ſuppoſe that the grantor of the future payment of one pound to the purchaſer mentioned in the queſtion does not confine himſelf to that ſingle grant, but makes at the ſame time $P-1$ more ſuch grants to as many different purchaſers of them, all of the age of N years; ſo that, including the grant to the purchaſer mentioned in the queſtion, he makes in all P ſuch grants to P different purchaſers, all of the age of N years. And we muſt further ſuppoſe that all theſe P purchaſers have the ſame chance, one with the other, of living any given number of years, or that there is no apparent reaſon for ſuppoſing that any one of them is more likely to live to any given future age than any other. And we muſt likewiſe ſuppoſe them to die off, in the courſe of the n years which are to elapſe before the payments become due, in the proportion in which perſons of the ſame age are repreſented to die off in the ſame time in the table of the probabilities of the duration of human life adopted for the calculation. We muſt therefore look into that table to find how many perſons out of P perſons living at the age of N years are repreſented as living to the age of $N+n$ years. And this number we will call p. Then it is evident that out of all the P purchaſers of the future ſums of one pound each, payable at the end of n years, only p perſons will be alive at the end of the ſaid n years to claim their reſpective payments. Therefore at the end of the ſaid n years the grantor of theſe future payments will have p ſums of one pound each to pay to the ſaid ſurviving purchaſers. Therefore, to the end that the ſaid grantor, when theſe payments become due, may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive juſt p times the preſent value of one pound payable at the end of n years, when the intereſt of money is that which is denoted by the proportion of r to 1, or p times the ſum which, being improved continually at compound intereſt during the ſpace of n years at the ſaid rate of intereſt, will at the end of the ſaid term amount to one pound: becauſe in that caſe, if he ſo improves the money he then receives, it will amount at the end of the ſaid term to exactly p times one pound, ſo as to enable him to ſatisfy the juſt demands of the p ſurviving purchaſers. Now the preſent value

value of one pound to be received at the end of n years, when the intereſt of money is that which is expreſſed by the proportion of r to 1, is juſt as much leſs than one pound as one pound is leſs than the amount of one pound improved at compound intereſt during n years at the ſame rate of intereſt. But the amount of one pound improved at compound intereſt for n years, at the rate of intereſt expreſſed by the proportion of r to 1, is r^n, or the nth power of r. Therefore the preſent value of one pound to be received at the end of n years, at the ſaid rate of intereſt, is to 1*l.* as 1*l.* is to r^n. And conſequently the ſaid preſent value of one pound is equal to $\frac{1}{r^n}$ of a pound. Therefore p times $\frac{1}{r^n}$ of a pound, or $\frac{p}{r^n}$£ is the ſum which the ſaid grantor ought to receive at the time of making the grants aforeſaid from all the P purchaſers of them. Therefore the ſum which he ought to receive at that time from each of the ſaid purchaſers is the Pth part of $\frac{p}{r^n}$£, or is equal to $\frac{p}{P \times r^n}$£. And conſequently, by Art. 21, when the ſaid grantor makes only one ſuch grant to a purchaſer of N years of age, he ought likewiſe to receive for it the ſame ſum of $\frac{p}{P \times r^n}$£. QEI.

XXIX. This concluſion may be expreſſed in words in the manner following.

Find the preſent value of one pound certain, to be received at the end of the given number of years, according to the given rate of intereſt, by the help of Mr. Smart's ſecond table of compound intereſt, or otherwiſe. Then find in the given table of the probabilities of the duration of human life at the ſeveral different ages of it, the number of perſons living at the age of the purchaſer. Then add to the age of the purchaſer the number of years at the end of which the ſum of one pound is made payable to him, and find in the ſaid table the number of perſons living at the ſaid greater age. Then multiply the aforeſaid preſent value of one pound certain by this latter number of perſons living; and divide the product thence ariſing by the number of perſons living at the age of the purchaſer. And the quotient will be the value of this contingent ſum of one pound, which is payable to the purchaſer at the end of the given number of years, if he is then alive, expreſſed in decimal parts of a pound ſterling.

XXX. COROLL. 1. If the ſum of money to be received by the purchaſer at the end of the given number of years is greater or leſs than one pound, it is evident that the price he ought to pay for it to the grantor will be greater or leſs than the price of the future payment of one pound in

the ſame proportion. Therefore if the ſum to be received at the end of n years is denoted by the letter S, the price he ought to pay for it will be $\frac{p}{P} \times \frac{S}{r^n}$. For as £1 is to S£, ſo is $\frac{p}{P \times r^n}$, or the price of a future payment of £1, to $\frac{p \times S}{P \times r^n}$, or $\frac{p}{P} \times \frac{S}{r^n}$.

XXXI. COROLL. 2. By finding in the method deſcribed in the ſolution of the foregoing problem the ſeveral values of one pound ſterling to be received by a purchaſer of a given age at the end of every future year of the whole ſpace of time through which it is poſſible that his life may be extended, provided he is living at the time when every ſuch payment becomes due, and adding theſe values all together, we ſhall obtain the value of a ſet of equal payments of one pound each to be made to the purchaſer at the end of every future year, or of an annuity of one pound a year for his life. And this is the manner in which tables of the values of life-annuities are, or ought to be, computed.

An example of the calculation of a life-annuity according to the method deſcribed in the foregoing articles.

XXXII. As an example of this method of computing the values of life-annuities, we will here compute at length the value of an annuity of one pound for the life of a purchaſer of 80 years of age, upon a ſuppoſition that the intereſt of money is 3 per cent. and that the probabilities of the duration of human life are ſuch as they are repreſented to be in Mr. de Parcieux's table.

Now it appears by Mr. de Parcieux's table that, out of 118 perſons of the age of 80 years, all living at the ſame time, 101 will live to the age of 81 years. And by Mr. Smart's ſecond table of compound intereſt, page 60, it appears that the preſent value of one pound certain, to be received at the end of one year, when the intereſt of money is 3 per cent. is .9708 of a pound. (For though he carries this value to eight places of figures, I omit the four laſt figures, as unneceſſary in the preſent computation.) Therefore by Art. 28, or 29, $\frac{101 \times 9708}{118}$£, or $\frac{98.0508}{118}$£, or .8309£, is the preſent value of the firſt year's payment of the annuity of one pound, which is to be made to the purchaſer when he ſhall be 81 years old.

In

In like manner it appears by Mr. de Parcieux's table that of the original number of 118 perſons of the age of 80 years, all living at the ſame time, there will be

	85	perſons living	at the age of 82 years,
and	71		at the age of 83,
and	59		at the age of 84,
and	48		at the age of 85,
and	38		at the age of 86,
and	29		at the age of 87,
and	22		at the age of 88,
and	16		at the age of 89,
and	11		at the age of 90,
and	7		at the age of 91,
and	4		at the age of 92,
and	2		at the age of 93,
and	1		at the age of 94,
and	none at all		at the age of 95.

And by Mr. Smart's ſecond table of compound intereſt, page 60, it appears that, when the intereſt of money is 3 per cent. the preſent value of one pound certain, payable at the end of

			£
	2 years,	is	.9425;
and at the end of	3 years,		.9151;
and at the end of	4 years,		.8884;
and at the end of	5 years,		.8626;
and at the end of	6 years,		.8374;
and at the end of	7 years,		.8130;
and at the end of	8 years,		.7894;
and at the end of	9 years,		.7664;
and at the end of	10 years,		.7440;
and at the end of	11 years,		.7224;
and at the end of	12 years,		.7013;
and at the end of	13 years,		.6809;
and at the end of	14 years,		.6611.

Therefore, by Art. 28 or 29, the value of the ſecond year's rent, or payment of one pound, which is to be received by the purchaſer when he is 82 years old, is $\frac{85 \times .9425}{118}$ £, or $\frac{80.1125}{118}$ £, or .6789 £.

And

And the value of the third year's rent, which is to be paid him when he is 83 years old, is $\frac{\text{£}71\text{x}.9151}{118}$, or $\frac{\text{£}65.0721}{118}$, or £.5514.

And that of the fourth year's rent is $\frac{\text{£}59\text{x}.8884}{118} = \frac{\text{£}52.4156}{118} = \text{£}.4442.$

And that of the fifth year's rent is $\frac{\text{£}48\text{x}.8626}{118} = \frac{\text{£}41.4048}{118} = \text{£}.3508.$

And that of the sixth year's rent is $\frac{\text{£}38\text{x}.8374}{118} = \frac{\text{£}31.8212}{118} = \text{£}.2612.$

And that of the seventh year's rent is $\frac{\text{£}29\text{x}.8130}{118} = \frac{\text{£}23.5770}{118} = \text{£}.1998.$

And that of the eighth year's rent is $\frac{\text{£}22\text{x}.7894}{118} = \frac{\text{£}17.3668}{118} = \text{£}.1472.$

And that of the ninth year's rent is $\frac{\text{£}16\text{x}.7664}{118} = \frac{\text{£}12.2624}{118} = \text{£}.1039.$

And that of the tenth year's rent is $\frac{\text{£}11\text{x}.7440}{118} = \frac{\text{£}8.1840}{118} = \text{£}.0693.$

And that of the eleventh year's rent is $\frac{\text{£}7\text{x}.7224}{118} = \frac{\text{£}5.0568}{118} = \text{£}.0428.$

And

And that of the twelfth year's rent is $\frac{\text{£}4\text{x}.7013}{118} = \frac{\text{£}2.8052}{118} = \text{£}.0237.$

And that of the thirteenth year's rent is $\frac{\text{£}2\text{x}.6809}{118} = \frac{\text{£}1.3618}{118} = \text{£}.0115.$

And that of the fourteenth year's rent is $\frac{\text{£}1\text{x}.6611}{118} = \frac{\text{£}.6611}{118} = \text{£}.0056.$

Add all these values of the several yearly rents together; and their sum total, £3.7212, or 3*l.* 14*s.* 5*d.* will be the value of an annuity of one pound for the whole life of the person of 80 years of age who was supposed to be desirous of purchasing it. QE I.

£	£	£	£
.8309	.2612	.0428	2.8562
.6789	.1998	.0237	.7814
.5514	.1472	.0115	.0836
.4442	.1039	.0056	
.3508	.0693		3.7212
		.0836	
2.8562	.7814		

XXXIII. The foregoing computation might have been somewhat shortened by first adding together all the several products £98.0508, £80.1125, £65.0721, £52.4156, &c. that arise from the multiplication of the numbers of persons living at the several ages of 81, 82, 83, 84, &c. years into the corresponding present values of the sum of one pound, and then dividing their sum total by 118, or the number of persons living at the age of 80 years, which is the age of the purchaser of the annuity. For as all these products are to be divided by 118, it is evident that the conclusion will be the same whether we first divide each of them by 118, and then add the quotients together, as has been done in the foregoing computation, or whether we first add the products themselves together, and then divide their sum by 118. And by this latter method of proceeding we shall have only one operation of division instead of several, which will considerably diminish the labour of the calculation. The addition of these products is as follows.

98.0508

£	£	£
98.0508	31.8212	5.0568
80.1125	23.5770	2.8052
65.0721	17.3668	1.3618
52.4156	12.2624	.6611
41.4048	8.1840	9.8849
337.0558	93.2114	
93.2114		
9.8849		
440.1521		

Therefore £440.1521 is the ſum of all thoſe products; which being divided by 118 gives us £3.7301, or 3*l.* 14*s.* 7*d.*½, for the value of an annuity of one pound for the life of a perſon of 80 years of age, when the intereſt of money is 3 per cent. Q E I.

N. B. This reſult is greater than the former by the ſmall difference of the 100th part of a pound, or about 2*d.*½; which is owing, I imagine, to our not loſing any part of the aforeſaid products by this addition of them, whereas in dividing each of them by 118, according to the former method, we obtained quotients which were for the moſt part ſomewhat leſs than the truth. But this difference is not worth attending to.

XXXIV. The reader will obſerve that in adding together the fourteen products in the laſt article, and the quotients of the diviſion of thoſe products by 118 in Art. 32, I have made uſe of a round-about method of proceeding to obtain thoſe ſums; to wit, by firſt ſeparating the ſaid products and quotients, which were to be added together, into three different parcels, two of which conſiſt of five terms and the third of four, and finding the ſums of theſe ſeveral parcels of terms, and then adding theſe three ſums together to obtain the ſum total of the whole fourteen terms. My reaſon for doing this was becauſe I find it eaſier to obtain the ſum total of a great number of quantities in this manner than by adding them all up together in one operation; and I think there is leſs danger of making an error in the computation. But this is a matter in which every perſon, engaged in arithmetical calculations, muſt be governed by his own experience.

XXXV. The foregoing example of the computation of the value of a life-annuity for the life of a purchaſer of fourſcore years of age will do as well to illuſtrate the method of computing ſuch annuities delivered in Coroll. 2d, Art. 31, as if we had taken a much younger life. And as this

this was the ſole intent of inſerting any example of this kind, an old life was pitched upon, in preference to a young one of only 20 or 30 years, for the ſake of avoiding the much greater number of multiplications and diviſions that would have been neceſſary in computing the value of an annuity for ſuch younger life, which might have appeared tedious to the reader.

XXXVI. COROLL. 3. The value of a remote life-annuity for the life of a purchaſer of a given age, that is, of an annuity that is to take place only at the end of a given number of years, and to continue from that time to the end of the purchaſer's life, according to any given rate of intereſt and any given table of the probabilities of human life, may likewiſe be computed by means of the foregoing problem. Of the value of a remote life-annuity.

For ſuch a remote life-annuity is a ſet of equal ſums of money which are to be paid to the purchaſer at the end of every year from the time when the annuity is to take place, and which are to continue from that time forwards to the end of the purchaſer's life. And conſequently, by finding the values of the ſaid equal contingent future payments from the time when the annuity is to commence to the end of the whole ſpace of time through which it is poſſible the annuitant's life may be prolonged, (that is, according to Mr. de Parcieux's table, to the end of that number of years which, being added to the purchaſer's preſent age, will make 94 years,) in the method deſcribed in the foregoing problem, and adding theſe values together, we ſhall obtain the value of the ſaid remote life-annuity which is to commence after a given number of years.

XXXVII. Thus, if a perſon of 80 years of age purchaſes a life-annuity of one pound, to commence when he ſhall be 85 years of age, or whereof he is to receive the firſt payment when he is 86 years old, the intereſt of money being 3 per cent. and the probabilities of the duration of human life ſuch as they are repreſented to be in Mr. de Parcieux's table, and it is required to find the value of ſuch annuity, we muſt find by the help of the foregoing problem the values of the ſeveral future payments of one pound each which are to be made to him at the end of his 86th, 87th, 88th, 89th, 90th, 91ſt, 92d, 93d, and 94th years, if he attains to thoſe reſpective ages, according to the ſaid rate of intereſt and table of probabilities. And the ſum of theſe values will be the value of the propoſed diſtant life-annuity, that was required to be found. Theſe values we have already found to be as follows. An example of the calculation of the value of ſuch an annuity.

The value of one pound a year to be received by a perſon of 80 years of age, according to the rate of intereſt and table of probabilities above-mentioned, when he ſhall be 86 years old, or

		£
at the end of 6 years,	is	.2612;
And the value of £1 to be received by him at the end of 7 years,	is	.1998;
And at the end of 8 years,	is	.1472;
And at the end of 9 years,	is	.1039;
And at the end of 10 years,	is	.0693;
And at the end of 11 years,	is	.0428;
And at the end of 12 years,	is	.0237;
And at the end of 13 years,	is	.0115;
And at the end of 14 years,	is	.0056.
And the ſum of all theſe values is - - -		.8650.

Therefore when the intereſt of money is 3 per cent. £.8650, or 17*s*. 3*d*. ½, is the value of an annuity of one pound for the life of a man of fourſcore years of age, to commence when he ſhall be 85 years old, or whereof the firſt payment is to be made him when he ſhall be compleatly 86 years old. Q E I.

SCHOLIUM.

Concerning the bill for eſtabliſhing certain remote life-annuities in pariſhes, which paſſed the Houſe of Commons in the ſpring of the year 1773.

XXXVIII. In this manner may be computed all thoſe tables of remote life-annuities which were prepared in the ſpring of the year 1773, and annexed to a bill at that time in the Houſe of Commons, for enabling pariſhes to grant ſuch annuities to their poor and induſtrious inhabitants. This bill was brought into the Houſe of Commons by Mr. Dowdeſwell, and ſeconded by Mr. George Rice, the member for Caermarthenſhire, and ſupported by Sir George Savile, Sir Richard Sutton, Mr. Edmund Burke, Mr. Cornwall, Mr. Jackſon, counſel to the Board of Trade, Mr. Thomas Townſhend, junior, and many other members of parliament of eminent abilities. And it paſſed in that houſe upon a diviſion, after a debate, by a majority of about two votes to one of all the members preſent. But it was thrown out by the Houſe of Lords. As great pains had been taken in the framing this bill by Mr. Dowdeſwell, (who brought it into the houſe,) Mr. Rice, and Sir George Savile, and many other gentlemen; who had often met together, for ſeveral hours at a time, at Sir George Savile's houſe in Leiceſter-Square to conſider the ſeveral clauſes of it; it may not be amiſs to give my readers the following general account of it.

XXXIX. The

XXXIX. The deſign of this bill was to encourage the lower ranks of people to induſtry and frugality, by laying before them a ſafe and eaſy method of employing ſome part of the money they could ſave out of their wages, or daily earnings, in a manner that would be moſt ſtrikingly for their benefit. It was obſerved that their wanting opportunities of this kind was probably one very principal cauſe of their neglecting ſo obvious a piece of prudence.—That they knew, for the moſt part, but little of the public funds; and that, when it happened that they were acquainted with them, the ſmallneſs of the ſums they would be intitled to receive as the intereſt of the money they could afford to lay out in them, was no encouragement to them to diſpoſe of it in that way. For what inducement, for inſtance, can it be to a poor man who has ſaved ten pounds out of his year's wages to inveſt it in the 3 per cent. bank-annuities, to conſider that it will produce him about ſix or ſeven ſhillings a year? It is but the wages of three days labour. And, if they lend their money to tradeſmen of their acquaintance, as they ſometimes do, it happens not unfrequently that their creditor becomes a bankrupt, and the money they had truſted him with is loſt for ever; which diſcourages others of them from ſaving their money at all, and makes them reſolve to ſpend it in the enjoyment of preſent pleaſure.—But that, if they ſaw an eaſy method of employing the money they could ſpare in ſuch a manner as would procure them a conſiderable income in return for it in ſome future period of their lives, without any ſuch hazard of loſing it by another man's folly or misfortune, it was probable they would frequently embrace it: and thus a diminution of the poor's rate on the eſtates of the rich, an increaſe of preſent induſtry and ſobriety in the poor, and a more independant and comfortable ſupport of them in their old age than they can otherwiſe expect, would be the happy conſequences of ſuch an eſtabliſhment. To effect theſe uſeful purpoſes the bill provided as follows. The deſign of the ſaid bill.

1ſt, That in every pariſh in England or Wales, in which there were two churchwardens and two or more overſeers of the poor, or one churchwarden and three or more overſeers of the poor, that is, four, or more, pariſh-officers intruſted with the care of the poor, it ſhould be lawful for the body of the rateable inhabitants of ſuch pariſh, that is, of thoſe inhabitants who contributed to the poor's rate, to grant life-annuities, payable every quarter of a year, to ſuch of the inhabitants thereof as ſhould be willing to purchaſe them, at the prices ſet down in the tables annexed to the bill, which were computed upon a ſuppoſition that the intereſt of money was only 3 per cent. The principal proviſions of it.

2dly, That the money received from the purchaſers of theſe annuities ſhould be veſted in the 3 per cent. bank-annuities in the name of the pariſh which had granted it: and the dividends duely received by them every

half-year, and employed in the purchaſe of new ſtock, ſo as to be improved at compound intereſt, to the end that it may be able to anſwer the annuities bought with it when they ſhall become due.

3dly, That for the aforeſaid purpoſes of granting theſe life-annuities, and receiving the money paid for them, and holding the ſtock purchaſed with it in the bank-annuities, and the other purpoſes of this bill, the ſaid rateable inhabitants of every ſuch pariſh ſhould be made a body politick and corporate, and have a common ſeal.

4thly, That, if the pariſh-fund in the 3 per cent. bank-annuities ſhould, by the miſmanagement of it, or from any other cauſe, prove inſufficient to ſupply the life-annuities charged upon it, the poor's-rate ſhould be made a collateral ſecurity to the poor purchaſers of theſe annuities for the payment of them, and ſhould be increaſed to ſuch a degree as ſhould be ſufficient to make good the deficiencies.

5thly, That no ſuch annuity ſhould be granted to any one perſon of more than 20*l.* ſterling a year.

6thly, That no ſuch annuity ſhould be granted to any of the inhabitants of a pariſh but ſuch as were legally ſettled in it, or had a right to be relieved by it in caſe they became poor and helpleſs.

7thly, That no ſuch annuity granted to any man ſhould commence before he was compleatly 50 years of age; nor to any woman before ſhe was compleatly 35 years of age.

8thly, That no ſum leſs than five pounds ſhould be received by the managers of theſe annuities as the price of any ſuch annuity.

9thly, That the miniſters, and church-wardens and overſeers of the poor, ſhould be the managers of theſe annuities for the whole body politick and corporate of the rateable inhabitants of the pariſh, and ſhould receive the money from the purchaſers of them and veſt it in the 3 per cent. bank-annuities, and receive the dividends, and employ them in the purchaſe of freſh ſtock, and pay the annuities to the purchaſers when they became due: and that for the tranſacting of the ſaid buſineſs at the Bank they ſhould give a power of attorney to ſome perſon reſiding in London.

10thly, That nevertheleſs the ſaid managers ſhould not have the power of granting any of theſe annuities without the conſent of the rateable inhabitants of the pariſh, who ſhould be aſſembled in veſtry for that purpoſe after public notice of ſuch intended meeting given in the pariſh-church

church on two Sundays immediately after divine ſervice. And in theſe meetings of the pariſhioners it ſhould be neceſſary not only that the majority of them in number ſhould conſent to the granting the annuity propoſed, but that thoſe who ſo conſented ſhould have paid more than half the laſt poor's-rate paid by all the rateable inhabitants ſo aſſembled.

This reſtraint was intended to prevent the renters of ſmall tenements in the pariſh from involving the pariſh in the contingent burthen on the poor's-rate that might ariſe from theſe annuities, againſt the will of the more ſubſtantial inhabitants.

And it was further provided that no ſuch annuity ſhould be granted unleſs there were preſent at the meeting, in which it was granted, at leaſt twelve of the ſaid rateable inhabitants of the pariſh, except in pariſhes where the whole number of rateable inhabitants was leſs than nineteen; and that in that caſe it ſhould be neceſſary that at leaſt two third parts of the whole number of inhabitants ſhould be preſent at it.

And, in the 11th and laſt place, it was provided that the purchaſers of theſe annuities ſhould not be permitted to alienate them without firſt making an offer of them to the pariſh at the price they were worth at the time of ſuch offer according to the tables annexed to the bill, or at ſome lower price: and that ſuch of them as ſhould, at the time of purchaſing them, conſent to a clauſe that ſhould declare them to be abſolutely unalienable, ſhould, in conſequence of ſuch conſent, be incapable of alienating them at all.

The reaſon of this reſtraint upon the alienation of theſe annuities was to guard the poor owners of them againſt their own folly and weakneſs, by making it impoſſible for them to ſell their annuities for a ſmall part of their true value, over a pot of ale and without a proper degree of deliberation.

The reaſon of computing the values of theſe life-annuities upon the ſuppoſition of ſo low a rate of intereſt as 3 per cent. was to make the fund ariſing from the money paid for them be amply ſufficient to anſwer them when they ſhould become due; ſo that it ſhould be almoſt impoſſible, without great negligence in the management of this fund, that there ſhould ever be a neceſſity of having recourſe to an augmentation of the poor's-rate to make good its deficiencies. Yet even at this low rate of intereſt the purchaſers of theſe annuities would uſually get nine or ten per cent. for their money, if they purchaſed them only five years before the time of their commencement; and 30 or 40 per cent. if they would be content to wait for them 25 or 30 years; which men under 30 years of age might do without any inconvenience. And the hope of this, it was preſumed, might be

be a ſufficient inducement to them to employ ſome part of their money in this way, and to be diligent in their callings, and frugal in their expences, with that view.

The principal objection that ſeemed likely to be made to the ſaid project.

XL. The only objection that ſeemed likely to be made to this project was the difficulty of carrying it into execution, ariſing from the inability of the miniſter and church-wardens and overſeers of the poor of the pariſh to manage the money received from the purchaſers of theſe annuities without an agent in London for that purpoſe; who would probably be, for the moſt part, either ſome ſtock-broker, or banker, or banker's clerk, or other man of buſineſs that dealt in money tranſactions, whom it might be difficult to engage in an employment of this kind without paying him for his trouble in a manner that the pariſh-fund could hardly afford. But this objection is not ſo ſtrong as it appears to be: becauſe the buſineſs of this kind to be done in London would not be ſo much as might at firſt be apprehended; and the price of brokerage upon buying and ſelling the pariſh-ſtock in the bank-annuities and receiving the dividends of it when they became due, is no great matter. But how far this difficulty was likely to hinder the execution of the bill could not be known with any tolerable degree of certainty without giving it a trial. And therefore the Houſe of Commons paſſed it. But that the experiment might be as little hazardous as poſſible, and pariſhes might not be involved by it againſt their wills in the danger of theſe remote incumbrances, the bill was made intirely optional, and the rateable inhabitants of every pariſh were left at liberty to grant or not grant any of theſe annuities, as they ſhould think fit, and even, after they had granted ſome ſuch, to deſiſt from granting any more. And this full liberty of proceeding herein according to their own judgements and inclinations was thought by the gentlemen who ſupported the bill in the Houſe of Commons to be a compleat anſwer to the objection above-mentioned ariſing from the ſuppoſed difficulty of carrying it into execution.

In order to remove the ſaid objection, the bill provided that every pariſh might chuſe whether it would, or would not, adopt the ſaid project.

An ingenious obſervation made in ſupport of the ſaid objection.

XLI. There was however a very ingenious and ſubtle obſervation made upon this anſwer, and in ſupport of the foregoing objection, by a noble lord, of diſtinguiſhed abilities, and who formerly filled the higheſt ſtation in the law with great reputation. This was, "that the option above-mentioned was not given to the right perſons, or to thoſe who were moſt likely to be affected by the burthens which the granting theſe annuities might hereafter bring upon the pariſhes.—For that the option was given to the rateable inhabitants of the pariſh, who were, for the moſt part, only renters of the lands they occupied; whereas the burthen upon the poor's-rate ariſing from the ſuppoſed deficiency of the annuity-fund was not likely to be felt till many years after the granting of the annuities, when

when the leaſes of the renters who had voted for the granting them, would be at an end, or, if they were renewed, would have been renewed at a lower rent than before, in conſideration of the approaching and probable increaſe of the poor's rate ariſing from the ſaid ſuppoſed deficiency; which would be an injury to the freeholders of the land, who were poſſeſſed of the permanent property of it:—and that therefore the conſent of the ſaid freeholders ought to be obtained to every act by which the lands of the pariſh might be expoſed to the danger of ſuch a future burthen."

This obſervation ſeems to be ſomewhat refined; but will admit, as I conceive, of the following anſwer. The rateable inhabitants of pariſhes are of the three following ſorts; either owners of the houſes and lands which they occupy; or renters of them under long leaſes for 21 years, or for three lives, and oft n with a right of renewal; or renters of them under ſhort leaſes for one or two years, or merely at the will of the owners without any leaſes. If they are of the firſt ſort, they are the very perſons in whom the noble author of the obſervation thinks the option of granting, or refuſing to grant, theſe life-annuities ought to have been veſted. If they are of the ſecond ſort, that is, renters of the lands they occupy under long leaſes, they then are more likely to feel the burthen brought upon the pariſh by the ſuppoſed augmentation of the poor's rate than the freeholder or owner of the reverſion, and therefore are fitter than he is, according to the principle of the obſervation, to be truſted with the power of bringing this contingent burthen upon the pariſh. And laſtly, if they are renters of the lands they occupy under ſhort leaſes or at will, (which is the caſe ſuppoſed in the objection,) they are, in conſequence of the precarioufneſs of ſuch a tenure, ſo much under the influence of their landlord, that, if he ſhould but ſignify his pleaſure to them, by his ſteward or by a letter, that he does not chuſe that any of theſe annuities ſhould be granted in the pariſh, leſt his lands ſhould be expoſed to ſuch a future increaſe of the poor's rate, they will be ſure to give their votes againſt them. So that in all theſe caſes the intereſts of the perſons who are moſt likely to be affected by the apprehended burthen on the poor's rate, are ſufficiently protected by the proviſion that veſts this option in the rateable inhabitants of the pariſh. And beſides, experience ſhews that the inhabitants of pariſhes in general, as well thoſe who rent lands and houſes by the year, or at will, as thoſe who have more permanent intereſts in them, are wonderfully averſe to every thing that has even a remote tendency to increaſe the poor's rate. And conſequently there is no reaſon to apprehend that they would conſent to grant any of theſe pariſh-annuities whenever there was the ſmalleſt danger of their being ill managed, and producing, in conſequence thereof, an augmentation of the poor's rate to make good the deficiencies of their proper fund.

An anſwer to the ſaid obſervation.

Some

Some other objections have alſo been made to this bill, which have been anſwered in a pamphlet, intitled, "*Conſiderations on the Bill now depending in the Houſe of Commons for enabling pariſhes to grant life-annuities to poor perſons, upon purchaſe, in certain circumſtances, and under certain reſtrictions; being an appendix to the pamphlet intitled, "A propoſal for eſtabliſhing life-annuities in pariſhes for the benefit of the induſtrious poor." Sold by B. White, in Fleet-Street, 1773;*" to which I refer the reader.

I have this further to ſay in favour of the foregoing propoſal, (which I hope will one day or other be again brought into parliament, and with better ſucceſs;) that it has been carefully examined, and fully approved, by the learned and publick-ſpirited Dr. Price, of Newington-Green near Iſlington, the author of the "*Obſervations on reverſionary payments*" above-mentioned; and by the very acute and judicious Dr. Benjamin Franklyn, and by Mr. Wedderburn, who at the time of this bill's paſſing the Houſe of Commons, was his Majeſty's ſollicitor-general, and now (in Auguſt, 1780,) is lord chief juſtice of the court of Common Pleas and baron Loughborough.

Of the value of an immediate, but imperfect, life-annuity.

XLII. COROLL. 4. The value of an immediate, but imperfect, life-annuity for the life of a purchaſer of a given age; that is, of an annuity that is to commence immediately, (ſo that the firſt payment thereof ſhall be received at the diſtance of only one year from the time of purchaſing it,) and to continue only during a certain number of years, which is leſs than that during which it is poſſible the purchaſer's life may be prolonged, and then to ceaſe, notwithſtanding the purchaſer ſhould live on to a later period; may likewiſe be computed by the help of the foregoing problem.

For we need only in this caſe compute the values of the ſeveral future payments that are to be made to the purchaſer at the ends of the ſeveral years during which the annuity is to continue, and add them together into one ſum. And this ſum will be the value of the propoſed annuity. QEI.

An example of the calculation of the value of ſuch an annuity.

Thus, if a man of fourſcore years of age wanted to purchaſe an annuity of one pound a year for the five following years of his life, and no longer, according to the rate of intereſt and table of probabilities mentioned in Coroll. 2, the price he ought to pay for it would be the ſum total of the ſeveral ſums following, to wit, £.8309, £.6789, £.5514, £.4442, and £.3508, which are the values of the ſeveral payments of one pound which would become due to him at the end of the 81ſt, 82d, 83d, 84th, and 85th years of his age; which ſum is £2.8562, or 2*l.* 17*s.* 1*d.*½.

Note.

Note. An annuity of this kind might be uſeful to a man who had a reverſionary intereſt in a good eſtate that was to fall in to him at the end of a certain, not very long, term of years. Thus, for example, if a man was the ground-landlord of a number of houſes that paid him only a trifling ſum in ground-rents, but which were to fall into his poſſeſſion by the expiration of the leaſes in ſeven or eight years time, and then would afford him an ample income, an annuity of this kind, which ſhould continue only till the ſaid leaſes ſhould expire, would be of great convenience to him.

Of the value of a diſtant and imperfect life-annuity.

XLIII. COROLL. 5. And in like manner the value of a diſtant and imperfect life-annuity for the life of a purchaſer of a given age; or of an annuity, depending on the purchaſer's life, but that is not to commence till the end of a given number of years after the time of purchaſing it, and is to continue only during a certain period of time leſs than the whole ſpace through which it is poſſible the purchaſer's life may be prolonged, and then is to ceaſe, although the purchaſer ſhould live beyond it; may be computed by the help of the foregoing problem.

For we need only compute the values of the ſeveral future payments that are to be made to the purchaſer at the ends of the ſeveral years during which the annuity is to continue. And the ſum total of theſe values will be the value of the propoſed annuity. Q E I.

An example of the calculation of the value of ſuch an annuity.

Thus, if a man of fourſcore years of age wants to purchaſe an annuity of one pound a year, that ſhall begin at the end of five years, and continue during the five following years, if he lives ſo long, and then ceaſe; ſo that he ſhall be intitled to receive the firſt payment of it at the end of the 86th year of his age, and to receive the other four payments at the ends of his 87th, 88th, 89th, and 90th years reſpectively, but nothing afterwards though he ſhould live to a greater age; according to the rate of intereſt and table of probabilities mentioned in Coroll. 2; the price he ought to pay for ſuch a diſtant and imperfect life-annuity would be the ſum total of the ſeveral following ſums, to wit, £.2612, £.1998, £.1472, £.1039, and £.0693, which are the values of the ſeveral payments of one pound which would be due to him at the ends of the 86th, 87th, 88th, 89th, and 90th years of his life reſpectively: which ſum is £.7814, or 15*s.* 6*d.* $\frac{1}{2}$.

Thus much may ſuffice for the computation of the values of annuities depending upon one life. We now proceed to conſider thoſe which depend upon the joint continuance of two lives.

PROBLEM III.

XLIV. To find the ſum of money which the purchaſer of a future payment of one pound ſterling, to be received at the end of any given number of years, provided the ſaid purchaſer and another perſon, (who may be called *his companion*,) ſhall be then living, ought to pay for ſuch future ſum; the ages of the ſaid purchaſer and his companion, and the rate of intereſt of money, and the probabilities of the duration of human life, being all given.

A ſolution of this problem in the caſe of a particular example.

Let the rate of intereſt of money be ſuppoſed to be 3 per cent. and the probabilities of the duration of human life to be ſuch as they are repreſented to be in Monſieur de Parcieux's table above-mentioned. And let the number of years at the end of which the ſaid ſum of one pound is to be paid to the purchaſer of it, in caſe not only the ſaid purchaſer himſelf, but likewiſe his companion aforeſaid, ſhall be then alive, be 30: and the age of the ſaid purchaſer 25 years; and that of his ſaid companion 20 years.

Then, in the firſt place, we muſt look into Mr. de Parcieux's table to ſee how many perſons of 25 years of age are there repreſented as all living at the ſame time. This number is 774. We muſt therefore ſuppoſe that the grantor of the one pound to the purchaſer propoſed in the queſtion makes at the ſame time 773 more ſuch grants of one pound to as many different perſons, all of the ſame age of 25 years, to be paid them at the end of 30 years, or when they ſhall be 55 years old, if, not only the grantees themſelves ſhall then be living, but certain other perſons, who may be called *their companions*, who are of the ſame age of 20 years with the companion of the purchaſer mentioned in the queſtion: that is, we muſt ſuppoſe that the grantor makes 774 ſuch grants in all, including that of the purchaſer propoſed in the queſtion. And we muſt likewiſe ſuppoſe that all theſe 774 purchaſers of theſe future payments of one pound have the ſame chance, one with another, of living any given number of years, or that there is no apparent reaſon for ſuppoſing that any one of them is more likely to live to any given future age than any other. This done, we muſt inquire how many of theſe 774 purchaſers of one pound each will be alive at the end of 30 years, ſuppoſing them to die off in the proportion mentioned in Mr. de Parcieux's table. Now it appears by Mr. de Parcieux's table that out of 774 perſons of the age of 25 years, all living at the ſame time, 526 will be alive at the age of 55 years, or at the end of 30 years. Therefore out of the ſaid 774 purchaſers of theſe future payments

payments of one pound each, only 526 will live to the end of the 30 years. And of theſe 526 ſurviving purchaſers only ſome part will be intitled to demand theſe payments, to wit, thoſe whoſe companions, who were of the age of 20 years at the time of making the grants, are likewiſe living at the end of the ſaid 30 years. For the other ſurviving purchaſers, whoſe companions are then dead, will, by the conditions of this problem, have no right to them. We muſt therefore, in the next place, inquire how many of the companions of the ſaid 526 ſurviving purchaſers will alſo be alive at the end of the ſaid 30 years. Now the companions of theſe 526 ſurviving purchaſers were at the time of making the grants juſt as many as thoſe purchaſers themſelves, that is, 526. We muſt therefore inquire by Mr. de Parcieux's table how many of theſe 526 companions of the ſaid 526 ſurviving purchaſers, who were all living and of the age of 20 years at the time of making the grants, will be alive at the end of the ſaid 30 years. Now it appears by Mr. de Parcieux's table that out of 814 perſons of the age of 20 years, all living at the ſame time, only 581 will be living at the age of 50 years; and conſequently out of 526 perſons of the age of 20 years, all living at the ſame time, only $526 \times \frac{581}{814}$, or 375 will be alive at the age of 50 years. Therefore of the 526 companions of the 526 ſurviving purchaſers only 375 will be living at the end of the ſaid 30 years. Therefore only 375 out of the ſaid 526 ſurviving purchaſers will be intitled to receive the ſaid payments of one pound each. Therefore at the end of the ſaid 30 years the grantor of the ſaid future payments will have only 375 ſums of one pound each to pay to the ſurviving purchaſers; which will be due to thoſe 375 of them whoſe companions will be then alive. And conſequently, to the end that the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive at the time of making the ſaid grants 375 times the preſent value of one pound payable at the end of 30 years, when the intereſt of money is 3 per cent. or 375 times the ſum which, being continually improved at compound intereſt during the ſaid term of 30 years at that rate of intereſt, will, at the end of that time, amount to one pound; becauſe in that caſe, if he improves the ſaid ſum of 375 times the preſent value of one pound, ſo received, at compound intereſt at the ſaid rate of 3 per cent. during the whole 30 years, it will in that time increaſe to juſt 375 pounds, which is the ſum which he will then be obliged to pay to the 375 ſurviving purchaſers who, by the continuance of the lives of their reſpective companions, will be intitled to their ſeveral payments of one pound a piece. Now, by Mr. Smart's ſecond table of compound intereſt, the preſent value of one pound payable at the end of 30 years. when the intereſt of money is 3 per cent. is .4119 of a pound. Therefore 375 times £.4119, or £154.4625, is the ſum which the ſaid grantor of theſe future payments ought to receive, at the time of making the ſaid grants, from all the 774 purchaſers of

them. Therefore the ſum which he ought then to receive from each of the ſaid purchaſers is the 774th part of £154.4625, that is, £.1995, or, nearly, £.2, or 4 ſhillings. And conſequently, by Art. 21, when he makes only one ſuch grant of one pound, payable at the end of 30 years, to a purchaſer of 25 years of age, provided a companion of the purchaſer, who is of the age of 20 years at the time of making the grant, ſhall alſo be living at the end of the ſaid 30 years, he ought to receive for it the ſame ſum of $\frac{2}{10}$ of a pound, or four ſhillings. Q E I.

XLV. I now proceed to give a ſolution of this problem in general terms, as I did before of problem 2d, in Art. 28.

A general ſolution of the foregoing problem.

Let 1 be to r as one pound ſterling, or any other ſum of money whatſoever ſuppoſed to be put out at intereſt at the rate ſuppoſed in the queſtion, is to the amount of the ſame ſum at the end of a year, or to the ſum total of the ſaid ſum and its intereſt for one year. And let n denote the number of years at the end of which the ſum of one pound is to be paid to the purchaſer, if both he and his companion are then alive. And let N denote the number of years in the age of the purchaſer at the time of his making the purchaſe of this future payment of one pound; and conſequently $N+n$ the number of years contained in his age at the time he becomes intitled to receive it. And let M denote the number of years in the age of the companion of the purchaſer at the time of making the purchaſe; and conſequently $M+n$ the number of years contained in his age at the time when the payment becomes due.

Then, in the firſt place, we muſt look into the table of the probabilities of the duration of human life by which the calculation is to be governed, and which I ſhall ſuppoſe to be that of Mr. de Parcieux, to find how many perſons of N years of age are there repreſented as all living at the ſame time. This number we will call P. We muſt then ſuppoſe that the grantor of the future payment of one pound to the purchaſer mentioned in the queſtion does not confine himſelf to that ſingle grant, but makes at the ſame time $P-1$ more ſuch grants of one pound each to as many different purchaſers of them; ſo that, including that to the purchaſer in the queſtion, he makes in all P ſuch grants to P different purchaſers. And we muſt further ſuppoſe that all theſe P purchaſers have the ſame chance, one with the other, of living any given number of years, or that there is no apparent reaſon for ſuppoſing that any one of them is more likely to live to any given future age than any other. And we muſt likewiſe ſuppoſe them to die off, in the courſe of the n years which are to elapſe before

before the payments become due, in the ſame proportion in which perſons of the ſame age are repreſented to have died off in the ſame number of years in the table of the probabilities of the duration of human life that is adopted for the calculation. We muſt therefore look into that table to find how many perſons out of P perſons living at the age of N years are repreſented as living to the age of $N+n$ years. And this number we will call p. Then it is evident that out of all the P purchaſers of the future ſums of one pound each, payable at the end of n years, only p perſons will be alive at the end of the ſaid n years. And of theſe p ſurviving purchaſers only ſome part will be intitled to demand theſe payments, to wit, thoſe whoſe companions, who were of the age of M years at the time of making the grants, are likewiſe living at the end of the ſaid n years. For the other ſurviving purchaſers, whoſe companions are then dead, will, by the conditions of this problem, have no right to them. We muſt therefore, in the next place, inquire how many of the companions of the ſaid p ſurviving purchaſers will be alive at the end of the ſaid n years; without concerning ourſelves about the fate of the companions of the deceaſed purchaſers, becauſe it is immaterial to the preſent queſtion how many of them, or whether all, or any, of them, are ſtill alive. Now the companions of theſe p ſurviving purchaſers were, at the time of making the grants, juſt as many as thoſe purchaſers themſelves, that is, juſt p in number. We muſt therefore inquire how many of theſe p companions of the ſaid p ſurviving purchaſers will be alive at the end of the ſaid n years. Theſe companions of the ſaid purchaſers were of the age of M years at the time of making the ſaid grants. We muſt therefore look into the table of the probabilities of the duration of human life adopted in the calculation, to find how many perſons are there repreſented as living at the ſaid age of M years, and how many at the ſubſequent age of $M+n$ years. Let the former of theſe numbers be denoted by Q, and the latter by q. Then, ſince the p companions of the p ſurviving purchaſers of theſe future payments of one pound are ſuppoſed to die off, in the courſe of theſe n years, in the ſame proportion as the Q perſons of the ſame age of M years are repreſented to have done in the ſaid table of probabilities, the number of perſons out of the ſaid p companions of the ſaid ſurviving purchaſers that will be alive at the end of the ſaid n years will be to the number of thoſe companions who are alive at the beginning of the ſaid time in the ſame proportion as q is to Q; and conſequently will be equal to $\frac{p \times q}{Q}$, or $\frac{pq}{Q}$. Therefore only $\frac{pq}{Q}$ out of the whole number p of the ſaid ſurviving purchaſers will be intitled to theſe payments of one pound each at the end of the ſaid n years. Therefore at the end of the ſaid n years the grantor of the ſaid future payments will have $\frac{pq}{Q}$ ſums, of one pound each,

to

to pay to theſe $\frac{pq}{Q}$ more fortunate ſurviving purchaſers, whoſe companions will have lived, as well as themſelves, to the end of the ſaid n years. Therefore, to the end that the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive, at the time of making the ſaid grants, $\frac{pq}{Q}$ times the preſent value of one pound payable at the end of n years, when the intereſt of money is that which is expreſſed by the proportion of r to 1, or $\frac{pq}{Q}$ times the ſum which, being continually improved at compound intereſt during the ſaid term of n years at that rate of intereſt, will, at the end of that time, amount to one pound: becauſe in that caſe, if he improves the ſaid ſum of $\frac{pq}{Q}$ times the preſent value of one pound, ſo received, at compound intereſt at the ſaid rate of intereſt during the whole term of n years, he will thereby have augmented it at the end of that time to juſt $\frac{pq}{Q}$ times one pound, or $\frac{pq}{Q}$ pounds, which is the ſum which he will then be obliged to pay to the $\frac{pq}{Q}$ fortunate ſurviving purchaſers of theſe future payments, whoſe companions will be then alive. Now the preſent value of one pound payable at the end of n years is $\frac{1}{r^n}$ of a pound. Therefore $\frac{pq}{Q}$ times $\frac{1}{r^n}$ of a pound, or $\frac{pq}{Qr^n}$ £, is the ſum of money which the ſaid grantor of theſe future payments ought to receive, at the time of making the ſaid grants, from all the P purchaſers of them. Therefore the ſum which he ought then to receive from each of the ſaid purchaſers is the Pth part of $\frac{pq}{Qr^n}$ £, or is equal to $\frac{pq}{PQr^n}$ £, or $\frac{pq}{PQ} \times \frac{1}{r^n}$ £. And conſequently, by Art. 21, when he makes only one ſuch grant of one pound, payable at the end of n years, to a purchaſer of N years of age, provided a companion of the purchaſer, who is of the age of M years at the time of making the grant, ſhall alſo be living at the end of the ſaid n years, he ought to receive for it from the ſaid ſingle purchaſer the ſame ſum of $\frac{pq}{PQr^n}$ £, or $\frac{pq}{PQ} \times \frac{1}{r^n}$ of a pound. Q E I.

XLVI. This

XLVI. This concluſion may be expreſſed in words in the manner following.

Find the preſent value of one pound certain, to be received at the end of the number of years (n) given in the problem, according to the rate of intereſt therein alſo given, and expreſſed by the proportion of r to 1; which preſent value will be $\frac{1}{r^n}$ of a pound. Then find in the given table of the probabilities of the duration of human life the number of perſons living at the age of the purchaſer; and call it P. Then add to the age of the purchaſer the number of years at the end of which the payment will become due; and find in the ſaid table the number of perſons living at the ſaid greater age; and call it p. Then find in the ſaid table of probabilities the number of perſons living at the age of the purchaſer's companion; and call it Q. Then add to the age of the ſaid companion of the purchaſer the number of years at the end of which the payment will become due; and find in the ſaid table the number of perſons that are there repreſented to be living at the ſaid greater age; and call it q. Then ſay as the number Q is to the number q, ſo is the number p to a fourth number; which will therefore be equal to $\frac{pq}{Q}$. Then multiply this fourth number into $\frac{1}{r^n}$ of a pound, or $\frac{1}{r^n}$ £, or the preſent value of one pound certain, payable at the end of the given number of years. And laſtly, divide the product, $\frac{pq}{Q} \times \frac{1}{r^n}$ £, thence ariſing, by P, the number of perſons repreſented in the given table of probabilities to be living at the age of the purchaſer. And the quotient of this diviſion, or the quantity $\frac{pq}{PQ} \times \frac{1}{r^n}$ £, will be the value of the future contingent payment of one pound, which is payable to the purchaſer at the end of the given number of years, provided both he and his companion are then alive, expreſſed in decimal parts of a pound ſterling.

The foregoing concluſion expreſſed in words.

XLVII. COROLL. 1. If the ſum of money to be received by the purchaſer at the end of the given number of years is greater or leſs than one pound, it is evident that the price he ought to pay for it to the grantor will be greater or leſs than the price of the future payment of one pound in the ſame proportion. Therefore, if the ſum to be received at the end of n years is denoted by S£, the price which the purchaſer ought to pay for it, upon the conditions ſuppoſed in the foregoing problem, will be $\frac{pq}{PQ} \times \frac{S}{r^n}$ £. For as as £1 is to S£, ſo is $\frac{pq}{PQ} \times \frac{1}{r^n}$ £, (which is the price of a future payment of one pound,) to $\frac{pq}{PQ} \times \frac{S}{r^n}$ £; which is therefore the price of a future payment of S pounds. QED.

XLVIII. CO-

Of the value of an annuity for the joint lives of two perſons.

XLVIII. COROLL. 2. By finding in the method deſcribed in the ſolution of the foregoing problem the ſeveral values of one pound ſterling, to be received by a purchaſer of a given age at the end of every future year of the whole ſpace of time during which it is poſſible that his life and that of his companion may be prolonged, provided the ſaid purchaſer and his companion, whoſe age is likewiſe given, ſhall be both living at the time when every ſuch payment becomes due; and then adding theſe values all together into one ſum; we ſhall obtain the value of a ſet of equal payments of one pound each to be received by the purchaſer at the end of every future year during the continuance of the joint lives of the ſaid purchaſer and his companion, or of an annuity for their joint lives. And this is the way in which tables of the values of annuities for the joint lives of two perſons of given ages ought to be computed.

An example of the calculation of an annuity for two joint lives according to the method deſcribed in the foregoing articles.

XLIX. As an example of this method of computing the values of annuities for two joint lives, let it be required to compute the value of an annuity for the joint lives of a purchaſer aged 80 years and a companion aged 75 years, upon a ſuppoſition that the intereſt of money is 3 per cent. and the probabilities of the duration of human life ſuch as they are repreſented to be in Mr. de Parcieux's table.

Now it appears by Mr. de Parcieux's table that out of 118 perſons of the age of 80 years, all living at the ſame time, 101 will live to the end of one year, or to the age of 81 years; and out of 211 perſons of the age of 75 years, all living at the ſame time, 192 will live to the end of one year, or to the age of 76 years. And by Mr. Smart's ſecond table of compound intereſt, page 60, it appears that, when the intereſt of money is 3 per cent. or r is $= 1.03$, the preſent value of one pound certain, to be received at the end of a year, is .9708 of a pound, or £.9703. Therefore, in order to find the value of the firſt payment of this annuity for the joint lives of theſe two perſons, we have $P = 118$, $p = 101$, $Q = 211$, $q = 192$, and $\frac{1}{r^n} = \frac{1}{r^n} =$ £.9708, and conſequently $\frac{pq}{PQ} \times \frac{1}{r^n}$, or $\frac{pq}{PQ} \times \frac{1}{r^1}$, $= \frac{101 \times 192}{118 \times 211} \times$ £.9708 $= \frac{19392}{24898} \times$ £.9708 $= \frac{£18825.7536}{24898} =$ £.7561. Therefore £.7561 is the value of the firſt year's payment of the annuity of one pound, which is to be made to him at the end of one year from the time of granting it, provided both he and his companion are then alive; at which time the purchaſer himſelf will be 81 years old, and his companion 76.

In

In like manner it appears by Monſieur de Parcieux's table that of the original number of 118 perſons of the age of 80 years, all living at the ſame time, there will be

	85 perſons living	at the age of 82 years,
and	71	at the age of 83,
and	59	at the age of 84,
and	48	at the age of 85,
and	38	at the age of 86,
and	29	at the age of 87,
and	22	at the age of 88,
and	16	at the age of 89,
and	11	at the age of 90,
and	7	at the age of 91,
and	4	at the age of 92,
and	2	at the age of 93,
and	1	at the age of 94,
and	none at all	at the age of 95.

And by the ſame table it appears that of the original number of 211 perſons of the age of 75 years, all living at the ſame time, there will be

	173 perſons living	at the age of 77 years,
and	154	at the age of 78,
and	136	at the age of 79,
and	118	at the age of 80,
and	101	at the age of 81,
and	85	at the age of 82,
and	71	at the age of 83,
and	59	at the age of 84,
and	48	at the age of 85,
and	38	at the age of 86,
and	29	at the age of 87,
and	22	at the age of 88,
and	16	at the age of 89 years.

It is needleſs to inquire how many perſons out of the aforeſaid original number of 211 perſons of the age of 75 years, all living at the ſame time, will live beyond the age of 89 years; becauſe, when the companion of the purchaſer is more than 89 years old, that is, when more than 14 years ſhall have elapſed after the time of granting the annuity, the purchaſer himſelf, who was 80 years old at the time of making the grant, will, according to Mr. de Parcieux's table, be dead, and conſequently the annuity will be at an end.

And by Mr. Smart's ſecond table of compound intereſt, page 60, it appears that, when the intereſt of money is 3 per cent. the preſent value of one pound certain, payable at the end of

			£
	2 years,	is	.9425;
and at the end of	3 years,		.9151;
and at the end of	4 years,		.8884;
and at the end of	5 years,		.8626;
and at the end of	6 years,		.8374;
and at the end of	7 years,		.8130;
and at the end of	8 years,		.7894;
and at the end of	9 years,		.7664;
and at the end of	10 years,		.7440;
and at the end of	11 years,		.7224;
and at the end of	12 years,		.7013;
and at the end of	13 years,		.6809;
and at the end of	14 years,		.6611.

Therefore, by Art. 46, the value of the ſecond year's rent, or payment of one pound, which is to be received by the purchaſer when he is 82 years old, provided his companion is then alive, is $= \frac{85 \times 173}{118 \times 211} \times £.9425 =$

$$\frac{14705 \times .9425}{118 \times 211} = \frac{14705 \times .9425}{24898} = \frac{13859.4625}{24898} = £.5566.$$

And the value of the third year's rent, which is to be paid him when he is 83 years old, if his companion is then alive, is $\frac{71 \times 154}{118 \times 211} \times £.9151 =$

$$\frac{10934}{24898} \times £.9151 = \frac{10{,}005.7034}{24898} = £.4018.$$

And the value of the fourth year's rent, which is to be paid him when he is 84 years old, if his companion is then alive, is $\frac{59 \times 136}{118 \times 211} \times £.8884 =$

$$\frac{8024}{24898} \times £.8884 = \frac{7288.4336}{24898} = £.2927.$$

And

And the value of the fifth year's rent is $\frac{48\times118}{118\times211} \times £.8626 = \frac{48}{211} \times £.8626 = £.1962$.

And that of the sixth year's rent is $\frac{38\times101}{118\times211} \times £.8374 = \frac{3838}{24898} \times £.8374 = £.1291$.

And that of the seventh year's rent is $\frac{29\times85}{118\times211} \times £.8130 = \frac{2465}{24898} \times £.8130 = \frac{£\ 2004.0450}{24898} = £.0805$.

And that of the eighth year's rent is $\frac{22\times71}{118\times211} \times £.7894 = \frac{1562}{24898} \times £.7894 = \frac{£\ 1233.0428}{24898} = £.0495$.

And that of the ninth year's rent is $\frac{16\times59}{118\times211} \times £.7664 = \frac{944}{24898} \times £.7664 = \frac{£\ 723.4816}{24898} = £.0290$.

And that of the tenth year's rent is $\frac{11\times48}{118\times211} \times £.7440 = \frac{528}{24898} \times £.7440 = \frac{£\ 392.8320}{24898} = £.0157$.

And that of the eleventh year's rent is $\frac{7\times38}{118\times211} \times £.7224 = \frac{266}{24898} \times £.7224 = \frac{£\ 192.1584}{24894} = £.0077$.

And that of the twelfth year's rent is $\frac{4\times29}{118\times211} \times £.7013 = \frac{116\times£.7013}{24898} = \frac{£\ 81.3508}{24898} = £.0032$.

And that of the thirteenth year's rent is $\frac{2\times22}{118\times211} \times £.6809 = \frac{44}{24898}$

$\times £.6809 = \frac{£}{\frac{29.9596}{24898}} = £.0012.$

And that of the fourteenth and laſt year's rent is $\frac{1\times16}{118\times211} \times £.6611$

$= \frac{£}{\frac{16\times.6611}{24898}} = \frac{£}{\frac{10.5776}{24898}} = £.0004.$

The ſum total of theſe values is £2.5197, or, nearly, £2.52, or 2*l.* 10s. 5*d.* which is therefore the value of the annuity propoſed for the joint lives of the purchaſer aged 80 years and his companion aged 75. QE I.

£
.7561
.5566
.4018
.2927
.1962
.1291
.0805
.0495
.0290
.0157
.0077
.0032
.0012
.0004
2.5197

Of the value of a remote life-annuity for two joint lives.

L. COROLL. 3. The value of a remote life-annuity, that is to begin at the diſtance of a given number of years, and to continue during the joint lives of the purchaſer and his companion, whoſe ages are given, may be eaſily computed by means of the foregoing problem.

For we need only compute the values of the ſeveral payments that are to be received at the end of every year after the ſaid given number of years during the longeſt poſſible continuance of the joint lives of the ſaid purchaſer

purchaſer and his companion, that is, during the longeſt poſſible continuance of the life of the older of them, according to the table of the probabilities of the duration of human life, that is, according to Mr. de Parcieux's table, till he is 94 years old. And the ſum of theſe values will be the value of the propoſed remote annuity. Q E I.

Of the value of an immediate, but imperfect, life-annuity for two joint lives.

LI. COROLL. 4. And in like manner the value of an immediate, but imperfect, life-annuity, that is to begin immediately, (or whereof the firſt payment is to be received at the end of a year,) but is to continue only during a certain number of years leſs than the utmoſt poſſible extent of the older of the two lives, provided both the purchaſer and his companion continue to live during the ſaid term, may be computed by means of the foregoing problem, by computing the values of the ſeveral payments that are to be received at the end of every year during the time that the ſaid annuity is to continue, and adding up thoſe values into one ſum.

Of the value of a remote and imperfect life-annuity for two joint lives.

LII. COROLL. 5. And in like manner we may compute the value of a remote and imperfect life-annuity depending on the joint lives of the purchaſer and his companion; that is, of an annuity depending on their joint lives, which is to commence at the end of a given number of years, and is to continue only during a certain ſpace of time leſs than the utmoſt poſſible extent of the older of the two lives, and then to ceaſe, though both the purchaſer and his companion ſhould be ſtill alive.

For this value will be the ſum of the values of the ſeveral payments that are to be received at the end of every year during the time that the annuity is to continue. Q E I.

LIII. COROLL. 6. It is eaſy to ſee that the method of reaſoning uſed in the ſolution of the foregoing problem may be extended to the valuation of a future payment, to be received at the end of a given number of years, depending upon the continuance of three lives, or any greater number of lives whatſoever. In the caſe of three lives the additions to the two ſolutions in Art 44 and 45 will be as follows.

An extenſion of the particular ſolution of the foregoing problem to the valuation of a future payment of one pound depending on three joint lives.

LIV. In the particular example ſolved in Art. 44 let the right of the purchaſer of 25 years of age to the future payment of one pound, at the end of 30 years, depend upon the continuance of two lives beſides his own,

own, inſtead of one; to wit, upon the life of a companion of the age of 20 years, as is ſuppoſed in that example, and upon the life of a ſecond companion of the age of 10 years. And let 774 ſuch grants of one pound each, to be paid at the end of 30 years, be made to as many different purchaſers; as is likewiſe ſuppoſed in the ſaid example: and let each of theſe 774 purchaſers have two companions; the one of 20, the other of 10, years of age, upon the continuance of whoſe lives at the end of the ſaid 30 years his right to the ſaid future payment of one pound ſhall depend.

Then it is evident that the number of the ſurviving purchaſers that will be intitled, at the end of the ſaid 30 years, to theſe payments of one pound, will be leſs than it was before; becauſe ſome of thoſe whoſe firſt companions (who were aged 20 years at the time of making the grants) are ſtill alive, will have loſt their ſecond companions, (who were 10 years of age at the time of making the grants,) and with them their right to this future payment. We muſt therefore inquire how many of the ſaid ſurviving purchaſers there will be whoſe firſt and ſecond companions will be both alive at the end of the ſaid 30 years. Now we have ſeen that the whole number of purchaſers who will themſelves be living at the end of the ſaid 30 years is 526; and that of theſe 526 ſurviving purchaſers there will be 375, whoſe firſt companions (who were 20 years of age at the time of making the grants,) will be alſo living at the end of the ſaid term. But there will be only a part of theſe 375 purchaſers whoſe ſecond, as well as firſt, companions will be living at that time. We muſt therefore now inquire how many ſuch there will be, that is, how many of the ſecond companions of theſe 375 purchaſers will be alive at the end of the ſaid 30 years. Now theſe ſecond companions of theſe 375 purchaſers were alſo 375 in number at the time of making the grants; each of the ſaid purchaſers having one ſecond, as well as one firſt, companion. We muſt therefore inquire how many out of theſe 375 ſecond companions of the ſaid 375 purchaſers will be living at the end of the ſaid 30 years. Now theſe ſecond companions of the ſaid purchaſers were ſuppoſed to be 10 years of age at the time of making the ſaid grants. And it appears that out of 880 perſons of 10 years of age, all living at the ſame time, 657 will be living at the age of 40 years, or at the end of a term of 30 years. Therefore out of 375 perſons of the ſame age of 10 years, all living at the ſame time, there will be $375 \times \frac{657}{880}$, or 280, who will live to the age of 40, or to the end of 30 years. Therefore only 280 of the ſecond companions of the ſaid 375 ſurviving purchaſers will be alive at the end of the ſaid 30 years. Therefore out of the whole number of 526 purchaſers who will ſurvive to the end of the ſaid 30 years, there will be only 280 whoſe firſt and ſecond companions will both be living at that time; and who will conſequently

conſequently be intitled to receive their reſpective payments of one pound each. Therefore 280 pounds is the ſum which the grantor of theſe future payments will have then to pay to theſe ſurviving purchaſers And conſequently 280 times the preſent value of one pound, payable at the end of 30 years, when the intereſt of money is 3 per cent. that is, 280 times .4119 of a pound, or £115.3320, is the ſum which he ought to receive, at the time of making the grants, from all the 774 purchaſers of theſe future payments. Therefore the ſum he ought to receive from each of the ſaid purchaſers at the time of making the grants, is the 774th part of £115.3320, or £.1490, or, nearly, £.15, or 3 ſhillings. And conſequently, by Art. 21, the ſame ſum of 3 ſhillings is the price which he ought to receive for ſuch a future payment of one pound, payable at the end of 30 years, provided the purchaſer and both his ſaid companions are then alive, when he makes only one ſuch grant. Q.E.I.

An extenſion of the general ſolution of the ſame problem to the valuation of a future payment of one pound depending on three joint lives.

LV. In the general ſolution of the foregoing problem given in Art. 45 let the right of the purchaſer to the future payment of one pound, to be received at the end of n years, depend upon the continuance of the lives of two companions inſtead of one, beſides his own life. And let the age of the purchaſer himſelf be N years, and that of his firſt companion M years, as before; and that of his ſecond companion L years. Further, let it be ſuppoſed, as before, that the grantor makes P ſuch grants of future ſums of one pound, to be received at the end of n years, to P different purchaſers, each of whom has two companions, the one of the age of M years, and the other of the age of L years, upon the continuance of both whoſe lives to the end of the ſaid n years his right to the ſaid future payment will depend.

Then it is evident that the number of the ſurviving purchaſers that will be intitled, at the end of the ſaid n years, to theſe payments of one pound each, will be leſs than it was before: becauſe ſome of thoſe whoſe firſt companions will be ſtill living at the end of that term, will have loſt their ſecond companions before that time, and with them their right to this future payment. We muſt therefore inquire how many of the ſaid ſurviving purchaſers there will be whoſe firſt and ſecond companions will be both living at the end of the ſaid n years. Now the whole number of purchaſers who will themſelves be living at the end of the ſaid n years, is p. And of theſe p purchaſers it has been ſhewn in Art. 45 that there will be $p \times \frac{q}{2}$, whoſe firſt companions (who were of the age of M years at the time

of

of making the grants,) will be living at the end of the ſaid term. But of theſe only a part will have their ſecond, as well as their firſt, companions living at the end of the ſaid term. We muſt therefore now inquire how great a part this will be of the ſaid $p \times \frac{q}{2}$ purchaſers whoſe firſt companions will be then alive, that is, how many of the ſecond companions of theſe $p \times \frac{q}{2}$ purchaſers will be alive at the end of the ſaid n years.

Now it is evident that theſe ſecond companions of the ſaid $p \times \frac{q}{2}$ purchaſers were alſo $p \times \frac{q}{2}$ in number at the time of making the ſaid grants; becauſe each of the ſaid purchaſers had a ſecond, as well as a firſt, companion. We muſt therefore inquire how many of theſe $p \times \frac{q}{2}$ ſecond companions of the ſaid $p \times \frac{q}{2}$ purchaſers will be living at the end of the ſaid n years, upon a ſuppoſition that they will die off in the courſe of the ſaid n years in the proportion repreſented in the table of the probabilities of the duration of human life by which the calculation is to be governed. Now theſe ſecond companions of the ſaid purchaſers were ſuppoſed to be L years of age at the time of making the grants. We muſt therefore look into the table of probabilities to ſee how many perſons are there repreſented to be living at the age of L years. Call this number T. We muſt then look again into the table to ſee how many perſons are there repreſented to be living at the age of $L+n$ years. Call this number t. Then, ſince out of T perſons of the age of L years, all living at the ſame time, only t perſons will live to the age of $L+n$ years, or to the end of n years, it follows that out of the $p \times \frac{q}{2}$ ſecond companions of the aforeſaid $p \times \frac{q}{2}$ purchaſers, who were all living at the time of making the ſaid grants, and were then L years old, only $p \times \frac{q}{2} \times \frac{t}{T}$ will be alive at the end of the ſaid n years. Therefore only $p \times \frac{q}{2} \times \frac{t}{T}$ of the ſaid ſurviving purchaſers will be intitled at the end of the ſaid n years to demand their reſpective payments of one pound. Therefore the ſum which the grantor of the ſaid payments will be obliged to pay to the ſaid purchaſers at the end of the ſaid n years, will be $p \times \frac{q}{2} \times \frac{t}{T}$ pounds. Therefore the ſum which

which he ought to receive, at the time of making the ſaid grants, from all the P purchaſers of the ſaid future payments, is $p \times \frac{q}{Q} \times \frac{t}{T}$ times the preſent value of one pound certain, payable at the end of n years, that is, $p \times \frac{q}{Q} \times \frac{t}{T}$ times $\frac{1}{r^n}$ of a pound, or $p \times \frac{q}{Q} \times \frac{t}{T} \times \frac{1}{r^n}$ £. Therefore the ſum which he ought to receive from each of the ſaid purchaſers, at the time of making the ſaid grants, as the price of his future payment of one pound, is the Pth part of $p \times \frac{q}{Q} \times \frac{t}{T} \times \frac{1}{r^n}$ £, or is equal to $\frac{p}{P} \times \frac{q}{Q} \times \frac{t}{T} \times \frac{1}{r^n}$ £. And conſequently, by Art. 21, the price which he ought to receive from a ſingle purchaſer of ſuch a future payment, when he makes only one ſuch grant, is likewiſe equal to $\frac{p}{P} \times \frac{q}{Q} \times \frac{t}{T} \times \frac{1}{r^n}$ £, or $\frac{pqt}{PQT} \times \frac{1}{r^n}$ £. QEI.

LVI. COROLL. 7. By finding, in the method deſcribed in the laſt corollary, the ſeveral values of one pound ſterling, to be received by a purchaſer of a given age at the end of every future year of the whole ſpace of time during which it is poſſible that his life and thoſe of his two companions, whoſe ages are likewiſe given, may be prolonged, provided that the ſaid purchaſer and both his ſaid companions ſhall be living at the time when every ſuch payment becomes due; and then adding theſe values all together into one ſum; we ſhall obtain the value of a ſet of equal payments of one pound each, to be received by the purchaſer at the end of every year during the continuance of the joint lives of the ſaid purchaſer and his two companions, or of an annuity for their joint lives. And this is the way in which tables of the values of annuities for the joint lives of three perſons of given ages ought to be computed.

Of the value of an annuity for three joint lives.

Of the value of future payments depending on the continuance of the longeſt of two or more given lives.

LVII. We come now in the laſt place to conſider the value of a future ſum of money depending upon the continuance of any one of two or more lives whoſe ages are given.

PROBLEM IV.

Of the value of a future payment of one pound ſterling, to be made at the end of a given number of years in caſe of the continuance of either of two given lives.

To find the ſum of money which a purchaſer ought to pay for a future ſum of one pound ſterling, to be received at the end of any given number of years, if either the ſaid purchaſer or another certain perſon (who may be called his companion,) ſhall be then living: the ages of the ſaid purchaſer and his companion, and the rate of intereſt of money, and the probabilities of the duration of human life, being all given.

A ſolution of this problem in the caſe of a particular example.

LVIII. Let the rate of intereſt of money be ſuppoſed to be 3 per cent. and the probabilities of the duration of human life to be ſuch as they are repreſented to be in Mr. de Parcieux's table above-mentioned. And let the ſpace of time at the end of which the ſaid ſum of one pound is to be paid to the purchaſer of it, if he is then living, or to his companion, if the purchaſer himſelf is then deceaſed, and his ſaid companion is ſtill alive, be 30 years: and the age of the ſaid purchaſer 25 years; and that of his ſaid companion 20 years.

Then in the firſt place we muſt look into Mr. de Parcieux's table to ſee how many perſons of 25 years of age are there repreſented to be all living at the ſame time. This number is 774. We muſt therefore ſuppoſe that the grantor of the future payment of one pound to the purchaſer propoſed in the queſtion, makes at the ſame time 773 more ſuch grants of one pound to as many different purchaſers, all of the ſame age of 25 years, to be paid to them at the end of 30 years, or when they ſhall be 55 years old, if they ſhall then be living, and if they ſhall then be dead, but certain other perſons, (who may be called their companions,) who are of the ſame age of 20 years with the companion of the purchaſer mentioned in the queſtion, ſhall be then alive, to be paid to their ſaid companions reſpectively: that is, we muſt ſuppoſe that the grantor makes 774 ſuch grants in all, including that to the purchaſer propoſed in the queſtion. And we muſt likewiſe ſuppoſe that all theſe 774 purchaſers of theſe future payments of one pound have the ſame chance, one with another, of living any given number of years, or that there is no apparent reaſon for ſuppoſing that any one of them is more likely to live to any given future age than any other. This done, we muſt inquire how many of theſe 774 purchaſers of theſe remote payments will be alive at the end of 30 years, ſuppoſing them to die off in that interval of time in the proportion mentioned in Mr. de Parcieux's table. Now it appears by Mr. de Parcieux's table that out of 774 perſons of the age of 25 years, all living at the ſame time, 526 will be alive at the age of 55 years, or at the end of 30 years. Therefore, out of the ſaid 774 purchaſers

purchaſers of theſe future payments of one pound each, 526 will live to the end of the ſaid 30 years; and conſequently 248 will have died in the mean time. But by the conditions of this problem, (which differ widely from thoſe of the laſt problem,) all theſe 526 ſurviving purchaſers of theſe future payments will be intitled to receive them, and likewiſe all the ſurviving companions of the deceaſed 248 purchaſers. We muſt therefore inquire, by the means of Mr. de Parcieux's table, how many of the companions of the ſaid deceaſed 248 purchaſers will be alive at the end of the ſaid 30 years. Now the number of the companions of the ſaid deceaſed 248 purchaſers, at the time of making the grants, was 248, each of the ſaid purchaſers having had one companion. And their age, at the time of making the grants, was 20 years. Now it appears from Mr. de Parcieux's table that out of 814 perſons of the age of 20 years, all living at the ſame time, 581 will be living at the age of 50 years, or at the end of 30 years. Therefore out of the 248 companions of the deceaſed purchaſers, (who were all living at the time of making the ſaid grants, and were then 20 years of age,) $248 \times \frac{581}{814}$, or 177, will be living at the end of the ſaid 30 years. Therefore the grantor at the end of the ſaid 30 years will have 526 pounds to pay to the 526 ſurviving purchaſers, and 177 pounds to pay to the ſurviving 177 companions of the 248 deceaſed purchaſers; that is, he will have, in all, 526 and 177 pounds, or 703 pounds, to pay to both. To the end therefore that the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive from all the purchaſers of the ſaid future payments, at the time of making the ſaid grants, 703 times the preſent value of one pound, payable at the end of 30 years, when the intereſt of money is 3 per cent. that is, 703 times .4119 of a pound, or £289.5657. Therefore the ſum which he ought to receive, at the time of making the ſaid grants, from each of the ſaid purchaſers, who are 774 in number, is the 774th part of £289.5657, or .3741 of a pound. Therefore, by Art. 21, the ſum which he ought to receive from a ſingle purchaſer, as the price of ſuch a future payment of one pound, when he makes only one ſuch grant, is likewiſe £.3741, or 7*s*. 5*d*.$\frac{3}{4}$. QEI.

A general ſolution of the foregoing problem.

LIX. Let 1 be to r as the ſum of one pound ſterling, or any other ſum of money whatſoever, that is put out to intereſt at the rate ſuppoſed in the queſtion, is to the amount of the ſame ſum at the end of a year, or to the ſum total of the ſaid ſum and its intereſt for a year. And let n denote the number of years at the end of which the ſum of one pound is to be paid to the purchaſer, if he is then alive, or, if he is then dead but his companion is ſtill alive, to his companion. And let N denote the number of years in the age of the purchaſer, at the time of his purchaſing this future

payment of one pound, and M the number of years in the age of his companion at the ſame time: whence it is evident that $N+n$ will be the number of years in the age of the purchaſer, and $M+n$ the number of years in the age of his companion, at the end of the ſaid n years, or when the payment becomes due.

Then, in the firſt place, we muſt look into the table of probabilities of the duration of human life by which the calculation is to be governed, to find how many perſons of N years of age are there repreſented to be living at the ſame time. This number we will call P. We muſt then ſuppoſe that the grantor of the ſaid future payment of one pound to the purchaſer mentioned in the queſtion does not confine himſelf to that ſingle grant, but makes at the ſame time $P-1$ more ſuch grants, of one pound each, to as many different purchaſers of them, all of the ſame age of N years; ſo that, including the grant to the purchaſer in the queſtion, he makes, in all, P ſuch grants to P different purchaſers. And we muſt further ſuppoſe that all theſe P purchaſers have the ſame chance, one with another, of living any given number of years, or that there is no apparent reaſon for ſuppoſing that any one of them is more likely to live to any given future age than any other. And we muſt likewiſe ſuppoſe them to die off, in the courſe of the ſaid n years which are to elapſe before the ſaid payments become due, in the ſame proportion in which perſons of the ſame age are repreſented to have died off in the ſame number of years in the table of the probabilities of the duration of human life that is adopted for the calculation. We muſt therefore look into that table to find how many perſons out of P perſons of the age of N years, all living at the ſame time, are repreſented as living to the age of $N+n$ years. And this number we will call p. Then it is evident that out of all the P purchaſers of the future ſums of one pound each, payable at the end of n years, only p perſons will be alive at the end of the ſaid n years. Therefore the number of theſe purchaſers who will have died in the ſaid n years is $P-p$. But by the conditions of this problem (which differ widely from thoſe of the laſt problem,) all the ſaid p ſurviving purchaſers will be intitled to receive their reſpective payments of one pound, and likewiſe all the ſurviving companions of the $P-p$ deceaſed purchaſers. We muſt therefore inquire, by means of the aforeſaid table of probabilities, how many of the companions of the ſaid deceaſed $P-p$ purchaſers will be alive at the end of the ſaid n years. Now the number of the companions of the ſaid deceaſed $P-p$ purchaſers, at the time of making the grants, was the ſame with that of the ſaid deceaſed purchaſers, that is, $P-p$; becauſe each of theſe deceaſed purchaſers was ſuppoſed to have one companion. And the age of theſe companions, at the time of making the grants, was M years. We muſt therefore look into the table of probabilities aforeſaid, to find how many perſons are there repreſented to be living at the age of M years, and how many at the ſubſequent age of $M+n$ years. Let the former of theſe numbers be denoted by Q, and the latter

latter by q. Then, since the $P-p$ companions of the $P-p$ deceased purchasers of these future payments are supposed to die off, in the course of these n years, in the same proportion as the Q persons of the same age of M years are represented to have done in the said table of probabilities, the number of persons out of the said $P-p$ companions of the said $P-p$ deceased purchasers, that will be alive at the end of the said n years, will be to the number of those companions who were alive at the beginning of the said time, in the same proportion as q is to Q; and consequently will be equal to $\overline{P-p} \times \frac{q}{Q}$. Therefore the grantor of the said future payments, will, at the end of the said n years, be obliged to pay p sums of one pound each to the p surviving purchasers themselves, and likewise $\overline{P-p} \times \frac{q}{Q}$ sums of one pound each to the $\overline{P-p} \times \frac{q}{Q}$ surviving companions of the $P-p$ deceased purchasers; that is, he will in all have $p+\overline{P-p} \times \frac{q}{Q}$ sums of one pound to pay to them both. Therefore, to the end that he may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive from all the purchasers of the said future payments, at the time of making the said grants, $p+\overline{P-p} \times \frac{q}{Q}$ times the present value of one pound, payable at the end of n years, when the interest of money is that which is expressed by the proportion of r to 1, that is, $p+\overline{P-p} \times \frac{q}{Q}$ times $\frac{1}{r^n}$ of a pound, or $p+\frac{Pq-pq}{Q}$ times $\frac{1}{r^n}$£, or $\overline{\frac{pQ+Pq-pq}{Q}} \times \frac{1}{r^n}$£. Therefore the sum which he ought to receive, at the time of making the said grants, from each of the said purchasers, whose number is P, is the Pth part of $\overline{\frac{pQ+Pq-pq}{Q}} \times \frac{1}{r^n}$£, or is equal to $\overline{\frac{pQ+Pq-pq}{PQ}} \times \frac{1}{r^n}$£. And consequently, by Art. 21, the sum which he ought to receive from a single purchaser, as the price of such a future payment of one pound, upon the conditions above-mentioned, when he makes only one such grant, is likewise equal to this same quantity, $\overline{\frac{pQ+Pq-pq}{PQ}} \times \frac{1}{r^n}$£, $\left(\text{or } \overline{\frac{pQ}{PQ}+\frac{Pq}{PQ}-\frac{pq}{PQ}} \times \frac{1}{r^n}\text{£},\right)$ or $\overline{\frac{p}{P}+\frac{q}{Q}-\frac{pq}{PQ}} \times \frac{1}{r^n}$£. QEI.

LX. CO-

Of the relation of the ſaid value to the values of the ſame payment of one pound ſterling to be made at the end of the ſame number of years in caſe of the continuance of the ſingle lives of the ſame two perſons and of the continuance of their joint lives.

LX. COROLL. 1. The value found by the foregoing problem, that is, the value of a future payment of one pound, to be received at the end of a given number of years denoted by n, provided that either of two perſons of given ages ſhall be then alive, is equal to the exceſs of the value of the ſame payment, to be received at the ſame time, in caſe the firſt of the ſaid perſons ſhall be then alive, added to the value of it, to be received at the ſame time, in caſe the ſecond of the ſaid perſons ſhall be then alive, above the value of it, to be received at the ſame time, in caſe both the ſaid perſons ſhall be then alive.

For the value found by the foregoing problem is $\overline{\frac{p}{P}+\frac{q}{Q}-\frac{pq}{PQ}} \times \frac{1}{r^n}£$, or $\frac{p}{p} \times \frac{1}{r^n}£ + \frac{q}{Q} \times \frac{1}{r^n}£ - \frac{pq}{PQ} \times \frac{1}{r^n}$. And it has been ſhewn in Problem 2d, Art. 28, that $\frac{p}{P} \times \frac{1}{r^n}£$ is the value of a future payment of one pound, to be received at the end of n years in caſe a purchaſer of the age of N years ſhall be then alive, and that $\frac{q}{Q} \times \frac{1}{r^n}£$ is the value of a future payment of one pound, to be received at the end of n years in caſe a purchaſer of the age of M years ſhall be then alive: and it has been ſhewn in Problem 3d, Art. 45, that $\frac{pq}{PQ} \times \frac{1}{r^n}£$ is the value of a future payment of one pound, to be received at the end of n years in caſe a purchaſer of the age of N years and a companion of the age of M years, ſhall be both living at that time. Therefore the value found in the foregoing problem is equal to the exceſs of the ſum of the two former values above the third, or laſt, value; or the value of a future payment of one pound, to be received at the end of n years in caſe that either of two given lives ſhall be then in being, is equal to the exceſs of the ſum, of the two values of the ſame future payment, to be received at the ſame time, in caſe each of the ſame two lives ſhall be then in being, above the value of the ſame future payment, to be received at the ſame time, in caſe both the ſaid lives ſhall be then in being. Q E D.

Of the value of a future payment of S pounds depending on the ſame contingencies.

LXI. COROLL. 2. If the ſum of money to be received by the purchaſer at the end of the given number of years is greater or leſs than one pound, it is evident that the price he ought to pay for it will be greater or leſs than the price of the future payment of one pound in the ſame proportion Therefore for a future payment of S pounds, to be received at the end of n years in caſe either himſelf or his companion, whoſe ages are N and M years, ſhall be then alive, he ought to pay the ſum of $\overline{\frac{p}{P}+\frac{q}{Q}-\frac{pq}{PQ}} \times \frac{S}{r^n}£$.

LXII. CO-

LXII. COROLL. 3. By finding in the method deſcribed in the ſolution of the foregoing problem the ſeveral values of one pound ſterling, to be received by a purchaſer of a given age at the end of every future year of the whole ſpace of time during which it is poſſible that either his life or that of his companion, whoſe age is alſo ſuppoſed to be given, may be prolonged, in caſe that either the ſaid purchaſer or his ſaid companion ſhall be living at the time when every ſuch payment becomes due; and then adding theſe values all together into one ſum; we ſhall obtain the value of a ſet of equal payments of one pound each, to be received by the purchaſer at the end of every future year during the lives of the ſaid purchaſer and his companion and the life of the longer liver of them, that is, of an annuity of one pound for the lives of the ſaid purchaſer and his companion and the life of the longer liver of them.

Of the value of an annuity of one pound ſterling for the life of the longer liver of two perſons of given ages.

LXIII. COROLL. 4. The value of an annuity of one pound for the lives of two perſons of given ages and the life of the longer liver of them, is equal to the exceſs of the ſum of the values of two ſeparate annuities of one pound each, for the ſingle lives of the ſame perſons, above the value of an annuity of one pound for their joint lives.

Of the relation of the ſaid value to the values of the like annuities of one pound for the ſingle lives of the ſame two perſons and for their joint lives.

In the foregoing articles P denoted the number of perſons repreſented, in the table of the probabilities of the duration of human life adopted for the calculation, to be living at the age of the purchaſer, or at the age of N years; and $\mathcal{Q}$ denoted the number of perſons therein repreſented to be living at the age of the purchaſer's companion, or at the age of M years. Now let P' denote the number of perſons therein repreſented to be living at the age of $N+1$, and P'' thoſe living at the age of $N+2$, and P''' thoſe living at the age of $N+3$, and P^{IV} thoſe living at the age of $N+4$; and ſo on for the following years; the Roman numeral figures placed over every new letter P denoting the number of years by which every new age, correſponding to it, exceeds the age of N years, or the age of the purchaſer. And, in like manner, let $\mathcal{Q}'$ denote the number of perſons repreſented in the ſaid table to be living at the age of $M+1$ years; and $\mathcal{Q}''$ the number of perſons living at the age of $M+2$ years; and $\mathcal{Q}'''$ thoſe living at the age of $M+3$ years; and $\mathcal{Q}^{IV}$ thoſe living at the age of $M+4$ years; and ſo on for the following years; the Roman numeral figures placed over every new letter $\mathcal{Q}$ denoting the number of years by which every new age correſponding to it, exceeds the age of M years, or the age of the purchaſer's companion.

Then it is evident from the foregoing problem that the value of a ſum of one pound, to be received at the end of one year, in caſe either

either the purchaſer or his companion ſhall be then alive, will be equal to $\left|\frac{P^{\prime}}{P}+\frac{Q^{\prime}}{Q}-\frac{P^{\prime}\,Q^{\prime}}{P\;Q}\right| \times \frac{1}{r}\mathcal{L}$; and the value of the ſame ſum, to be received at the end of two years, ſubject to the ſame contingency, will be equal to $\left|\frac{P^{\prime\prime}}{P}+\frac{Q^{\prime\prime}}{Q}-\frac{P^{\prime\prime}\;Q^{\prime\prime}}{P\;Q}\right| \times \frac{1}{r^2}\mathcal{L}$; and the value of the ſame ſum, to be received at the end of three years, ſubject to the ſame contingency, will be $\left|\frac{P^{\prime\prime\prime}}{P}+\frac{Q^{\prime\prime\prime}}{Q}-\frac{P^{\prime\prime\prime}}{P}\frac{Q^{\prime\prime\prime}}{Q}\right| \times \frac{1}{r^3}\mathcal{L}$; and the value of the ſame ſum, to be received at the end of four years, ſubject to the ſame contingency, will be $\left|\frac{P^{\text{IV}}}{P}+\frac{Q^{\text{IV}}}{Q}-\frac{P^{\text{IV}}\;Q^{\text{IV}}}{P\;Q}\right| \times \frac{1}{r^4}\mathcal{L}$; and ſo on for the following years.

Therefore the ſum of theſe values continued to the utmoſt poſſible extent of the younger of theſe two lives, that is, the value of a life-annuity of one pound for the lives of the purchaſer and his companion and the life of the longer liver of them, is equal to the following ſet of values; to wit,

$$\frac{P^{\prime}}{P}+\frac{Q^{\prime}}{Q}-\frac{P^{\prime}\;Q^{\prime}}{P\;Q} \times \frac{1}{r}\mathcal{L}$$

$$+\left|\frac{P^{\prime\prime}}{P}+\frac{Q^{\prime\prime}}{Q}-\frac{P^{\prime\prime}\;Q^{\prime\prime}}{P\;Q}\right| \times \frac{1}{r^2}\mathcal{L},$$

$$+\left|\frac{P^{\prime\prime\prime}}{P}+\frac{Q^{\prime\prime\prime}}{Q}-\frac{P^{\prime\prime\prime}\;Q^{\prime\prime\prime}}{P\;Q}\right| \times \frac{1}{r^3}\mathcal{L},$$

$$+\left|\frac{P^{\text{IV}}}{P}+\frac{Q^{\text{IV}}}{Q}-\frac{P^{\text{IV}}\;Q^{\text{IV}}}{P\;Q}\right| \times \frac{1}{r^4}\mathcal{L},$$

$$+\left|\frac{P^{\text{V}}}{P}+\frac{Q^{\text{V}}}{Q}-\frac{P^{\text{V}}\;Q^{\text{V}}}{P\;Q}\right| \times \frac{1}{r^5}\mathcal{L},$$

+&c. continued to the utmoſt extent of human life;

And

And conſequently to the following ſet of quantities; to wit,

$$-\frac{P^{\prime}}{P}\times\frac{1}{r}\mathcal{L}\quad+\frac{Q^{\prime}}{Q}\times\frac{1}{r}\mathcal{L}\quad-\frac{P^{\prime}\,Q^{\prime}}{P\;Q}\times\frac{1}{r}\mathcal{L}$$

$$+\frac{P^{\prime\prime}}{P}\times\frac{1}{r^{2}}\mathcal{L}\quad+\frac{Q^{\prime\prime}}{Q}\times\frac{1}{r^{2}}\mathcal{L}\quad-\frac{P^{\prime\prime}\,Q^{\prime\prime}}{P\;Q}\times\frac{1}{r^{2}}\mathcal{L}$$

$$+\frac{P^{\prime\prime\prime}}{P}\times\frac{1}{r^{3}}\mathcal{L}\quad+\frac{Q^{\prime\prime\prime}}{Q}\times\frac{1}{r^{3}}\mathcal{L}\quad-\frac{P^{\prime\prime\prime}\,Q^{\prime\prime\prime}}{P\;Q}\times\frac{1}{r^{3}}\mathcal{L}$$

$$+\frac{P^{\mathrm{IV}}}{P}\times\frac{1}{r^{4}}\mathcal{L}\quad+\frac{Q^{\mathrm{IV}}}{Q}\times\frac{1}{r^{4}}\mathcal{L}\quad-\frac{P^{\mathrm{IV}}\,Q^{\mathrm{IV}}}{P\;Q}\times\frac{1}{r^{4}}\mathcal{L}$$

$$+\frac{P^{\mathrm{V}}}{P}\times\frac{1}{r^{5}}\mathcal{L}\quad+\frac{Q^{\mathrm{V}}}{Q}+\frac{1}{r^{5}}\mathcal{L}\quad-\frac{P^{\mathrm{V}}\,Q^{\mathrm{V}}}{P\;Q}\times\frac{1}{r^{5}}\mathcal{L}$$

+&c. continued to the utmoſt extent of human life:

And conſequently to the three following ſets of quantities, which are the ſame with the quantities laſt mentioned, but only are placed in a different order, (thoſe which before were placed in perpendicular columns, one under the other, being now placed in ſeparate horizontal lines,) to wit,

1ſt, $\frac{P^{\prime}}{P}\times\frac{1}{r}\mathcal{L}+\frac{P^{\prime\prime}}{P}\times\frac{1}{r^{2}}\mathcal{L}+\frac{P^{\prime\prime\prime}}{P}\times\frac{1}{r^{3}}\mathcal{L}+\frac{P^{\mathrm{IV}}}{P}\times\frac{1}{r^{4}}\mathcal{L}+\frac{P^{\mathrm{V}}}{P}\times\frac{1}{r^{5}}\mathcal{L}$

+&c. continued to the utmoſt extent of human life;

And 2dly, $\frac{Q^{\prime}}{Q}\times\frac{1}{r}\mathcal{L}+\frac{Q^{\prime\prime}}{Q}\times\frac{1}{r^{2}}\mathcal{L}+\frac{Q^{\prime\prime\prime}}{Q}\times\frac{1}{r^{3}}\mathcal{L}+\frac{Q^{\mathrm{IV}}}{Q}\times\frac{1}{r^{4}}\mathcal{L}$

$+\frac{Q^{\mathrm{V}}}{Q}\times\frac{1}{r^{5}}\mathcal{L}$+&c. continued to the utmoſt extent of human life;

And 3dly, $-\frac{P^{\prime}\,Q^{\prime}}{P\;Q}\times\frac{1}{r}\mathcal{L}-\frac{P^{\prime\prime}\,Q^{\prime\prime}}{P\;Q}\times\frac{1}{r^{2}}\mathcal{L}-\frac{P^{\prime\prime\prime}\,Q^{\prime\prime\prime}}{P\;Q}\times\frac{1}{r^{3}}\mathcal{L}$

$-\frac{P^{\mathrm{IV}}\,Q^{\mathrm{IV}}}{P\;Q}\times\frac{1}{r^{4}}\mathcal{L}-\frac{P^{\mathrm{V}}\,Q^{\mathrm{V}}}{P\;Q}\times\frac{1}{r^{5}}\mathcal{L}$—&c. continued to the utmoſt extent of human life;

that is, the value of the life-annuity of one pound for the lives of the ſaid purchaſer and his companion and the life of the longer liver of them, is equal to the exceſs of the ſum of the firſt and ſecond of the three laſt ſets of quantities above the third ſet.

Now it appears from Problem 2d, Coroll. 2, Art. 31, that the firſt of theſe three ſets of quantities, to wit, $\frac{P^{\text{I}}}{P} \times \frac{1}{r}£ + \frac{P^{\text{II}}}{P} \times \frac{1}{r^2}£ + \frac{P^{\text{III}}}{P} \times \frac{1}{r^3}£ + \frac{P^{\text{IV}}}{P} \times \frac{1}{r^4}£ + \frac{P^{\text{V}}}{P} \times \frac{1}{r^5}£ +$&c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the life of the purchaſer, who was ſuppoſed to be of the age of N years, which correſponds to the number P in the table.

And it appears, in like manner, that the 2d of theſe 3 ſets of quantities, to wit, $\frac{Q^{\text{I}}}{Q} \times \frac{1}{r}£ + \frac{Q^{\text{II}}}{Q} \times \frac{1}{r^2}£ + \frac{Q^{\text{III}}}{Q} \times \frac{1}{r^3}£ + \frac{Q^{\text{IV}}}{Q} \times \frac{1}{r^4}£ + \frac{Q^{\text{V}}}{Q} \times \frac{1}{r^5}£$ +&c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the life of the ſaid purchaſer's companion, who was ſuppoſed to be of the age of M years, which correſponds to the number Q in the table.

And it appears from Prob. 3, Coroll. 2, Art. 48, that the laſt of the ſaid 3 ſets of quantities, to wit, $\frac{P^{\text{I}}\,Q^{\text{I}}}{P\;Q} \times \frac{1}{r}£ + \frac{P^{\text{II}}\,Q^{\text{II}}}{P\;Q} \times \frac{1}{r^2}£ + \frac{P^{\text{III}}\,Q^{\text{III}}}{P\;Q} \times \frac{1}{r^3}£ + \frac{P^{\text{IV}}\,Q^{\text{IV}}}{P\;Q} \times \frac{1}{r^4}£ + \frac{P^{\text{V}}\,Q^{\text{V}}}{P\;Q} \times \frac{1}{r^5}£ +$&c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the joint lives of the ſaid purchaſer and his companion, whoſe ages N and M correſpond to the numbers P and Q in the ſaid table.

Therefore the value of a life-annuity of one pound for the lives of the ſaid purchaſer and his companion, and the life of the longer liver of them, is equal to the exceſs of the ſum of the values of two ſeparate life-annuities of one pound each, for the ſingle lives of the ſaid purchaſer and his companion, above the value of an annuity of one pound for their joint lives. Q E D.

LXIV. Therefore when tables of the values of life-annuities for ſingle lives and for two joint lives have been computed and prepared for uſe, it ſeems to be unneceſſary, and hardly worth the labour of it, to compute a third table of the values of life-annuities for the lives of two perſons of given ages, and the life of the longer liver: becauſe the values of theſe latter annuities may be ſo eaſily derived, by means of the foregoing corollary, from the values of the correſponding annuities of the two former kinds, which are ſuppoſed to be ſet down in the tables.

LXV.

LXV. COROLL. 5. The value of a remote life-annuity of one pound, that is to begin at the diſtance of a given number of years, and to continue during the lives of the purchaſer and his companion and the life of the longer liver of them, whoſe ages are given, is equal to the exceſs of the ſum of two ſeparate remote annuities of one pound each, for the ſingle lives of the ſame two perſons, that are to begin at the ſame future period, above the value of a third remote annuity of one pound, that is alſo to begin at the ſame future period and to continue during the joint lives of both the ſaid perſons.

Of the value of a remote annuity of one pound for the life of the longer liver of two perſons of given ages.

For all the reaſoning uſed in the foregoing corollary applies equally to theſe remote life-annuities and to thoſe compleat and immediate life-annuities that are the ſubject of that corollary; of which compleat annuities the former remote annuities are only the latter parts. This I take to be ſo evident, that a repetition of that reaſoning to apply it to this caſe would probably be tedious to the reader. And therefore I omit it.

LXVI. COROLL. 6. And in like manner the value of an immediate, but imperfect, life-annuity of one pound, that is to begin immediately (or whereof the firſt payment is to be received at the end of a year,) but is to continue only during a certain number of years, leſs than the utmoſt extent of human life, if either the purchaſer or his companion, whoſe ages are given, continues to live through that period, is equal to the exceſs of the ſum of the values of two ſeparate, immediate, and imperfect life-annuities of one pound each, for the ſingle lives of the purchaſer and his companion, to continue during the ſame limited period, if each of them continues to live ſo long, above the value of a like immediate and imperfect annuity of one pound for the joint lives of the ſaid purchaſer and his companion during the ſame term of years, if they both continue to live ſo long.

Of the value of an immediate, but imperfect, life-annuity of one pound for a certain number of years, if the life of either of two perſons of given ages ſhall ſo long continue.

This alſo may be eaſily collected from the reaſoning uſed in Coroll. 4, theſe annuities being only the firſt parts of the compleat life-annuities therein mentioned.

LXVII. COROLL. 7. And in like manner the value of a remote and imperfect life-annuity of one pound, for the lives of the purchaſer and his companion, whoſe ages are given, and the life of the longer liver of them; that is, of an annuity that is to begin at the end of a given number of years, and is to continue only during a certain ſpace of time, leſs than the utmoſt poſſible extent of the life of the younger of them, if either of them lives to the end of the ſaid period, and then to ceaſe, though one, or both, of them ſhould be ſtill alive; is equal to the exceſs of the ſum of the values of

Of the value of a remote and imperfect life-annuity of one pound for the life of the longer liver of two perſons whoſe ages are given.

of two ſeparate annuities, of one pound each, for the ſingle lives of the ſaid purchaſer and his companion, that ſhould commence at the end of the ſame given number of years as the former, and continue only during the ſame time, in caſe each of them ſhould continue to live ſo long, and then ceaſe, though they ſhould be ſtill alive, above the value of a like remote and imperfect life-annuity for the joint lives of the ſame perſons, that ſhould commence at the end of the ſame number of years, and continue during the ſame ſpace of time, if they both ſhall continue to live ſo long.

This alſo may be eaſily collected from the reaſoning uſed in Coroll. 4; theſe remote and imperfect life-annuities for the lives of the purchaſer and his companion and the life of the ſurvivor of them, being only the middle parts of the compleat life-annuities of the ſame kind therein mentioned. And to this corollary I therefore refer the reader.

Of the value of a future payment depending on the continuance of any one of three given lives.

LXVIII. COROLL. 8. The reaſoning uſed in the ſolution of the foregoing problem may be extended to the valuation of a future payment, to be received at the end of a given number of years in caſe any one of three perſons, or more, whoſe ages are given, ſhall be then alive. In the caſe of three lives the additions neceſſary to be made to the two ſolutions in Art. 58 and 59, will be as follows.

An inveſtigation of the ſaid value in the caſe of a particular example.

LXIX. In the particular example ſolved in Art. 58 let the right of the purchaſer, of 25 years of age, to the future payment of one pound, at the end of 30 years, be extended to two other perſons beſides himſelf, inſtead of one; ſo that, if either the purchaſer himſelf or either of his ſaid companions ſhall be then alive, the ſaid future ſum ſhall be payable by the grantor of it to one of the ſaid three perſons. And to make the caſe more clear and definite, let it be ſuppoſed that the older of theſe two perſons is called his *firſt companion* and the younger his *ſecond companion*; and that, if the purchaſer himſelf is alive at the end of the ſaid 30 years, the ſaid ſum of one pound ſhall be payable to him alone, though either or both his ſaid companions ſhould be alſo living at the ſame time; and that, if he is then dead, but his companions are both alive, it ſhall be paid to the elder of the two, or his firſt companion; and that, if only one of them is then alive, it ſhall be paid to the ſaid only ſurviving companion. And let the age of the ſaid purchaſer's older, or firſt, companion, be 20 years,

as

as was ſuppoſed in the foregoing example; and that of his younger, or ſecond, companion 10 years; at the time of making the grant. And further, let it be ſuppoſed that the grantor of the ſaid future payment of one pound makes 774 ſuch grants of one pound each, to be received at the end of 30 years, to as many different purchaſers, all of the ſame age of 25 years as the purchaſer propoſed in the queſtion; and that the grant to the purchaſer propoſed in the queſtion is one of thoſe 774 grants; and that each of theſe purchaſers has two companions, to wit, an older, or firſt, companion of 20 years of age, and a younger, or ſecond, companion of 10 years of age; and that each of the ſums ſo granted is to be paid by the ſaid grantor, at the end of the ſaid ſpace of 30 years, provided the purchaſer himſelf, or either of his two companions, is then alive; namely, to the purchaſer himſelf, if he is then alive; or otherwiſe to the older of his two companions, if they are then both alive; or, if only one of them is then alive, to the ſaid only ſurviving companion.

Theſe things being ſuppoſed, it is evident that the number of perſons that will be intitled to receive theſe payments of one pound each at the end of the ſaid 30 years, will be greater than in the caſe ſuppoſed in the ſolution of the foregoing problem, Art. 58: becauſe, not only all the 526 ſurviving purchaſers themſelves, together with the 177 ſurviving firſt companions of the 248 deceaſed purchaſers, making in all 703 perſons, will be intitled to theſe payments, as they were in that ſolution, but there will alſo be ſome ſurviving ſecond companions of the deceaſed purchaſers who will alſo have a right to them. What the number of theſe will be, we muſt now proceed to inquire.

Now it is evident, in the firſt place, from the conditions of this queſtion, that the ſurviving ſecond companions of the 526 ſurviving purchaſers can have no claim to theſe payments of one pound; becauſe they are to be made to the ſaid ſurviving purchaſers themſelves. And, for the like reaſon, it is evident, in the ſecond place, that the ſecond companions of thoſe deceaſed purchaſers whoſe firſt companions are alive at the end of the ſaid 30 years, can have no claim to theſe payments; becauſe it is provided that when both the companions of a deceaſed purchaſer are alive at the time his payment becomes due, it ſhall be made to the ſaid purchaſer's firſt companion, and not to his ſecond companion. Now it has been ſhewn in Art. 58 that out of the 248 purchaſers, who will have died in the courſe of the ſaid 30 years, there will be 177, whoſe firſt companions (who were 20 years old at the time of making the grants,) will be alive at the end of the ſaid 30 years. Therefore the number of the ſaid deceaſed purchaſers, whoſe firſt companions will alſo be dead before the end of the ſaid 30 years, is the exceſs of 248 above 177 perſons, that is, 71 perſons. There are therefore 71 deceaſed purchaſers, whoſe ſecond companions will have a right

to

to receive theſe payments of one pound each, if they live to the end of the ſaid 30 years. We muſt therefore inquire how many of the ſecond companions of theſe 71 deceaſed purchaſers will live to the end of the ſaid 30 years, ſuppoſing them to die off in the proportion ſet forth in Mr. de Parcieux's table of the probabilities of the duration of human life. Now theſe companions are evidently 71 in number, becauſe each of the ſaid 71 deceaſed purchaſers had one ſecond as well as one firſt companion. And the age of theſe ſecond companions, at the time of making the grants, is ſuppoſed to be 10 years. Now it appears from Mr. de Parcieux's table that out of 880 perſons of the age of 10 years, all living at the ſame time, 657 will live to the age of 40 years, or to the end of a term of 30 years. Therefore out of the ſaid 71 ſecond companions of the ſaid deceaſed purchaſers there will be $71 \times \frac{657}{880}$, or 53, who will live to the end of the ſaid 30 years. Therefore the whole number of perſons intitled to receive the ſaid payments of one pound each, at the end of the ſaid 30 years, will be, firſt, the 526 ſurviving purchaſers, and, ſecondly, the 177 ſurviving firſt companions of the 248 deceaſed purchaſers, and, thirdly, the ſaid 53 ſurviving ſecond companions of thoſe 71 of the ſaid 248 deceaſed purchaſers whoſe firſt companions will have died before the end of the ſaid 30 years; that is, in all, 756 perſons. Therefore at the end of the ſaid 30 years the aforeſaid grantor will be obliged to pay to the ſaid ſurviving purchaſers and the ſaid firſt and ſecond companions of the purchaſers that are deceaſed, the ſum of 756 pounds. Therefore, to the end that, when the ſaid payments become due, the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive, at the time of making the ſaid grants, 756 times the preſent value of one pound certain, payable at the end of 30 years, when the intereſt of money is 3 per cent. that is, 756 times .4119 of a pound, or £311.3964, from all the 774 purchaſers of theſe future payments. Therefore the ſum which each of the ſaid purchaſers ought then to pay him, as the price of the ſaid future payment of one pound to be received at the end of the ſaid 30 years, is the 774th part of £311.3964, or £.4023, or 8*s.* $\frac{1}{2}$*d.* And conſequently, by Art. 21, this ſum of 8*s.* $\frac{1}{2}$*d.* is likewiſe the price which a ſingle purchaſer ought to pay for a grant of ſuch a future payment of one pound, to be received at the end of 30 years, if either himſelf or either of his two companions aforeſaid ſhall be then alive, when the grantor of it makes only one ſuch grant. Q E I.

LXX. In the foregoing article it was ſuppoſed that, where both the companions of the deceaſed purchaſers happened to live to the end of the ſaid 30 years, the ſaid ſums of one pound were to be paid to their firſt, or older, companions, who were 20 years of age at the time of making the grants

grants of them, in preference to their ſecond, or younger, companions, who were only 10 years old at that time. But the values of theſe future ſums of one pound will be exactly the ſame, if we ſhould ſuppoſe that they are to be paid to the ſaid ſecond, or younger, companions of the ſaid deceaſed purchaſers in preference to the firſt, or older. This is what we might reaſonably expect and ſuppoſe to be true without a particular demonſtration. But, to remove all doubt from the reader's mind, we ſhall prove it diſtinctly by applying the reaſoning uſed in the foregoing article to this new ſuppoſition, in the manner following.

Upon this new ſuppoſition it is evident that the perſons who will be intitled to receive the ſaid payments of one pound at the end of the ſaid 30 years, will be; in the firſt place, thoſe of the 774 purchaſers themſelves who will then be alive, whoſe number has been ſhewn to be 526; and, in the ſecond place, thoſe of the ſecond companions of the 248 deceaſed purchaſers who will then be living; and, in the third place, after ſubtracting from the ſaid 248 deceaſed purchaſers thoſe of them whoſe ſecond companions will have lived to the end of the ſaid 30 years, thoſe of the firſt companions of the remaining deceaſed purchaſers (that is, of thoſe deceaſed purchaſers whoſe ſecond companions have died in the courſe of the ſaid 30 years,) who will alſo then be living. We muſt therefore inquire by the means of Mr. de Parcieux's table, in the firſt place, how many of the ſecond companions of the ſaid 248 deceaſed purchaſers will be alive at the end of the ſaid 30 years: and then, ſubtracting this number from that of all the deceaſed purchaſers, that is, from 248, in order to obtain the number of the ſaid deceaſed purchaſers whoſe ſecond companions will have died in the courſe of the ſaid 30 years, we muſt inquire, in the next place, by means of the ſaid table, how many of the firſt companions of the ſaid remaining number of the ſaid deceaſed purchaſers will be alive at the end of the ſaid 30 years.

Now it is evident that the ſecond companions of the 248 deceaſed purchaſers are alſo 248 in number, each of the ſaid purchaſers having had one ſecond as well as one firſt companion. And theſe ſecond companions are ſuppoſed to have been 10 years old at the time of making the grants. Now it appears by Mr. de Parcieux's table above-mentioned that out of 880 perſons of 10 years of age, all living at the ſame time, 657 will be alive at the age of 40 years, or at the end of 30 years. Therefore out of the aforeſaid 248 ſecond companions of the 248 deceaſed purchaſers there will be $248 \times \frac{657}{880}$, or 185, perſons alive at the end of the ſaid 30 years. Therefore the number of the ſaid ſecond companions of the ſaid 248 deceaſed purchaſers who will have died in the courſe of the ſaid 30 years, will be the exceſs of 248 above 185, that is, 63 perſons. Therefore there

are

are 63 perſons out of the ſaid 248 deceaſed purchaſers whoſe ſecond companions will have died in the courſe of the ſaid 30 years. But, though the ſecond companions of theſe 63 deceaſed purchaſers will be dead at the end of the ſaid 30 years, yet ſome of their firſt companions, who were of the age of 20 years at the time of making the grants, will probably be ſtill alive at that time; and, if they are ſo, will be intitled to theſe payments of one pound each. We muſt therefore inquire, by Mr. de Parcieux's table, how many of the ſaid firſt companions of theſe remaining 63 deceaſed purchaſers will be alive at the end of the ſaid 30 years. Now theſe firſt companions of the ſaid 63 deceaſed purchaſers are likewiſe 63 in number at the time of making the ſaid grants; becauſe each of the ſaid purchaſers is ſuppoſed to have had at that time two companions. And the age of theſe firſt companions at the time of making the ſaid grants, was ſuppoſed to be 20 years. Now it appears by Mr. de Parcieux's table that out of 814 perſons of the age of 20 years, all living at the ſame time, 581 will be alive at the age of 50 years, or at the end of 30 years. Therefore of the ſaid 63 firſt companions of the ſaid 63 deceaſed purchaſers, there will be $63 \times \frac{581}{814}$, or 45, perſons living at the end of the ſaid 30 years, and will be intitled to receive their reſpective payments of one pound each. Therefore the whole number of perſons intitled to receive the ſaid payments, at the end of the ſaid 30 years, will be, firſt, the aforeſaid 526 ſurviving purchaſers, and, ſecondly, the 185 ſurviving ſecond companions of the 248 deceaſed purchaſers, and thirdly, the 45 ſurviving firſt companions of the 63 deceaſed purchaſers whoſe ſecond companions will have died in the courſe of the ſaid 30 years; that is, in all, 756 perſons, as before. Therefore at the end of the ſaid 30 years the aforeſaid grantor will be obliged to pay to the ſaid ſurviving purchaſers, and the ſaid firſt and ſecond companions of the purchaſers that are deceaſed, the ſum of 756 pounds. Therefore, to the end that, at the expiration of the ſaid 30 years, the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive, at the time of making the ſaid grants, 756 times the value of one pound certain, payable at the end of 30 years, when the intereſt of money is 3 per cent. that is, 756 times .4119 of a pound, or £311.3964, from all the 774 purchaſers of the ſaid future payments. Therefore the ſum which he ought then to receive from each of the ſaid purchaſers as the price of his ſaid future ſum of one pound, is the 774th part of £311.3964, or 4023*l.* or 8*s.* $\frac{1}{2}$*d.* And conſequently, by Art. 21, this ſum of £.4023, or 8*s.* $\frac{1}{2}$*d.* is likewiſe the price which a purchaſer ought to pay the grantor for ſuch a future payment, when the latter makes only one ſuch grant; which is the ſame price which was to be paid for it on the former ſuppoſition of the firſt, or older, companion's being intitled to receive the ſaid payment in preference to the younger. Q E D.

LXXI. In

LXXI. In the two foregoing articles it is ſhewn that the price to be paid for the future ſum of one pound, to be received at the end of 30 years, if the purchaſer or either of his two companions ſhall be then alive, is exactly the ſame, whether, in caſe of the purchaſer's death and the ſurvivorſhip of both his companions at the end of the ſaid time, the purchaſer's firſt, or older, companion be intitled to receive it in preference to his ſecond, or younger, companion, or his ſecond, or younger, companion, be intitled to receive it in preference to the firſt: for that in both caſes, if we ſuppoſe 774 ſuch grants of future ſums of one pound to be made to as many different purchaſers upon the conditions above-mentioned, the number of perſons living at the end of the ſaid 30 years who will be intitled to receive the ſaid ſums, will be the ſame, to wit, 756 perſons. Now that this is not accidental, or peculiar to the ages and numbers that occur in this particular example, but would have been true, if any other ages than thoſe of 25, 20, and 10 years, had been pitched upon, and any other number of years than 30 had been ſuppoſed to intervene before the ſaid future payments became due, will appear by ſetting down the ſeveral factors and diviſors by whoſe multiplication and diviſion of each other the ſaid reſulting number of 756 claimants of theſe ſums of one pound is obtained in both caſes: which may be done in the manner following.

In Art. 69 the number of the perſons intitled to receive the ſaid payments of one pound each at the end of the ſaid 30 years was compoſed of the whole number of purchaſers who were then alive, which was 526, and of the 177 firſt, or older, companions of the deceaſed 248 purchaſers, and of the 53 ſurviving ſecond companions of thoſe (248—177. or) 71 deceaſed purchaſers whoſe firſt companions had died in the courſe of the ſaid 30 years.

Now the ſaid number, 177, of the ſurviving firſt companions of the deceaſed 248 purchaſers, is $= 248 \times \frac{581}{814} = \overline{774-526} \times \frac{581}{814} = \frac{774 \times 581}{814} - \frac{526 \times 581}{814}$.

And 53, the number of the ſurviving ſecond companions of the ſaid 71 deceaſed purchaſers, whoſe firſt companions will have died in the courſe of the ſaid 30 years, is $= 71 \times \frac{657}{880} = \overline{248-177} \times \frac{657}{880} = \frac{248 \times 657}{880} - \frac{177 \times 657}{880}$

$= \frac{\overline{774-526} \times 657}{880} - \frac{177 \times 657}{880} = \frac{774 \times 657}{880} - \frac{526 \times 657}{880} - \frac{177 \times 657}{880}$

$= \frac{774 \times 657}{880} - \frac{526 \times 657}{880} - \frac{774 \times 581}{814} \times \frac{657}{880} + \frac{526 \times 581}{814} \times \frac{657}{880} = \frac{774 \times 657}{880}$

$- \frac{526 \times 657}{880} - \frac{774 \times 581 \times 657}{814 \times 880} + \frac{526 \times 581 \times 657}{814 \times 880}$.

Therefore $526+177+53$, or the whole number of perſons intitled to receive theſe payments at the end of the ſaid 30 years, is $= 526 + \frac{774\times581}{814} - \frac{526\times581}{814} + \frac{774\times657}{880} - \frac{526\times657}{880} - \frac{774\times581\times657}{814\times880} + \frac{526\times581\times657}{814\times880}$.

And in Art. 70 the number of perſons intitled to receive the ſaid payments of one pound each at the end of the ſaid 30 years, is compoſed of the ſaid 526 ſurviving purchaſers, and of the 185 ſurviving ſecond, or younger, companions of the 248 deceaſed purchaſers, and of the 45 ſurviving firſt, or older, companions of thoſe (248—185, or) 63 deceaſed purchaſers whoſe ſecond companions had died in the courſe of the ſaid 30 years.

Now the ſaid number, 185, of the ſecond companions of the 248 deceaſed purchaſers, is $= 248 \times \frac{657}{880} = \overline{774-526} \times \frac{657}{880} = \frac{774\times657}{880} - \frac{526\times657}{880}$.

And 45, the number of the ſurviving firſt companions of thoſe 63 purchaſers whoſe ſecond companions have died in the courſe of the ſaid 30 years, is $= 63 \times \frac{581}{814} = \overline{248-185} \times \frac{581}{814} = \frac{248\times581}{814} - \frac{185\times581}{814} = \overline{\frac{774-526}{814}\times581} - \frac{185\times581}{814} = \frac{774\times581}{814} - \frac{526\times581}{814} - \frac{185\times581}{814} = \frac{774\times581}{814} - \frac{526\times581}{814} - \frac{774\times657}{880}\times\frac{581}{814} + \frac{526\times657}{880}\times\frac{581}{814} = \frac{774\times581}{814} - \frac{526\times581}{814} - \frac{774\times657\times581}{880\times814} + \frac{526\times657\times581}{880\times814}$.

Therefore $526+185+45$, or the whole number of perſons intitled to receive theſe payments at the end of the ſaid 30 years is $= 526 + \frac{774\times657}{880} - \frac{526\times657}{880} + \frac{774\times581}{814} - \frac{526\times581}{814} - \frac{774\times657\times581}{880\times814} + \frac{526\times657\times581}{880\times814}$.

Now

Now it is eaſy to obſerve that this expreſſion conſiſts of the ſame number of terms, to wit, ſeven, as the expreſſion found above for the number of claimants of theſe ſums of one pound upon the former ſuppoſition of preferring the ſurviving firſt companions of the purchaſers to their ſurviving ſecond companions; and that the terms of both the expreſſions are the ſame, being compoſed of the ſame factors and diviſors, though placed in a ſomewhat different order. For in both expreſſions the firſt term is 526; and the ſecond term of the firſt expreſſion is $\frac{774\times581}{814}$, which makes the fourth term in the ſecond expreſſion; and the third term of the firſt expreſſion is $\frac{526\times581}{814}$, with the mark of ſubtraction prefixed to it, which makes the fifth term in the ſecond expreſſion, where it has the ſame mark of ſubtraction; and the fourth term of the firſt expreſſion is $\frac{774\times657}{880}$, which makes the ſecond term of the ſecond expreſſion; and the fifth term of the firſt expreſſion is $\frac{526\times657}{880}$, with the mark of ſubtraction prefixed to it, which makes the third term in the ſecond expreſſion, where it has the ſame mark of ſubtraction prefixed to it; and the ſixth term of the firſt expreſſion is $\frac{774\times581\times657}{814\times880}$, with the mark of ſubtraction prefixed to it; and $\frac{774\times657\times581}{880\times814}$ with the mark of ſubtraction prefixed to it, is the ſixth term of the ſecond expreſſion, which is equal to the ſixth term of the former expreſſion, becauſe it is compoſed of the ſame factors both in the numerator and denominator, though placed in a different order; and the ſeventh term in the firſt expreſſion is $\frac{526\times581\times657}{814\times880}$, which is compoſed of the ſame factors with, and therefore is equal to, the ſeventh term of the ſecond expreſſion, or $\frac{526\times657\times581}{880\times814}$. Therefore the whole firſt expreſſion is equal to the whole ſecond expreſſion, or the number of perſons intitled to receive theſe payments at the end of the ſaid 30 years is the ſame upon both the ſaid ſuppoſitions. Q E D.

A general inveſtigation of the value of a future payment of one pound depending on the continuance of any one of three given lives.

LXXII. We proceed now to extend the general ſolution of the foregoing problem given in Art. 59 to the caſe of a grant of a future payment of one pound made to a purchaſer, to be received at the end of a given number of years, in caſe either the purchaſer himſelf, or either of two other perſons named by him, and who may be called his *companions*, ſhall be then alive; the ages of the purchaſer and his two companions, and the rate of intereſt of money, and the table of the probabilities of the duration of human life, being all given.

Let the rate of intereſt of money be expreſſed, as before, by the proportion of r to 1. And let n denote the number of years at the end of which the ſaid ſum of one pound is to be received; and N the number of years in the age of the purchaſer himſelf; and M the number of years in the age of the purchaſer's firſt, or older, companion; and L the number of years in the age of his ſecond, or younger, companion. And let the ſaid ſum of one pound be payable, at the end of the ſaid n years, to the ſaid purchaſer himſelf, if he is then alive; and, if he is then dead, but both his companions are alive, to his firſt, or older, companion; and, if he alſo is then dead, but his ſecond, or younger, companion is then alive, to his ſaid ſecond, or younger, companion. And let P denote the number of perſons repreſented in the given table of probabilities to be living at the age of N years, or the age of the purchaſer; and $\mathcal{Q}$ the number of perſons therein repreſented to be living at the age of M years, or the age of the purchaſer's firſt, or older, companion; and $\mathcal{T}$ the number of perſons therein repreſented to be living at the age of L years, or the age of the purchaſer's ſecond, or younger, companion; and p, q, and t the numbers of perſons therein repreſented to be living at the ages of $N+n$, $M+n$, and $L+n$ reſpectively. And let it be ſuppoſed that the grantor of the ſaid future ſum of one pound, to be received at the end of n years, makes not only one ſuch grant, but P ſuch grants to P different purchaſers, each of whom has two companions, an older, or firſt, companion of the age of M years, and a younger, or ſecond, companion of the age of L years; of which P grants that to the purchaſer in the queſtion is one: and that each of theſe grants is ſubject ro the ſame conditions as the grant to the purchaſer in the queſtion.

Then it is evident that the number of perſons that will be intitled to receive theſe payments of one pound each at the end of the ſaid n years, will be greater than in the caſe ſuppoſed in the ſolution of the foregoing problem, Art. 59; becauſe not only all the p purchaſers who will be living at the end of the ſaid 30 years, together with the $\frac{\overline{P-p} \times q}{\mathcal{Q}}$ ſurviving firſt

companions

companions of the $P-p$ deceaſed purchaſers, making in all $p+\overline{P-p}\times\frac{q}{Q}$ perſons, will be intitled to theſe payments, as they were in that ſolution, but there will alſo be ſome ſurviving ſecond companions of the ſaid deceaſed purchaſers who will have a right to them. What the number of theſe will be, we muſt now proceed to inquire.

Now it is evident, in the firſt place, from the conditions of this queſtion that the ſurviving ſecond companions of the p purchaſers, who will be alive at the end of the ſaid n years, can have no claim to the ſaid ſums of one pound; becauſe they are to be paid to the ſaid ſurviving purchaſers themſelves. And, for the like reaſon, it is evident, in the ſecond place, that the ſecond companions of thoſe deceaſed purchaſers whoſe firſt companions are alive at the end of the ſaid n years, can have no claim to theſe payments; becauſe it is provided that, when both the companions of a deceaſed purchaſer are alive at the time the payment becomes due, it ſhall be received by the ſaid purchaſer's firſt, or older, companion in preference to the ſecond. Now it has been ſhewn in Art. 58 that out of the $P-p$ purchaſers who will have died in the courſe of the ſaid n years, there will be $\overline{P-p}\times\frac{q}{Q}$ perſons whoſe firſt, or older, companions, who were M years of age at the time of making the grants, will be alive at the end of the ſaid n years. Therefore the number of the ſaid deceaſed purchaſers whoſe firſt, or older, companions will alſo be dead at the end of the ſaid n years, is the exceſs of $P-p$ above $\overline{P-p}\times\frac{q}{Q}$, or $\frac{Pq-pq}{Q}$, perſons, or is equal to $P-p-\left|\overline{\frac{Pq-pq}{Q}}\right. = \frac{PQ-pQ-Pq+pq}{Q}$. There are therefore $\frac{PQ-pQ-Pq+pq}{Q}$ deceaſed purchaſers whoſe ſecond companions will have a right to receive theſe payments of one pound each, if they live to the end of the ſaid n years. We muſt therefore inquire how many of the ſecond companions of theſe $\frac{PQ-pQ-Pq+pq}{Q}$ deceaſed purchaſers will live to the end of the ſaid n years, ſuppoſing them to die off in the proportion ſet forth in the given table of the probabilities of the duration of human life. Now theſe ſecond companions of the ſaid deceaſed purchaſers are evidently juſt as many as the ſaid deceaſed purchaſers themſelves, that is, $\frac{PQ-pQ-Pq+pq}{Q}$; becauſe each of the ſaid deceaſed purchaſers had one ſecond as well as one firſt companion. And the age of theſe ſecond companions at the time of making the grants, is ſuppoſed to be L years. Now the numbers of perſons repreſented in the given table of probabilities to be living at the ages of L years and $L+n$ years are ſuppoſed to be T and t. Therefore, ſince out of T perſons of L years of age, all living at the ſame time, t perſons

perſons will be alive at the end of n years, it is evident that of the aforeſaid $\frac{P\mathcal{Q}-p\mathcal{Q}-Pq+pq}{\mathcal{Q}}$ ſecond companions of the ſaid deceaſed purchaſers, who are all living, and of the age of L years, at the time of making the grants, there will be $\frac{t}{\mathcal{T}} \times \frac{P\mathcal{Q}-p\mathcal{Q}-Pq+pq}{\mathcal{Q}}$, or $\frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pqt}{\mathcal{Q}\mathcal{T}}$, alive at the end of the ſaid n years. Therefore the whole number of perſons intitled to receive the ſaid payments of one pound each at the end of the ſaid n years, will be, firſt, the p ſurviving purchaſers themſelves, and, ſecondly, the $\overline{P-p} \times \frac{q}{\mathcal{Q}}$, or $\frac{Pq-pq}{\mathcal{Q}}$, ſurviving firſt companions of the $P-p$ deceaſed purchaſers, and, thirdly, the ſaid $\frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pqt}{\mathcal{Q}\mathcal{T}}$ ſurviving ſecond companions of the ſaid $\frac{P\mathcal{Q}-p\mathcal{Q}-Pq+pq}{\mathcal{Q}}$ deceaſed purchaſers, whoſe firſt companions will have died in the courſe of the ſaid n years: that is, the number of the perſons intitled to receive theſe payments will be, in all, $p + \frac{Pq-pq}{\mathcal{Q}} + \frac{P\mathcal{Q}t-p\mathcal{Q}t-Pqt+pqt}{\mathcal{Q}\mathcal{T}}$, or $p + \frac{Pq}{\mathcal{Q}} - \frac{pq}{\mathcal{Q}} + \frac{Pt}{\mathcal{T}} - \frac{pt}{\mathcal{T}} - \frac{Pqt}{\mathcal{Q}\mathcal{T}} + \frac{pqt}{\mathcal{Q}\mathcal{T}}$. Therefore at the end of the ſaid n nears the aforeſaid grantor will be obliged to pay to the ſaid ſurviving purchaſers and the ſaid firſt and ſecond companions of the purchaſers that are deceaſed, the ſum of $p + \frac{Pq}{\mathcal{Q}} - \frac{pq}{\mathcal{Q}} + \frac{Pt}{\mathcal{T}} - \frac{pt}{\mathcal{T}} - \frac{Pqt}{\mathcal{Q}\mathcal{T}} + \frac{pqt}{\mathcal{Q}\mathcal{T}}$ pounds.

Therefore, to the end that the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive, at the time of making the ſaid grants, $p + \frac{Pq}{\mathcal{Q}} - \frac{pq}{\mathcal{Q}} + \frac{Pt}{\mathcal{T}} - \frac{pt}{\mathcal{T}} - \frac{Pqt}{\mathcal{Q}\mathcal{T}} + \frac{pqt}{\mathcal{Q}\mathcal{T}}$ times the preſent value of one pound certain, payable at the end of n years, when the intereſt of money is expreſſed by the proportion of r to 1; that is, $p + \frac{Pq}{\mathcal{Q}} - \frac{pq}{\mathcal{Q}} + \frac{Pt}{\mathcal{T}} - \frac{pt}{\mathcal{T}} - \frac{Pqt}{\mathcal{Q}\mathcal{T}} + \frac{pqt}{\mathcal{Q}\mathcal{T}}$ times $\frac{1}{r^n}$ of a pound, or $\frac{1}{r^n}$ £, from all the P purchaſers of theſe future payments. Therefore the ſum which he ought then to receive from each of the ſaid purchaſers is the Pth part of $p + \frac{Pq}{\mathcal{Q}} - \frac{pq}{\mathcal{Q}} + \frac{Pt}{\mathcal{T}} - \frac{pt}{\mathcal{T}} - \frac{Pqt}{\mathcal{Q}\mathcal{T}} + \frac{pqt}{\mathcal{Q}\mathcal{T}}$ times $\frac{1}{r^n}$ £, or is equal to $\frac{p}{P} + \frac{q}{\mathcal{Q}} - \frac{pq}{P\mathcal{Q}} + \frac{t}{\mathcal{T}} - \frac{pt}{P\mathcal{T}} - \frac{qt}{\mathcal{Q}\mathcal{T}} + \frac{pqt}{P\mathcal{Q}\mathcal{T}}$ times $\frac{1}{r^n}$ £. And conſequently, by Art. 21, the price which a purchaſer ought to pay for ſuch a grant of a future

future ſum of one pound, to be received at the end of n years, upon the conditions above-mentioned, when the grantor of it makes only one ſuch grant, is likewiſe equal to $\frac{p}{P} + \frac{q}{Q} - \frac{pq}{PQ} + \frac{t}{T} - \frac{pt}{PT} - \frac{qt}{QT} + \frac{pqt}{PQT}$ times $\frac{1}{r^n}$ £. QED.

LXXIII. In the foregoing article it was ſuppoſed that when both the companions of the deceaſed purchaſers were alive at the end of the ſaid n years, the ſaid ſums of one pound were to be paid to their firſt, or older, companions, who were of the age of M years at the time of making the grants, in preference of their ſecond, or younger, companions, who were at that time of the age of L years. But the values of theſe future ſums of one pound will be exactly the ſame, if we ſhould ſuppoſe that they are to be paid to the ſaid ſecond, or younger, companions of the ſaid deceaſed purchaſers in preference to the firſt or older. This may be ſhewn in the manner following.

Upon this new ſuppoſition it is evident that the perſons who will be intitled to receive the ſaid ſums of one pound at the end of the ſaid n years, will be compoſed, in the firſt place, of the p ſurviving purchaſers themſelves, and, in the ſecond place, of the ſurviving ſecond, or younger, companions of the $P-p$ deceaſed purchaſers, and, in the third place, of the ſurviving firſt, or older, companions of thoſe of the ſaid $P-p$ deceaſed purchaſers whoſe ſecond companions will have died in the courſe of the ſaid n years. Now, ſince of T perſons of the age of L years, or of the age of the ſecond companions of the ſaid purchaſers at the time of making the grants, t perſons are found in the given table of probabilities to be living at the end of n years, it follows, that of the $P-p$ ſecond companions of the $P-p$ deceaſed purchaſers, who were living at the time of making the grants, there will be $\overline{P-p} \times \frac{t}{T}$, or $\frac{Pt-pt}{T}$, perſons alive at the end of the ſaid n years. Therefore the number of deceaſed purchaſers whoſe ſecond companions will die in the courſe of the ſaid n years is $P-p-\left|\frac{Pt-pt}{T}\right.$, or $\frac{PT-pT-Pt+pt}{T}$; and conſequently the number of the firſt companions of theſe deceaſed purchaſers is likewiſe equal to $\frac{PT-pT-Pt+pt}{T}$. But the age of theſe firſt companions is that of M years. And it appears by the table that of Q perſons of the age of M years, all living at the ſame time,

time, there will be q perſons living at the end of n years. Therefore of the ſaid $\frac{PT-pT-Pt+pt}{T}$ firſt companions of the ſaid deceaſed purchaſers, who were living at the time of making the grants, there will be living at the end of the ſaid n years only $\left|\frac{PT-pT-Pt+pt}{T}\right| \times \frac{q}{Q}$, or $\frac{PTq-pTq-Ptq+ptq}{TQ}$, perſons. Therefore the whole number of perſons intitled to receive the ſaid payments of one pound each at the end of the ſaid n years is $p+\frac{Pt-pt}{T}+\frac{PTq-pTq-Ptq+ptq}{TQ}$, or $p+\frac{Pt}{T}-\frac{pt}{T}+\frac{Pq}{Q}-\frac{pq}{Q}-\frac{Ptq}{TQ}+\frac{ptq}{TQ}$. Therefore this will be the number of pounds which the grantor of the ſaid future payment will have to pay at the end of the ſaid n years. Therefore at the time of making the grants he ought to receive from all the P purchaſers of theſe future payments a ſum equal to $p+\frac{Pt}{T}-\frac{pt}{T}+\frac{Pq}{Q}-\frac{pq}{Q}-\frac{Ptq}{TQ}+\frac{ptq}{TQ}$ times the preſent value of one pound payable at the end of n years, or $p+\frac{Pt}{T}-\frac{pt}{T}+\frac{Pq}{Q}-\frac{pq}{Q}-\frac{Ptq}{TQ}+\frac{ptq}{TQ}$ times $\frac{1}{r^n}$ of a pound; and conſequently from each of the ſaid purchaſers the Pth part of the former ſum, or $\frac{p}{P}+\frac{t}{T}-\frac{pt}{PT}+\frac{q}{Q}-\frac{pq}{PQ}-\frac{tq}{TQ}+\frac{ptq}{PTQ}$ times $\frac{1}{r^n}$ of a pound.

Therefore, by Art. 21, this is likewiſe the ſum which a purchaſer ought to pay to the grantor for a grant of ſuch a future ſum of one pound, when he makes only one ſuch grant. Now this ſum of $\frac{p}{P}+\frac{t}{T}-\frac{pt}{PT}+\frac{q}{Q}-\frac{pq}{PQ}-\frac{tq}{TQ}+\frac{ptq}{PTQ}$ times $\frac{1}{r^n}$ of a pound, is evidently equal to the ſum which was found in the laſt article to be the price of ſuch a grant of a future payment of one pound to be received at the end of n years, in caſe the purchaſer himſelf who was aged N years, or either of his two companions, who were aged M years and L years at the time of making the grant, ſhould be then alive, upon a ſuppoſition that his firſt, or older, companion ſhould be intitled to the ſaid payment in preference to his ſecond, or younger, companion, namely, the ſum of $\frac{p}{P}+\frac{q}{Q}-\frac{pq}{PQ}+\frac{t}{T}-\frac{pt}{PT}-\frac{qt}{QT}+\frac{pqt}{PQT}$ times $\frac{1}{r^n}$ of a pound; becauſe theſe two expreſſions conſiſt of the ſame terms connected together by addition and ſubtraction in the ſame manner, and differing from each other only in the order in which they are placed, which cannot

cannot affect their magnitude. It is therefore true, as was before affirmed, that whether the future ſum of one pound be made payable to the firſt, or older, companion of the purchaſer in preference to his ſecond, or younger, companion, or to the ſecond companion in preference to the firſt, or older, the price to be paid to the grantor for ſuch future ſum, will in both caſes be the ſame. Q E D.

Of the relation of the value of a future payment of one pound depending on the continuance of the life of any one of three perſons of given ages to the value of the ſame payment to be made at the ſame future time in caſe of the continuance of the ſingle lives of each of the ſame three perſons, and in caſe of the joint continuance of the lives of every two of them, and in caſe of the joint continuance of the lives of all the three.

LXXIV. COROLL. 9. It follows from the laſt corollary that the value of a future payment of one pound, to be received at the end of n years if either the purchaſer himſelf or either of his two companions, whoſe ages are given, ſhall be then alive, is equal to the exceſs of the ſum of the three values of the like future payment of one pound, to be received at the end of the ſaid n years in caſe each of the ſaid three perſons, to wit, the purchaſer and his two companions, ſhall be then alive, and the value of the like future payment to be received at the end of the ſaid time in caſe all the ſaid three perſons ſhall be then alive, above the ſum of the three values of the like future payment of one pound in caſe the purchaſer himſelf and his firſt companion ſhall be then alive, and in caſe the purchaſer himſelf and his ſecond companion ſhall be then alive, and in caſe his firſt and ſecond companion ſhall both be then alive.

For it has been ſhewn that the ſaid value of a future payment of one pound, to be received at the end of n years if either the purchaſer himſelf or either of his two companions ſhall be then alive, is equal to $\frac{p}{P}+\frac{q}{Q}-\frac{pq}{PQ}+\frac{t}{T}-\frac{pt}{PT}-\frac{qt}{QT}+\frac{pqt}{PQT}$ times $\frac{1}{r^n}$ of a pound. And conſequently it is equal to $\frac{p}{P}\times\frac{1}{r^n}£+\frac{q}{Q}\times\frac{1}{r^n}£+\frac{t}{T}\times\frac{1}{r^n}£+\frac{pqt}{PQT}\times\frac{1}{r^n}£-\frac{pq}{PQ}\times\frac{1}{r^n}£-\frac{pt}{PT}\times\frac{1}{r^n}£-\frac{qt}{QT}\times\frac{1}{r^n}£$. But, by Prob. 2, Art. 28, $\frac{p}{P}\times\frac{1}{r^n}£$ is the value of a future payment of one pound, to be received at the end of n years in caſe a purchaſer of the age of N years, which correſponds to the number P in the table of the probabilities of the duration of human life, ſhall be then alive; and $\frac{q}{Q}\times\frac{1}{r^n}£$ is the value of a like future payment of one pound, to be received at the ſame time, in caſe a perſon of the age of M years, which correſponds to the number Q in the table, or of the age of the aforeſaid purchaſer's firſt, or older, companion, ſhall be then alive; and $\frac{t}{T}\times\frac{1}{r^n}£$ is the value of a like future payment

of one pound, to be received at the ſame time in caſe a perſon of the age of L years, which correſponds to the number T in the ſaid table, or of the age of the aforeſaid purchaſer's ſecond, or younger, companion, ſhall be then alive. And, by Prob. 3, Coroll. 6, Art. 55, $\frac{pqt}{PQT} \times \frac{1}{r^n}$ £ is the value of a future payment of one pound, to be received at the end of n years in caſe a purchaſer of the age of N years, which correſponds to the number P in the table, and two companions of the ſaid purchaſer, one of the age of M years, which correſponds to the number Q in the table, and the other of the age of L years, which correſponds to the number T in the table, ſhall all three be living at the end of the ſaid time. And, by Prob. 3, Art. 45, $\frac{pq}{PQ} \times \frac{1}{r^n}$ £ is the value of a future payment of one pound, to be received at the end of n years if both a purchaſer of the age of N years, which correſponds to the number P in the table, and a companion of the age of M years, which correſponds to the number Q in the table, ſhall be living at that time: and $\frac{pt}{PT} \times \frac{1}{r^n}$ £ is the value of one pound, to be received at the end of n years if two perſons of the ages of N and L years, which correſpond to the numbers P and T in the table, ſhall be then alive: and $\frac{qt}{QT} \times \frac{1}{r^n}$ £ is the value of a future payment of one pound, to be received at the end of n years, if two perſons of the ages of M and L years, which correſpond to the numbers Q and T in the table, ſhall be then alive. Therefore the value of a future payment of one pound, to be received at the end of n years if either the purchaſer aforeſaid, aged N years, or either of his two companions, aged M years and L years, ſhall be then alive, is equal to the exceſs of the ſum of the four following values, to wit, the values of the ſame payment, to be received at the ſame time, depending upon the ſingle lives of the ſaid purchaſer and his two companions, and the value of it, to be received at the ſame time in caſe of the joint continuance of the lives of all the three, above the ſum of the three following values, to wit, the values of the ſame payment, to be received at the ſame time, in caſe of the joint continuance of the two lives of the purchaſer and his firſt, or older, companion, and in caſe of the joint continuance of the lives of the purchaſer and his ſecond, or younger, companion, and in caſe of the joint continuance of the lives of his two companions. Q.E.D.

LXXV. CO-

LXXV. COROLL. 10. By finding in the method deſcribed in the two laſt corollaries the ſeveral values of one pound ſterling, to be received at the end of every year of the whole ſpace of time during which it is poſſible that either a purchaſer of a given age, or either of his two companions, whoſe ages are likewiſe given, may be prolonged, in caſe that either the ſaid purchaſer or either of his ſaid companions ſhall be living at the time when every ſuch payment becomes due; and then adding theſe values all together into one ſum; we ſhall obtain the value of a ſet of equal payments of one pound each, to be received by the purchaſer or his companions at the end of every future year during the lives of either of them, that is, the value of an annuity of one pound for the lives of the ſaid purchaſer and his two companions and the life of the longeſt liver of them.

Of the value of an annuity of one pound ſterling for the life of the longeſt liver of three perſons of given ages.

LXXVI. COROLL. 11. The value of an annuity of one pound ſterling for the lives of three perſons of given ages and the life of the longeſt liver of them, is equal to the exceſs of the ſum of the four following values, to wit, the value of an annuity of one pound for the life of the firſt of the ſaid three perſons, the value of a like annuity for the life of the ſecond of them, and the value of a like annuity for the life of the third of them, and the value of a like annuity for the joint lives of all the three, above the ſum of the three following values, to wit, the value of a like annuity of one pound during the joint lives of the firſt and ſecond perſons, and the value of a like annuity during the joint lives of the firſt and third perſons, and the value of a like annuity during the joint lives of the ſecond and third perſons.

Of the relation of the ſaid value to the values of the like annuities of one pound for the ſingle lives of the ſame three perſons, and for the joint lives of every two of them, and for the joint lives of all the three.

Let the firſt of theſe three perſons be ſuppoſed, as before, to be the purchaſer of the ſaid annuity of one pound, and the other two be called his companions. And let N be the number of years in the age of the ſaid purchaſer; and M the number of years in the age of the ſaid purchaſer's firſt, or older, companion; and L the number of years in the age of his ſecond, or younger, companion. And let P denote, as before, the number of perſons repreſented in the table of the probabilities of the duration of human life adopted for the calculation, to be living at the age of the purchaſer, or at the age of N years; and $\mathfrak{Q}$ denote the number of perſons therein repreſented to be living at the age of the purchaſer's firſt, or older, companion, or the age of M years; and T the number of perſons therein repreſented to be living at the age of the purchaſer's ſecond, or younger, companion, or the age of L years. And further, let P' denote the number of perſons repreſented in the ſaid table to be living at the age of $N+1$ years; and P'' the number of thoſe living at the age of $N+2$ years; and P''' the number of thoſe living at the age of $N+3$ years; and P^{IV} the number of thoſe living at the age of $N+4$ years; and ſo on for the following

years; the Roman numeral figures placed over every new letter P denoting the number of years by which every new age, corresponding to it, exceeds the age of N years, or the age of the purchaſer. And, in like manner, let $\mathcal{Q}^{I}$ denote the number of perſons repreſented in the ſaid table to be living at the age of $M+1$ years; and $\mathcal{Q}^{II}$ the number of perſons living at the age of $M+2$ years; and $\mathcal{Q}^{III}$ the number of perſons living at the end of $M+3$ years; and $\mathcal{Q}^{IV}$ the number of perſons living at the end of $M+4$ years; and ſo on, for the following years; the Roman numeral figures placed over every new letter $\mathcal{Q}$ denoting the number of years by which every new age correſponding to it, exceeds the age of M years, or of the purchaſer's firſt, or older, companion. And, laſtly, let $\mathcal{T}^{I}$ denote the number of perſons repreſented in the ſaid table to be living at the age of $L+1$ years; and $\mathcal{T}^{II}$ the number of perſons living at the age of $L+2$ years; and $\mathcal{T}^{III}$ the number of perſons living at the age of $L+3$ years; and $\mathcal{T}^{IV}$ the number of perſons living at the age of $L+4$ years; and ſo on for the following years; the Roman numeral figures placed over every new letter $\mathcal{T}$ denoting the number of years by which every new age, correſponding to it, exceeds the age of L years, or of the purchaſer's ſecond, or younger, companion.

Then it is evident from Coroll. 8. that the value of a ſum of one pound to be received at the end of one year, in caſe either the purchaſer or one of his two companions ſhall be then alive, will be equal to

$$\left| \frac{P^{I}}{P} + \frac{\mathcal{Q}^{I}}{\mathcal{Q}} + \frac{\mathcal{T}^{I}}{\mathcal{T}} - \frac{P^{I}\,\mathcal{Q}^{I}}{P\,\mathcal{Q}} - \frac{P^{I}\,\mathcal{T}^{I}}{P\,\mathcal{T}} - \frac{\mathcal{Q}^{I}\,\mathcal{T}^{I}}{\mathcal{Q}\,\mathcal{T}} + \frac{P^{I}\,\mathcal{Q}^{I}\,\mathcal{T}^{I}}{P\,\mathcal{Q}\,\mathcal{T}} \right| \times \frac{1}{r} \text{ of a}$$

pound; becauſe in this caſe n is $= 1$, and p, q, t are reſpectively equal to P^{I}, $\mathcal{Q}^{I}$, and $\mathcal{T}^{I}$.

And in like manner it is evident that the value of the ſame ſum of one pound, to be received at the end of two years, ſubject to the ſame contingency, is equal to

$$\left| \frac{P^{II}}{P} + \frac{\mathcal{Q}^{II}}{\mathcal{Q}} + \frac{\mathcal{T}^{II}}{\mathcal{T}} - \frac{P^{II}\,\mathcal{Q}^{II}}{P\,\mathcal{Q}} - \frac{P^{II}\,\mathcal{T}^{II}}{P\,\mathcal{T}} - \frac{\mathcal{Q}^{II}\,\mathcal{T}^{II}}{\mathcal{Q}\,\mathcal{T}} + \frac{P^{II}\,\mathcal{Q}^{II}\,\mathcal{T}^{II}}{P\,\mathcal{Q}\,\mathcal{T}} \right|$$

$\times \frac{1}{r^2}$ of a pound; becauſe in this caſe n is equal to 2, and p, q, t are reſpectively equal to P^{II}, $\mathcal{Q}^{II}$, and $\mathcal{T}^{II}$.

And in like manner the value of the ſame ſum, to be received at the end of three years, ſubject to the ſame contingency, is equal to

$$\left| \frac{P^{III}}{P} + \frac{\mathcal{Q}^{III}}{\mathcal{Q}} + \frac{\mathcal{T}^{III}}{\mathcal{T}} - \frac{P^{III}\,\mathcal{Q}^{III}}{P\,\mathcal{Q}} - \frac{P^{III}\,\mathcal{T}^{III}}{P\,\mathcal{T}} - \frac{\mathcal{Q}^{III}\,\mathcal{T}^{III}}{\mathcal{Q}\,\mathcal{T}} + \frac{P^{III}\,\mathcal{Q}^{III}\,\mathcal{T}^{III}}{P\,\mathcal{Q}\,\mathcal{T}} \right|$$

$\times \frac{1}{r^3}$ of a pound; becauſe in this caſe n is $= 3$, and p, q, t, are equal to P^{III}, $\mathcal{Q}^{III}$, $\mathcal{T}^{III}$, reſpectively.

And

And in like manner the value of the ſame ſum, to be received at the end of four years, ſubject to the ſame contingency, is equal to

$$\left|\frac{P^{IV}}{P}+\frac{Q^{IV}}{Q}+\frac{T^{IV}}{T}-\frac{P^{IV}Q^{IV}}{P\ Q}-\frac{P^{IV}T^{IV}}{P\ T}-\frac{Q^{IV}T^{IV}}{Q\ T}+\frac{P^{IV}Q^{IV}T^{IV}}{P\ Q\ T}\right|\times\frac{1}{r^4}$$

of a pound; becauſe in this caſe n is $= 4$, and p, q, t, are equal to P^{IV}, Q^{IV}, T^{IV}, reſpectively.

And in the ſame manner we may find the values of one pound, to be received at the end of the fifth, ſixth, ſeventh, and every following year of the whole ſpace of time during which it is poſſible that the lives of the aforeſaid purchaſer, or either of his two companions, may be prolonged.

Therefore the ſum of theſe values, continued to the utmoſt poſſible extent of the youngeſt of thoſe three lives, that is, the value of a life-annuity of one pound for the lives of the purchaſer and his two companions, and the life of the longeſt liver of them, is equal to the following ſet of values, to wit,

$$\left|\frac{P^{I}}{P}+\frac{Q^{I}}{Q}+\frac{T^{I}}{T}-\frac{P^{I}Q^{I}}{P\ Q}-\frac{P^{I}T^{I}}{P\ T}-\frac{Q^{I}T^{I}}{Q\ T}+\frac{P^{I}Q^{I}T^{I}}{P\ Q\ T}\right|\times\frac{1}{r}\ \text{£},$$

$$+\left|\frac{P^{II}}{P}+\frac{Q^{II}}{Q}+\frac{T^{II}}{T}-\frac{P^{II}Q^{II}}{P\ Q}-\frac{P^{II}T^{II}}{P\ T}-\frac{Q^{II}T^{II}}{Q\ T}+\frac{P^{II}Q^{II}T^{II}}{P\ Q\ T}\right|\times\frac{1}{r^2}\text{£},$$

$$+\left|\frac{P^{III}}{P}+\frac{Q^{III}}{Q}+\frac{T^{III}}{T}-\frac{P^{III}Q^{III}}{P\ Q}-\frac{P^{III}T^{III}}{P\ T}-\frac{Q^{III}T^{III}}{Q\ T}+\frac{P^{III}Q^{III}T^{III}}{P\ Q\ T}\right|$$

$$\times\frac{1}{r^3}\text{£},$$

$$+\left|\frac{P^{IV}}{P}+\frac{Q^{IV}}{Q}+\frac{T^{IV}}{T}-\frac{P^{IV}Q^{IV}}{P\ Q}-\frac{P^{IV}T^{IV}}{P\ T}-\frac{Q^{IV}T^{IV}}{Q\ T}-\frac{P^{IV}Q^{IV}T^{IV}}{P\ Q\ T}\right|\times\frac{1}{r^4}\text{£},$$

$$+\left|\frac{P^{V}}{P}+\frac{Q^{V}}{Q}+\frac{T^{V}}{T}-\frac{P^{V}Q^{V}}{P\ Q}-\frac{P^{V}T^{V}}{P\ T}-\frac{Q^{V}T^{V}}{Q\ T}+\frac{P^{V}Q^{V}T^{V}}{P\ Q\ T}\right|\times\frac{1}{r^5}\text{£},$$

+&c. continued to the utmoſt extent of human life;

And

And conſequently to the following ſet of quantities, to wit,

$$\frac{P^{\prime}}{P}\times\frac{1}{r}\mathcal{L}\quad+\frac{Q^{\prime}}{Q}\times\frac{1}{r}\mathcal{L}\quad+\frac{T^{\prime}}{T}\times\frac{1}{r}\mathcal{L}\quad-\frac{P^{\prime}Q^{\prime}}{PQ}\times\frac{1}{r}\mathcal{L}\quad-\frac{P^{\prime}T^{\prime}}{PT}\times\frac{1}{r}\mathcal{L}$$
$$-\frac{Q^{\prime}T^{\prime}}{QT}\times\frac{1}{r}\mathcal{L}\quad+\frac{P^{\prime}Q^{\prime}T^{\prime}}{PQT}\times\frac{1}{r}\mathcal{L},$$

$$+\frac{P^{\prime\prime}}{P}\times\frac{1}{r^{2}}\mathcal{L}\quad+\frac{Q^{\prime\prime}}{Q}\times\frac{1}{r^{2}}\mathcal{L}\quad+\frac{T^{\prime\prime}}{T}\times\frac{1}{r^{2}}\mathcal{L}\quad-\frac{P^{\prime\prime}Q^{\prime\prime}}{PQ}\times\frac{1}{r^{2}}\mathcal{L}$$
$$-\frac{P^{\prime\prime}T^{\prime\prime}}{PT}\times\frac{1}{r^{2}}\mathcal{L}\quad-\frac{Q^{\prime\prime}T^{\prime\prime}}{QT}\times\frac{1}{r^{2}}\mathcal{L}\quad+\frac{P^{\prime\prime}Q^{\prime\prime}T^{\prime\prime}}{PQT}\times\frac{1}{r^{2}}\mathcal{L},$$

$$+\frac{P^{\prime\prime\prime}}{P}\times\frac{1}{r^{3}}\mathcal{L}\quad+\frac{Q^{\prime\prime\prime}}{Q}\times\frac{1}{r^{3}}\mathcal{L}\quad+\frac{T^{\prime\prime\prime}}{T}\times\frac{1}{r^{3}}\mathcal{L}\quad-\frac{P^{\prime\prime\prime}Q^{\prime\prime\prime}}{PQ}\times\frac{1}{r^{3}}\mathcal{L}$$
$$-\frac{P^{\prime\prime\prime}T^{\prime\prime\prime}}{PT}\times\frac{1}{r^{3}}\mathcal{L}\quad-\frac{Q^{\prime\prime\prime}T^{\prime\prime\prime}}{QT}\times\frac{1}{r^{3}}\mathcal{L}\quad+\frac{P^{\prime\prime\prime}Q^{\prime\prime\prime}T^{\prime\prime\prime}}{PQT}\times\frac{1}{r^{3}}\mathcal{L},$$

$$+\frac{P^{\mathrm{IV}}}{P}\times\frac{1}{r^{4}}\mathcal{L}\quad+\frac{Q^{\mathrm{IV}}}{Q}\times\frac{1}{r^{4}}\mathcal{L}\quad+\frac{T^{\mathrm{IV}}}{T}\times\frac{1}{r^{4}}\mathcal{L}\quad-\frac{P^{\mathrm{IV}}Q^{\mathrm{IV}}}{PQ}\times\frac{1}{r^{4}}\mathcal{L}$$
$$-\frac{P^{\mathrm{IV}}T^{\mathrm{IV}}}{PT}\times\frac{1}{r^{4}}\mathcal{L}\quad-\frac{Q^{\mathrm{IV}}T^{\mathrm{IV}}}{QT}\times\frac{1}{r^{4}}\mathcal{L}\quad+\frac{P^{\mathrm{IV}}Q^{\mathrm{IV}}T^{\mathrm{IV}}}{PQT}\times\frac{1}{r^{4}}\mathcal{L},$$

$$+\frac{P^{\mathrm{V}}}{P}\times\frac{1}{r^{5}}\mathcal{L}\quad+\frac{Q^{\mathrm{V}}}{Q}\times\frac{1}{r^{5}}\mathcal{L}\quad+\frac{T^{\mathrm{V}}}{T}\times\frac{1}{r^{5}}\mathcal{L}\quad-\frac{P^{\mathrm{V}}Q^{\mathrm{V}}}{PQ}\times\frac{1}{r^{5}}\mathcal{L}$$
$$-\frac{P^{\mathrm{V}}T^{\mathrm{V}}}{PT}\times\frac{1}{r^{5}}\mathcal{L}\quad-\frac{Q^{\mathrm{V}}T^{\mathrm{V}}}{QT}\times\frac{1}{r^{5}}\mathcal{L}\quad+\frac{P^{\mathrm{V}}Q^{\mathrm{V}}T^{\mathrm{V}}}{PQT}\times\frac{1}{r^{5}}\mathcal{L},$$

+&c. continued to the utmoſt extent of human life.

And conſequently to the ſeven following ſets of quantities, which are the ſame with the quantities laſt-mentioned, but only are placed in a different order, (thoſe which before were placed in perpendicular columns, one under the other, being now placed in ſeparate horizontal lines;) to wit,

1ſt, $\frac{P^{\prime}}{P}\times\frac{1}{r}\mathcal{L}\quad+\frac{P^{\prime\prime}}{P}\times\frac{1}{r^{2}}\mathcal{L}\quad+\frac{P^{\prime\prime\prime}}{P}\times\frac{1}{r^{3}}\mathcal{L}\quad+\frac{P^{\mathrm{IV}}}{P}\times\frac{1}{r^{4}}\mathcal{L}$

$+\frac{P^{\mathrm{V}}}{P}\times\frac{1}{r^{5}}\mathcal{L}$ +&c. continued to the utmoſt extent of human life:

And

And 2dly, $\frac{Q'}{Q} \times \frac{1}{r}\mathcal{L} + \frac{Q''}{Q} \times \frac{1}{r^2}\mathcal{L} + \frac{Q'''}{Q} \times \frac{1}{r^3}\mathcal{L} + \frac{Q^{iv}}{Q} \times \frac{1}{r^4}\mathcal{L}$ $+\frac{Q^{v}}{Q} \times \frac{1}{r^5}\mathcal{L}$ +&c. continued to the utmoſt extent of human life:

And 3dly, $\frac{T'}{T} \times \frac{1}{r}\mathcal{L} + \frac{T''}{T} \times \frac{1}{r^2}\mathcal{L} + \frac{T'''}{T} \times \frac{1}{r^3}\mathcal{L} + \frac{T^{iv}}{T} \times \frac{1}{r^4}\mathcal{L}$ $+\frac{T^{v}}{T} \times \frac{1}{r^5}\mathcal{L}$ +&c. continued to the utmoſt extent of human life:

And 4thly, $-\frac{P' Q'}{P\ Q} \times \frac{1}{r}\mathcal{L} - \frac{P'' Q''}{P\ Q} \times \frac{1}{r^2}\mathcal{L} - \frac{P''' Q'''}{P\ Q} \times \frac{1}{r^3}\mathcal{L}$ $-\frac{P^{iv}\ Q^{iv}}{P\ Q} \times \frac{1}{r^4}\mathcal{L} - \frac{P^{v}\ Q^{v}}{P\ Q} \times \frac{1}{r^5}\mathcal{L}$ —&c. continued to the utmoſt extent of human life:

And 5thly, $-\frac{P' T'}{P\ T} \times \frac{1}{r}\mathcal{L} - \frac{P'' T''}{P\ T} \times \frac{1}{r^2}\mathcal{L} - \frac{P''' T'''}{P\ T} \times \frac{1}{r^3}\mathcal{L}$ $-\frac{P^{iv}\ T^{iv}}{P\ T} \times \frac{1}{r^4}\mathcal{L} - \frac{P^{v}\ T^{v}}{P\ T} \times \frac{1}{r^5}\mathcal{L}$ —&c. continued to the utmoſt extent of human life:

And 6thly, $-\frac{Q' T'}{Q\ T} \times \frac{1}{r}\mathcal{L} - \frac{Q'' T''}{Q\ T} \times \frac{1}{r^2}\mathcal{L} - \frac{Q''' T'''}{Q\ T} \times \frac{1}{r^3}\mathcal{L}$ $-\frac{Q^{iv}\ T^{iv}}{Q\ T} \times \frac{1}{r^4}\mathcal{L} - \frac{Q^{v}\ T^{v}}{Q\ T} \times \frac{1}{r^5}\mathcal{L}$ —&c. continued to the utmoſt extent of human life:

And 7thly, $\frac{P' Q' T'}{P\ Q\ T} \times \frac{1}{r}\mathcal{L} + \frac{P'' Q'' T''}{P\ Q\ T} \times \frac{1}{r^2}\mathcal{L} + \frac{P''' Q''' T'''}{P\ Q\ T} \times \frac{1}{r^3}\mathcal{L}$ $+\frac{P^{iv}\ Q^{iv}\ T^{iv}}{P\ Q\ T} \times \frac{1}{r^4}\mathcal{L} + \frac{P^{v}\ Q^{v}\ T^{v}}{P\ Q\ T} \times \frac{1}{r^5}\mathcal{L}$ +&c. continued to the utmoſt extent of human life;

thus

that is, the value of a life-annuity of one pound for the lives of the ſaid purchaſer and his two companions, and the life of the longeſt liver of them, is equal to the exceſs of the ſum of the firſt, ſecond, third, and ſeventh, of the ſeven laſt ſets of quantities above the ſum of the fourth, fifth, and ſixth ſets.

Now it appears from Prob. 2, Coroll. 2, Art. 31, that the firſt of theſe ſeven ſets of quantities, to wit, $\frac{P^{\text{I}}}{P} \times \frac{1}{r}£$ $+\frac{P^{\text{II}}}{P} \times \frac{1}{r^2}£$ $+\frac{P^{\text{III}}}{P} \times \frac{1}{r^3}£$ $+\frac{P^{\text{IV}}}{P} \times \frac{1}{r^4}£$ $+\frac{P^{\text{V}}}{P} \times \frac{1}{r^5}£$ +&c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the life of the purchaſer, who was ſuppoſed to be of the age of N years, which correſponds to the number P in the table.

And, in like manner, the ſecond of theſe ſeven ſets of quantities, to wit, $\frac{\mathcal{Q}^{\text{I}}}{\mathcal{Q}} \times \frac{1}{r}£$ $+\frac{\mathcal{Q}^{\text{II}}}{\mathcal{Q}} \times \frac{1}{r^2}£$ $+\frac{\mathcal{Q}^{\text{III}}}{\mathcal{Q}} \times \frac{1}{r^3}£$ $+\frac{\mathcal{Q}^{\text{IV}}}{\mathcal{Q}} \times \frac{1}{r^4}£$ $+\frac{\mathcal{Q}^{\text{V}}}{\mathcal{Q}} \times \frac{1}{r^5}£$ +&c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the life of the ſaid purchaſer's firſt, or older, companion, who was ſuppoſed to be of the age of M years, which correſponds to the number $\mathcal{Q}$ in the table.

And, in like manner, the third of theſe ſeven ſets of quantities, to wit, $\frac{\mathcal{T}^{\text{I}}}{\mathcal{T}} \times \frac{1}{r}£$ $+\frac{\mathcal{T}^{\text{II}}}{\mathcal{T}} \times \frac{1}{r^2}£$ $+\frac{\mathcal{T}^{\text{III}}}{\mathcal{T}} \times \frac{1}{r^3}£$ $+\frac{\mathcal{T}^{\text{IV}}}{\mathcal{T}} \times \frac{1}{r^4}£$ $+\frac{\mathcal{T}^{\text{V}}}{\mathcal{T}} \times \frac{1}{r^5}£$ +&c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the life of the ſaid purchaſer's ſecond, or younger, companion, who was ſuppoſed to be of the age of L years, which correſponds to the number $\mathcal{T}$ in the table.

And

And it appears from Prob. 3, Coroll. 2, Art. 48, that the fourth of the ſaid ſeven ſets of quantities, to wit, $\frac{P^{\prime}\,\mathcal{Q}^{\prime}}{P\ \mathcal{Q}} \times \frac{1}{r}\mathcal{L} + \frac{P^{\prime\prime}\,\mathcal{Q}^{\prime\prime}}{P\ \mathcal{Q}} \times \frac{1}{r^2}\mathcal{L}$ $+\frac{P^{\prime\prime\prime}\,\mathcal{Q}^{\prime\prime\prime}}{P\ \mathcal{Q}} \times \frac{1}{r^3}\mathcal{L} + \frac{P^{\mathrm{IV}}\,\mathcal{Q}^{\mathrm{IV}}}{P\ \mathcal{Q}} \times \frac{1}{r^4}\mathcal{L} + \frac{P^{\mathrm{V}}\,\mathcal{Q}^{\mathrm{V}}}{P\ \mathcal{Q}} \times \frac{1}{r^5}\mathcal{L}$ +&c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the joint lives of the ſaid purchaſer and his ſaid firſt, or older, companion, whoſe ages N and M correſpond to the letters, or numbers, P and $\mathcal{Q}$ in the ſaid table.

And in like manner the fifth of the ſaid ſeven ſets of quantities, to wit, $\frac{P^{\prime}\,\mathcal{T}^{\prime}}{P\ \mathcal{T}} \times \frac{1}{r}\mathcal{L} + \frac{P^{\prime\prime}\,\mathcal{T}^{\prime\prime}}{P\ \mathcal{T}} \times \frac{1}{r^2}\mathcal{L} + \frac{P^{\prime\prime\prime}\,\mathcal{T}^{\prime\prime\prime}}{P\ \mathcal{T}} \times \frac{1}{r^3}\mathcal{L} + \frac{P^{\mathrm{IV}}\,\mathcal{T}^{\mathrm{IV}}}{P\ \mathcal{T}} \times \frac{1}{r^4}\mathcal{L}$ $+\frac{P^{\mathrm{V}}\,\mathcal{T}^{\mathrm{V}}}{P\ \mathcal{T}} \times \frac{1}{r^5}\mathcal{L}$ +&c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the joint lives of the ſaid purchaſer and his ſaid ſecond, or younger, companion, whoſe ages N and L correſpond to the numbers P and $\mathcal{T}$ in the ſaid table.

And in like manner the ſixth of the ſaid ſeven ſets of quantities, to wit, $\frac{\mathcal{Q}^{\prime}\,\mathcal{T}^{\prime}}{\mathcal{Q}\ \mathcal{T}} \times \frac{1}{r}\mathcal{L} + \frac{\mathcal{Q}^{\prime\prime}\,\mathcal{T}^{\prime\prime}}{\mathcal{Q}\ \mathcal{T}} \times \frac{1}{r^2}\mathcal{L} + \frac{\mathcal{Q}^{\prime\prime\prime}\,\mathcal{T}^{\prime\prime\prime}}{\mathcal{Q}\ \mathcal{T}} \times \frac{1}{r^3}\mathcal{L} + \frac{\mathcal{Q}^{\mathrm{IV}}\,\mathcal{T}^{\mathrm{IV}}}{\mathcal{Q}\ \mathcal{T}} \times \frac{1}{r^4}\mathcal{L}$ $+\frac{\mathcal{Q}^{\mathrm{V}}\,\mathcal{T}^{\mathrm{V}}}{\mathcal{Q}\ \mathcal{T}} \times \frac{1}{r^5}\mathcal{L}$ +&c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the joint lives of the ſaid purchaſer's two companions, whoſe ages M and L correſpond to the numbers $\mathcal{Q}$ and $\mathcal{T}$ in the ſaid table.

And, laſtly, it appears from Prob. 3d, Coroll. 7, Art. 56, that the ſeventh, or laſt, of the ſaid ſeven ſets of quantities, to wit, $\frac{P^{\prime}\,\mathcal{Q}^{\prime}\,\mathcal{T}^{\prime}}{P\ \mathcal{Q}\ \mathcal{T}} \times \frac{1}{r}\mathcal{L} + \frac{P^{\prime\prime}\,\mathcal{Q}^{\prime\prime}\,\mathcal{T}^{\prime\prime}}{P\ \mathcal{Q}\ \mathcal{T}} \times \frac{1}{r^2}\mathcal{L} + \frac{P^{\prime\prime\prime}\,\mathcal{Q}^{\prime\prime\prime}\,\mathcal{T}^{\prime\prime\prime}}{P\ \mathcal{Q}\ \mathcal{T}} \times \frac{1}{r^3}\mathcal{L}$ $+\frac{P^{\mathrm{IV}}\,\mathcal{Q}^{\mathrm{IV}}\,\mathcal{T}^{\mathrm{IV}}}{P\ \mathcal{Q}\ \mathcal{T}} \times \frac{1}{r^4}\mathcal{L} + \frac{P^{\mathrm{V}}\,\mathcal{Q}^{\mathrm{V}}\,\mathcal{T}^{\mathrm{V}}}{P\ \mathcal{Q}\ \mathcal{T}} \times \frac{1}{r^5}\mathcal{L}$ +&c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the joint lives of the ſaid purchaſer and his two companions, whoſe ages N, M, and L correſpond to the numbers P, $\mathcal{Q}$, and $\mathcal{T}$.

Therefore the value of an annuity of one pound for the three lives of the ſaid purchaſer and his two companions, and the life of the longeſt liver of them, is equal to the exceſs of the ſum of the values of three ſeparate life-annuities of one pound each for the ſingle lives of the ſaid purchaſer and his two companions, together with the value of an annuity of the ſame ſum for the joint lives of all the three, above the ſum of the three values of a like annuity for the joint lives of the ſaid purchaſer and his firſt, or older, companion, and for the joint lives of the ſaid purchaſer and his ſecond, or younger, companion, and for the joint lives of both his ſaid companions. Q.E.D.

Short and general Expreſſions for the Values of the Annuities that are the Subject of the foregoing Problems.

LXXVII. Having now explained, in as full and clear a manner as I was able, the grounds and reaſons of the methods above delivered for the calculation of the values of annuities, ſo far as relates to annuities for given numbers of years, not depending upon the continuance of any life, or upon any other contingent event, and to annuities for given numbers of years depending upon the continuance of a ſingle life of a given age, or upon the joint continuance of two, or of three, lives of given ages, or upon the continuance of any one of two, or of three, lives of given ages, the intereſt of money, in all theſe caſes, being likewiſe given; it will now be convenient to recapitulate the ſubſtance of the concluſions that have been obtained in the foregoing articles, by exhibiting ſhort and general expreſſions of the values of theſe ſeveral kinds of annuities, to which the reader, whenever he ſhall have occaſion to compute any of theſe annuities, may readily have recourſe. This may be done in the manner following.

A ſhort

A ſhort expreſſion of the value of an annuity of one pound per annum for a given number of years, not depending on the continuance of any life or on any other uncertain event, the intereſt of money being alſo given.

LXXVIII. Let r denote the value of one pound ſterling together with its intereſt for one year at any given rate of intereſt: and let n be the number of years during which an annuity of one pound per annum is to continue.

Then by Art. 23, pages 21 and 22, it is evident that the preſent value of this annuity will be $=$ £1 $\times$ the ſeries $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. continued to the term $\frac{1}{r^n}$, or to n terms.

A method of obtaining another and ſtill ſhorter expreſſion of the ſame value.

LXXIX. As this ſeries $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. is a geometrical progreſſion, the ſum of its terms may be found in a ſingle ſhort expreſſion without taking the trouble of computing them all ſeparately, and then adding them into one ſum total. This may be done by means of the following Lemma.

A LEMMA.

LXXX. The ſum of the terms of every decreaſing geometrical progreſſion is equal to the quotient that ariſes by dividing the exceſs of the ſquare of its firſt, or greateſt, term above the product, or rectangle under its ſecond term and its leaſt term, by the exceſs of its firſt, or greateſt, term above its ſecond term.

Of the ſum of the terms of a decreaſing geometrical progreſſion.

For, if A, B, C, D, E, are a ſeries of terms in geometrical proportion, we ſhall have $A : B :: B : C$,
and $A : B :: C : D$,
and $A : B :: D : E$.

Therefore, by El. 5, 12, the ſum of all the antecedents A, B, C, D, will be to the ſum of all the conſequents B, C, D, E, as the firſt antecedent A is to the firſt conſequent B. But the ſum of all the antecedents A, B, C, D, is the ſum of all the terms of the ſeries except the laſt term E;

and the ſum of all the conſequents B, C, D, E, is the ſum of all the terms of the ſeries except the firſt term A. Therefore, if the ſum of all the terms of the ſeries be called S, we ſhall have $S-E : S-A :: A : B$. Therefore $B \times \overline{S-E}$ will be $= A \times \overline{S-A}$, or $BS-BE$ will be $= AS-AA$. Therefore, adding AA to both ſides, we ſhall have $AA-BE+BS=AS$; and, ſubtracting BS (which is leſs than AS, becauſe B is leſs than A, and which therefore is alſo leſs than the other ſide of the equation, to wit, $AA-BE+BS$) from both ſides, we ſhall have $AA-BE=AS-BS=S \times \overline{A-B}$. Therefore S, or the ſum of all the terms, A, E, C, D, E, will be equal to $\frac{AA-BE}{A-B}$, that is, to the quotient that ariſes by dividing the exceſs of AA, the ſquare of the firſt, or greateſt, term A, above BE, the product, or rectangle under the ſecond term B and the leaſt term E, by the exceſs of the firſt, or greateſt, term A, above the ſecond term B. And it is eaſy to ſee that the ſame reaſonings will take place, and conſequently that the ſame concluſion will follow from them, if the ſeries A, B, C, D, &c. ſhould conſiſt of any other number of terms whatever, as well as when it conſiſts of five terms. QED.

LXXXI. Now in the ſeries $\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}+\frac{1}{r^5}+\frac{1}{r^6}+\frac{1}{r^7}$ + &c. $+\frac{1}{r^n}$, $\frac{1}{r}$ is $= A$, and $\frac{1}{rr}$ is $= B$, and $\frac{1}{r^n}$ is $= E$. Therefore AA is $= \frac{1}{rr}$, and BE is $= \frac{1}{rr} \times \frac{1}{r^n} = \frac{1}{r^{n+2}}$, and conſequently $AA-BE$ is $= \frac{1}{rr} - \frac{1}{r^{n+2}} = \frac{r^{n+2}-r^2}{r^{n+4}} = \frac{r^n-1}{r^{n+2}}$. And $A-B$ is $= \frac{1}{r} - \frac{1}{r^2} = \frac{r^2-r}{r^3} = \frac{r-1}{rr}$. And conſequently $\frac{AA-BE}{A-B}$ is $= \frac{\frac{r^n-1}{r^{n+2}}}{\frac{r-1}{r^2}} = \frac{r^n-1}{r^{n+2}} \times \frac{r^2}{r-1} = \frac{r^n-1}{r^n \times r-1} = \frac{r^n}{r^n \times r-1} - \frac{1}{r^n \times r-1} = \frac{1}{r-1} - \frac{1}{r^n \times r-1}$. Therefore the ſum of all the terms of the ſeries $\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}+\frac{1}{r^5}+\frac{1}{r^6}+\frac{1}{r^7}$ + &c. continued to n terms, is $= \frac{1}{r-1} - \frac{1}{r^n \times r-1}$, and conſequently the value of an annuity of one pound per annum for n years is $= £1 \times \overline{\frac{1}{r-1} - \frac{1}{r^n \times r-1}}$. QEI.

A ſecond, and very ſhort, expreſſion of the value of an annuity of one pound a year for a given number of years certain.

LXXXII. I ſhall

LXXXII. I ſhall now proceed to give an example of the computation of the value of an annuity of one pound for a given number of years by means of both theſe expreſſions, that is, the expreſſion $\frac{£}{1} \times$ the ſeries $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. $+ \frac{1}{r^n}$ and the expreſſion $\frac{£}{1} \times \left| \frac{1}{r-1} - \frac{1}{r^n \times r-1} \right.$.

An example of the computation of the value of an annuity of one pound for a given number of years by means of the expreſſion $\frac{£}{1} \times$ *the ſeries* $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. $+ \frac{1}{r^n}$.

LXXXIII. Let n, or the given number of years during which the annuity is to continue, be 30 years; and let the rate of the intereſt of money be $3\frac{1}{2}$ per cent.

Then will r be $= 1.035$, and $\frac{1}{r}$ will be $= \frac{1}{1.035} = .966,184$. And

$\frac{1}{r^2}$ will be $= .933,511$, and $\frac{1}{r^3} = .901,943$,

and $\frac{1}{r^4} = .871,442$, and $\frac{1}{r^5} = .841,973$,

and $\frac{1}{r^6} = .813,501$, and $\frac{1}{r^7} = .785,991$,

and $\frac{1}{r^8} = .759,412$, and $\frac{1}{r^9} = .733,731$,

and $\frac{1}{r^{10}} = .708,919$, and $\frac{1}{r^{11}} = .684,946$,

and $\frac{1}{r^{12}} = .661,783$, and $\frac{1}{r^{13}} = .639,404$,

and $\frac{1}{r^{14}} = .617,782$, and $\frac{1}{r^{15}} = .596,891$,

and

and $\frac{1}{r^{16}}$ will be = .576,706, and $\frac{1}{r^{17}}$ = .557,204,

and $\frac{1}{r^{18}}$ = .538,361, and $\frac{1}{r^{19}}$ = .520,156,

and $\frac{1}{r^{20}}$ = .502,566, and $\frac{1}{r^{21}}$ = .485,571,

and $\frac{1}{r^{22}}$ = .469,151, and $\frac{1}{r^{23}}$ = .453,286,

and $\frac{1}{r^{24}}$ = .437,957, and $\frac{1}{r^{25}}$ = .423,147,

and $\frac{1}{r^{26}}$ = .408,838, and $\frac{1}{r^{27}}$ = .395,012,

and $\frac{1}{r^{28}}$ = .381,654, and $\frac{1}{r^{29}}$ = .368,748,

and $\frac{1}{r^{30}}$ = .356,278. Therefore the series $\frac{1}{r} + \frac{1}{r^2}$

$+\frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \frac{1}{r^8} + \frac{1}{r^9} + \frac{1}{r^{10}} + \frac{1}{r^{11}} + \frac{1}{r^{12}}$

$+\frac{1}{r^{13}} + \frac{1}{r^{14}} + \frac{1}{r^{15}} + \frac{1}{r^{16}} + \frac{1}{r^{17}} + \frac{1}{r^{18}} + \frac{1}{r^{19}} + \frac{1}{r^{20}} + \frac{1}{r^{21}}$

$+\frac{1}{r^{22}} + \frac{1}{r^{23}} + \frac{1}{r^{24}} + \frac{1}{r^{25}} + \frac{1}{r^{26}} + \frac{1}{r^{27}} + \frac{1}{r^{28}} + \frac{1}{r^{29}} + \frac{1}{r^{30}}$ is =

.966,184,	+ .933,511,	+ .901,943,	+ .871,442,
+ .841,973,	+ .813,501,	+ .785,991,	+ .759,412,
+ .733,731,	+ .708,919,	+ .684,946,	+ .661,783,
+ .639,404,	+ .617,782,	+ .596,891,	+ .576,706,
+ .557,204,	+ .538,361,	+ .520,156,	+ .502,566,
+ .485,571,	+ .469,151,	+ .453,286,	+ .437,957,
+ .423,147,	+ .408,838,	+ .395,012,	+ .381,654,
+ .368,748,	+ .356,278	=	
+ 5.015,962,	+ 4.846,341,	+ 4.338,225,	+ 4.191,520,

= 18.392,048; and consequently the value of an annuity of one pound a year for 30 years certain, when the interest of money is $3\frac{1}{2}$ per cent. is = £1 × 18.392,048, or £18.392,048, or 18*l.* 7*s.* 10*d.*$\frac{1}{2}$. QEI.

A con-

A computation of the ſame annuity for 30 *years by means of the expreſſion* £1 × $\frac{1}{r-1} - \frac{1}{r^n \times r-1}$.

LXXXIV. Here r is = 1.035; and conſequently $\frac{1}{r}$ is = $\frac{1}{1.035}$ = .966,184; and $\frac{1}{r^n}$ is = $\frac{1}{r^{30}}$ = .356,278; and $r-1$ is = .035 = $\frac{35}{1000}$; and $\frac{1}{r-1}$ is = $\frac{1}{\frac{35}{1000}}$ = 1 × $\frac{1000}{35}$ = 28.571,428; and $\frac{1}{r^n \times r-1}$, or $\frac{1}{r^n} \times \frac{1}{r-1}$, is = .356,278 × $\frac{1000}{35}$ = $\frac{356.278}{35}$ = 10.179,37. Therefore $\frac{1}{r-1} - \frac{1}{r^n \times r-1}$ is = 28.571,42 — 10.179,37 = 18.392,05. Therefore the value of the propoſed annuity of one pound per annum for 30 years, when the intereſt of money is $3\frac{1}{2}$ per cent. is £1 × 18.392,05, or £18.392,05, or 18*l.* 7*s.* 10*d.*$\frac{1}{2}$, as before. QE I.

LXXXV. Note. A table of the values of annuities of this kind, ſhewing the preſent value of an annuity of one pound a year for any number of years certain, not exceeding 90 years, when the intereſt of money is at 5, 4, and 3, per cent. has been given us by the late learned Mr. Thomas Simpſon, of Woolwich, in the 114th and 115th pages of his comprehenſive little treatiſe, called, "*The doctrine of annuities and reverſions, &c.*" in ſmall octavo, which was publiſhed in the year 1742. And another table of the ſame kind had been publiſhed before by Mr. John Smart in the year 1726 in his very uſeful work above-mentioned in Art. 25, intitled "*Tables of Intereſt, &c.*" This latter table is computed to eight places of decimal fractions; and for the twelve different rates of intereſt mentioned above in Art. 24, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. and for every number of years, and even half-years, in the ſpace of an hundred years. So that nothing can be expected, or need be deſired, more compleatly convenient for the purpoſe of eſtimating the values of annuities for terms of years certain, than this table. It is Mr. Smart's fourth table of compound intereſt, pages 76, 77, 78, 79, 80, 81, 82, 83, of his valuable book above-mentioned.

A ſhort

A ſhort expreſſion of the value of an annuity of one pound per annum for a given number of years, depending on the continuance of one life of a given age, the rate of the intereſt of money being alſo given.

LXXXVI. Let r be, as, before, the ſum of one pound together with its intereſt for one year according to the given rate of intereſt. And let N be the number of years in the age of the perſon on whoſe life the annuity is to depend, and E be the greateſt number of years through which it is ſuppoſed to be poſſible for human life to be extended, according to the table of the probabilities of the duration of human life adopted for the calculation; which number is in Monſieur de Parcieux's table 94 years. Then will $E-N$ be the greateſt number of years through which, according to ſuch ſuppoſition, it is poſſible that the life of the annuitant of the given age of N years can be extended. Let n be any number of years not greater than $E-N$ years. And let the annuity of one pound per annum be granted for the term of n years, provided the ſaid annuitant of the age of N years ſhall ſo long live, but otherwiſe to ceaſe at his death. Let P be the number of perſons of the ſaid age of N years repreſented to be all living at the ſame time in Monſieur de Parcieux's table of the probabilities of the duration of human life, or in ſuch other table of theſe probabilities as is thought proper by the calculator to be adopted as the ground of his calculation. And let P' be the number of perſons repreſented in the ſaid table to be living at the age of $N+1$ years; and P'' the number living at the age of $N+2$ years; and P''' the number living at the age of $N+3$ years; and P^{IV}, P^{V}, P^{VI}, P^{VII}, P^{VIII}, P^{IX}, P^{X}, &c. the numbers living at the ſeveral following ages of $N+4$ years, $N+5$ years, $N+6$ years, $N+7$ years, $N+8$ years, $N+9$ years, $N+10$ years, &c. reſpectively.

Then will the preſent value of an annuity of one pound a year for the ſpace of n years, in caſe the ſaid life of N years ſhall ſo long continue, be equal to the following expreſſion, to wit, £1 × the ſeries $\frac{P'}{Pr} + \frac{P''}{Pr^2} + \frac{P'''}{Pr^3} + \frac{P^{IV}}{Pr^4} + \frac{P^{V}}{Pr^5} + \frac{P^{VI}}{Pr^6} + \frac{P^{VII}}{Pr^7}$ +&c. continued to n terms, or to the term $\frac{P^n}{Pr^n}$, or to the expreſſion £$\frac{1}{P}$ × the ſeries $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7}$ +&c. continued to n terms, or to the term $\frac{P^n}{r^n}$.

This is evident from Problem 2, and its ſecond corollary, Art. 26, 27, 28, 29, 31, pages 24, 25, 26, 27, 28.

LXXXVII. If

LXXXVII. If n years is the greateſt number of years through which it is poſſible (according to the table of probabilities of the duration of human life adopted in the calculation,) for the given life of N years to be extended, the ſaid expreſſion $\frac{\pounds 1}{P} \times$ the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7}$ + &c. continued to n terms, or to the term $\frac{P^{n}}{r^n}$, will be the value of an annuity of one pound *per annum* for the whole life of a perſon of N years of age: but, if n is leſs than the ſaid complement of N years to the utmoſt duration of human life, the ſaid expreſſion will be leſs than the value of an annuity of one pound *per annum* for the whole life of a perſon of the age of N years, and will be the value of an immediate, but imperfect, life-annuity during n years of the life of a perſon of that age. This is evident from Art. 31, 32, 33, pages 28, 29, 30, 31, 32.

An example of the computation of the value of an immediate and compleat life-annuity of one pound per annum for the whole life of a perſon of a given age, by means of the foregoing expreſſion.

LXXXVIII. Let it be required to find the value of an annuity of one pound *per annum* for the life of a man of fourſcore years of age, according to Monſieur de Parcieux's table of the probabilities of the duration of human life, and upon a ſuppoſition that the intereſt of money is 3 per cent.

Here n, or the number of years through which the annuity is to continue in caſe the life of fourſcore years of age ſhall laſt ſo long, is the greateſt poſſible number of years through which, according to Monſieur de Parcieux's table, a life of fourſcore years of age can be extended, that is, 94—80 years, or 14 years. Therefore the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7}$ + &c. in the foregoing expreſſion muſt be continued to 14 terms; which may be computed as follows.

P is $= 118$, $P^{I} = 101$, $P^{II} = 85$, $P^{III} = 71$, $P^{IV} = 59$, $P^{V} = 48$, $P^{VI} = 38$, $P^{VII} = 29$, $P^{VIII} = 22$, $P^{IX} = 16$, $P^{X} = 11$, $P^{XI} = 7$, $P^{XII} = 4$, $P^{XIII} = 2$, and $P^{XIV} = 1$. And r is $= 1.03$, and $\frac{1}{r} = \frac{1}{1.03} = .9708$,

and $\frac{1}{r^2} = .9425$, $\frac{1}{r^3} = .9151$, $\frac{1}{r^4} = .8884$, $\frac{1}{r^5} = .8626$, $\frac{1}{r^6} = .8374$, $\frac{1}{r^7} = .8130$, $\frac{1}{r^8} = .7894$, $\frac{1}{r^9} = .7664$, $\frac{1}{r^{10}} = .7440$, $\frac{1}{r^{11}} = .7224$, $\frac{1}{r^{12}} = .7013$, $\frac{1}{r^{13}} = .6809$, and $\frac{1}{r^{14}} = .6611$. Therefore the expreſſion $\frac{\text{£}1}{P} \times$ the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7} + \frac{P^{VIII}}{r^8} + \frac{P^{IX}}{r^9} + \frac{P^{X}}{r^{10}} + \frac{P^{XI}}{r^{11}} + \frac{P^{XII}}{r^{12}} + \frac{P^{XIII}}{r^{13}} + \frac{P^{XIV}}{r^{14}}$ will be equal to

$\frac{\text{£}1}{118} \times$ the ſeries $101 \times .9708 + 85 \times .9425 + 71 \times .9151 + 59 \times .8884 + 48 \times .8626 + 38 \times .8374 + 29 \times .8130 + 22 \times .7894 + 16 \times .7664 + 11 \times .7440 + 7 \times .7224 + 4 \times .7013 + 2 \times .6809 + 1 \times .6611$

$= \frac{\text{£}1}{118} \times$ the ſeries $98.0508 + 80.1125 + 65.0721 + 52.4156 + 41.4048 + 31.8212 + 23.5770 + 17.3668 + 12.2624 + 8.1840 + 5.0568 + 2.8052 + 1.3618 + .6611$

$= \frac{\text{£}1}{118} \times 440.1521 = \text{£}1 \times \frac{440.1521}{118} = \text{£}1 \times 3.7301 = \text{£}3.7301 = \text{£}3,\ 14\text{s.},\ 7\frac{1}{2}\text{d.}$

Conſequently the value of an annuity of one pound per annum for the whole life of a man of fourſcore years of age, according to Monſieur de Parcieux's table of the probabilities of the duration of human life, and when the intereſt of money is 3 per cent. is 3*l.* 14*s.* 7$d.\frac{1}{2}$. Q E I.

An example of the computation of the value of an immediate, but imperfect, life-annuity, depending on the life of a perſon of a given age, by means of the ſame expreſſion.

LXXXIX. Let it be required to find the value of an annuity of one pound *per annum* for the firſt five years of the life of a man of fourſcore years of age, according to the ſame table of the probabilities of the duration of human life and the ſame rate of intereſt as before.

For

For this purpoſe we need only take the firſt five terms of the foregoing ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} +$ &c. and multiply their ſum into the fraction $\frac{1}{P}$ £; and the product will be the value of the propoſed annuity. Theſe terms are $101 \times .9708 + 85 \times .9425 + 71 \times .9151 + 59 \times .8884 + 48 \times .8626$, which are equal to $98.0508 + 80.1125 + 65.0721 + 52.4156 + 41.4048 = 337.0558$; which, being multiplied into the fraction $\frac{1}{P}$ £, or $\frac{1}{118}$ £, is $= £1 \times \frac{337.0558}{118} = £1 \times 2.8564 = £2.8564 = 2l.\ 17s.\ 1d.\frac{1}{2}$. Therefore the value of an annuity of one pound *per annum* for the firſt five years of the life of a man of fourſcore years of age, according to the foregoing ſuppoſitions of the duration of human life and the intereſt of money, is $2l.\ 17s.\ 1d.\frac{1}{2}$. QEI.

XC. And, if the annuity to be purchaſed is to be a remote one, or to be paid at the diſtance of more than one year, a ſhort expreſſion of its value, ſimilar to the foregoing expreſſions, may be found as follows.

A ſhort and general expreſſion of the value of a remote annuity of one pound per annum *for a given number of years, depending on a life of a given age, when the intereſt of money is alſo given.*

Let r be, as before, the value of one pound together with its intereſt for a year at the given rate of the intereſt of money. And let m be the number of years at the end of which the annuity is to commence, ſo that the firſt payment of it ſhall be made at the end of $m+1$ years. And let N be the number of years in the age of the purchaſer of the annuity at the time of purchaſing it; and let E, as before, be the whole number of years through which it is poſſible, according to the table of probabilities adopted in the calculation, for human life to be extended; which in Monſieur de Parcieux's table is 94 years. Then will $N+m$ be the number of years in the age of the ſaid purchaſer at the time of the commencement of the ſaid annuity, and $N+m+1$ the number of years in his age at the time when the firſt payment of the ſaid annuity will become due; and $E-\overline{|N+m}$, or $E-N-m$, will be the greateſt poſſible number of years through which the life of the ſaid purchaſer can be extended after he ſhall have attained the age of $N+m$ years and the annuity ſhall have commenced. Let n be any number of years not greater than $E-N-m$; and let P^{m+1}

denote the number of perſons of the age of $N+m+1$ years repreſented, in the table of probabilities of life adopted in the calculation, to be all living at the ſame time; and P^{m+II} the number of perſons repreſented in the ſaid table to be living at the age of $N+m+2$ years; and P^{m+III} the number of perſons repreſented to be living at the age of $N+m+3$ years; and P^{m+IV} the number of perſons living at the age of $N+m+4$ years; and P^{m+V} the number of perſons living at the age of $N+m+5$ years; and ſo on for all the following ages in the table.

Then will the value of an annuity of one pound per annum to commence at the diſtance of m years, (ſo that the firſt payment of it ſhall be made at the end of $m+1$ years,) and to continue during n years, provided a perſon aged N years at the time of the purchaſe ſhall ſo long live, but to ceaſe as ſoon as ſuch perſon ſhall be dead, be equal to the following expreſſion, to wit,

$$\frac{\pounds 1}{P} \times \text{the ſeries } \frac{P^{m+I}}{r^{m+1}} + \frac{P^{m+II}}{r^{m+2}} + \frac{P^{m+III}}{r^{m+3}} + \frac{P^{m+IV}}{r^{m+4}} + \frac{P^{m+V}}{r^{m+5}} + \frac{P^{m+VI}}{r^{m+6}} + \frac{P^{m+VII}}{r^{m+7}} + \&c.$$

continued to n terms, or to the term $\frac{P^{m+n}}{r^{m+n}}$. This is evident from Art. 36 and 37, pages 33, 34.

An example of the computation of the value of a remote annuity, depending upon a life of a given age, by means of the foregoing expreſſion.

XCI. Let it be required to find the value of an annuity of one pound *per annum* for the life of a perſon of fourſcore years of age, but which ſhall not commence till five years after the purchaſe of it, ſo that the firſt payment of it ſhall be made to the ſaid perſon of fourſcore years of age at the end of ſix years, or when he ſhall be fourſcore and ſix years of age, if he ſhall be then living, and which ſhall continue during the whole remainder of the life of the ſaid purchaſer; the intereſt of money being 3 per cent. (as in the laſt example,) and the probabilities of the duration of human life ſuch as they are repreſented to be in Monſieur de Parcieux's table.

Here N, or the number of years in the age of the purchaſer of the annuity, is 80; and m, the number of years before the annuity is to commence, is 5; and conſequently $N+m$, or the number of years in the age of the annuitant at the time the annuity is to commence, is 85. E, the greateſt number of years through which human life can be extended, is, according to Monſieur de Parcieux's table of probabilities, 94 years; and conſequently $E-N-m$, or the greateſt number of years through which it is

is poſſible that the life of the propoſed annuitant can be extended after the annuity ſhall have commenced, is 94 — 80 — 5, or 94 — 85, or 9, years. And, n, or the number of years during which it is poſſible the annuity may continue, will conſequently be equal to 9 years, becauſe the annuity, when once it has taken place, is ſuppoſed to continue during the whole remaining part of the annuitant's life; and conſequently the ſeries $\frac{P^{m+1}}{r^{m+1}} + \frac{P^{m+11}}{r^{m+2}} + \frac{P^{m+111}}{r^{m+3}} + \frac{P^{m+1v}}{r^{m+4}} + \frac{P^{m+v}}{r^{m+5}}$ + &c. will conſiſt of nine terms. Theſe terms may be computed as follows.

Since m is $= 5$, we ſhall have $m + 1 = vi$, and $m + 11 = vii$, and $m + 111 = viii$, and $m + 1v = ix$, and $m + v = x$, and $m + vi = xi$, and $m + vii = xii$, and $m + viii = xiii$, and $m + ix = xiv$; and, in like manner, $m + 1 = 6$, and $m + 2 = 7$, and $m + 3 = 8$, and $m + 4 = 9$, and $m + 5 = 10$, and $m + 6 = 11$, and $m + 7 = 12$, and $m + 8 = 13$, and $m + 9 = 14$. Therefore the ſeries $\frac{P^{m+1}}{r^{m+1}} + \frac{P^{m+11}}{r^{m+2}} + \frac{P^{m+111}}{r^{m+3}} + \frac{P^{m+1v}}{r^{m+4}} + \frac{P^{m+v}}{r^{m+5}} + \frac{P^{m+vi}}{r^{m+6}} + \frac{P^{m+vii}}{r^{m+7}} + \frac{P^{m+viii}}{r^{m+8}} + \frac{P^{m+ix}}{r^{m+9}}$ is $= \frac{P^{vi}}{r^6} + \frac{P^{vii}}{r^7} + \frac{P^{viii}}{r^8} + \frac{P^{ix}}{r^9} + \frac{P^{x}}{r^{10}} + \frac{P^{xi}}{r^{11}} + \frac{P^{xii}}{r^{12}} + \frac{P^{xiii}}{r^{13}} + \frac{P^{xiv}}{r^{14}}$.

But r is, as before, $= 1.03$; and conſequently $\frac{1}{r}$ is $= \frac{1}{1.03} = .9708$, and $\frac{1}{r^6}$ is $= .8374$, and $\frac{1}{r^7}$ is $= .8130$, and $\frac{1}{r^8}$ is $= .7894$, and $\frac{1}{r^9}$ is $= .7664$, and $\frac{1}{r^{10}}$ is $= .7440$, and $\frac{1}{r^{11}}$ is $= .7224$, and $\frac{1}{r^{12}}$ is $= .7013$, and $\frac{1}{r^{13}}$ is $= .6809$, and $\frac{1}{r^{14}}$ is $= .6611$. And P is $= 118$, and $P^{vi} = 38$, and $P^{vii} = 29$, and $P^{viii} = 22$, and $P^{ix} = 16$, and $P^{x} = 11$, and $P^{xi} = 7$, and $P^{xii} = 4$, and $P^{xiii} = 2$, and $P^{xiv} = 1$. Therefore the expreſſion $\frac{£1}{P}$ x the ſeries $\frac{P^{vi}}{r^6} + \frac{P^{vii}}{r^7} + \frac{P^{viii}}{r^8} + \frac{P^{ix}}{r^9} + \frac{P^{x}}{r^{10}} + \frac{P^{xi}}{r^{11}} + \frac{P^{xii}}{r^{12}} + \frac{P^{xiii}}{r^{13}} + \frac{P^{xiv}}{r^{14}}$ is equal to $\frac{£1}{118}$ x the ſeries $38 \times .8374 + 29 \times .8130$

$+29\times.8130\ +22\times.7894\ +16\times.7664\ +11\times.7440\ +7\times.7224$
$+4\times.7013\ +2\times.6809\ +1\times.6611 = \frac{\pounds 1}{118} \times$ the ſeries 31.8212
$+23.5770\ +17.3668\ +12.2624\ +8.1840\ +5.0568\ +2.8052$
$+1.3618\ +.6611 = \frac{\pounds 1}{118} \times 103.0963 = \pounds 1 \times \frac{103.0963}{118} = \pounds 1\times.8736$
$= \pounds.8736 = 17s.\ 5d.\frac{1}{2}$. Therefore $17s.\ 5d.\frac{1}{2}$ is the value of an annuity of one pound *per annum* for the life of a man of fourſcore years of age, to commence at the diſtance of five years, or when he ſhall be fourſcore and five years old, ſo that the firſt payment of it ſhall be made to him when he ſhall be 86 years old, according to Monſieur de Parcieux's table of the probabilities of life and when the intereſt of money is 3 per cent. Q E I.

XCII. N. B. This value of this annuity is a little greater than that which is found for it above in Art. 37, page 34, which is $17s.\ 3d.\frac{1}{2}$. The reaſon of the difference is a ſmall miſtake which I have diſcovered to have been made in one of the arithmetical operations in Art. 32, page 30. The quotient of the diviſion of 31.8212 by 118 is there made to be .2612, whereas it ought to be .2696, which is greater than .2612 by .0084. This difference .0084, being added to the number .8650 in Art. 37, page 34, will make it equal to .8734, which agrees in its three higheſt figures with the number .8736, juſt now found for the value of this annuity. The diſcovery of this miſtake, if I had made it ſooner, would have prevented the inſertion of the note at the end of Art. 33, page 32, which now appears to be ill-grounded.

XCIII. The examples that have been given, in the ſeveral foregoing articles, of the computation of the values of life-annuities, both immediate and remote, are the ſame which were given above in the ſeveral articles that come after the ſolution of Problem 11. The reaſon of repeating them in this place was that they might ſerve to illuſtrate the ſhort and general expreſſions of thoſe values which have been ſet forth in this latter part of the preſent tract, and might thereby enable the reader, if he choſe it, to become more familiarly acquainted with the practice of computing the values of life-annuities by means of thoſe expreſſions. With the ſame view I ſhall here ſubjoin another example of the computation of the values of life-annuities by means of thoſe general expreſſions, and ſhall chuſe for that purpoſe a younger life than thoſe in the foregoing examples. This will occaſion a long and tedious calculation, which would, I thought, have too much interrupted the chain of the reaſonings uſed in Problem 11 and its corollaries,

corollaries, which are intended to explain the theory, or the grounds and reaſons, of theſe computations; but in this part of the work, which is intended to illuſtrate the practice of theſe computations, I conceive that the exhibition of ſuch a calculation may be of uſe.

A calculation of the value of an annuity of one pound ſterling for the life of a perſon of 10 years of age; upon a ſuppoſition that the intereſt of money is 3 and an half per cent. and that the probabilities of the duration of human life are ſuch as they are repreſented to be in Monſieur de Parcieux's table.

XCIV. The number of perſons of the age of 10 years, repreſented in Monſieur de Parcieux's table to be all living at the ſame time, is 880; that is, according to the notation of Art. 86, P is $= 880$. And it appears by the ſame table that the number living at 11 years of age is 872, and the number living at 12 years of age is 866, and thoſe living at 13, 14, 15, and all the following years of age up to 94 years, are 860, 854, 848, 842, 835, 828, 821, 814, 806, 798, 790, 782, 774, 766, 758, 750, 742, 734, 726, 718, 710, 702, 694, 686, 678, 671, 664, 657, 650, 643, 636, 629, 622, 615, 607, 599, 590, 581, 571, 560, 549, 538, 526, 514, 502, 489, 476, 463, 450, 437, 423, 409, 395, 380, 364, 347, 329, 310, 291, 271, 251, 231, 211, 192, 173, 154, 136, 118, 101, 85, 71, 59, 48, 38, 29, 22, 16, 11, 7, 4, 2, and 1; that is, according to the notation of Art. 86, P^{I} is $= 872$, and P^{II} is $= 866$, and P^{III}, P^{IV}, P^{V}, &c. are equal to 860, 854, 848, and the other numbers juſt now mentioned, reſpectively.

And it appears by Mr. Smart's ſecond table of Compound Intereſt, pages 60—67, that, when the intereſt of money is $3\frac{1}{2}$ per cent. r is $= 1.035$, and $\frac{1}{r} = \frac{1}{1.035} = .96618$, and $\frac{1}{r^2} = .93351$, and $\frac{1}{r^3} = .90194$, and $\frac{1}{r^4} = .87144$, and $\frac{1}{r^5} = .84197$, and $\frac{1}{r^6} = .81350$, and $\frac{1}{r^7} = .78599$, and $\frac{1}{r^8} = .75941$, and $\frac{1}{r^9} = .73373$, and $\frac{1}{r^{10}} = .70891$, and $\frac{1}{r^{11}} = .68494$, and $\frac{1}{r^{12}} = .66178$, and $\frac{1}{r^{13}} = .63940$, and $\frac{1}{r^{14}} = .61778$, and $\frac{1}{r^{15}} = .59689$, and $\frac{1}{r^{16}} = .57670$, and $\frac{1}{r^{17}} = .55720$, and $\frac{1}{r^{18}} = .53836$,

$= .53836$, and $\frac{1}{r^{19}} = .52015$, and $\frac{1}{r^{20}} = .50256$, and $\frac{1}{r^{21}} = .48557$, and $\frac{1}{r^{22}} = .46915$, and $\frac{1}{r^{23}} = .45328$, and $\frac{1}{r^{24}} = .43795$, and $\frac{1}{r^{25}} = .42314$, and $\frac{1}{r^{26}} = .40883$, and $\frac{1}{r^{27}} = .39501$, and $\frac{1}{r^{28}} = .38165$, and $\frac{1}{r^{29}} = .36874$, and $\frac{1}{r^{30}} = .35627$, and $\frac{1}{r^{31}} = .34423$, and $\frac{1}{r^{32}} = .33258$, and $\frac{1}{r^{33}} = .32134$, and $\frac{1}{r^{34}} = .31047$, and $\frac{1}{r^{35}} = .29997$, and $\frac{1}{r^{36}} = .28983$, and $\frac{1}{r^{37}} = .28003$, and $\frac{1}{r^{38}} = .27056$, and $\frac{1}{r^{39}} = .26141$, and $\frac{1}{r^{40}} = .25257$, and $\frac{1}{r^{41}} = .24403$, and $\frac{1}{r^{42}} = .23577$, and $\frac{1}{r^{43}} = 22780$, and $\frac{1}{r^{44}} = .22010$, and $\frac{1}{r^{45}} = .21265$, and $\frac{1}{r^{46}} = .20546$, and $\frac{1}{r^{47}} = .19851$, and $\frac{1}{r^{48}} = .19180$, and $\frac{1}{r^{49}} = .18532$, and $\frac{1}{r^{50}} = .17905$, and $\frac{1}{r^{51}} = .17299$, and $\frac{1}{r^{52}} = .16714$, and $\frac{1}{r^{53}} = .16149$, and $\frac{1}{r^{54}} = .15603$, and $\frac{1}{r^{55}} = .15075$, and $\frac{1}{r^{56}} = .14566$, and $\frac{1}{r^{57}} = .14073$, and $\frac{1}{r^{58}} = .13597$, and $\frac{1}{r^{59}} = .13137$, and $\frac{1}{r^{60}} = .12693$, and $\frac{1}{r^{61}} = .12264$, and $\frac{1}{r^{62}} = .11849$, and $\frac{1}{r^{63}} = .11448$, and $\frac{1}{r^{64}} = 11061$, and $\frac{1}{r^{65}} = .10687$, and $\frac{1}{r^{66}} = .10326$, and $\frac{1}{r^{67}} = .09976$, and $\frac{1}{r^{68}} = .09639$, and $\frac{1}{r^{69}} = .09313$, and $\frac{1}{r^{70}} = .08998$, and $\frac{1}{r^{71}} = .08694$, and $\frac{1}{r^{72}} = .08400$, and $\frac{1}{r^{73}} = .08116$, and $\frac{1}{r^{74}} = .07841$, and $\frac{1}{r^{75}} = .07576$, and $\frac{1}{r^{76}} = .07320$, and $\frac{1}{r^{77}} = .07072$, and $\frac{1}{r^{78}} = .06833$, and $\frac{1}{r^{79}} = .06602$, and $\frac{1}{r^{80}} = .06379$, and $\frac{1}{r^{81}} = .06163$,

$= .06163$, and $\frac{1}{r^{82}} = .05955$, and $\frac{1}{r^{83}} = .05753$, and $\frac{1}{r^{84}} = .05559$.

Therefore the expreſſion $\frac{\pounds 1}{P}$ x the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4}$

$+ \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7}$ + &c. to $\frac{P^{LXXXIV}}{r^{84}}$ is in this caſe equal to $\frac{\pounds 1}{880}$

x the ſeries $872 \times .96618 + 866 \times .93351 + 860 \times .90194$
$+ 854 \times .87144 + 848 \times .84197 + 842 \times .81350 + 835 \times .78599$
$+ 828 \times .75941 + 821 \times .73373 + 814 \times .70891 + 806 \times .68494$
$+ 798 \times .66178 + 790 \times .63940 + 782 \times .61778 + 774 \times .59689$
$+ 766 \times .57670 + 758 \times .55720 + 750 \times .53836 + 742 \times .52015$
$+ 734 \times .50256 + 726 \times .48557 + 718 \times .46915 + 710 \times .45328$
$+ 702 \times .43795 + 694 \times .42314 + 686 \times .40883 + 678 \times .39501$
$+ 671 \times .38165 + 664 \times .36874 + 657 \times .25627 + 650 \times .34423$
$+ 643 \times .33258 + 636 \times .32134 + 629 \times .31047 + 622 \times .29997$
$+ 615 \times .28983 + 607 \times .28003 + 599 \times .27056 + 590 \times .26141$
$+ 581 \times .25257 + 571 \times .24403 + 560 \times .23577 + 549 \times .22780$
$+ 538 \times .22010 + 526 \times .21265 + 514 \times .20546 + 502 \times .19851$
$+ 489 \times .19180 + 476 \times .18532 + 463 \times .17905 + 450 \times .17299$
$+ 437 \times .16714 + 423 \times .16149 + 409 \times .15603 + 395 \times .15075$
$+ 380 \times .14566 + 364 \times .14073 + 347 \times .13597 + 329 \times .13137$
$+ 310 \times .12693 + 291 \times .12264 + 271 \times .11849 + 251 \times .11448$
$+ 231 \times .11061 + 211 \times .10687 + 192 \times .10326 + 173 \times .09976$
$+ 154 \times .09639 + 136 \times .09313 + 118 \times .08998 + 101 \times .08694$
$+ 85 \times .08400 + 71 \times .08116 + 59 \times .07841 + 48 \times .07576$
$+ 38 \times .07320 + 29 \times .07072 + 22 \times .06833 + 16 \times .06602$
$+ 11 \times .06379 + 7 \times .06163 + 4 \times .05955 + 2 \times .05753$
$+ 1 \times .05559 = \frac{\pounds 1}{880}$ x the ſeries $842.50896 + 808.41966$
$+ 775.66840 + 744.20976 + 713.99056 + 684.96700$
$+ 656.30165 + 628.79148 + 602.39233 + 577.05274$
$+ 552.06164 + 528.10044 + 505.12600 + 483.10396$
$+ 461.99286 + 441.75220 + 422.35760 + 403.77000$
$+ 385.95130 + 368.87904 + 352.52382 + 336.84970$
$+ 321.82880$

+ 321.82880	+ 307.44090	+ 293.65916	+ 280.45738
+ 267.81678	+ 256.08715	+ 244.84336	+ 234.06939
+ 223.74950	+ 213.84894	+ 204.37224	+ 195.28563
+ 186.58134	+ 178.24545	+ 169.97821	+ 162.06544
+ 154.23190	+ 146.74317	+ 139.34113	+ 132.03120
+ 125.06220	+ 118.41380	+ 111.85390	+ 105.60644
+ 99.65202	+ 93.79020	+ 88.21232	+ 82.90015
+ 77.84550	+ 73.04018	+ 68.31027	+ 63.81627
+ 59.54625	+ 55.35080	+ 51.22572	+ 47.18159
+ 43.22073	+ 39.34830	+ 35.68824	+ 32.11079
+ 28.73448	+ 25.55091	+ 22.54957	+ 19.82592
+ 17.25848	+ 14.84406	+ 12.66568	+ 10.61764
+ 8.78094	+ 7.14000	+ 5.76236	+ 4.62619
+ 3.63628	+ 2.78160	+ 2.05088	+ 1.50326
+ 1.05632	+ .70169	+ .43141	+ .23820
+ .11506	+ .05559	= $\frac{£1}{880}$	× 18250.54036 = £1 ×

$\frac{18250.54036}{880}$ = £1 × 20.73925 = £20.73925, or 20*l.* 14*s.* 9*d.* $\frac{1}{4}$.

Therefore the value of an annuity of one pound *per annum* for the life of a perſon of the age of 10 years, when the intereſt of money is $3\frac{1}{2}$ per cent. is, according to Monſieur de Parcieux's table of probabilities, 20*l.* 14*s.* 9*d.* $\frac{1}{4}$. QE I.

XCV. The addition of the terms of the foregoing ſeries may be performed as follows.

842.50896	552.06164	352.52382	223.74950	139.34113
808.41966	528.10044	336.84970	213.84894	132.03120
775.66840	505.12600	321.82880	204.37224	125.06220
744.20976	483.10396	307.44090	195.28563	118.41380
713.99056	461.99286	293.65916	186.58134	111.85390
684.96700	441.75220	280.45738	178.24545	105.60644
656.30165	422.35760	267.81678	169.97821	99.65202
628.79143	403.77000	256.08715	162.06544	93.79020
602.39233	385.95130	244.84336	154.23190	88.21232
577.05274	368.87904	234.06939	146.74317	82.90015
7054.30254	4553.09504	2895.57644	1835.10182	1096.86336

77.84550	35.68824	8.78094	7034.30254
73.04018	32.11079	7.14000	4553.09504
68.31027	28.73448	5.76236	2895.57644
63.81627	25.55091	4.62619	1835.10182
59.54625	22.54957	3.63628	1096.86336
53.35080	19.82592	2.78160	576.88561
51.22572	17.25848	2.05088	219.84577
47.18159	14.84406	1.50326	38.86978
43.22073	12.66368	1.05632	18250.54036
39.34830	10.61764	.70169	
576.88561	219.84577	.43141	
		.23820	
		.11506	
		.05559	
		38.86978	

XCVI. It appears from the foregoing calculation that, when the intereſt of money is $3\frac{1}{2}$ per cent. an annuity for the life of a perſon of 10 years of age is worth near 21 years purchaſe: and, if the annuity, or rather in this caſe we ſhould ſay, the *penſion*, was to be paid every half-year, inſtead of every year, ſo that the firſt payment of it ſhould be made at the end of half a year after the time of granting it, or when the grantee is ten years and a half old, the value of it would be greater than before by almoſt one half a year's payment, and conſequently would be worth more than 21 years purchaſe. And, if we were to make the like calculation upon a ſuppoſition that the intereſt of money was only 3 per cent. we ſhould find that ſuch an annuity would be worth about 23 years purchaſe. But theſe are greater prices than, I believe, any life-annuities have ever been ſold for.

The real values of annuities for lives are greater than they are generally thought to be.

XCVII. It appears from the operations in the foregoing calculation, that, when the intereſt of money is $3\frac{1}{2}$ per cent. the value of an annuity of one pound a year for 30 years depending on the life of a perſon of 10 years of age, or that ſhall continue for 30 years in caſe the ſaid perſon ſhall ſo long live, but ſhall ceaſe upon his death, is, according to Monſieur de Parcieux's table of probabilities of the duration of human life, equal to

Of the value of an annuity for 30 years depending on the life of a perſon of the age of 10 years.

$$\frac{£1}{880} \times \text{the ſeries } 7034.30254 + 4553.09504 + 2895.57644 =$$

$$\frac{£1}{880} \times 14482.97402 = £1 \times \frac{14482.97402}{880} = £1 \times 16.45792 =$$

£16.45792,

Of the value of a remote annuity for the life of a perſon of the age of 10 years, to commence at the end of 30 years.

£16.45792, or 16*l.* 9*s.* 2*d.* And it appears likewiſe that the value of a remote annuity of one pound a year for the life of a perſon of ten years of age, to commence at the end of 30 years, (or whereof the firſt payment ſhall be made at the end of 31 years, or when the ſaid perſon ſhall be compleatly 41 years of age,) and to continue during the whole remainder of his life, will, (upon the ſame ſuppoſitions of the intereſt of money and the probabilities of the duration of human life,) be equal to

$$\frac{\text{£}1}{880} \times \text{the ſeries } 1835.10182 + 1096.86336 + 576.88561 + 219.84577 + 38.86978 = \frac{\text{£}1}{880} \times 3768.46634 = \text{£}1 \times \frac{3768.46634}{880} = \text{£}1 \times 4.28234 = \text{£}4.28234,$$

or 4*l.* 5*s.* 7*d.* $\frac{3}{4}$. Therefore any life-annuity for the life of a perſon of 10 years of age, to commence at the diſtance of 30 years, is (upon the ſuppoſitions here made,) worth about four years and a quarter's purchaſe.

Of the different intereſts that might be made of money by veſting it in the publick funds in the years 1773 and 1780.

XCVIII. The intereſt that might be made of money by veſting it in the publick funds in January, 1773, was ſomething leſs than $3\frac{1}{2}$ per cent. For 100 pounds ſtock in the conſolidated 3 per cent. annuities might then be bought for 87 pounds; and, when it is worth 85*l.* 14*s.* 3*d.*$\frac{1}{2}$, or £85 7142, or, when £85.7142 will purchaſe a perpetual annuity of 3 pounds *per annum*, £100 will purchaſe a perpetual annuity of $\frac{\text{£}3 \times 100}{85.7142}$, or $3\frac{1}{2}$ pounds, *per annum*. Therefore, if the Government had then wanted to borrow a moderate ſum of money of the people, (as, for inſtance, a million or two of pounds ſterling,) they could, probably, have borrowed it at the rate of $3\frac{1}{2}$ per cent. This therefore was at that time the moſt uſeful rate of intereſt, and the fitteſt to be attended to and made the ground of calculations of the values of annuities either for terms of years or for lives. But now, in October, 1780, £100 ſtock in the 3 per cent. conſolidated annuities, or a perpetual annuity of 3 pounds *per annum*, is worth only about 60 pounds; and conſequently £100, employed in the purchaſe of thoſe annuities, would purchaſe a perpetual annuity of 5 pounds a year. Therefore 5 per cent. or, perhaps, $5\frac{1}{2}$, or even 6, per cent. is now the rate of intereſt which moſt deſerves to be conſidered in the eſtimation of the value of annuities, and to be made the foundation of our calculations concerning them. In order therefore to make proper eſtimates of the values of annuities in different periods of national wealth and proſperity, it will be moſt convenient to have tables of the values of annuities adapted

Expedience of having ſeveral different tables of life-annuities adapted to different rates of intereſt.

to

to ſeveral different rates of intereſt; as we have in Mr. Smart's excellent book above-mentioned for the values of annuities for terms of years at the twelve different rates of intereſt mentioned above in Art. 24, page 23, to wit, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, 8, 9, and 10, per cent. And accordingly I ſhall inſert in the following part of this work the like tables for annuities for ſingle lives of all ages from 3 years to 93 years, at the ſame twelve different rates of intereſt and according to Monſieur de Parcieux's table of the probabilities of the duration of human life; which have all been faithfully calculated, under the inſpection and direction of the learned and worthy Dr. Price above-mentioned, upon the principles that have been above explained.

XCIX. In peruſing the foregoing calculation of the value of an annuity for the life of a perſon of 10 years of age, in Art. 94, the reader will, I doubt not, have entertained a high idea of the difficulty of compoſing a whole table of the values of life-annuities for every year of human life as far as 93 years, if the ſame method is to be taken for the computation of every one of thoſe values as is there purſued for that of an annuity for a life of the age of ten years, He will therefore, probably, be glad to be informed that the repetition of this laborious proceſs for every year of human life is not neceſſary, but that the value of an annuity for a life of any age may be deduced from the value of an annuity for a life that is one year older by a very ſhort and eaſy proceſs, inſomuch that a whole table of the values of life-annuities for every year of human life may be computed with very little more labour than has been employed above, in Art. 94, in computing the value of only that one annuity for the life of a perſon of ten years of age. The method of doing this was communicated to me by the learned Dr. Price above-mentioned, and may be explained as follows.

A ſhort and eaſy method of deducing from the value of an annuity of one pound for any given life the value of a like annuity for a life one year younger than the former.

C. We have ſeen in Art. 94 that, if the intereſt of money be $3\frac{1}{2}$ per cent. and the probabilities of the duration of human life ſuch as they are repreſented in Monſieur de Parcieux's table, the value of an annuity of one pound for the life of a perſon of 10 years of age is £20.73925, or 20*l.* 14*s.* 9*d.* $\frac{1}{4}$. Now from this value we may derive that of a like annuity of one pound for the life of a perſon of the age of 9 years in the following manner.

An explanation of the ſaid method in the caſe of a particular example, or age of human life.

CI. In

CI. In Monſieur de Parcieux's table of probabilities there are 890 perſons repreſented to be living at the age of 9 years. Now let us ſuppoſe that a man were to make 890 grants of annuities, of one pound a year each, to all theſe 890 perſons of the age of 9 years, for their reſpective lives. And let the payments which the ſaid grantor would thereby oblige himſelf to make to the ſaid grantees be divided into the two following parts, or claſſes, to wit, firſt, the payments he is to make at the end of the firſt year to ſuch of the 890 grantees as ſhall be then alive, and, ſecondly, all the other payments which he will be obliged to make to ſuch of them as ſhall ſurvive beyond the ſaid firſt year, and which will become due at the ends of the ſecond, third, fourth, fifth, and other following years reſpectively, till all the ſaid grantees ſhall be dead. And let us inveſtigate ſeparately the prices he ought to receive from the ſaid grantees for theſe two ſets, or claſſes, of payments which he thus binds himſelf to make.

Now it appears from Monſieur de Parcieux's table that of theſe 890 grantees, all of the age of 9 years, 880 will live to the end of one year, or till they are 10 years old. Therefore at the end of the ſaid firſt year the ſaid grantor will have 880 payments, of one pound each, to make to the ſaid 880 ſurviving grantees. He ought therefore to receive, as the fair price of thoſe payments, 880 times the preſent value of one pound to be received at the end of a year, or 880 times $\frac{1}{r}$ of a pound, or $880 \times \frac{1}{r}$, or $\frac{1}{r} \times 880 \times £1$.

Secondly, it is evident that the ſubſequent payments which the ſaid grantor will have to make, at the ends of the ſecond, third, fourth, fifth, and other following years, to ſuch of the ſaid 880 grantees (who are living at the end of the firſt year,) as ſhall further ſurvive to the ends of the ſaid ſecond, third, fourth, fifth, and other following years, will be preciſely the ſame as he would have had to make to them at the ſame times reſpectively if he had poſt-poned making any of theſe grants for the ſpace of a year, or till the ſaid 880 perſons were 10 years old, and had then made them 880 grants, of one pound a year each, for their reſpective lives. Thus, for example, the payment he will have to make at the end of the ſecond year, or when the grantees are 11 years old, will be in both caſes £872, becauſe there will be 872 of the ſaid grantees then alive; and the payment he will have to make at the end of the third year, or when the grantees are 12 years old, will be in both caſes £866, becauſe there will be 866 of the ſaid grantees alive at that time; and the ſame is true of the payments at the ends of all the following years, to wit, that they will be the ſame in both caſes. If therefore the prices of theſe ſeveral payments were to be paid when the ſaid grantees were 10 years old, they muſt be preciſely the ſame as if the grants had been made when the grantees were 10 years old; and

and conſequently the ſum total of all theſe prices would be preciſely equal to the value of 880 life-annuities of one pound a year granted to 880 perſons of the age of 10 years, that is, (as appears by Art. 94) to 880 times £20.73925. But the prices of theſe ſeveral payments of £872, £866, &c. are ſuppoſed in the preſent caſe to be paid to the grantor when the grantees are only 9 years old, or a year ſooner than on the laſt ſuppoſition. Therefore (by Problem 1, Art XXIII, page 21,) they muſt be leſs than thoſe other prices of the ſame payments, reſpectively, in the proportion of $\frac{1}{r}$ to 1. Therefore the ſum total of all the ſaid prices muſt be leſs than the ſum total of all thoſe other prices of the ſame payments in the ſame proportion of $\frac{1}{r}$ to 1, and conſequently muſt be equal to $\frac{1}{r} \times 880 \times £20.73925$; that is, the price to be paid to the grantor by all the 890 grantees of the age of 9 years, on account of all the ſaid future payments of the ſecond claſs, which are to be made at the ends of the ſecond, third, fourth, fifth, and other following years from the time of making the grants, is $\frac{1}{r} \times 880 \times £20.73925$.

But it was before ſhewn that the price to be paid to the ſaid grantor by the ſaid 890 grantees on account of the payments to be made to them at the end of the firſt year, is $\frac{1}{r} \times 880 \times £1$.

Therefore the price to be paid to the ſaid grantor by the ſaid 890 grantees on both the ſaid accounts, or for the whole life-annuities of one pound that are thus granted to them at the age of 9 years, is $\frac{1}{r} \times 880 \times £1 + \frac{1}{r} \times 880 \times £20.73925$. And conſequently the price that ought to be paid by each of the ſaid 890 grantees for his annuity is the 890th part of the ſaid ſum, or $\frac{1}{r} \times \frac{880}{890} \times £1 + \frac{1}{r} \times \frac{880}{890} \times £20.73925$, or $\frac{1}{r} \times \frac{880}{890} \times \overline{|£1 + £20.73925|}$. Therefore by Art. 21, if only one ſuch grant is made to a perſon of the age of 9 years, the price of the ſaid ſingle annuity ought alſo to be $\frac{1}{r} \times \frac{880}{890} \times \overline{|£1 + £20.73925|}$; that is, (becauſe $\frac{1}{r}$ is in this caſe $= \frac{1}{1.035}$,) the price of the ſaid annuity will be $\frac{1}{1.035} \times \frac{880}{890} \times \overline{|1£ + 20.73925£}$ $(= \frac{1}{1.035} \times \frac{880}{890} £21.73925 = \frac{1}{1.035} \times £21.494988) = £20.76810$, or 20*l.* 15*s.* 4*d.* $\frac{1}{4}$. Q E I.

CII. Thus

CII. Thus it appears that from the value of an annuity of one pound for a life of the age of 10 years we may, by very eaſy arithmetical operations, deduce that of a like annuity for a life of the age of 9 years, to wit, by only adding £1 to the former value, and then multiplying the ſum thence ariſing, firſt, into the fraction $\frac{880}{890}$, (of which the numerator expreſſes the number of perſons repreſented in Monſieur de Parcieux's table of probabilities to be living at the age of 10 years, and the denominator expreſſes the number of perſons that are therein repreſented to be living at the age of 9 years,) and, ſecondly, into the fraction $\frac{1}{r}$, which bears the ſame proportion to 1 as the preſent value of one pound to be received at the end of a year bears to one pound. And, as the reaſonings employed in making this deduction are not peculiar to the ages of 9 and 10 years, but may be applied to any other two ages of human life that differ from each other only by a year, we may ſafely conclude in general terms, that, if $\overset{£}{V}$ denotes the value of an annuity of one pound a year for the life of a perſon of the age of N years, and P is the number of perſons repreſented in Monſieur de Parcieux's table of the probabilities of the duration of human life, (or in ſuch other table of thoſe probabilities as is adopted as the ground of the calculation,) as living at the age of N years, and $P+d$ is the number of perſons repreſented there as living at the age of $N-1$ years, the value of an annuity of one pound a year for the life of a perſon of the age of $N-1$ years will be equal to $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|£$. QEI.

An extenſion of the ſaid method to all other ages of human life, and a general expreſſion of the value of an annuity of one pound for the younger of the two lives.

Other examples of the foregoing method of deducing the value of an annuity for any propoſed life from that of a like annuity for a life that is one year older.

CIII. Let it be ſuppoſed that the value of an annuity of one pound a year for the life of a perſon of 9 years of age is already known, and that it is (as we have juſt now found it to be by the computation of it made above in Art. 101,) equal to £20.76810; the intereſt of money being $3\frac{1}{2}$ per cent. and the probabilities of the duration of human life ſuch as they are repreſented in Monſieur de Parcieux's table. And let it be required to deduce from this value of an annuity of one pound for a life of a perſon of the age of 9 years the value of a like annuity for the life of a perſon of the age of 8 years, in the method above explained, or by means of the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|£$.

Here

Here P is $= 890$, and $P+d$ is $= 902$, and $\overset{\pounds}{V}$ is $= £20.76810$, and consequently $\overline{1+V}|\pounds$ is $= £21.76810$; and $\frac{1}{r}$ is (as before) $= \frac{1}{1.035}$. Therefore $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|\pounds$ is $= \frac{1}{1.035} \times \frac{890}{902} \times £21.76810 = \frac{1}{1.035} \times £21.47850 = £20.75217$. Consequently the value of an annuity of one pound for the life of a perſon of the age of 8 years is $£20.75217$, or 20*l.* 15*s.* $\frac{1}{2}$*d.* QEI.

CIV. In like manner by putting $\overset{\pounds}{V} = £20.75217$, and $P = 902$, and $P+d = 915$, and computing the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|\pounds$, or $\frac{1}{1.035} \times \frac{902}{915} \times £21.75217$, we ſhall find the value of an annuity of one pound for a life of 7 years to be equal to $\left(\frac{1}{1.035} \times £21.44312, \text{ or}\right)$ $£20.71799$, or 20*l.* 14*s.* 4*d.* $\frac{1}{4}$.

CV. And in like manner an annuity of one pound for a life of the age of 6 years will be found to be worth $\frac{1}{1.035} \times \frac{915}{930} \times £21.71799$ $\left(= \frac{1}{1.035} \times £21.367,699,8\right) = £20.64512$, or 20*l.* 12*s.* 10*d.* $\frac{3}{4}$.

And an annuity of one pound for a life of the age of 5 years will be worth $\frac{1}{1.035} \times \frac{930}{948} \times £21.64512 \left(= \frac{1}{1.035} \times £21.23413\right) = £20.51606$, or 20*l.* 10*s.* 3*d.* $\frac{3}{4}$.

And an annuity of one pound for a life of the age of 4 years will be worth $\frac{1}{1.035} \times \frac{948}{970} \times £21.51606 \left(= \frac{1}{1.035} \times £21.028,06\right) =$ $£20.31696$, or 20*l.* 6*s.* 4*d.*

And an annuity of one pound for a life of the age of 3 years will be worth $\frac{1}{1.035} \times \frac{970}{1000} \times £21.31696$ $(= \frac{1}{1.035} \times £20.67745) = £19.97821$, or 19*l.* 19*s.* 6*d.* $\frac{3}{4}$.

CVI. Therefore, if no miſtakes have been made in the foregoing calculations, we may conclude that the values of an annuity of one pound for the lives of children of 3, 4, 5, 6. 7, 8, 9, and 10 years of age, when the intereſt of money is $3\frac{1}{2}$ per cent. are (according to Monſieur de Parcieux's table of the probabilities of the duration of human life,) reſpectively, equal to the following ſums of money, to wit,

	£ 19.97821,	or	£ 19.	19*s.*	6*d.* $\frac{3}{4}$.
And	£ 20.31696,	or	£ 20.	6*s.*	4*d.*
And	£ 20.51606,	or	£ 20.	10*s.*	3*d.* $\frac{3}{4}$.
And	£ 20.64512,	or	£ 20.	12*s.*	10*d.* $\frac{3}{4}$.
And	£ 20.71799,	or	£ 20.	14*s.*	4*d.* $\frac{1}{4}$.
And	£ 20.75217,	or	£ 20.	15*s.*	$\frac{1}{2}$*d.*
And	£ 20.76810,	or	£ 20.	15*s.*	4*d.* $\frac{1}{2}$.
And	£ 20.73925,	or	£ 20.	14*s.*	9*d.* $\frac{1}{4}$.

CVII. All the other values of a compleat table of life-annuities grounded upon the foregoing ſuppoſitions that the intereſt of money is $3\frac{1}{2}$ per cent. and that the life of man decays in the manner repreſented in Monſieur de Parcieux's table of probabilities,) may be computed in the ſame manner, one from another, by beginning with the oldeſt life in the table and proceeding downwards to the next younger life till we have obtained the value of an annuity of one pound for a life of 10 years of age, which we have already computed in Art. 94, and found to be £20.73925. And the trouble of computing the values of the annuities of one pound belonging to all theſe different ages will not be very much greater than that of computing, by the method uſed in Art. 94, the value of only that one annuity for the life of a perſon of the age of 10 years, which is computed in that article. The operations neceſſary for this purpoſe are as follows.

A com-

A computation of the values of an annuity of one pound for every different year of human life from the age of 93 years to the age of 10 years, inclusively; upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. and that the probabilities of the duration of human life are such as they are represented to be in Monsieur de Parcieux's table.

CVIII. It appears by Monsieur de Parcieux's table that of two persons aged 93 years, living at the same time, only one will live another year, or to the age of 94, and that the said survivor at 94 years of age will die before he is 95. If therefore a man were to grant to 2 old persons of the age of 93 years annuities of one pound a year each for their lives, he would, at the end of the first year, have only one payment of one of the said annuities to make, to wit, that which would be then due to the survivor of the said two grantees; and he would never after be obliged to make another payment of either of the annuities, because both the said grantees would be dead before the end of the second year, or before the second payment would become due. To the end therefore that the grantor of the said annuities might be neither a gainer nor a loser by granting them, it would be necessary that he should receive from both the said grantees, at the time of making the said grants, the present value of the said only payment of one pound which he would have to make to the survivor of the said two grantees at the end of the first year; which present value is the sum of $\frac{\pounds 1}{r}$ of a pound, or $\frac{\pounds 1}{r}$. And consequently he ought to receive from each of the said two grantees one half of that sum, or $\frac{1}{2} \times \frac{\pounds 1}{r}$, or (because $\frac{\pounds 1}{r}$ is in this case $= \frac{\pounds 1}{1.035}$, the interest of money being $3\frac{1}{2}$ per cent.) $\frac{1}{2} \times \frac{\pounds 1}{1.035}$, or $\frac{1}{2} \times \pounds.96618$, or $\pounds.48309$. Therefore, by Art. 21, if the grantor were to make only one such grant of an annuity of one pound a year to a person of the age of 93 years for his life, he ought to receive, as the price of such single grant, the same sum of $\pounds.48309$; or, in other words, the value of an annuity of one pound a year for a life of 93 years is $\pounds.48309$, or 9*s.* 8*d.* QED.

CIX. Having thus found the value of an annuity of one pound a year for a life of 93 years, the values of the like annuities for all younger lives may be found by the continual application of the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$, or $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$, in the manner following.

CX. To find the value of an annuity of one pound for a life of 92 years, we muſt put $\overset{\mathcal{L}}{V} = \mathcal{L}.48309$, and $P = 2$, and $P+d = 4$; becauſe there are 2 perſons living, according to Monſ. de Parcieux's table, at the age of 93 years, and 4 perſons living at the age of 92 years. And we ſhall have $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L} = \frac{1}{1.035} \times \frac{2}{4} \times \overline{1+.48309}|\mathcal{L} = \frac{1}{1.035} \times \frac{2}{4} \times \mathcal{L}1.48309 = \frac{1}{1.035} \times \mathcal{L}.741,545 = \mathcal{L}.716,468$. Therefore the value of an annuity of one pound for a life of 92 years is $\mathcal{L}$ 716,468, or 14*s.* 4*d.* QEI.

CXI. To find the value of an annuity of one pound for a life of 91 years, we muſt put $\overset{\mathcal{L}}{V} = \mathcal{L}.716,468$, and conſequently $\overline{1+V}|\mathcal{L} = \mathcal{L}1.716,468$. And, by the table of probabilities, the numbers of perſons living at the ages of 91 and 92 years are 7 and 4; that is, P is $= 4$, and $P+d$ is $= 7$. Therefore $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$ is $= \frac{1}{1.035} \times \frac{4}{7} \times \mathcal{L}1.716,468 = \frac{1}{1.035} \times \mathcal{L}\ 980,838 = .947,669$. Conſequently the value of an annuity of one pound for the life of a perſon of the age of 91 years is .947,669, or 18*s.* 11*d.* $\frac{1}{2}$.

CXII. Again, put $\overset{\mathcal{L}}{V} = \mathcal{L}.947,669$, and conſequently $\overline{1+V}|\mathcal{L} = \mathcal{L}1.947,669$; and let P be $= 7$, and $P+d = 11$, which are the numbers of perſons repreſented in the table as living at the ages of 91 and 90 years. And we ſhall have $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L} = \frac{1}{1.035} \times \frac{7}{11} \times \mathcal{L}1.947,669 = \frac{1}{1.035} \times \mathcal{L}1.239,425 = \mathcal{L}1.197,512$. Therefore the value of an annuity of one pound for a life of the age of 90 years is $= \mathcal{L}1.197,512$, or 1*l.* 3*s.* 11*d.* $\frac{1}{2}$.

CXIII. And

CXIII. And in the ſame manner we ſhall find that the value of an annuity of one pound for a life of the age of 89 years is $\left(= \frac{1}{1.035} \times \frac{11}{16} \times \overline{1+1.197512}|\mathcal{L} = \frac{1}{1.035} \times \frac{11}{16} \times \mathcal{L}2.197,512 = \frac{1}{1.035} \times \mathcal{L}1.510,789 =\right)$ $\mathcal{L}1.459,699$, or 1*l.* 9*s.* 2*d.* $\frac{1}{4}$.

And that the value of the like annuity for a life of the age of 88 years is $\left(= \frac{1}{1.035} \times \frac{16}{22} \times \mathcal{L}2.459,699 = \frac{1}{1.035} \times \mathcal{L}1.788,872 =\right) \mathcal{L}1.728,378$, or 1*l.* 14*s.* 6*d.* $\frac{3}{4}$.

And that the value of the like annuity for a life of 87 years is $\left(= \frac{1}{1.035} \times \frac{22}{29} \times \mathcal{L}2.728,378 = \frac{1}{1.035} \times \mathcal{L}2.069,804 =\right) \mathcal{L}1.999,810$, or 2*l.* 0*s.* 0*d.*

And that the value of the like annuity for a life of 86 years is $\left(= \frac{1}{1.035} \times \frac{29}{38} \times \mathcal{L}2.999,810 = \frac{1}{1.035} \times \mathcal{L}2.289,328 =\right) \mathcal{L}2.211,911$, or 2*l.* 4*s.* 2*d.* $\frac{3}{4}$.

And that the value of the like annuity for a life of 85 years is $\left(= \frac{1}{1.035} \times \frac{38}{48} \times \mathcal{L}3.211,911 = \frac{1}{1.035} \times \mathcal{L}2.542,762 =\right) \mathcal{L}2.456,774$, or 2*l.* 9*s.* 1*d.* $\frac{1}{2}$.

And that the value of the like annuity for a life of 84 years is $\left(= \frac{1}{1.035} \times \frac{48}{59} \times \mathcal{L}3.456,774 = \frac{1}{1.035} \times \mathcal{L}2.812,290 =\right) \mathcal{L}2.717,188$, or 2*l.* 14*s.* 4*d.*

And that the value of the like annuity for a life of 83 years is $\left(= \frac{1}{1.035} \times \frac{59}{71} \times \mathcal{L}3.717,188 = \frac{1}{1.035} \times \mathcal{L}3.088,930 =\right) \mathcal{L}2.984,473$, or 2*l.* 19*s.* 8*d.* $\frac{1}{4}$.

And

And that the value of the like annuity for a life of 82 years is $\left(= \frac{1}{1.035} \times \frac{71}{85} \times £3.984,473 = \frac{1}{1.035} \times £3.328,206 = \right) £3.215,658$, or 3*l.* 4*s.* 3*d.* $\frac{1}{4}$.

And that for a life of 81 years it is $\left(= \frac{1}{1.035} \times \frac{85}{101} \times £4.215,658 = \frac{1}{1.035} \times £3.547,831 = \right) £3.427,856$, or 3*l.* 8*s.* 6*d.* $\frac{1}{4}$.

And that for a life of 80 years it is $\left(= \frac{1}{1.035} \times \frac{101}{118} \times £4.427,856 = \frac{1}{1.035} \times £3.789,944 = \right) £3.661,781$, or 3*l.* 13*s.* 2*d.* $\frac{1}{4}$.

CXIV. And for a life of 79 years it is $\left(= \frac{1}{1.035} \times \frac{118}{136} \times £4.661,781 = \frac{1}{1.035} \times £4.044,780 = \right) £3.908,000$, or 3*l.* 18*s.* 2*d.*

And for a life of 78 years it is $\left(= \frac{1}{1.035} \times \frac{136}{154} \times £4.908,000 = \frac{1}{1.035} \times £4.334,337 = \right) £4.187,758$, or 4*l.* 3*s.* 9*d.*

And for a life of 77 years it is $\left(= \frac{1}{1.035} \times \frac{154}{173} \times £5.187,758 = \frac{1}{1.035} \times £4.618,004 = \right) £4.461,839$, or 4*l.* 9*s.* 2*d.* $\frac{3}{4}$.

And for a life of 76 years it is $\left(= \frac{1}{1.035} \times \frac{173}{192} \times £5.461,839 = \frac{1}{1.035} \times £4.921,344 = \right) £4.754,921$, or 4*l.* 15*s.* 1*d.* $\frac{1}{4}$.

And

And for a life of 75 years it is $\left(= \frac{1}{1.035} \times \frac{192}{211} \times £5.754,921 = \frac{1}{1.035} \times £5.236,705 =\right)$ £5.059,623, or 5*l.* 1*s.* 2*d.* $\frac{1}{4}$.

And for a life of 74 years it is $\left(= \frac{1}{1.035} \times \frac{211}{231} \times £6.059,623 = \frac{1}{1.035} \times £5.534,980 =\right)$ £5.347,806, or 5*l.* 6*s.* 11*d.* $\frac{1}{2}$.

And for a life of 73 years it is $\left(= \frac{1}{1.035} \times \frac{231}{251} \times £6.347,806 = \frac{1}{1.035} \times £5.842,004 =\right)$ £5.644,448, or 5*l.* 12*s.* 10*d.* $\frac{1}{2}$.

And for a life of 72 years it is $\left(= \frac{1}{1.035} \times \frac{251}{271} \times £6.644,448 = \frac{1}{1.035} \times £6.154,082 =\right)$ £5.945,972, or 5*l.* 18*s.* 11*d.*

And for a life of 71 years it is $\left(= \frac{1}{1.035} \times \frac{271}{291} \times £6.945,972 = \frac{1}{1.035} \times £6.468,585 =\right)$ £6.249,840, or 6*l.* 5*s.* 0*d.*

And for a life of 70 years it is $\left(= \frac{1}{1.035} \times \frac{291}{310} \times £7.249,840 = \frac{1}{1.035} \times £6.805,495 =\right)$ £6.575,357, or 6*l.* 11*s.* 6*d.*

And for a life of 69 years it is $\left(= \frac{1}{1.035} \times \frac{310}{329} \times £7.575,357 = \frac{1}{1.035} \times £7.137,874 =\right)$ £6.896,496, or 6*l.* 17*s.* 11*d.* $\frac{1}{4}$.

And

And for a life of 68 years it is $\left(= \frac{1}{1.035} \times \frac{329}{347} \times £7.896,496 = \frac{1}{1.035} \times £7.486,879 =\right)$ £7.233,699, or 7*l.* 4*s.* 8*d.*

And for a life of 67 years it is $\left(= \frac{1}{1.035} \times \frac{347}{364} \times £8.233,699 = \frac{1}{1.035} \times £7.849,158 =\right)$ £7.583,727, or 7*l.* 11*s.* 8*d.*

And for a life of 66 years it is $\left(= \frac{1}{1.035} \times \frac{364}{380} \times £8.583,727 = \frac{1}{1.035} \times £8.222,307 =\right)$ £7.944,258, or 7*l.* 18*s.* 10*d.* $\frac{1}{2}$.

And for a life of 65 years it is $\left(= \frac{1}{1.035} \times \frac{380}{395} \times £8.944,258 = \frac{1}{1.035} \times £8.604,602 =\right)$ £8.313,625, or 8*l.* 6*s.* 3*d.* $\frac{1}{4}$.

And for a life of 64 years it is $\left(= \frac{1}{1.035} \times \frac{395}{409} \times £9.313,625 = \frac{1}{1.035} \times £8.994,821 =\right)$ £8.690,648, or 8*l.* 13*s.* 9*d.* $\frac{3}{4}$.

And for a life of 63 years it is $\left(= \frac{1}{1.035} \times \frac{409}{423} \times £9.690,648 = \frac{1}{1.035} \times £9.369,917 =\right)$ £9.053,059, or 9*l.* 1*s.* 0*d.* $\frac{3}{4}$.

And for a life of 62 years it is $\left(= \frac{1}{1.035} \times \frac{423}{437} \times £10.053,059 = \frac{1}{1.035} \times £9.730,993 =\right)$ £9.401,925, or 9*l.* 8*s.* 0*d.* $\frac{1}{2}$.

And

And for a life of 61 years it is $\left(= \frac{1}{1.035} \times \frac{437}{450} \times £10.401,925 = \frac{1}{1.035} \times £10.101,424 =\right)$ £9.759,829, or 9*l.* 15*s.* 2*d.* $\frac{1}{4}$.

And for a life of 60 years it is $\left(= \frac{1}{1.035} \times \frac{450}{463} \times £10.759,829 = \frac{1}{1.035} \times £10.457,717 =\right)$ £10.104,074, or 10*l.* 2*s.* 1*d.*

And for a life of 59 years it is $\left(= \frac{1}{1.035} \times \frac{463}{476} \times £11.104,074 = \frac{1}{1.035} \times £10.800,811 =\right)$ £10.435,566, or 10*l.* 8*s.* 8*d.* $\frac{1}{2}$.

And for a life of 58 years it is $\left(= \frac{1}{1.035} \times \frac{476}{489} \times £11.435,566 = \frac{1}{1.035} \times £11.131,553 =\right)$ £10.755,123, or 10*l.* 15*s.* 1*d.* $\frac{1}{4}$.

And for a life of 57 years it is $\left(= \frac{1}{1.035} \times \frac{489}{502} \times £11.755,123 = \frac{1}{1.035} \times £11.450,707 =\right)$ £11.063,485, or 11*l.* 1*s.* 3*d.* $\frac{1}{4}$.

And for a life of 56 years it is $\left(= \frac{1}{1.035} \times \frac{502}{514} \times £12.063,485 = \frac{1}{1.035} \times £11.781,847 =\right)$ £11.383,427, or 11*l.* 7*s.* 8*d.*

And for a life of 55 years it is $\left(= \frac{1}{1.035} \times \frac{514}{526} \times £12.383,427 = \frac{1}{1.035} \times £12.100,915 =\right)$ £11.691,801, or 11*l.* 13*s.* 10*d.*

And for a life of 54 years it is $\left(= \frac{1}{1.035} \times \frac{526}{538} \times £12.691,801 = \frac{1}{1.035} \times £12.408,712 =\right)$ £11.989,093, or 11*l.* 19*s.* 9*d.* $\frac{1}{4}$.

And for a life of 53 years it is $\left(= \frac{1}{1.035} \times \frac{538}{549} \times £12.989,093 = \frac{1}{1.035} \times £12.728,838 =\right)$ £12.298,386, or 12*l.* 5*s.* 11*d.* $\frac{1}{4}$.

And for a life of 52 years it is $\left(= \frac{1}{1.035} \times \frac{549}{560} \times £13.298,386 = \frac{1}{1.035} \times £13.037,167 =\right)$ £12.596,296, or 12*l.* 11*s.* 11*d.*

And for a life of 51 years it is $\left(= \frac{1}{1.035} \times \frac{560}{571} \times £13.596,296 = \frac{1}{1.035} \times £13.334,370 =\right)$ £12.883,449, or 12*l.* 17*s.* 8*d.*

And for a life of 50 years it is $\left(= \frac{1}{1.035} \times \frac{571}{581} \times £13.883,449 = \frac{1}{1.035} \times £13.644,491 =\right)$ £13.183,083, or 13*l.* 3*s.* 8*d.*

And for a life of 49 years it is $\left(= \frac{1}{1.035} \times \frac{581}{590} \times £14.183,083 = \frac{1}{1.035} \times £13.966,730 =\right)$ £13.494,425, or 13*l.* 9*s.* 10*d.* $\frac{1}{2}$.

And for a life of 48 years it is $\left(= \frac{1}{1.035} \times \frac{590}{599} \times £14.494,425 = \frac{1}{1.035} \times £14.276,645 =\right)$ £13.793,859, or 13*l.* 15*s.* 10*d.* $\frac{1}{2}$.

And

And for a life of 47 years it is $\left(=\frac{1}{1.035}\times\frac{599}{607}\times £14.793,859\right.$
$\left.=\frac{1}{1.035}\times £14,598,882=\right)$ £14.105,200, or 14*l.* 2*s.* 1*d.* $\frac{1}{4}$.

And for a life of 46 years it is $\left(=\frac{1}{1.035}\times\frac{607}{615}\times £15.105,200\right.$
$\left.=\frac{1}{1.035}\times £14.908,709=\right)$ £14.404,549, or 14*l.* 8*s.* 1*d.*

And for a life of 45 years it is $\left(=\frac{1}{1.035}\times\frac{615}{622}\times £15.404,549\right.$
$\left.=\frac{1}{1.035}\times £15.231,185=\right)$ £14.716,120, or 14*l.* 14*s.* 3*d.* $\frac{3}{4}$.

And for a life of 44 years it is $\left(=\frac{1}{1.035}\times\frac{622}{629}\times £15.716,120\right.$
$\left.=\frac{1}{1.035}\times £15.541,218=\right)$ £15.015,669, or 15*l.* 0*s.* 3*d.* $\frac{3}{4}$.

And for a life of 43 years it is $\left(=\frac{1}{1.035}\times\frac{629}{636}\times £16.015,669\right.$
$\left.=\frac{1}{1.035}\times £15.839,395=\right)$ £15.303,763, or 15*l.* 6*s.* 1*d.*

And for a life of 42 years it is $\left(=\frac{1}{1.035}\times\frac{636}{643}\times £16.303,763\right.$
$\left.=\frac{1}{1.035}\times £16.126,272=\right)$ £15.580,939, or 15*l.* 11*s.* 7*d.* $\frac{1}{2}$.

And for a life of 41 years it is $\left(=\frac{1}{1.035}\times\frac{643}{650}\times £16.580,939\right.$
$\left.=\frac{1}{1.035}\times £16.448,528=\right)$ £15.892,297, or 15*l.* 17*s.* 10*d.*

And for a life of 40 years it is $\left(= \frac{1}{1.035} \times \frac{650}{657} \times £16.892,297 = \frac{1}{1.035} \times £16.712,318 =\right)$ £16.147,167, or 16*l.* 2*s.* 11*d.* $\frac{1}{4}$.

And for a life of 39 years it is $\left(= \frac{1}{1.035} \times \frac{657}{664} \times £17.147,167 = \frac{1}{1.035} \times £16.966,398 =\right)$ £16.392,655, or 16*l.* 7*s.* 10*d.* $\frac{1}{4}$.

And for a life of 38 years it is $\left(= \frac{1}{1.035} \times \frac{664}{671} \times £17.392,655 = \frac{1}{1.035} \times £17.211,211 =\right)$ £16.629,189, or 16*l.* 12*s.* 7*d.*

And for a life of 37 years it is $\left(= \frac{1}{1.035} \times \frac{671}{678} \times £17\,629,189 = \frac{1}{1.035} \times £17.447,176 =\right)$ £16.857,174, or 16*l.* 17*s.* 1*d.* $\frac{3}{4}$.

And for a life of 36 years it is $\left(= \frac{1}{1.035} \times \frac{678}{686} \times £17.857,174 = \frac{1}{1.035} \times £17.648,927 =\right)$ £17.052,103, or 17*l.* 1*s.* $\frac{1}{2}$*d.*

And for a life of 35 years it is $\left(= \frac{1}{1.035} \times \frac{686}{694} \times £18.052,103 = \frac{1}{1.035} \times £17.844,009 =\right)$ £17.240,588, or 17*l.* 4*s.* 9*d.* $\frac{3}{4}$.

And for a life of 34 years it is $\left(\frac{1}{1.035} \times \frac{694}{702} \times £18.240,588 = \frac{1}{1.035} \times £18.032,718 =\right)$ £17.422,915, or 17*l.* 8*s.* 5*d.* $\frac{1}{2}$.

And

And for a life of 33 years it is $\left(=\frac{1}{1.035}\times\frac{702}{710}\times £18.422,915\right.$
$=\frac{1}{1.035}\times £18.215,332=\left.\right)$ £17.599,354, or 17*l.* 11*s.* 11*d*$\frac{3}{4}$.

And for a life of 32 years it is $\left(=\frac{1}{1.035}\times\frac{710}{718}\times £18.599,354\right.$
$=\frac{1}{1.035}\times £18.392,118=\left.\right)$ £17.770,162, or 17*l.* 15*s.* 2*d.*$\frac{3}{4}$.

And for a life of 31 years it is $\left(=\frac{1}{1.035}\times\frac{718}{726}\times £18.770,162\right.$
$=\frac{1}{1.035}\times £18.563,328=\left.\right)$ £17.935,582, or 17*l.* 18*s.* 8*d.*$\frac{1}{2}$.

And for a life of 30 years it is $\left(=\frac{1}{1.035}\times\frac{726}{734}\times £18.935,582\right.$
$=\frac{1}{1.035}\times £18.729,199=\left.\right)$ £18.095,844, or 18*l.* 1*s.* 11*d.*

And for a life of 29 years it is $\left(=\frac{1}{1.035}\times\frac{734}{742}\times £19.095,844\right.$
$=\frac{1}{1.035}\times £18.889,958=\left.\right)$ £18.251,167, or 18*l.* 5*s.* 0*d.*$\frac{1}{4}$.

And for a life of 28 years it is $\left(=\frac{1}{1.035}\times\frac{742}{750}\times £19.251,167\right.$
$=\frac{1}{1.035}\times £19.045,821=\left.\right)$ £18.401,759, or 18*l.* 8*s.* $\frac{1}{2}$*d.*

And for a life of 27 years it is $\left(=\frac{1}{1.035}\times\frac{750}{758}\times £19.401,759\right.$
$=\frac{1}{1.035}\times £19.196,991=\left.\right)$ £18.547,817, or 18*l.* 10*s.* 11*d.*$\frac{1}{2}$.

And

And for a life of 26 years it is $\left(= \frac{1}{1.035} \times \frac{758}{766} \times £19.547,817 = \frac{1}{1.035} \times £19.343,662 =\right)$ £18.689,528, or 18*l.* 13*s.* 9*d.* $\frac{1}{2}$.

And for a life of 25 years it is $\left(= \frac{1}{1.035} \times \frac{766}{774} \times £19.689,528 = \frac{1}{1.035} \times £19.486,018 =\right)$ £18.827,070, or 18*l.* 16*s.* 6*d.* $\frac{1}{2}$.

And for a life of 24 years it is $\left(= \frac{1}{1.035} \times \frac{774}{782} \times £19.827,070 = \frac{1}{1.035} \times £19.624,235 =\right)$ £18.960,613, or 18*l.* 19*s.* 2*d.* $\frac{1}{2}$.

And for a life of 23 years it is $\left(= \frac{1}{1.035} \times \frac{782}{790} \times £19.960,613 = \frac{1}{1.035} \times £19.758,480 =\right)$ £19.090,318, or 19*l.* 1*s.* 9*d.* $\frac{1}{2}$.

And for a life of 22 years it is $\left(= \frac{1}{1.035} \times \frac{790}{798} \times £20,090,318 = \frac{1}{1.035} \times £19.888,911 =\right)$ £19.216,339, or 19*l.* 4*s.* 4*d.*

And for a life of 21 years it is $\left(= \frac{1}{1.035} \times \frac{798}{806} \times £20.216,339 = \frac{1}{1.035} \times £20.015,680 =\right)$ £19.338,821, or 19*l.* 6*s.* 9*d.* $\frac{1}{4}$.

And for a life of 20 years it is $\left(= \frac{1}{1.035} \times \frac{806}{814} \times £20.338,821 = \frac{1}{1.035} \times £20.138,930 =\right)$ £19.457,903, or 19*l.* 9*s.* 1*d.* $\frac{3}{4}$.

And

And for a life of 19 years it is $\left(= \frac{1}{1.035} \times \frac{814}{821} \times £20.457,903 = \frac{1}{1.035} \times £20.283,475 =\right)$ £19.597,560, or 19*l.* 11*s.* 11*d.* $\frac{1}{2}$.

And for a life of 18 years it is $\left(= \frac{1}{1.035} \times \frac{821}{828} \times £20.597,560 = \frac{1}{1.035} \times £20.423,426 =\right)$ £19.732,778, or 19*l.* 14*s.* 7*d.* $\frac{3}{4}$.

And for a life of 17 years it is $\left(= \frac{1}{1.035} \times \frac{828}{835} \times £20.732,778 = \frac{1}{1.035} \times £20.558,970 =\right)$ £19.863,739, or 19*l.* 17*s.* 3*d.* $\frac{1}{4}$.

And for a life of 16 years it is $\left(= \frac{1}{1.035} \times \frac{835}{842} \times £20.863,739 = \frac{1}{1.035} \times £20.690,287 =\right)$ £19.990,615, or 19*l.* 19*s.* 9*d.* $\frac{3}{4}$.

And for a life of 15 years it is $\left(= \frac{1}{1.035} \times \frac{842}{848} \times £20.990,615 = \frac{1}{1.035} \times £20.842,096 =\right)$ £20.137,194, or 20*l.* 2*s.* 9*d.*

And for a life of 14 years it is $\left(= \frac{1}{1.035} \times \frac{848}{854} \times £21.137,194 = \frac{1}{1.035} \times £20.988,677 =\right)$ £20.278,914, or 20*l.* 5*s.* 7*d.*

And for a life of 13 years it is $\left(= \frac{1}{1.035} \times \frac{854}{860} \times £21.278,914 = \frac{1}{1.035} \times £21.130,456 =\right)$ £20.415,899, or 20*l.* 8*s.* 3*d.* $\frac{3}{4}$.

And

And for a life of 12 years it is $(= \frac{1}{1.035} \times \frac{860}{866} \times £21.415,899$ $= \frac{1}{1.035} \times £21.267,520 =)$ £20.548,338, or 20*l.* 10*s.* 11*d.* $\frac{1}{2}$.

And for a life of 11 years it is $(= \frac{1}{1.035} \times \frac{866}{872} \times £21.548,338$ $= \frac{1}{1.035} \times £21.400,069 =)$ £20.676,395, or 20*l.* 13*s.* 6*d.* $\frac{1}{4}$.

And for a life of 10 years it is $(= \frac{1}{1.035} \times \frac{872}{880} \times £21.676,395$ $= \frac{1}{1.035} \times £21.479,336 =)$ £20.752,981, or 20*l.* 15*s.* 0*d.* $\frac{3}{4}$.

CXV. This value of an annuity of one pound for a life of 10 years is ſomewhat greater than that which was found for it in Art. 94, which was £20.73925. But the difference between them is but ſmall, being only .013,731 of a pound, or (240 × .013,731, or) 3.295,440 of a penny, or about three pence, farthing, upon the ſum of twenty pounds, fifteen ſhillings. They may therefore be conſidered as being equal to each other, and conſequently as affording a proof of the agreement of the two different methods of calculation by which they have been obtained, and a confirmation of their truth. Which of theſe two values is the more exact, I do not know; but I am inclined to think it is this laſt, which we have juſt now obtained by means of the repeated application of the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|£$, to wit, £20.752,981. And, if this number does exhibit the value of the ſaid annuity more exactly than the other number £20.73925, it is evident that the values of the remaining annuities for lives of the ages of 9, 8, 7, 6, 5, 4, and 3 years, will be ſomewhat greater than they are found to be in Art. 103, 104, 105, and 106. In order, therefore, to make the table of theſe values (which we have here computed,) uniform and regular throughout, I ſhall compute the values of an annuity of one pound for lives of 9, 8, 7, 6, 5, 4, and 3 years of age

a ſecond

a ſecond time by means of the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$, upon a ſuppoſition that the firſt value of $V\mathcal{L}$, or the value of an annuity of one pound for a life of the age of 10 years, is $= \mathcal{L}20.752,981$. This may be done in the manner following.

CXVI. If $\overset{\mathcal{L}}{V}$ is $= \mathcal{L}20.752,981$, we ſhall have $\overline{1+V}|\mathcal{L} = \mathcal{L}21.752,981$. And in this caſe P is $= 880$, and $P+d$ is $= 890$. Therefore the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$ will be $= \frac{1}{1.035} \times \frac{880}{890} \times \mathcal{L}21.752,981$ $\left(= \frac{1}{1.035} \times \mathcal{L}21.508,563\right) = \mathcal{L}20.781,222$; or the value of an annuity of one pound for a life of 9 years is $\mathcal{L}20.781,222$, or 20*l.* 15*s.* 7*d.*$\frac{1}{2}$.

CXVII. And in like manner the value of an annuity of one pound for a life of 8 years will be $\left(= \frac{1}{1.035} \times \frac{890}{902} \times \mathcal{L}21.781,222 = \frac{1}{1.035} \times \mathcal{L}21.491,449 =\right) \mathcal{L}20.764,685$, or 20*l.* 15*s.* 3*d.*$\frac{1}{2}$.

And for a life of 7 years it is $\left(= \frac{1}{1.035} \times \frac{902}{915} \times \mathcal{L}21.764,685 = \frac{1}{1.035} \times \mathcal{L}21.455,459 =\right) \mathcal{L}20.729,912$, or 20*l.* 14*s.* 7*d.*

And for a life of 6 years it is $\left(= \frac{1}{1.035} \times \frac{915}{930} \times \mathcal{L}21.729,912 = \frac{1}{1.035} \times \mathcal{L}21.379,322 =\right) \mathcal{L}20.656,349$, or 20*l.* 13*s.* 1*d.*$\frac{1}{2}$.

And for a life of 5 years it is $\left(= \frac{1}{1.035} \times \frac{930}{948} \times \mathcal{L}21.656,349 = \frac{1}{1.035} \times \mathcal{L}21.245,152 =\right) \mathcal{L}20.526,716$, or 20*l.* 10*s.* 6*d.*$\frac{1}{2}$.

And for a life of 4 years it is $(= \frac{1}{1.035} \times \frac{948}{970} \times £21.526,716$ $= \frac{1}{1.035} \times £21.038,481 =)$ £20.327,034, or 20*l.* 6*s.* 6*d.* $\frac{1}{2}$.

And for a life of 3 years it is $(= \frac{1}{1.035} \times \frac{970}{1000} \times £21.327,034$ $= \frac{1}{1.035} \times £20.687,222 =)$ £19.987,654, or 19*l.* 19*s.* 9*d.*

CXVIII. Having now compleated the computation of the values of an annuity of one pound a year for every year of human life from the age of 93 years down to the age of 3 years, inclufively, I fhall proceed to range them in a regular feries or table, and fhall expref them (as I have hitherto done) both in decimal parts of a pound fterling, and in pounds, fhillings, and pence. And, that the rate, or degree, at which they increafe, or decreafe, in the different periods of human life, may appear the more readily, I fhall fubjoin to them a table of their differences, expreffed likewife in a double manner, to wit, in decimal parts of a pound fterling and in pounds, fhillings, and pence. This table is as follows.

TABLE

TABLE III.

Containing the values of an annuity of one pound ſterling a year for the lives of perſons of all ages from the age of 3 years to the age of 93 years, incluſively; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent.

Years of Age.	*Values of an annuity of one pound, in decimal parts of a pound.*	*Differences of the ſaid values.*	*Values of the ſame annuity, in pounds, ſhillings, and pence.*	*Differences of the ſaid values.*
	£	£	£ s. d.	£ s. d.
3	19.987,654		19 19 9	
		0.339,380		0 6 $9\frac{1}{2}$
4	20.327,034		20 6 $6\frac{1}{2}$	
		0.199,682		0 4 0
5	20.526,716		20 10 $6\frac{1}{2}$	
		0.129,633		0 2 7
6	20.656,349		20 13 $1\frac{1}{2}$	
		0.073,563		0 1 $5\frac{1}{2}$
7	20.729,912		20 14 7	
		0.034,773		0 0 $8\frac{1}{2}$
8	20.764,685		20 15 $3\frac{1}{2}$	
		0.016,537		0 0 4
9	20.781,222		20 15 $7\frac{1}{2}$	
		0.028,241		0 0 $6\frac{3}{4}$
10	20 752,981		20 15 $C\frac{3}{4}$	
		0.076,586		0 1 $6\frac{1}{2}$
11	20.676,395		20 13 $6\frac{1}{4}$	
		0.128,057		0 2 $6\frac{3}{4}$
12	20.548,338		20 10 $11\frac{1}{2}$	
		0.132,439		0 2 $7\frac{3}{4}$
13	20.415,899		20 8 $3\frac{3}{4}$	
		0.136,985		0 2 $8\frac{3}{4}$
14	20.278,914		20 5 7	
		0.141,720		0 2 10
15	20.137,194		20 2 9	
		0.146,579		0 2 $11\frac{1}{4}$

Years of Age.	*Values of an annuity of one pound, in decimal parts of a pound.*	*Differences of the said values.*	*Values of the same annuity, in pounds, shillings, and pence.*	*Differences of the said values.*
	£	£	£ s. d.	£ s. d.
16	19.990,615		19 19 9¾	
		0.126,876		0 2 6½
17	19.863,739		19 17 3¼	
		0.130,961		0 2 7½
18	19.732,778		19 14 7¾	
		0.135,218		0 2 8¾
19	19.597,560		19 11 11½	
		0.138,657		0 2 9¾
20	19.457,903		19 9 1¾	
		0.119,082		0 2 4½
21	19.338,821		19 6 9¼	
		0.122,482		0 2 5¼
22	19.216,339		19 4 4	
		0.126,021		0 2 6½
23	19.090,318		19 1 9½	
		0.129,705		0 2 7
24	18.960,613		18 19 2½	
		0.133,543		0 2 8
25	18.827,070		18 16 6½	
		0.137,542		0 2 9
26	18.689,528		18 13 9½	
		0.141,711		0 2 10
27	18.547,817		18 10 11½	
		0.146,058		0 2 11
28	18.401,759		18 8 0½	
		0.150,592		0 3 0¼
29	18.251,167		18 5 0¼	
		0.155,323		0 3 1¼
30	18.095,844		18 1 11	
		0.160,262		0 3 2½
31	17.935,582		17 18 8½	
		0.165,420		0 3 5¾
32	17.770,162		17 15 2¾	
		0.170,808		0 3 3
33	17.599,354		17 11 11¾	
		0.176,439		0 3 6¼
34	17.422,915		17 8 5½	
		0.182,327		0 3 7¾
35	17.240,588		17 4 9¾	
		0.188,485		0 3 9¼

Years of Age.	*Values of an annuity of one pound, in decimal parts of a pound.*	*Differences of the said values.*	*Values of the same annuity, in pounds, shillings, and pence.*	*Differences of the said values.*
	£	£	£ s. d.	£ s. d.
36	17.052,103	0.194,929	17 1 $0\frac{1}{2}$	0 3 $10\frac{3}{4}$
37	16.857,174	0.227,985	16 17 $1\frac{3}{4}$	0 4 $6\frac{3}{4}$
38	16.629,189	0.236,534	16 12 7	0 4 $8\frac{3}{4}$
39	16.392,655	0.245,488	16 7 $10\frac{1}{4}$	0 4 11
40	16.147,167	0.254,870	16 2 $11\frac{1}{4}$	0 5 $1\frac{3}{4}$
41	15.892,297	0.311,358	15 17 10	0 6 $2\frac{1}{2}$
42	15.580,939	0.277,176	15 11 $7\frac{1}{2}$	0 5 $6\frac{1}{2}$
43	15.303,763	0.288,094	15 6 1	0 5 $9\frac{1}{4}$
44	15.015,669	0.299,549	15 0 $3\frac{3}{4}$	0 6 0
45	14.716,120	0.311,571	14 14 $3\frac{3}{4}$	0 6 $2\frac{3}{4}$
46	14.404,549	0.299,349	14 8 1	0 5 $11\frac{3}{4}$
47	14.105,200	0.311,341	14 2 $1\frac{1}{4}$	0 6 $2\frac{3}{4}$
48	13.793,859	0.299,434	13 15 $10\frac{1}{2}$	0 6 0
49	13.494,425	0.311,342	13 9 $10\frac{1}{2}$	0 6 $2\frac{1}{2}$
50	13.183,083	0.299,634	13 3 8	0 6 0
51	12.883,449	0 287,153	12 17 8	0 5 9
52	12.596,296	0.297,910	12 11 11	0 5 $11\frac{1}{4}$
53	12.298,386	0.309,293	12 5 $11\frac{1}{2}$	0 6 $2\frac{1}{4}$
54	11.989,093	0.297,292	11 19 $9\frac{1}{4}$	0 5 $11\frac{1}{4}$
55	11.691,801	0.308,374	11 13 10	0 6 2

Years of Age.	*Values of an annuity of one pound, in decimal parts of a pound.*	*Differences of the said values.*	*Values of the same annuity, in pounds, shillings, and pence.*	*Differences of the said values.*
	£	£	£ s. d.	£ s. d.
56	11.383,427		11 7 8	
		0.319,942		0 6 4$\frac{3}{4}$
57	11.063,485		11 1 3$\frac{1}{4}$	
		0.308,362		0 6 2
58	10.755,123		10 15 1$\frac{1}{4}$	
		0.319,557		0 6 4$\frac{3}{4}$
59	10.435,566		10 8 8$\frac{1}{2}$	
		0.331,492		0 6 7$\frac{1}{2}$
60	10.104,074		10 2 1	
		0.344,245		0 6 10$\frac{3}{4}$
61	9.759,829		9 15 2$\frac{1}{4}$	
		0.357,904		0 7 1$\frac{3}{4}$
62	9.401,925		9 8 0$\frac{1}{2}$	
		0.348,866		0 6 11$\frac{3}{4}$
63	9.053,059		9 1 0$\frac{3}{4}$	
		0.362,411		0 7 3
64	8.690,648		8 13 9$\frac{3}{4}$	
		0.377,023		0 7 6$\frac{1}{2}$
65	8.313,625		8 6 3$\frac{1}{4}$	
		0.369,367		0 7 4$\frac{3}{4}$
66	7.944,258		7 18 10$\frac{1}{2}$	
		0.360,531		0 7 2$\frac{1}{2}$
67	7 583,727		7 11 8	
		0.350,028		0 7 0
68	7.233,699		7 4 8	
		0.337,203		0 6 8$\frac{3}{4}$
69	6.896,496		6 17 11$\frac{1}{4}$	
		0.321,139		0 6 5$\frac{1}{4}$
70	6.575,357		6 11 6	
		0.325,517		0 6 6
71	6.249,840		6 5 0	
		0.303,868		0 6 1
72	5.945,972		5 18 11	
		0.301,524		0 6 1$\frac{1}{2}$
73	5.644,448		5 12 10$\frac{1}{2}$	
		0.296,642		0 5 11
74	5.347,806		5 6 11$\frac{1}{2}$	
		0.283,183		0 5 9$\frac{1}{4}$
75	5.059,623		5 1 2$\frac{1}{4}$	
		0.304,702		0 6 1

Years of Age.	*Values of an annuity of one pound, in decimal parts of a pound.*	*Differences of the said values.*	*Values of the same annuity, in pounds, shillings, and pence.*	*Differences of the said values.*
	£	£	£ s. d.	£ s. d.
76	4.754,921		4 15 $1\frac{1}{4}$	
		0.293,082		0 5 $10\frac{1}{2}$
77	4.461,839		4 9 $2\frac{3}{4}$	
		0.274,081		0 5 $5\frac{3}{4}$
78	4.187,758		4 3 9	
		0.279,753		0 5 7
79	3.908,000		3 18 2	
		0.246,219		0 4 $11\frac{3}{4}$
80	3.661,781		3 13 $2\frac{3}{4}$	
		0.233,925		0 4 8
81	3.427,856		3 8 $6\frac{3}{4}$	
		0.212,198		0 4 3
82	3.215,658		3 4 $3\frac{3}{4}$	
		0.231,185		0 4 $7\frac{1}{2}$
83	2.984,473		2 19 $8\frac{1}{4}$	
		0.267,285		0 5 $4\frac{1}{4}$
84	2.717,188		2 14 4	
		0.260,414		0 5 $2\frac{1}{2}$
85	2.456,774		2 9 $1\frac{1}{2}$	
		0.244,863		0 4 $10\frac{3}{4}$
86	2.211,911		2 4 $2\frac{3}{4}$	
		0.212,101		0 4 $2\frac{3}{4}$
87	1.999,810		2 0 0	
		0.271,432		0 5 $5\frac{1}{4}$
88	1.728,378		1 14 $6\frac{3}{4}$	
		0.268,679		0 5 $4\frac{1}{2}$
89	1.459,699		1 9 $2\frac{1}{4}$	
		0.262,187		0 5 $2\frac{3}{4}$
90	1.197,512		1 3 $11\frac{1}{2}$	
		0.249,843		0 5 0
91	0.947.669		0 18 $11\frac{1}{2}$	
		0.231,201		0 4 $7\frac{1}{2}$
92	0.716,468		0 14 4	
		0.233,378		0 4 8
93	0.48309		0 9 8	

CXIX. In

CXIX. In the foregoing table it may be obſerved that the differences of the values of an annuity of one pound for the ſeveral ſucceſſive years of age in human life, (which are ſet down in pounds, ſhillings, and pence in the fifth column of the ſaid table,) vary gradually, by increaſing or decreaſing, within the limits of 4 pence on the one hand, and 7 ſhillings, and 6 pence, half-penny, on the other; the leaſt of them, (which is the difference of the ſaid values for the ages of 8 and 9 years,) being 4 pence, and the greateſt of them (which is the difference of the ſaid values for the ages of 64 and 65 years,) being 7*s.* 6*d.*$\frac{1}{2}$. And, between theſe limits, they vary, in ſome places by increaſing, and in others by decreaſing, (while the years of life increaſe,) by the ſmall differences of a penny, two pence, or three pence, for the moſt part, and in a few places by ſix pence, or ſeven pence, or, at moſt, eleven pence, excepting only the beginning of the table, between the ages of 3 and 12 years, in which they decreaſe (while the years of life increaſe,) by differences of 2*s.* 9*d.*$\frac{1}{2}$, ſeventeen pence, and 13 pence, halfpenny.

CXX. This obſervation may be of uſe in computing other tables of this kind according to other rates of intereſt, by enabling the computer to detect in good time any error he may happen to have fallen into in the courſe of his arithmetical operations. Thus, for example, if in computing the value of an annuity for a life of 60 years by means of the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$, or $\frac{1}{1.035} \times \frac{450}{463} \times \overline{\mathcal{L}1+\mathcal{L}9.759,829}$, or $\frac{1}{1.035} \times \frac{450}{463} \times \mathcal{L}10.759,829$, I had found the ſaid value to be $\mathcal{L}$10.154,074, inſtead of $\mathcal{L}$10.104,074, that is, 10*l.* 3*s.* 1*d.* inſtead of 10*l.* 2*s.* 1*d.* I might have ſoon diſcovered that I had made a miſtake in the calculation, by ſubtracting from this value the value of the like annuity of one pound for a life of 61 years, which I had before found to be $\mathcal{L}$9.759,829, or 9*l.* 15*s.* 2*d.*$\frac{1}{4}$, and then comparing the remainder, or difference, with the foregoing differences between the values of the ſame annuity for lives of 61, 62, 63 and 64 years, all which I had before computed. For I ſhould then have found that this laſt difference, (or the exceſs of 10*l.* 3*s.* 1*d.* above 9*l.* 15*s.* 2*d.*$\frac{1}{4}$,) was 7*s.* 10*d.*$\frac{3}{4}$; which exceeds the difference next preceeding it, or that between the values of two annuities of one pound each for lives of 61 and 62 years, (which is 7*s.* 1*d.*$\frac{1}{4}$,) by no leſs a ſum than 9 pence: whereas that difference (7*s.* 1*d* $\frac{3}{4}$,) exceeds the next preceeding difference (which is 6*s.* 11*d.*$\frac{3}{4}$) by only 2 pence; and that difference (6*s.* 11*d.*$\frac{3}{4}$) is exceeded by the next preceeding difference, to wit, 7*s.* 3*d.* by only 3*d.*$\frac{1}{4}$; and this laſt difference, (7*s.* 3*d*) is exceeded by the next preceeding difference (which is 7*s.* 6*d.*$\frac{1}{2}$,) by only 3*d.*$\frac{1}{2}$. Now each of theſe three differences of the differences

differences ſet down in the table, to wit, 2*d.* 3*d.* $\frac{1}{4}$, and 3*d.* $\frac{1}{2}$, is much leſs than 9 pence, which is the exceſs of the new difference, 7*s.* 10*d.* $\frac{3}{4}$, above the next preceeding difference, 7*s.* 1*d* $\frac{3}{4}$. I ſhould therefore, upon obſerving the uncommon magnitude of this laſt-mentioned exceſs, have been led to ſuſpect that the laſt-found difference, 7*s.* 10*d.* $\frac{1}{4}$, was greater than the truth, aud conſequently that the number found for the value of an annuity of one pound for a life of 60 years, to wit, 10*l.* 3*s.* 1*d.* or £10.154,074, was too great. And upon this ſuſpicion, ſo ariſing from the compariſon of the ſaid differences in the fifth column of the foregoing table with each other, I ſhould have reviſed the calculation of the value of the ſaid annuity by means of the expreſſion $\frac{1}{1.035} \times \frac{450}{463} \times$ £10.759,829, and ſhould have diſcovered my miſtake, and have found that the true value of the ſaid annuity for a life of 60 years was not £10.154,074, but £10.104,074, or 10*l.* 2*s.* 1*d.* And this number there is no reaſon to ſuſpect of being erroneous, becauſe it exceeds the value of the like annuity for a life of 61 years (which is 9*l.* 15*s.* 2*d.* $\frac{1}{4}$) by only 6*s.* 10*d.* $\frac{1}{4}$, which is exceeded by the next preceeding difference, to wit, 7*s.* 1*d.* $\frac{3}{4}$, by only 3 pence. And in fact I did in this manner diſcover and ſet right ſome miſtakes I had fallen into in computing the foregoing table.

CXXI. There is, however, another and more accurate method of diſcovering the errors that may have been made in computing ſuch tables of the values of life-annuities by means of the expreſſion above-mentioned, as ſoon as they ariſe, and conſequently of confirming the truth of the computations, when they have been rightly performed. But it requires conſiderably more labour than is neceſſary to the making of ſuch a table of differences as is above ſet down. Yet its uſefulneſs in aſcertaining the truth of theſe computations is ſo great, that, I think, all perſons who undertake to compute tables of the values of life-annuities by means of the expreſſion above-mentioned, would do well to undergo that labour, and to examine and correct their calculations by it. It is given, and explained, and illuſtrated by examples, by Mr. William Morgan, actuary to the Society of Equitable Aſſurances on Lives and Survivorſhips, in the 58th, 59th, 60th, &c.—73d, and the 212th and 213th, pages of his uſeful and learned treatiſe, intitled, *The Doctrine of Annuities and Aſſurances on Lives and Survivorſhips, ſtated and explained*; which was publiſhed in octavo in the year 1779, and printed for T. Cadell in the Strand. And I propoſe to make farther mention of it in the ſubſequent part of this diſcourſe.

Of certain irregularities in the increaſe and decreaſe of the differences in the foregoing table.

CXXII. But, though the obſervation made in Art. 119 concerning the differences ſet down in the fifth column of the foregoing table, to wit, that they increaſe, or decreaſe, in different parts of the table, by ſmall and almoſt equal increments or decrements, be true in general, yet it is liable to ſeveral exceptions. For it may alſo be obſerved, that, in ſeveral places in the ſaid table, the ſaid differences, after having increaſed or decreaſed almoſt regularly, or by nearly equal increments or decrements, for ſeveral years together, do, on a ſudden, increaſe, or decreaſe, by a much larger increment, or decrement, than before; after which they again increaſe, or decreaſe, as before, in a nearly regular manner, or by nearly equal increments, or decrements, for ſeveral more years together; and then again receive an uncommonly large increment, or decrement. Of this it will be proper to give ſome examples: and in examining theſe differences with this view, it will be moſt convenient to proceed in the order in which the values of the annuities ſet down in the foregoing table were computed, that is, from the older lives to the younger.

Now, if we begin this examination from the age of 41 years and continue it upwards to the age of 19 years, we ſhall find the variations of theſe differences to be as follows.

From the age of 41 years to the age of 37 years theſe differences are 5*s.* 1*d.* $\frac{1}{4}$, 4*s.* 11*d.* 4*s.* 8*d* $\frac{3}{4}$, and 4*s.* 6*d.* $\frac{3}{4}$; which differ from each other by only 2*d.* $\frac{1}{4}$, 2*d.* $\frac{1}{4}$ and 2*d:* and then, from the age of 37 years to the age of 36 years, the difference is 3*s.* 10*d.* $\frac{3}{4}$; which is leſs than the next preceeding difference, 4*s.* 6*d* $\frac{3}{4}$, by the much larger ſum of 8 pence, inſtead of 2*d.* or 2*d.* $\frac{1}{4}$. After this, from the age of 36 years to the age of 32 years, the differences are 3*s.* 9*d.* $\frac{1}{4}$, 3*s.* 7*d.* $\frac{3}{4}$, 3*s.* 6*d.* $\frac{1}{4}$, and 3*s.* *d.* which differ from the next preceeding difference, 3*s.* 10*d.* $\frac{3}{4}$, and from each other by the ſmall ſums of 1*d.* $\frac{1}{2}$, 1*d.* $\frac{1}{2}$, 1*d* $\frac{1}{2}$, and 3*d.* $\frac{1}{4}$. And thus far, that is, from the age of 41 years to the age of 32 years, the ſaid differences have continually decreaſed from 5*s.* 1*d.* $\frac{1}{4}$, to 3*s.* 3*d* But here they increaſe for one year, to wit, from the age of 32 years to the age of 31 years, and then decreaſe again for the next 11 years, or from the age of 31 years to the age of 20 years, and then the ſaid difference increaſes again from the age of 20 years to the age of 19 years. From the age of 32 years to the age of 31 years the ſaid difference increaſes from 3*s.* 3*d.* to 3*s.* 5*d.* $\frac{3}{4}$, and therefore differs from the value it would have had if it had gone on decreaſing as before, (which would have been about 3*s.*) by about 5*d.* $\frac{3}{4}$. And from the age of 31 years to the age of 20 years, the ſaid differences are 3*s.* 2*d.* $\frac{1}{2}$, 3*s.* 1*d.* $\frac{1}{4}$, 3*s.* 0*d.* $\frac{1}{4}$, 2*s.* 11*d.* 2*s.* 10*d.* 2*s.* 9*d.* 2*s.* 8*d.* 2*s.* 7*d.* 2*s.* 6*d.* $\frac{1}{2}$, 2*s.* 5*d.* $\frac{1}{4}$, and 2*s.* 4*d.* $\frac{1}{2}$; which decreaſe continually, and differ from the next preceeding difference, 3*s.* 5*d.* $\frac{3}{4}$, and from each other, by the moderate decrements of 3*d.* $\frac{1}{4}$, 1*d.* $\frac{1}{4}$,

1*d*. $\frac{1}{4}$, 1*d*. 1*d*. $\frac{1}{4}$, 1*d*. 1*d*. 1*d*. 1*d*. o*d*. $\frac{1}{2}$, 1*d*. $\frac{1}{4}$, and o*d*. $\frac{3}{4}$. And, from the age of 20 years to the age of 19 years, the ſaid difference (inſtead of decreaſing, as it had done for the preceeding 11 years;) increaſes from 2*s*. 4*d*. $\frac{1}{2}$ to 2*s*. 9*d*. $\frac{3}{4}$, or by an increment of 5*d*. $\frac{1}{4}$, and therefore is greater than it would have been if it had gone on decreaſing as before, by about 6 pence, or 7 pence.

And the like irregularities, or inequalities, in the variation of theſe differences may be obſerved in other parts of the ſaid table.

CXXIII. This irregularity ſeems, at firſt ſight, ſtrange and unaccountable, and (agreeably to what has been obſerved in Art. 119, 120) may give room to a ſuſpicion that the values which increaſe, or decreaſe, by theſe uncommonly large increments, or decrements, have been erroneouſly computed. And yet I am perſuaded, from the care I took in the computation of them, that they are not erroneous. But, I believe, it will be generally found that this irregularity is owing to the variations in the decreaſe of the numbers of perſons who are repreſented, in Monſieur de Parcieux's table above-mentioned, as living at the ends of the ſeveral ſucceſſive years of human life. For it is remarkable that, in ſeveral parts of that table, the number of the perſons who die in the ſpace of a year continues the ſame for ſeveral years together, and then changes to ſome greater, or leſſer, number. Thus, for example, from the age of 11 years to the age of 16 years 6 perſons are repreſented in the ſaid table as dying every year; the numbers of perſons repreſented there as living at the ages of 11, 12, 13, 14, 15, and 16, years being 872, 866, 860, 854, 848, and 842, reſpectively; which numbers differ from each other by 6. And, in like manner, from the age of 16 years to the age of 20 years the number of thoſe who die every year is 7; the numbers of perſons repreſented to be living at the ſeveral ages of 16, 17, 18, 19, and 20, years being 842, 835, 828, 821, and 814, reſpectively; which numbers differ from each other by 7. And afterwards, for 17 years together, from the age of 20 years to the age of 37 years, the number of thoſe who die every year is 8; the numbers of perſons repreſented as living at the age of 20 years and the ſeveral ſubſequent ages of 21, 22, 23, 24, 25, &c. years, up to 37 years, being 814, 806, 798, 790, 782, 774, 766, 758, 750, 742, 734, 726, 718, 710, 702, 694, 686, and 678; which all differ from each other by 8. And for the next 9 years, or from the age of 37 years to the age of 46 years, the number of thoſe who die every year is again only 7; the numbers of perſons repreſented in the ſaid table as living at the ages of 37, 38, 39, 40, 41, 42, 43, 44, 45, and 46, years being 678, 671, 664, 657, 650, 643, 636, 629, 622, and 615, reſpectively; which differ from each other by 7. And the ſame thing may be obſerved in other.

Of the cauſe of many of thoſe irregularities.

parts of Monſieur de Parcieux's ſaid table, and likewiſe in Mr. Kerſſeboom's table above-mentioned and in Dr. Halley's Breſlaw table, and in all other tables of the probabilities of the duration of human life that have ever been publiſhed.

Now it will be found that the extraordinary decrements and increments above-mentioned in the differences which are ſet down in the fifth column of the foregoing table, happen, for the moſt part, in thoſe years of human life in which the number of perſons who die in the ſaid years increaſes or decreaſes, or varies from the number of thoſe who died in the year next preceeding. Thus, for example, we have obſerved that, from the age of 41 years to the age of 37 years, the ſaid differences decreaſe from 5*s.* 1*d.* $\frac{1}{4}$ to 4*s.* 6*d.* $\frac{3}{4}$ by the moderate decrements of 2*d* $\frac{1}{4}$, 2*d.* $\frac{1}{4}$, and 2*d.*; and during the ſame period we find in Monſieur de Parcieux's table of probabilities, that the number of thoſe who die every year is always the ſame, to wit, 7. But in the next year, or from the age of 37 years to the age of 36 years, the correſponding difference in the fifth column of the aforeſaid table is 3*s.* 10*d.* $\frac{3}{4}$; which is leſs than the next preceeding difference, 4*s.* 6*d.* $\frac{3}{4}$, by the much greater ſum of 8 pence: and in this ſame year the number of perſons that die in the courſe of a year (which from the age of 46 years to the age of 37 years had been conſtantly 7 in a year,) is increaſed to 8; which occaſions an uncommon increaſe in the value of d in the denominator of the fraction $\frac{P}{P+d}$ in the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$, which expreſſes the value of the next younger life. By this uncommon increaſe of d the whole denominator $P+d$ receives alſo a greater increaſe than uſual, and conſequently the whole fraction $\frac{P}{P+d}$, and therefore alſo the whole expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$, becomes leſs than it would have been if d, or the number of perſons dying in the year, had not increaſed to the number 8, but had continued to be 7, as it had been for the foregoing 9 years, or from the age of 46 years to the age of 37 years. Therefore the value of the annuity of one pound for a life of 36 years (which is equal to the ſaid expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$) will be leſs than it would have been if d had continued to be equal to 7 inſtead of becoming equal to 8. And hence it comes to paſs that the exceſs of this value above the value of the like annuity for a life of 37 years, is leſs than it would have been if d had continued to be equal to 7, or that the difference 3*s.* 10*d.* $\frac{3}{4}$ is leſs than it would have been, if d had continued to be equal to 7, and conſequently that it falls ſhort of the preceeding difference, 4*s.* 6*d.* $\frac{3}{4}$, by the uncommonly large decrement of 8 pence.

And

And in the ſame manner we have ſeen that from the age of 31 years to the age of 20 years the ſaid differences continually decreaſe from 3*s.* 5*d.*$\frac{3}{4}$ to 2*s.* 4*d.*$\frac{1}{2}$ by the moderate decrements of 3*d.*$\frac{1}{4}$, 1*d.*$\frac{1}{4}$, 1*d.* 1*d.*$\frac{1}{4}$, 1*d.* 1*d.* 1*d.* 1*d.* o*d.*$\frac{1}{2}$, 1*d.*$\frac{1}{4}$, and o*d.*$\frac{3}{4}$: and during the ſame period of time we find in Monſieur de Parcieux's table of probabilities, that the number of thoſe who die every year (and which is denoted by the letter *d* in the denominator of the fraction $\frac{P}{P+d}$ in the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{|1+V|}\,\pounds$) is always the ſame, to wit, 8. But in the next year, or from the age of 20 years to the age of 19 years, the correſponding difference in the fifth column of the aforeſaid table is 2*s.* 9*d.*$\frac{3}{4}$; which is not only not leſs than the preceeding difference, 2*s.* 4*d.*$\frac{1}{2}$, by about a penny, (as might have been expected from a view of the aforeſaid decrements 3*d.*$\frac{1}{4}$, 1*d.*$\frac{1}{4}$, 1*d.* 1*d*$\frac{1}{4}$, 1*d.* 1*d.* 1*d.* 1*d.* o*d.*$\frac{1}{2}$, 1*d.*$\frac{1}{4}$, and o*d.*$\frac{3}{4}$ of the next preceeding differences,) but exceeds it by 5*d.*$\frac{1}{4}$, and conſequently is greater than might have been expected by about 6 pence: and in this ſame year, or from the age of 20 years to the age of 19 years, the number of perſons who die in the courſe of a year (and which from the age of 37 years to the age of 20 years had conſtantly been 8 in a year) is diminiſhed to 7; which occaſions a leſs diminution than uſual in the denominator of the fraction $\frac{P}{P+d}$ in the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{|1+V|}\,\pounds$, by which every new value is derived from that of the annuity for the next preceeding life which is one year older. Hence it comes to paſs that the ſaid expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{|1+V|}\,\pounds$, (which is equal to the value of an annuity for a life of 19 years,) is greater than it would have been if *d* had not decreaſed to 7, but had continued to be equal to 8, as it had been for ſeveral years before; and conſequently the ſaid value of an annuity for a life of 19 years exceeds the value of an annuity for a life of 20 years by a greater difference than it would have done if *d* had continued to be equal to 8, and the ſaid difference, to wit, 2*s.* 9*d.*$\frac{3}{4}$, inſtead of being leſs than the preceeding difference 2*s.* 4*d.*$\frac{1}{2}$, by about a penny, exceeds it by 5*d.*$\frac{1}{4}$.

CXXIV. We have ſeen in the foregoing article that the uncommonly great increments and decrements of the aforeſaid differences, (which are ſet down in the fifth column of the foregoing table,) are often occaſioned by correſpondent changes in the number of perſons who are repreſented in Monſieur de Parcieux's table of probabilities as dying in a year, or in the number

number d in the denominator of the fraction $\frac{P}{P+d}$ in the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{|1+V}\, £$. But they alſo ſometimes happen without any ſuch change in the value of d. Thus, for example, from the age of 36 years to the age of 32 years, the differences in the ſaid fifth column are $3s.\ 9d.\frac{1}{4}$, $3s.\ 7d.\frac{3}{4}$, $3s.\ 6d.\frac{1}{4}$ and $3s.\ 3d.$ which all decreaſe by the moderate decrements of $1d.\frac{1}{2}$, $1d.\frac{1}{2}$, and $3d.\frac{1}{4}$. But in the next year, or from the age of 32 years to the age of 31 years, the difference is $3s.\ 5d.\frac{3}{4}$; which is not only not leſs than the next preceeding difference, $3s.\ 3d.$ by about $3d.$ (as might have been expected from the courſe of the foregoing decrements,) but exceeds it by the ſum of $2d.\frac{3}{4}$, and therefore is greater than it might have been naturally expected to be by about $5d.\frac{3}{4}$, or 6 pence. And yet the number of perſons who die between the ages of 32 and 31 years is the ſame with that of thoſe who die between the ages of 33 and 32 years, and between the ages of 34 and 33 years, and of 35 and 34 years, and of 36 and 35 years, and of 37 and 36 years, to wit, 8. How to account for this, and the like uncommonly great variations in the ſaid differences, when there is no correſpondent change in the number of perſons repreſented in the table of probabilities as dying in the ſpace of a year, I do not know. But thus much, I think, we may ſafely conclude, to wit, that, as it *often* happens that an uncommonly large decrement, or increment, of the differences in the ſaid fifth column of the foregoing table, is occaſioned by a change in the number of perſons dying in the year, correſponding to ſuch difference, there will always be reaſon, whenever ſuch uncommonly large decrement, or increment of the ſaid differences is obſerved, and there is no change in the number of perſons dying in the correſponding year of human life, to occaſion it; — I ſay, there will always be reaſon in theſe caſes to ſuſpect that ſome error has crept into the calculation, and it will therefore be prudent to re-examine the operations of it with great care.

Of the decrements of human life.

CXXV. The numbers of perſons repreſented, in a table of the probabilities of the duration of human life, as dying every year, are called by many writers on this ſubject *the decrements of human life.* And, becauſe theſe decrements continue the ſame for ſeveral years together, in ſeveral different periods of human life, it is ſaid by theſe writers that human life waſtes, or *decreaſes*, *uniformly*, for ſeveral years together, in ſeveral of its ſtages, or periods; the numbers of the perſons living at the ends of the ſeveral years in each of the ſaid ſtages, or periods, (if reckoned from the younger ages to the older,) forming a regular, decreaſing, arithmetical progreſſion. Thus, for example, the numbers of perſons living at the

ſeveral

ſeveral ages of 20 years, 21 years, 22 years, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, and 37, years according to Monſieur de Parcieux's table of probabilities, form the following regular, decreaſing, arithmetical progreſſion, in which the common difference of the terms is 8, to wit, $814 + 806 + 798 + 790 + 782 + 774 + 766 + 758 + 750 + 742 + 734 + 726 + 718 + 710 + 702 + 694 + 686 + 678$, or $814 + 814 - 8$, $+ 814 - 2 \times 8$, $+ 814 - 3 \times 8$, $+ 814 - 4 \times 8$ $+ 814 - 5 \times 8$, $+ 814 - 6 \times 8$, $+ 814 - 7 \times 8$, $+ 814 - 8 \times 8$, $+ 814 - 9 \times 8$, $+ 814 - 10 \times 8$, $+ 814 - 11 \times 8$, $+ 814 - 12 \times 8$, $+ 814 - 13 \times 8$, $+ 814 - 14 \times 8$, $+ 814 - 15 \times 8$, $+ 814 - 16 \times 8$, $+ 814 - 17 \times 8$.

CXXVI. To make the courſe of theſe decrements in the different periods of human life more apparent, it will be convenient to preſent again to the reader's view the above-mentioned tables of probabilities of Mr. Kerſſeboom and Monſieur de Parcieux, and to ſet down in a ſeparate column, adjoining to that which exhibits the numbers of the perſons living at the ends of the ſeveral ſucceſſive years of human life in each table, the numbers of the perſons who have died in the next preceeding year. Theſe tables will then be as follows.

TABLE

TABLE IV.

Being the foregoing table of probabilities of Mr. Kerſſeboom, above ſet down in page 4, together with the numbers of the perſons dying in every year, or the decrements of human life, ſet down in an adjoining column.

Years of Age.	*Perſons living.*	*Perſons dying in a year, or the decrements of human life.*	*Years of Age.*	*Perſons living.*	*Perſons dying in a year, or the decrements of human life.*
0	1400		15	856	
		275			7
1	1125		16	849	
		50			7
2	1075		17	842	
		45			7
3	1030		18	835	
		37			9
4	993		19	826	
		29			9
5	964		20	817	
		17			9
6	947		21	808	
		17			8
7	930		22	800	
		17			8
8	913		23	792	
		9			9
9	904		24	783	
		9			11
10	895		25	772	
		9			12
11	886		26	760	
		8			13
12	878		27	747	
		8			12
13	870		28	735	
		7			12
14	863		29	723	
		7			12

Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.	Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.
30	711	12	50	507	12
31	699	12	51	495	13
32	687	12	52	482	12
33	675	10	53	470	12
34	665	10	54	458	12
35	655	10	55	446	12
36	645	10	56	434	13
37	635	10	57	421	13
38	625	10	58	408	13
39	615	10	59	395	13
40	605	9	60	382	13
41	596	9	61	369	13
42	587	9	62	356	13
43	578	9	63	343	14
44	569	9	64	329	14
45	560	10	65	315	14
46	550	10	66	301	14
47	540	10	67	287	14
48	530	8	68	273	14
49	518	11	69	259	14

Years of Age.	*Persons living.*	*Persons dying in a year, or the decrements of human life.*	*Years of Age.*	*Persons living.*	*Persons dying in a year, or the decrements of human life.*
70	245	14	86	36	8
71	231	14	87	28	7
72	217	14	88	21	6
73	203	14	89	15	5
74	189	14	90	10	3
75	175	15	91	7	2
76	160	15	92	5	2
77	145	15	93	3	1
78	130	15	94	2	1
79	115	15	95	1	0.4
80	100	13	96	0.6	0.1
81	87	12	97	0.5	0.1
82	75	11	98	0.4	0.2
83	64	9	99	0.2	0.2
84	55	10	100	0.0	
85	45	9			

TABLE

TABLE V.

Being the foregoing table of probabilities of Monſieur de Parcieux, above ſet down in page 6, together with the numbers of the perſons dying in every year, or the decrements of human life, ſet down in an adjoining column.

Years of Age.	*Perſons living.*	*Perſons dying in a year, or the decrements of human life.*	*Years of Age.*	*Perſons living.*	*Perſons dying in a year, or the decrements of human life.*
3	1000		18	828	
		30			7
4	970		19	821	
		22			7
5	948		20	814	
		18			8
6	930		21	806	
		15			8
7	915		22	798	
		13			8
8	902		23	790	
		12			8
9	890		24	782	
		10			8
10	880		25	774	
		8			8
11	872		26	766	
		6			8
12	866		27	758	
		6			8
13	860		28	750	
		6			8
14	854		29	742	
		6			8
15	848		30	734	
		6			8
16	842		31	726	
		7			8
17	835		32	718	
		7			8

Years of Age.	*Persons living.*	*Persons dying in a year, or the decrements of human life.*	*Years of Age.*	*Persons living.*	*Persons dying in a year, or the decrements of human life.*
33	710	8	52	560	11
34	702	8	53	549	11
35	694	8	54	538	12
36	686	8	55	526	12
37	678	7	56	514	12
38	671	7	57	502	13
39	664	7	58	489	13
40	657	7	59	476	13
41	650	7	60	463	13
42	643	7	61	450	13
43	636	7	62	437	14
44	629	7	63	423	14
45	622	7	64	409	14
46	615	8	65	395	15
47	607	8	66	380	16
48	599	9	67	364	17
49	590	9	68	347	18
50	581	10	69	329	19
51	571	11	70	310	19

Years of Age.	*Perſons living.*	*Perſons dying in a year, or the decrements of human life.*	*Years of Age.*	*Perſons living.*	*Perſons dying in a year, or the decrements of human life.*
71	291	20	84	59	11
72	271	20	85	48	10
73	251	20	86	38	9
74	231	20	87	29	7
75	211	19	88	22	6
76	192	19	89	16	5
77	173	19	90	11	4
78	154	18	91	7	3
79	136	18	92	4	2
80	118	17	93	2	1
81	101	16	94	1	1
82	85	14	95	0	
83	71	12			

A remar-

A remark on the decrements of human life in the foregoing tables of Mr. Kerſſeboom and Monſieur de Parcieux.

CXXVII. It appears from the foregoing table of Mr. Kerſſeboom that in ſeveral different periods of human life, from the age of 12 years to the age of 80 years, the decrements of life, or the numbers of perſons dying in a year, continue the ſame for five, ſix, or ſeven, years together, and in one period for twelve years together, namely, from the age of 63 years to the age of 75 years, in each of which years the number of perſons dying is 14. And it appears in like manner from the foregoing table of Monſieur de Parcieux, that in ſeveral parts of human life, from the age of 11 years to the age of 80 years, the decrements of life, or the numbers of perſons dying in a year, continue the ſame for four, five, or eight, years together, and in one period for no leſs than ſeventeen years together, namely, from the age of 20 years to the age of 37 years, in each of which years the number of perſons dying is 8. Now by the help of this obſervation we may in ſome degree abridge the labour of computing the values of life-annuities, when thoſe values are not derived one from the other by means of the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times |\overline{1+V}|\,£$, but are computed ſeparately in the manner deſcribed in Art. 86 and exemplified in Art. 94. by the computation of the value of an annuity of one pound for a life of the age of 10 years. For it is there ſhewn that the value of an annuity of one pound a year during n years, in caſe a perſon aged N years ſhall ſo long live, is equal to $\frac{£1}{P} \times$ the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7}$ + &c. continued to n terms, or to the term $\frac{P^{n}}{r^n}$.

Application of the ſaid remark to the purpoſe of abridging the computations of life-annuities, when performed in the method deſcribed in Art. 86.

Now this ſeries may be divided into ſeveral different ſerieſes, in ſome of which the numerators of the terms will (as appears from the foregoing obſervation) decreaſe by equal differences, or form arithmetical progreſſions. And where this happens, the ſaid leſſer ſerieſes may be ſummed in a more eaſy and expeditious manner than by, firſt, computing the values of every ſingle term in them, and then adding them all together into one ſum, as was done in Art. 94. For the denominators of the terms of theſe ſerieſes are the terms of a geometrical progreſſion, and decreaſe continually in the proportion of $\frac{1}{r}$ to 1, or of 1 to r: and where the terms of a ſeries conſiſt of fractions whoſe numerators form an arithmetical, and their denominators a geometrical, progreſſion, the ſum of the terms of ſuch a ſeries may be found in a compendious manner, without an actual computation and addition of them all together, by a method not unlike that explained above in Art. 80 and 81 for finding the ſum of the terms of a ſimple

ſimple geometrical progreſſion, (ſuch as $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^n}$,) and which is derived from that method. The manner of deriving ſuch a method of ſumming theſe compound ſerieſes, conſiſting of terms whoſe numerators form an arithmetical, and denominators a geometrical, progreſſion, from the aforeſaid method of ſumming a ſimple geometrical progreſſion, I will now endeavour to explain.

Of the ſummation of a ſeries of fractions, conſiſting of a given number of terms, whoſe numerators form a decreaſing arithmetical progreſſion, and denominators an increaſing geometrical progreſſion.

CXXVIII. Let a, $+a-d$, $+a-2d$, $+a-3d$, $+a-4d$, $+a-5d$, $+a-6d$, $+\&c.$ be a decreaſing arithmetical progreſſion, conſiſting of any number of terms denoted by the letter n. Then, it is evident, its laſt term will be $a-\overline{|n-1|} \times d$. or $a-\overline{|nd-d}$, or $a-nd+d$.

In the next place let r be any quantity greater than 1, and let $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c.$ be a geometrical progreſſion conſiſting likewiſe of n terms, which, it is evident, will decreaſe in the common ratio of 1 to r. The laſt term of this ſeries will evidently be $\frac{1}{r^n}$.

Laſtly, let the terms of this latter, or geometrical, progreſſion, $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^n}$, be multiplied into the correſpondent terms of the arithmetical progreſſion, a, $+a-d$, $+a-2d$, $+a-3d$, $+a-4d$, $+a-5d$, $+a-6d$, $+\&c.$ $+a-nd+d$; and we ſhall thereby form a third ſeries, which will be as follows, to wit, $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7} + \&c. + \frac{a-nd+d}{r^n}$. Now the ſum of the terms of a compound ſeries of this form may be obtained without actually computing every ſeparate term in it and then adding them all up together, by a method derived from that given in Art. 80 for finding the ſum of the terms of the geometrical progreſſion $A+B+C+D+E$. This method may be explained as follows.

CXXIX. The

CXXIX. The foregoing compound ſeries may be divided into the following ſets of quantities, to wit,

$$\begin{array}{lllllllll}
\frac{a}{r} & +\frac{a}{r^2} & +\frac{a}{r^3} & +\frac{a}{r^4} & +\frac{a}{r^5} & +\frac{a}{r^6} & +\frac{a}{r^7} & +\&c. & +\frac{a}{r^n} \\
 & -\frac{d}{r^2} & -\frac{d}{r^3} & -\frac{d}{r^4} & -\frac{d}{r^5} & -\frac{d}{r^6} & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & -\frac{d}{r^3} & -\frac{d}{r^4} & -\frac{d}{r^5} & -\frac{d}{r^6} & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & & -\frac{d}{r^4} & -\frac{d}{r^5} & -\frac{d}{r^6} & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & & & -\frac{d}{r^5} & -\frac{d}{r^6} & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & & & & -\frac{d}{r^6} & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & & & & & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & & & & & & -\&c. & -\frac{d}{r^n} \\
 & & & & & & & & -\frac{d}{r^n}.
\end{array}$$

In this ſet of quantities it is evident in the 1ſt place that the upper line $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \&c.$ will conſiſt of n terms, and the ſecond line $-\frac{d}{r^2} - \frac{d}{r^3} - \frac{d}{r^4} - \&c.$ of one term leſs than the upper line, or of $n-1$ terms, and the third line of $n-2$ terms, and the fourth line of $n-3$ terms, and ſo on, every new line conſiſting of one term leſs than that which immediately preceeds it; and conſequently that the loweſt line will conſiſt of only one term, to wit, $-\frac{d}{r^n}$.

Secondly,

Secondly, the number of theſe horizontal lines of terms will be the ſame as the number of terms in the upper line $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4}$ + &c. that is, n.

Thirdly, the terms in all the lines, except the firſt, or upper, line, $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4}$ + &c. have the ſign — prefixed to them, and are to be ſubtracted from the upper line.

And, fourthly, all the lines of terms except the two laſt, (which conſiſt of only two terms and one term,) are geometrical progreſſions whoſe terms decreaſe in the common ratio of 1 to r. Conſequently they may be ſummed by the application of the expreſſion $\frac{AA - BE}{A - B}$ given in Art. 80 for the ſum of the terms of the geometrical progreſſion $A + B + C + D + E$.

CXXX. Now, if we apply this expreſſion to the firſt, or upper, ſeries, $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7}$ + &c. $+ \frac{a}{r^n}$, we ſhall have $A = \frac{a}{r}$, and $B = \frac{a}{r^2}$, and $E = \frac{a}{r^n}$. Therefore AA will be $= \frac{aa}{r^2}$; and BE will be $\frac{a}{r^2} \times \frac{a}{r^n} = \frac{aa}{r^{n+2}}$; and $AA - BE$ will be $= \frac{aa}{r^2} - \frac{aa}{r^{n+2}} \left(= \frac{r^{n+2}\,aa - r^2\,aa}{r^{n+4}}\right) = \frac{r^n\,aa - aa}{r^{n+2}}$; and $A - B$ will be $= \frac{a}{r} - \frac{a}{r^2} \left(= \frac{r^2 a - ra}{r^3}\right) = \frac{ra - a}{r^2}$; and conſequently $\frac{AA - BE}{A - B}$ will be $= \frac{\frac{r^n\,aa - aa}{r^{n+2}}}{\frac{ra - a}{r^2}} \left(= \frac{r^n\,aa - aa}{r^{n+2}} \times \frac{r^2}{ra - a} = \frac{r^n a - a}{r^n \times r - 1} = \frac{a}{r - 1} \times \frac{r^n - 1}{r^n}\right) = \frac{a}{r-1} \times \frac{1 - \frac{1}{r^n}}{1}$. Therefore the ſum of all the terms of

the ſeries $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7} +$ &c. $+ \frac{a}{r^n}$ is $= \frac{a}{r-1} \times \overline{1 - \frac{1}{r^n}}$. QEI.

CXXXI. And, in like manner, if the ſame expreſſion $\frac{AA - BE}{A - B}$ be applied to the ſecond ſeries $\frac{d}{r^2} + \frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7}$ + &c. $+ \frac{d}{r^n}$, we ſhall have $A = \frac{d}{r^2}$, and $B = \frac{d}{r^3}$, and $E = \frac{d}{r^n}$; and conſequently $AA = \frac{dd}{r^4}$, and $BE = \frac{dd}{r^{n+3}}$ and $AA - BE = \frac{dd}{r^4} - \frac{dd}{r^{n+3}} \left(= \frac{r^{n+3}\,dd - r^4\,dd}{r^{n+7}}\right) = \frac{r^{n-1}\,dd - dd}{r^{n+3}}$; and $A - B = \frac{d}{r^2} - \frac{d}{r^3} \left(= \frac{r^3 d - r^2 d}{r^5}\right) = \frac{rd - d}{r^3}$, and $\frac{AA - BE}{A - B} = \frac{\frac{r^{n-1}\,dd - dd}{r^{n+3}}}{\frac{rd - d}{r^3}} \left(= \frac{r^{n-1}\,dd - dd}{r^{n+3}} \times \frac{r^3}{rd - d} = \frac{r^{n-1}\,d - d}{r^n \times r - 1} = \frac{d}{r-1} \times \frac{r^{n-1} - 1}{r^n}\right) = \frac{d}{r-1} \times \overline{\frac{1}{r} - \frac{1}{r^n}}$. Therefore the ſum of all the terms of the ſaid ſecond ſeries, $\frac{d}{r^2} + \frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7}$ + &c. $+ \frac{d}{r^n}$, is $= \frac{d}{r-1} \times \overline{\frac{1}{r} - \frac{1}{r^n}}$. QEI.

CXXXII. The third ſeries $\frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7}$ + &c. $+ \frac{d}{r^n}$ is equal to the ſecond ſeries $\frac{d}{r^2} + \frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7}$ + &c. $+ \frac{d}{r^n}$ without the firſt term $\frac{d}{r^2}$, and therefore is $= \frac{d}{r-1}$

$\frac{d}{r-1} \times \left| \frac{1}{r} \quad \frac{1}{r^n} \right. - \frac{d}{r^2}$ $\left(= \frac{r^{n-1}d - d}{r^n \times r-1} - \frac{d}{r^2} = \frac{r^{n-1}d - d}{r^n \times r-1} \right.$

$- \frac{r^{n-2} \times r-1 \times d}{r^n \times r-1} = \frac{r^{n-1}d - d}{r^n \times r-1} - \frac{r^{n-2} \times rd - d}{r^n \times r-1} = \frac{r^{n-1}d - d}{r^n \times r-1}$

$- \frac{r^{n-1}d + r^{n-2}d}{r^n \times r-1} = \frac{r^{n-2}d - d}{r^n \times r-1} = \frac{d}{r-1} \times \frac{r^{n-2} - 1}{r^n} \left. \right) = \frac{d}{r-1}$

$\times \left| \frac{1}{rr} \quad \frac{1}{r^n} \right.$. QEI.

CXXXIII. And the fourth series, $\frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} +$ &c. $+ \frac{d}{r^n}$, is, in like manner, equal to the third series $\frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} +$ &c. $+ \frac{d}{r^n}$ without its first term $\frac{d}{r^3}$, and therefore is $= \frac{d}{r-1} \times \left| \frac{1}{rr} \quad \frac{1}{r^n} \right. - \frac{d}{r^3}$ $\left(= \frac{r^{n-2}d - d}{r^n \times r-1} - \frac{d}{r^3} = \frac{r^{n-2}d - d}{r^n \times r-1} \right.$

$- \frac{r^{n-3} \times r-1 \times d}{r^n \times r-1} = \frac{r^{n-2}d - d}{r^n \times r-1} - \frac{r^{n-3} \times rd - d}{r^n \times r-1} = \frac{r^{n-2}d - d}{r^n \times r-1}$

$- \frac{r^{n-2}d + r^{n-3}d}{r^n \times r-1} = \frac{r^{n-3}d - d}{r^n \times r-1} = \frac{d}{r-1} \times \frac{r^{n-3} - 1}{r^n} = \frac{d}{r-1}$

$\times \frac{r^{n-3} - 1}{r^n} \left. \right) = \frac{d}{r-1} \times \left| \frac{1}{r^3} \quad \frac{1}{r^n} \right.$. QEI.

CXXXIV. And by proceeding in the same manner we shall find that the fifth series, $\frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} +$ &c. $+ \frac{d}{r^n}$, is $= \frac{d}{r-1} \times \left| \frac{1}{r^4} \quad \frac{1}{r^n} \right.$, and that the sixth series, $\frac{d}{r^6} + \frac{d}{r^7} +$ &c. $+ \frac{d}{r^n}$, is $= \frac{d}{r-1} \times \left| \frac{1}{r^5} \quad \frac{1}{r^n} \right.$, and that the seventh series, $\frac{d}{r^7} + \frac{d}{r^8} +$ &c. $+ \frac{d}{r^n}$,

$+\frac{d}{r^n}$, is $= \frac{d}{r-1} \times \overline{\left|\frac{1}{r^6} - \frac{1}{r^n}\right.}$, and that, in general, the mth series, $\frac{d}{r^m} + \frac{d}{r^{m+1}} + \&c. + \frac{d}{r^n}$, is $= \frac{d}{r-1} \times \overline{\left|\frac{1}{r^{m-1}} - \frac{1}{r^n}\right.}$.

CXXXV. Consequently the value of all the serieſes in Art. 129, except the firſt, that is, of all the serieſes whoſe terms are marked with the sign — and are therefore to be ſubtracted from the firſt, or upper, series $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7} + \&c. + \frac{a}{r^t}$, will be $= \frac{d}{r-1} \times \overline{\left|\frac{1}{r} - \frac{1}{r^n}\right.} + \frac{d}{r-1} \times \overline{\left|\frac{1}{r^2} - \frac{1}{r^n}\right.} + \frac{d}{r-1} \times \overline{\left|\frac{1}{r^3} - \frac{1}{r^n}\right.}$ $+ \frac{d}{r-1} \times \overline{\left|\frac{1}{r^4} - \frac{1}{r^n}\right.} + \frac{d}{r-1} \times \overline{\left|\frac{1}{r^5} - \frac{1}{r^n}\right.} + \frac{d}{r-1} \times \overline{\left|\frac{1}{r^6} - \frac{1}{r^n}\right.}$ $+ \&c.$ continued to $n-1$ terms, and consequently is $= \frac{d}{r-1} \times$ the geometrical series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \&c.$ continued to $n-1$ terms, or to the term $\frac{1}{r^{n-1}}$, $-\frac{d}{r-1} \times \overline{n-1} \times \frac{1}{r^n}$ $= \frac{d}{r-1} \times$ the geometrical series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6}$ $+ \&c. + \frac{1}{r^{n-1}}$, $-\frac{d}{r-1} \times \overline{\left|\frac{n-1}{r^n}\right.}$. Now it has been ſhewn above in Art. 81, page 92, that the ſum of all the terms of the geometrical series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \&c.$ continued to the term $\frac{1}{r^n}$, is $= \frac{1}{r-1} - \frac{1}{r^n \times r-1}$. Therefore the ſum of all the terms of the ſame geometrical series continued to the term $\frac{1}{r^{n-1}}$, or to one term leſs than before, will be $= \frac{1}{r-1} - \frac{1}{r^n \times r-1} - \frac{1}{r^n}$ $\left(= \frac{1}{r-1} - \frac{1}{r^n \times r-1} - \frac{1 \times r-1}{r^n \times r-1}\right.$

$\frac{-1\times\overline{r-1}}{r^n\times r-1} = \frac{1}{r-1}\ \frac{-1}{r^n\times r-1}\ \frac{-r+1}{r^n\times r-1} = \frac{1}{r-1}\ \frac{-r}{r^n\times r-1}\Big)$ $= \frac{1}{r-1}\ \frac{-1}{r^{n-1}\times r-1}$. Therefore $\frac{d}{r-1}\times$ the geometrical series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} +$ &c. $+ \frac{1}{r^{n-1}}$ is $=$ $\frac{d}{r-1}\times\frac{1}{r-1} - \frac{d}{r-1}\times\frac{1}{r^{n-1}\times r-1}$. Conſequently the ſum of all the ſerieſes in Art. 129, except the firſt ſeries, $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4}$ $+$ &c. $+ \frac{a}{r^n}$ is $= \frac{d}{r-1}\times\frac{1}{r-1} - \frac{d}{r-1}\times\frac{1}{r^{n-1}\times r-1} - \frac{d}{r-1}\times\frac{n-1}{r^n}$ $= \frac{d}{\overline{r-1}^2} - \frac{d}{r^{n-1}\times\overline{r-1}^2} - \frac{nd+d}{r^n\times r-1}$. But it has been ſhewn above, in Art. 130, that the ſum of all the terms of the firſt, or higheſt, ſeries, $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7}$ $+$ &c. $+ \frac{a}{r^n}$, is $= \frac{a}{r-1}\times\overline{1-\frac{1}{r^n}}$. Therefore the exceſs of this firſt ſeries above the ſum of all the reſt is $= \frac{a}{r-1}\times\overline{1-\frac{1}{r^n}} - \frac{d}{\overline{r-1}^2} + \frac{d}{r^{n-1}\times\overline{r-1}^2}$ $+ \frac{nd-d}{r^n\times r-1}$. Therefore the ſum of all the terms of the compound ſeries $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7}$ $+$ &c. $+ \frac{a-nd+d}{r^n}$, (which is equal to the ſaid exceſs of the ſaid firſt ſeries in Art. 129 above the ſum of all the reſt,) is alſo equal to $\frac{a}{r-1}\times\overline{1-\frac{1}{r^n}} - \frac{d}{\overline{r-1}^2} + \frac{d}{r^{n-1}\times\overline{r-1}^2} + \frac{nd-d}{r^n\times r-1}$. QEI.

CXXXVI. The

CXXXVI. The foregoing expreſſion may be made ſomewhat more ſimple by means of the following ſubſtitutions.

A reduction of the value of the aforeſaid compound ſeries found in the laſt article to a ſhorter and ſimpler expreſſion.

Let G be equal to $\frac{1}{r-1} - \frac{1}{r^n \times r-1}$, or to the ſum of the terms of the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. continued to n terms, or to the term $\frac{1}{r^n}$; and let H be equal to $G - \frac{1}{r^n}$, or to the ſame geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. continued to the term $\frac{1}{r^{n-1}}$ inſtead of the term $\frac{1}{r^n}$. Then will the foregoing expreſſion $\frac{a}{r-1} \times \overline{1 - \frac{1}{r^n}} - \frac{d}{\overline{r-1}^2} + \frac{d}{r^{n-1} \times \overline{r-1}^2} + \frac{nd-d}{r^n \times r-1}$ be $= aG - \frac{dH}{r-1} + \frac{nd-d}{r^n \times r-1} = aG - \frac{d}{r-1} \times \frac{\overline{G-1}}{r^n} + \frac{nd-d}{r^n \times r-1} = aG - \frac{dG}{r-1} + \frac{d}{r^n \times r-1} + \frac{nd-d}{r^n \times r-1} = aG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$. Therefore the compound ſeries $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7} +$ &c. $+ \frac{a-nd+d}{r^n}$ is $= aG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$. QE I.

CXXXVII. I will now proceed to apply this expreſſion to the computation of a part of the ſeries $\frac{P^{\text{I}}}{r} + \frac{P^{\text{II}}}{r^2} + \frac{P^{\text{III}}}{r^3} + \frac{P^{\text{IV}}}{r^4} + \frac{P^{\text{V}}}{r^5} + \frac{P^{\text{VI}}}{r^6} + \frac{P^{\text{VII}}}{r^7} +$ &c. in the caſe of an annuity for a life of the age of 10 years according to Monſieur de Parcieux's table of the probabilities of the duration of human life; of the value of which annuity we have already exhibited the full computation above in Art. 94.

An

An example of the ſummation of a compound ſeries of this form, to wit, $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7} + \&c.$ *continued to n terms, or to the term* $\frac{a-nd+d}{r^n}$, *by means of the expreſſion* $aG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$; *in which the capital letter G is put for* $\frac{1}{r-1} \times \frac{1}{r^n \times r-1}$, *or the ſum of the terms of the geometrical progreſſion* $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c.$ *continued to n terms, or to the term* $\frac{1}{r^n}$.

CXXXVIII. The tenth, eleventh, twelfth, and other following terms of the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7}$ + &c. in Art. 94, as far as the 27th term, or $\frac{P^{XXVII}}{r^{27}}$, are as follows, to wit, $\frac{814}{r^{10}} + \frac{806}{r^{11}} + \frac{798}{r^{12}} + \frac{790}{r^{13}} + \frac{782}{r^{14}} + \frac{774}{r^{15}} + \frac{766}{r^{16}} + \frac{758}{r^{17}} + \frac{750}{r^{18}} + \frac{742}{r^{19}} + \frac{734}{r^{20}} + \frac{726}{r^{21}} + \frac{718}{r^{22}} + \frac{710}{r^{23}} + \frac{702}{r^{24}} + \frac{694}{r^{25}} + \frac{686}{r^{26}} + \frac{678}{r^{27}}$, in which the numerators 814, 806, 798, 790, &c. decreaſe continually by the ſame common difference, 8. This ſeries is equal to the product of $\frac{1}{r^9} \times$ the ſeries $\frac{814}{r} + \frac{806}{r^2} + \frac{798}{r^3} + \frac{790}{r^4} + \frac{782}{r^5} + \frac{774}{r^6} + \frac{766}{r^7} + \frac{758}{r^8} + \frac{750}{r^9} + \frac{742}{r^{10}} + \frac{734}{r^{11}} + \frac{726}{r^{12}} + \frac{718}{r^{13}} + \frac{710}{r^{14}} + \frac{702}{r^{15}} + \frac{694}{r^{16}} + \frac{686}{r^{17}} + \frac{678}{r^{18}}$; and this

this laſt ſeries is of the ſame form with the ſeries $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7} + \&c. + \frac{a-nd+d}{r^n}$, which is the ſubject of the foregoing articles; and conſequently it may be ſummed by means of the expreſſion $aG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$. This may be done in the following manner.

CXXXIX. Here r is $= 1.035$; and n, (or the number of terms in the ſeries,) is 18; and a, (the numerator of the firſt term,) is 814; and d, (the common difference of the numerators,) is 8. Therefore the expreſſion $aG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$ is $= 814G - \frac{8G}{.035} + \frac{18 \times 8}{1.035|^{18} \times .035}$ $= 814G - 228.571,428G + \frac{144}{1.035|^{18} \times .035} = 585.428,572G + \frac{4114.285,714}{1.035|^{18}}$. Now it appears from Mr. Smart's firſt table of compound intereſt, page 52, that the 18th power of 1.035 is 1.857,489; and it appears by his fourth table of compound intereſt, page 76, that G, or the value of the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^{18}}$, when r is $= 1.035$, is 13.189.681. Therefore $585.428,572G + \frac{4114.285,714}{1.035|^{18}}$ is $= 585.428,572 \times 13.189,681 + \frac{4114.285,714}{1.857,489} = 7721.616,112 + 2214.971,778 = 9936.587,890$. Therefore the ſeries $\frac{814}{r} + \frac{806}{r^2} + \frac{798}{r^3} + \frac{790}{r^4} + \frac{782}{r^5} + \frac{774}{r^6} + \frac{766}{r^7} + \frac{758}{r^8} + \frac{750}{r^9} + \frac{742}{r^{10}} + \frac{734}{r^{11}} + \frac{726}{r^{12}} + \frac{718}{r^{13}} + \frac{710}{r^{14}} + \frac{702}{r^{15}} + \frac{694}{r^{16}} + \frac{686}{r^{17}} + \frac{678}{r^{18}}$ is equal to 9936.587,890. Conſequently

quently the ſeries $\frac{814}{r^{10}} + \frac{806}{r^{11}} + \frac{798}{r^{12}} + \frac{790}{r^{13}} + \frac{782}{r^{14}} + \frac{774}{r^{15}}$ $+ \frac{766}{r^{16}} + \frac{758}{r^{17}} + \frac{750}{r^{18}} + \frac{742}{r^{19}} + \frac{734}{r^{20}} + \frac{726}{r^{21}} + \frac{718}{r^{22}} + \frac{710}{r^{23}}$ $+ \frac{702}{r^{24}} + \frac{694}{r^{25}} + \frac{686}{r^{26}} + \frac{678}{r^{27}}$ is $= 9936.587,890 \times \frac{1}{r^9} = \frac{9936.587,890}{\overline{1.035}|^9}$ = (by Mr. Smart's firſt table of compound intereſt, page 52,) $\frac{9936.587,890}{1.362,897} = 7290.784,182$. Q E I.

CXL. If the terms of this laſt ſeries be computed ſeparately, (as they have been above in Art. 94) they will be equal to the following numbers, to wit, 577.05274 + 552.06164 + 528.10044 + 505.12600 + 483.10396 + 461.99286 + 441.75220 + 422.35760 + 403.77000 + 385.95130 + 368.87904 + 352.52382 + 336.84970 + 321.82880 + 307.44090 + 293.65916 + 280.45738 + 267.81678; the ſum of which is 7290.72432, which agrees with the foregoing value of them found by means of the expreſſion $aG \frac{-dG}{r-1} + \frac{nd}{\overline{r^n \times r-1}}$ together with the multiplication by $\frac{1}{r^9}$, to wit, 7290.784,182, to five places of figures, and thereby confirms the truth of the ſaid expreſſion. See theſe numbers, 577.05274 + 552.06164 + &c. above in Art. 94, pages 105, 106.

CXLI. When n, (or the numerator of the firſt and greateſt term of the ſeries $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6}$ $+ \frac{a-6d}{r^7}$ + &c. $+ \frac{a-nd+d}{r^n}$) is an exact multiple of d (the common difference of the numerators of the ſeveral terms,) and the ſeries is continued ſo far as to make $a - nd + d$, or $a - \overline{|n-1|} \times d$, (the numerator of its laſt term) equal to a, or $\overline{n-1} \times d$ equal to a, the ſeries will

A particular caſe of the foregoing compound ſeries.

be as follows, to wit, $\frac{nd-d}{r} + \frac{nd-d-d}{r^2} + \frac{nd-d-2d}{r^3} + \frac{nd-d-3d}{r^4}$ $+ \frac{nd-d-4d}{r^5} + \frac{nd-d-5d}{r^6} + \frac{nd-d-6d}{r^7}$ + &c. $+ \frac{nd-d-nd+d}{r^n}$, or $\frac{nd-d}{r} + \frac{nd-2d}{r^2} + \frac{nd-3d}{r^3} + \frac{nd-4d}{r^4} + \frac{nd-5d}{r^5} + \frac{nd-6d}{r^6}$ $+ \frac{nd-7d}{r^7}$ + &c. $+ \frac{nd-nd}{r^n}$, or $+ \frac{o}{r^n}$; the laſt term $\frac{nd-nd}{r^n}$, being equal to o, and being here ſet down only for the ſake of analogy and uniformity, and to make the ſeries appear to conſiſt of n terms, as it did in the former caſe, though in reality it contains in this caſe only $n-1$ terms. For ſuch a preſervation of the general forms of expreſſions in caſes that do not ſtrictly come under them, is often found to be of uſe in theſe ſciences.

The expreſſion of its ſum in that caſe.

CXLII. Now, when a is thus $= nd-d$, the expreſſion $aG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$, obtained in Art. 136, will be $= \overline{nd-d} \times G - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$ $= ndG - dG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1} = nd \times \left| \frac{1}{r-1} - \frac{1}{r^n \times r-1} \right. - dG$ $- \frac{dG}{r-1} + \frac{nd}{r^n \times r-1} = \frac{nd}{r-1} - \frac{nd}{r^n \times r-1} - dG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$ $= \frac{nd}{r-1} - dG - \frac{dG}{r-1} = \frac{nd}{r-1} - \frac{dG \times \overline{r-1}}{r-1} - \frac{dG}{r-1} = \frac{nd}{r-1}$ $- \frac{rdG+dG}{r-1} - \frac{dG}{r-1} = \frac{nd-rdG}{r-1}$. Therefore the ſeries $\frac{nd-d}{r}$ $+ \frac{nd-2d}{r^2} + \frac{nd-3d}{r^3} + \frac{nd-4d}{r^4} + \frac{nd-5d}{r^5} + \frac{nd-6d}{r^6} + \frac{nd-7d}{r^7}$ + &c. $+ \frac{nd-nd}{r^n}$, or $+ \frac{o}{r^n}$, is equal to $\frac{nd-rdG}{r-1}$, or to $d \times \left| \frac{n-rG}{r-1} \right.$; which is a very ſhort and ſimple expreſſion.

CXLIII. And

CXLIII. And, if d, the common difference of the numerators of the terms in the laſt-mentioned ſeries $\frac{nd-d}{r} + \frac{nd-2d}{r^2} + \frac{nd-3d}{r^3} + \frac{nd-4d}{r^4} + \frac{nd-5d}{r^5} + \frac{nd-6d}{r^6} + \frac{nd-7d}{r^7} + \&c. + \frac{nd-nd}{r^n}$, is $= 1$, both the ſaid ſeries itſelf and the expreſſion $\frac{nd-rdG}{r-1}$ for the ſum of all its terms, will be thereby ſtill further ſimplified. For the ſeries will then be $= \frac{n-1}{r} + \frac{n-2}{r^2} + \frac{n-3}{r^3} + \frac{n-4}{r^4} + \frac{n-5}{r^5} + \frac{n-6}{r^6} + \frac{n-7}{r^7} + \&c. + \frac{n-n}{r^n}$; and the expreſſion $\frac{nd-rdG}{r-1}$ will be $= \frac{n-rG}{r-1}$. And conſequently the ſeries $\frac{n-1}{r} + \frac{n-2}{r^2} + \frac{n-3}{r^3} + \frac{n-4}{r^4} + \frac{n-5}{r^5} + \frac{n-6}{r^6} + \frac{n-7}{r^7} + \&c. + \frac{n-n}{r^n}$ is $= \frac{n-rG}{r-1}$.

A reduction of the ſeries obtained in Art. 141 to a more ſimple form.

The expreſſion of the ſum of the ſeries reduced ſeries.

CXLIV. Therefore, if we divide both the laſt ſeries $\frac{n-1}{r} + \frac{n-2}{r^2} + \frac{n-3}{r^3} + \frac{n-4}{r^4} + \frac{n-5}{r^5} + \frac{n-6}{r^6} + \frac{n-7}{r^7} + \&c. + \frac{n-n}{r^n}$ and the expreſſion $\frac{n-rG}{r-1}$, (to which it is equal) by n, the quotients thence ariſing will be equal; that is, the ſeries $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} + \&c. + \frac{n-n}{nr^n}$ is $\left(= \frac{\frac{n-r}{n}G}{r-1}\right) = \frac{1-\frac{r}{n}G}{r-1}$.

An expreſſion for the ſum of a ſeries derived from the ſeries in the laſt article by dividing its terms by n.

Mr. De Moivre's expreſſion for the ſum of the terms of the laſt ſeries.

CXLV. If the letter P be uſed, inſtead of the letter G, to denote the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^n}$, the expreſſion $\frac{1 - \frac{rP}{n}}{r-1}$ will be equal to the ſum of all the terms of the ſeries $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} + \&c. + \frac{n-n}{nr^n}$.

CXLVI. This laſt expreſſion $\frac{1 - \frac{rP}{n}}{r-1}$ is that which Mr. De Moivre has given us for the purpoſe of ſumming the laſt ſeries $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \&c. + \frac{n-n}{nr^n}$ in his treatiſe of Annuities on Lives, Part 2d, pages 310, 311, of the third edition publiſhed in quarto, at the end of his treatiſe on the doctrine of Chances, in the year 1756. And in the ſame place he has given us a ſynthetick demonſtration of the truth of this expreſſion, or of its equality with the ſaid ſeries $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \&c. + \frac{n-n}{nr^n}$. But he has not informed us by what ſteps, or in what manner, he diſcovered that this expreſſion was equal to the ſaid ſeries, but has alledged as an excuſe for this omiſſion, that *the reaſonings that led him to this expreſſion require ſomething more than an ordinary ſkill in the doctrine of ſerieſes*. He probably therefore had deduced it from ſome more abſtruſe propoſitions in that doctrine than that which we have had recourſe to for the ſame purpoſe in the foregoing articles, which is only the well-known theorem that was demonſtrated in Art. 80, that the ſum of the terms of a decreaſing geometrical progreſſion, $A + B + C + D + E$, is equal to the fraction $\frac{AA - BE}{A - B}$. His ſynthetick demonſtration of the truth of this expreſſion is for the moſt part very clear and ſatisfactory. But in the laſt ſtep

He has given us a ſynthetick demonſtration of the truth of the ſaid expreſſion.

Obſervations on the ſaid ſynthetick demonſtration.

ſtep of it, I think there is ſome obſcurity. For, inſtead of dividing the ſeries $\frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} - \frac{1}{nr^7}$ — &c. $- \frac{1}{nr^{n-1}}$ (which conſiſts of only a finite number of terms, to wit, n terms,) by $r-1$, as the courſe of the demonſtration requires, he multiplies it by the infinite ſeries $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. *ad infinitum*, which is equal to $\frac{1}{r-1}$. This multiplication is certainly equivalent to a diviſion by $r-1$; and, I imagine, Mr. De Moivre conſidered it as a ſomewhat ſimpler and eaſier operation. But in this he ſeems on the preſent occaſion to have been miſtaken. For it is hardly, if at all, eaſier than the diviſion by that very ſhort and convenient diviſor, $r-1$; and it is certainly much leſs ſimple, becauſe it introduces to the reader's notice a ſeries conſiſting of an infinite number of terms in a matter relating to a ſeries conſiſting of only a finite number of terms. And it is natural to ſuppoſe that the product ariſing by the multiplication of a finite ſeries by an infinite one muſt be an infinite ſeries: and ſo in truth it muſt, unleſs, after a certain number of terms in the firſt part of it, the ſeveral numbers, that compoſe each of the ſubſequent terms, are equal to each other and marked with contrary ſigns + and —, ſo as thereby to deſtroy one another and make the whole term equal to 0; in which caſe the ultimate product (when properly collected by the addition of all the ſeparate products, or horizontal lines of terms, of which it is compoſed,) may poſſibly be a ſeries conſiſting of a finite number of terms, notwithſtanding the number of the ſeveral products, of which it is compoſed, and conſequently the number of terms in thoſe products, may be infinite. And this is in truth the caſe with the ſeries that is the product of the multiplication of the finite ſeries $\frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6}$ — &c. $- \frac{1}{nr^{n-1}}$ by the infinite ſeries $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. *ad infinitum*. Though the number of the ſeparate products of which it is compoſed, and conſequently the number of terms in thoſe products, is infinite, it is nevertheleſs a finite ſeries, or conſiſts of only a finite number of terms; as will appear upon a cloſe and attentive examination of its nature and conſtitution. But Mr. De Moivre does not ſhew that it will be ſo, nor even aſſert that the number of its terms will not be infinite, though the number of thoſe of one of its factors is ſo: inſo-

much

much that a reader who does not conſider the ſubject with more than ordinary care, when he has gone through the whole demonſtration, and is come to the laſt ſeries $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} +$ &c. may very well be at a loſs to know, whether the number of its terms be finite or infinite. But, if Mr. De Moivre had added the neceſſary obſervations to explain this matter and to ſhew that the number of terms in the ſaid ultimate ſeries will only be finite, it would ſtill have been an impropriety to introduce an infinite ſeries in a demonſtration relating to a finite one, when it might be ſo eaſily avoided. I ſhall therefore, in preſenting my reader with Mr. De Moivre's ſynthetick demonſtration of this propoſition, take the liberty of changing that ſtep in it, and dividing the ſeries $\frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} -$ &c. $- \frac{1}{nr^{n-1}}$ by $r-1$, inſtead of multiplying it by the infinite ſeries $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. *ad infinitum*. With this ſingle change that demonſtration is as follows.

CXLVII. *A ſynthetick demonſtration that the expreſſion* $\frac{1 - \frac{rP}{n}}{r-1}$ (*in which P denotes the ſum of the terms of the geometrical progreſſion* $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. $+ \frac{1}{r^n}$) *is equal to the ſum of the terms of the ſeries* $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} +$ &c. $+ \frac{n-n}{nr^n}$.

Since P is $= \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. $+ \frac{1}{r^n}$, we ſhall have $rP = \frac{r}{r} + \frac{r}{r^2} + \frac{r}{r^3} + \frac{r}{r^4} + \frac{r}{r^5} + \frac{r}{r^6} + \frac{r}{r^7}$

$+\frac{r}{r^7}$ +&c. $+\frac{r}{r^n} = 1+\frac{1}{r} +\frac{1}{r^2} +\frac{1}{r^3} +\frac{1}{r^4} +\frac{1}{r^5} +\frac{1}{r^6}$ +&c. $+\frac{1}{r^{n-1}}$, and $\frac{r}{n}P = \frac{1}{n} +\frac{1}{nr} +\frac{1}{nr^2} +\frac{1}{nr^3} +\frac{1}{nr^4} +\frac{1}{nr^5} +\frac{1}{nr^6}$ +&c. $+\frac{1}{nr^{n-1}}$.

Therefore $1-\frac{rP}{n}$ is $= 1-\frac{1}{n} -\frac{1}{nr} -\frac{1}{nr^2} -\frac{1}{nr^3} -\frac{1}{nr^4} -\frac{1}{nr^5} -\frac{1}{nr^6}$ -&c. $-\frac{1}{nr^{n-1}} = \frac{n-1}{n} -\frac{1}{nr} -\frac{1}{nr^2} -\frac{1}{nr^3} -\frac{1}{nr^4} -\frac{1}{nr^5} -\frac{1}{nr^6}$ -&c. $-\frac{1}{nr^{n-1}}$; and, dividing both sides by $r-1$, $\frac{1-\frac{rP}{n}}{r-1} = \frac{n-1}{nr} +\frac{n-2}{nr^2} +\frac{n-3}{nr^3} +\frac{n-4}{nr^4} +\frac{n-5}{nr^5} +\frac{n-6}{nr^6} +\frac{n-7}{nr^7}$ +&c. $+\frac{n-n}{nr^n}$. QED.

CXLVIII. The diviſion of the ſeries $\frac{n-1}{n} -\frac{1}{nr} -\frac{1}{nr^2} -\frac{1}{nr^3} -\frac{1}{nr^4} -\frac{1}{nr^5} -\frac{1}{nr^6}$ -&c. $-\frac{1}{nr^{n-1}}$ by $r-1$ may be performed as follows.

$r-1$)

The operation of diviſion mentioned in the foregoing article.

$$r-1\Big)\ \frac{n-1}{n}\ -\frac{1}{nr}\ -\frac{1}{nr^2}\ -\frac{1}{nr^3}\ -\&c.\qquad \Big(\frac{n-1}{nr}+\frac{n-2}{nr^2}+\frac{n-3}{nr^3}+\frac{n-4}{nr^4}+\&c.$$

$$\frac{n-1}{n}\ -\Big|\frac{n-1}{nr}$$

$$*\ +\frac{n-2}{nr}\ -\frac{1}{nr^2}$$

$$+\frac{n-2}{nr}\ -\Big|\frac{n-2}{nr^2}$$

$$*\ +\frac{n-3}{nr^2}\ -\frac{1}{nr^3}$$

$$+\frac{n-3}{nr^2}\ -\Big|\frac{n-3}{nr^3}$$

$$*\ +\frac{n-4}{nr^3}$$

Of the uſe made of the foregoing expreſſion by Mr. De Moivre in the computation of life-annuities.

CXLIX. Mr. De Moivre, in his treatiſe of Life-annuities, makes great uſe of this expreſſion $\frac{1-rP}{r-1}$. For he employs it in the ſolution of his firſt and moſt important problem, or the computation of the value of an annuity upon a life of any given age. And with this view he adopts an hypotheſis, concerning the probabilities of the duration of human life, which is fitted to make that expreſſion applicable to his purpoſe, and which he conceives to be ſo little different from the real eſtimate of thoſe probabilities, as deduced from obſervations by Dr. Halley, Monſieur de Parcieux, and others, that it will make the values of life-annuities computed by it very nearly the ſame as they would be, if computed ſtrictly from tables founded on obſervations in the manner that has been above explained. This hypotheſis is as follows.

Of

Of Mr. De Moivre's Hypotheſis concerning the decrements of human life.

CL. We have ſeen above, in Art. 126, that, both in Mr. Kerſſeboom's and Monſieur de Parcieux's tables of the probabilities of the duration of human life, the numbers of perſons dying in a year, or the decrements of human life, (as they are called,) continue the ſame for ſeveral years together in ſeveral different ſtages, or periods, of life from the age of 11 or 12 years to the age of 75 or 80 years. And in this interval (from the age of 11 or 12 years to the age of 75 or 80 years,) the number of perſons dying in a year during one of theſe ſtages, or periods, is not either conſtantly greater, or conſtantly leſs, than the number of thoſe who die in a year during the next older period of life, but in ſome parts of human life is greater than the ſaid next number, and in other parts of it is leſs. Thus, for example, in Monſieur de Parcieux's table of probabilities, it appears that from the age of 11 years to the age of 16 years the number of perſons who die in every year is 6; and from the age of 16 years to the age of 20 years the number of perſons who die in every year is 7; and from the age of 20 years to the age of 37 years, the number of perſons who die in every year is 8: and thus far the number of perſons dying every year in theſe three ſucceſſive ſtages, or periods, of human life, is found to increaſe as the period of human life advances, or becomes older. But in the next period of life, namely, from the age of 37 years to the age of 46 years, the number of perſons dying in a year is only 7; after which, in the interval between the age of 46 years and the age of 75 years, it again increaſes to 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 20 perſons in a year; and then it decreaſes again to 19 and 18 perſons in the next five years, or from the age of 75 years to the age of 80 years. And the ſame variation, from a greater number to a leſſer, and from a leſſer number to a greater, may be obſerved in the decrements of human life, or the number of perſons dying in a year, in different ſtages, or periods, of it, in Mr. Kerſſeboom's table above-mentioned, and in Dr. Halley's table of probabilities deduced from the obſervations made at Breſlaw, and in ſeveral other tables of the ſame nature.

The decrements of human life in ſome periods of life become greater, and in others become leſs, as the age of life advances.

CLI. From this obſervation on the courſe of the decrements of human life Mr. De Moivre was led to conjecture that, if the ſaid decrements were ſuppoſed to be equal in every year, throughout the whole extent of life, as well as during particular periods of it, and the values of life-annuities were computed upon that ſuppoſition, they would be very nearly equal to the

Mr. De Moivre's conjectural ſuppoſition derived from the foregoing obſervation.

the true values of them when computed in ſtrict conformity to Dr. Halley's ſaid Breſlaw table, or any other table of the probabilities of the duration of human life that was grounded on actual obſervations. For, as, by ſuch a reduction of the irregular variation of the decrements of life to an abſolute equality of the ſaid decrements throughout life, the values of ſome of the future payments of an annuity for any given age, in ſome particular years of life, would be made greater than they ought to be, ſo, it is evident, the values of ſome others of the future payments of it, in other years of human life, would be made leſs than they ought to be; and theſe contrary variations in the values of different future payments of the annuity in different parts of human life, Mr. De Moivre ſuppoſed, would pretty nearly counter-balance each other, and make the ſum of all the ſaid future payments, or the value of the whole annuity throughout the whole extent of life, pretty nearly the ſame as it would have been if it had been regularly computed in ſtrict conformity to Dr. Halley's, or ſome other, table of the probabilities of human life derived from actual obſervations, in the manner above deſcribed and exemplified in Art. 94. And he tells us that, upon trial, he found this conjecture to be well-founded: for that, having computed the value of an annuity for a life of a certain age upon the ground of this hypotheſis, and compared it with the value of the ſame annuity for the ſame life, as computed by Dr. Halley in ſtrict conformity to the Breſlaw table of probabilities, he found theſe values to be ſo very little different from each other that, for all uſeful and practical purpoſes, they might well be conſidered as the ſame, and conſequently that, in the buſineſs of computing the values of life-annuities, he might ſafely neglect the tables of probabilities deduced from obſervations, and proceed upon the ground of his own Hypotheſis.

CLII. Mr. De Moivre does not inform us what was the age of the life for which he calculated an annuity upon the ground of his Hypotheſis, in order to compare it with Dr Halley's value of the ſame annuity computed ſtrictly from the Breſlaw table of probabilities. But it ſeems not unlikely to have been an annuity for a life of the age of 10 years, that being one of thoſe which Dr. Halley had computed and ſet down in his tract above-mentioned, and being in ſome degree fitter than the others there ſet down for the purpoſe of the ſaid compariſon. The ſeveral annuities there ſet down as having been accurately computed from the Breſlaw table of probabilities, are for lives of the ages of one year, 5 years, 10 years, 15 years, 20 years, 25, 30, 35, 40, 45, 50, 55, 60, 65, and 70, years. They were computed upon a ſuppoſition that the intereſt of money was 6 per cent. as at that time (in the year, 1692,) it generally was. This little table of the values of life-annuities is as follows.

Of Dr. Halley's table of the values of life-annuities for lives of certain ages, computed from the Breſlaw table of probabilities.

TABLE

TABLE VI.

Containing the values of an annuity of one pound a year for the lives of perſons of the ages of 1 year, 5 years, 10 years, 15 years, 20 years, and every following age of human life that exceeds the next preceeding age by 5 years, as far as the age of 70 years: computed by Dr. Halley from the Breſlaw table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 6 per cent.

Years of Age.	*Values of an annuity of one pound.*	*Years of Age.*	*Values of an annuity of one pound.*
	£		£
1	10.28	40	10.57
5	13.40	45	9.91
10	13.44	50	9.21
15	13.33	55	8.51
20	12.78	60	7.60
25	12.27	65	6.54
30	11.72	70	5.32
35	11.12		

This was, I believe, the firſt table of the values of life-annuities that ever was publiſhed.

Of the manner of comparing the conjectural probabilities of the duration of human life, reſulting from Mr. De Moivre's Hypotheſis, with the real probabilities of the ſame as exhibited in Monſieur de Parcieux's table.

CLIII. That we may be the better able to judge of the degree in which Mr. De Moivre's Hypotheſis above-mentioned, concerning the decrements of human life, approaches to, or varies from, the real decrements of life, as repreſented in Monſieur de Parcieux's table of probabilities, it will be proper to form a new table of the numbers of perſons that would be living at the ends of the ſeveral years of human life after the age of 3 years, (which is the youngeſt age in Monſieur de Parcieux's table,) out of the ſame original number of perſons living at the ſaid age of 3 years as are repreſented in Monſieur de Parcieux's table to be living at that age, that is, out of 1000 perſons of the age of 3 years, if the whole number were to die in the courſe of the ſame number of years in which they are repreſented to be all dead in Monſieur de Parcieux's table, that is, in the courſe of 92 years, but were to die in equal numbers in every year of that period, inſtead of dying in the numbers repreſented as dying in the ſeveral years of the ſame period in Monſieur de Parcieux's table. Now for this purpoſe it is neceſſary, in the firſt place, to divide the whole original number of perſons repreſented in Monſieur de Parcieux's table as living at the age of 3 years, that is, 1000, by 92, or the number of years in which all the ſaid 1000 lives are there repreſented to become extinct; becauſe, it is evident, the quotient of this diviſion will give us the number of perſons who would die in every year of the ſaid period of 92 years, upon Mr. De Moivre's ſuppoſition that that number were the ſame in every year. This quotient is 10.8697, which conſiſts of the whole number 10 and the decimal fraction .8697; which ſhews that it is not poſſible that the number of perſons dying every year in the ſaid period of 92 years, (from the age of 3 years to the age of 95 years,) out of the ſaid original number of 1000 perſons living at the age of 3 years, ſhould be always exactly the ſame. But, if we ſomewhat increaſe this original number, and make it 1012 inſtead of 1000, we ſhall thereby render it capable of an exact diviſion by 92, and the quotient will be 11. And this variation, I apprehend, will not be of much importance, notwithſtanding we do not ſuppoſe the other numbers in Monſieur de Parcieux's table of probabilities to be increaſed at the ſame time in the ſame proportion of 1012 to 1000, but to continue as before. For, if a table of the values of life-annuities were to be calculated from Monſieur de Parcieux's table of probabilities ſo altered in its firſt number only, all the ſaid values would be preciſely the ſame as before, except the value of an annuity for a life of the age of 3 years, which would be leſs than before in the proportion of $\frac{1}{1012}$ to $\frac{1}{1000}$, or 1000 to 1012. This variation, therefore, in the firſt number of Monſieur de Parcieux's ſaid table may well enough be made on the preſent occaſion for the ſake of avoiding the inconvenience and obſcurity that might ariſe from

from repreſenting the number of perſons who are ſuppoſed to die every year according to Mr. De Moivre's Hypotheſis by a number that involves a fraction, ſuch as the quotient 10.8697 above found.

CLIV. Let us therefore ſuppoſe this variation to be made in the firſt number of Monſieur de Parcieux's table of probabilities, but that all the other numbers in it continue as before. And from the firſt number of this table ſo increaſed, or from the number 1012, (which contains the number 11 exactly 92 times,) let us continually ſubtract the number 11, till the ſaid number 1012 be reduced to nothing. The table of numbers thence ariſing will repreſent the numbers of perſons ſuppoſed to be living at the ages of 4 years, 5 years, 6 years, and every following age of life that exceeds the age next preceeding by one year, as far as the age of 95 years, according to Mr. De Moivre's Hypotheſis. This table, together with Monſieur de Parcieux's table, with its firſt number increaſed from 1000 to 1012, as aforeſaid, will be as follows.

TABLE

TABLE VII.

Containing Monſieur de Parcieux's table of the probabilities of the duration of human life, from the age of 3 years to the age of 95 years, with the firſt number of it increaſed from 1000 to 1012; and likewiſe an artificial table fitted to the ſame original number 1012, in which the decrements of human life, or perſons dying in a year, are ſuppoſed to be always the ſame throughout the ſaid period of 92 years, or from the age of 3 years to the age of 95 years, agreeably to Mr. De Moivre's Hypotheſis.

Years of Age.	*Perſons living according to Monſieur de Parcieux's table, with an increaſe of the firſt number from 1000 to 1012.*	*Perſons living according to Mr. De Moivre's Hypotheſis.*	*Years of Age.*	*Perſons living according to Monſieur de Parcieux's table.*	*Perſons living according to Mr. De Moivre's Hypotheſis.*
3	1012	1012	26	766	759
4	970	1001	27	758	748
5	948	990	28	750	737
6	930	979	29	742	726
7	915	968	30	734	715
8	902	957	31	726	704
9	890	946	32	718	693
10	880	935	33	710	682
11	872	924	34	702	671
12	866	913	35	694	660
13	860	902	36	686	649
14	854	891	37	678	638
15	848	880	38	671	627
16	842	869	39	664	616
17	835	858	40	657	605
18	828	847	41	650	594
19	821	836	42	643	583
20	814	825	43	636	572
21	806	814	44	629	561
22	798	803	45	622	550
23	790	792	46	615	539
24	782	781	47	607	528
25	774	770	48	599	517

Years of Age.	Perſons living according to Monſieur de Parcieux's table.	Perſons living according to Mr. De Moivre's Hypotheſis.	Years of Age.	Perſons living according to Monſieur de Parcieux's table.	Perſons living according to Mr. De Moivre's Hypotheſis.
49	590	506	73	251	242
50	581	495	74	231	231
51	571	484	75	211	220
52	560	473	76	192	209
53	549	462	77	173	198
54	538	451	78	154	187
55	526	440	79	136	176
56	514	429	80	118	165
57	502	418	81	101	154
58	489	407	82	85	143
59	476	396	83	71	132
60	463	385	84	59	121
61	450	374	85	48	110
62	437	363	86	38	99
63	423	352	87	29	88
64	409	341	88	22	77
65	395	330	89	16	66
66	380	319	90	11	55
67	364	308	91	7	44
68	347	297	92	4	33
69	329	286	93	2	22
70	310	275	94	1	11
71	291	264	95	0	0
72	271	253			

CLV. It

An obſervation on the foregoing table of the real and conjectural probabilities of the duration of human life.

CLV. It appears from the foregoing table, that, from the age of 3 years to the age of 24 years, the numbers of perſons repreſented as living at the end of every year are greater in the arithmetical progreſſion 1012, 1001, 990, 979, &c. formed upon Mr. De Moivre's Hypotheſis, than in Monſieur de Parcieux's ſeries of numbers 1012, 970, 948, 930, 915, &c. and that at the age of 24 years they are nearly the ſame in both ſerieſes of numbers, being 781 and 782; and that, from the age of 24 years to the age of 74 years, the numbers in Mr. De Moivre's ſeries are leſs than thoſe in Monſieur de Parcieux's; and that at the age of 74 years they are equal in both ſerieſes, being 231 in both ſerieſes; and that then, after the age of 74 years, Mr. De Moivre's numbers become a ſecond time greater than Monſieur de Parcieux's, and continue ſo to the end of the table.

A concluſion from the foregoing obſervation concerning the value of an annuity for a life of the age of 74 years.

CLVI. It follows from the foregoing obſervation that, if the value of an annuity for a life of 74 years were to be computed according to both theſe ſerieſes of numbers, the value of it derived from Mr. De Moivre's numbers would be greater than the value of it derived from Monſieur de Parcieux's numbers. For the number P, or the number of perſons living at the given age, would be the ſame in both caſes, to wit, 231; and the numbers P^{I}, P^{II}, P^{III}, P^{IV}, P^{V}, P^{VI}, P^{VII}, &c. would be greater in the calculation grounded on Mr. De Moivre's ſeries than in that which was built on Monſieur de Parcieux's ſeries: and conſequently $\frac{\pounds 1}{P}$ x the ſeries

$$\frac{P^{\text{I}}}{r} + \frac{P^{\text{II}}}{r^2} + \frac{P^{\text{III}}}{r^3} + \frac{P^{\text{IV}}}{r^4} + \frac{P^{\text{V}}}{r^5} + \frac{P^{\text{VI}}}{r^6} + \frac{P^{\text{VII}}}{r^7} + \text{\&c.}$$

(which, by Art. 87, is equal to the value of the annuity for the given life,) would be greater in the former caſe than in the latter.

Concluſions from the ſame obſervation concerning the values of annuities for lives of other ages.

CLVII. But, if we were to compute the value of an annuity for 50 years, and no longer, dependent upon a life of the age of 24 years, by means of theſe two ſerieſes of numbers, the value of ſuch an annuity derived from Mr. De Moivre's numbers would be leſs than the other value of it derived from Monſieur de Parcieux's numbers. For in this caſe the value of P in the former calculation would be 781, and in the latter it would be 782, which is almoſt equal to 781; and the values of P^{I}, P^{II}, P^{III}, P^{IV}, P^{V}, P^{VI}, P^{VII}, &c. in the former calculation would be 770, 759, 748, 737, 726, 715, 704, &c. and in the latter calculation they

they would be 774, 766, 758, 750, 742, 734, 726, &c. which are reſpectively greater than the other numbers: and conſequently the expreſſion

$$\frac{\text{£}}{P} \times \text{the ſeries } \frac{P^{\text{I}}}{r} + \frac{P^{\text{II}}}{r^2} + \frac{P^{\text{III}}}{r^3} + \frac{P^{\text{IV}}}{r^4} + \frac{P^{\text{V}}}{r^5} + \frac{P^{\text{VI}}}{r^6} + \frac{P^{\text{VII}}}{r^7} + \&c.$$

when derived from Mr. De Moivre's numbers, will be leſs than the ſame expreſſion, when derived from Monſieur de Parcieux's numbers, or the value of an annuity for 50 years, dependent upon a life of the age of 24, will be leſs according to Mr. De Moivre's numbers than according to Monſieur de Parcieux's numbers.

CLVIII. But, if we continue the laſt annuity for a life of the age of 24 years to the end of life, or to 70 years from its commencement, inſtead of reſtraining it to 50 years, the values of the future contingent payments of it to be made in the laſt 20 years of its duration, or from the age of 74 to the age of 94 years, will be greater according to Mr. De Moivre's numbers than according to Monſieur de Parcieux's. And conſequently the value of a compleat life-annuity for a perſon of the age of 24 years, computed from Mr. De Moivre's numbers, will differ leſs from the value of it, computed from Monſieur de Parcieux's numbers, than the values of a limited annuity for only 50 years, depending upon the ſame life, computed according to the ſame two ſets of numbers, differ from each other; becauſe the exceſs of the values of the future payments to be made after the age of 74 years according to Mr. De Moivre's numbers, above the values of thoſe payments according to Monſieur de Parcieux's numbers, will, in ſome degree, make amends for the deficiencies of the values of the preceeding future payments, from the age of 24 years to the age of 74 years, according to Mr. De Moivre's numbers, below the values of the ſame payments according to Monſieur de Parcieux's numbers. But it will only do this *in ſome degree*, and not by any means compleatly; becauſe the values of the 20 laſt future payments of the annuity from the age of 74 years to the age of 94 years, are ſo much leſs (on account of the diſtance of time at which thoſe payments are to be made,) than the values of the 50 preceeding future payments of it from the age of 24 years to the age of 74 years, that it does not greatly ſignify towards obtaining the true value of the whole annuity for a life of the age of 24 years, by which of the two ſerieſes of numbers they are computed. And therefore, upon the whole, the value of a compleat life-annuity for a perſon of the age of 24 years, computed according to Mr. De Moivre's numbers, will ſtill be conſiderably leſs than its value, computed according to Monſieur de Parcieux's numbers, or than its true value. And, in general, it will be found that the value of an annuity for a life of any age under 45 years, computed according to Mr. De Moivre's numbers, will be leſs than its value computed from Monſieur

de Parcieux's numbers, or its true value. But, that we may be the better able to judge of this matter, I will now proceed to compute a ſmall table of the values of life-annuities (ſuch as that which was computed exactly from the Breſlaw table of probabilities by Dr. Halley, and which is inſerted above in Art. 152,) according to Mr. De Moivre's numbers, upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. which values we may afterwards compare with the values of the ſame annuities, as computed above (in Art. 108, 109, &c.—114) from the numbers of Monſieur de Parcieux's table.

The numbers of perſons living according to Mr. De Moivre's Hypotheſis in Table VII. may be reduced to ſmaller numbers.

CLIX. As the numbers in Mr. De Moivre's ſeries above-mentioned are, all of them, exact multiples of 11, they will all be diviſible by 11 without any remainders, and conſequently may be reduced to ſmaller numbers without altering their proportions to one another. Let them all be ſo divided by 11; and the numbers thence ariſing will be as follows, to wit, 92, 91, 90, 89, 88, 87, 86, 85, 84, 83, 82, 81, 80, 79, 78, 77, 76, 75, 74, 73, 72, 71, 70, 69, 68, 67, 66, 65, 64, 63, 62, 61, 60, 59, 58, 57, 56, 55, 54, 53, 52, 51, 50, 49, 48, 47, 46, 45, 44, 43, 42, 41, 40, 39, 38, 37, 36, 35, 34, 33, 32, 31, 30, 29, 28, 27, 26, 25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, which is an arithmetical progreſſion of the ſimpleſt nature poſſible, in which the terms continually decreaſe by the ſame common difference 1. Yet, as theſe numbers are to each other in exactly the ſame proportions as the former numbers 1012, 1001, 990, 979, 968, 957, 946, &c. (of which they are exact 11th parts,) it is evident that the values of any life-annuities of one pound deduced from the general expreſſion $\frac{\text{£ } 1}{P} \times$ the ſeries $\frac{P^{\text{I}}}{r} + \frac{P^{\text{II}}}{r^2} + \frac{P^{\text{III}}}{r^3} + \frac{P^{\text{IV}}}{r^4} + \frac{P^{\text{V}}}{r^5} + \frac{P^{\text{VI}}}{r^6} + \frac{P^{\text{VII}}}{r^7} +$ &c. by ſubſtituting in its terms, inſtead of P, P^{I}, P^{II}, P^{III}, P^{IV}, P^{V}, P^{VI}, P^{VII}, &c. any numbers taken from the ſeries 92, 91, 90, 89, 88, 87, 86, &c. will be the ſame with the values deduced from the ſame general expreſſion by ſubſtituting in its terms, inſtead of P, P^{I}, P^{II}, P^{III}, P^{IV}, P^{V}, P^{VI}, P^{VII}, &c. the correſponding numbers of the ſeries 1012, 1001, 990, 979, 968, 957, 946, &c.

CLX. If theſe new numbers are ſet down in a regular ſeries near the ages to which they reſpectively belong, and are ſuppoſed to begin from the age of one year, inſtead of beginning from the age of 3 years (as the numbers in the laſt table do,) they will be as follows.

TABLE

TABLE VIII.

Containing an artificial eſtimate of the probabilities of the duration of human life, ſimilar to that contained in the ſecond column of Table VII. *but expreſſed in the ſmalleſt numbers poſſible; derived from Mr. De Moivre's Hypotheſis that the numbers of perſons dying every year, out of any given original number, are always the ſame throughout the whole extent of human life, and formed upon a ſuppoſition that the utmoſt poſſible duration of human life is leſs than 95 years.*

Years of Age.	*Perſons living.*	*Years of Age.*	*Perſons living.*	*Years of Age.*	*Perſons living.*	*Years of Age.*	*Perſons living.*
1	94	25	70	49	46	73	22
2	93	26	69	50	45	74	21
3	92	27	68	51	44	75	20
4	91	28	67	52	43	76	19
5	90	29	66	53	42	77	18
6	89	30	65	54	41	78	17
7	88	31	64	55	40	79	16
8	87	32	63	56	39	80	15
9	86	33	62	57	38	81	14
10	85	34	61	58	37	82	13
11	84	35	60	59	36	83	12
12	83	36	59	60	35	84	11
13	82	37	58	61	34	85	10
14	81	38	57	62	33	86	9
15	80	39	56	63	32	87	8
16	79	40	55	64	31	88	7
17	78	41	54	65	30	89	6
18	77	42	53	66	29	90	5
19	76	43	52	67	28	91	4
20	75	44	51	68	27	92	3
21	74	45	50	69	26	93	2
22	73	46	49	70	25	94	1
23	72	47	48	71	24	95	0
24	71	48	47	72	23		

Of *the complement of Life.*

CLXI. In the foregoing table the number of perſons repreſented as living at any given age is always the ſame with the number of years which are neceſſary to be added to the number of years in the given age in order to make it equal to 95 years, or the period in the courſe of which it is ſuppoſed that all the perſons ſet down in the table as living at the ends of the ſeveral foregoing years, will be dead. Thus, the number of perſons living at the age of one year is 94; which, being added to one year, the age of the ſaid perſons, makes 95: and the number living at the age of 10 years is 85; which being added to 10, the number of years in the age of the ſaid perſons, is 95, as before. And the ſame obſervation is true throughout the table, to wit, that the number in the ſecond column, which expreſſes the perſons living at any given age, is always the complement of the number of years in the ſaid age to 95. This complement Mr. De Moivre calls *the complement of life:* ſo that, according to his Hypotheſis, the complement of life at any given age is always equal to the number of perſons living at the ſame age.

A computation of the values of an annuity of one pound a year for the lives of perſons of ſeveral different ages, according to the foregoing artificial table of the probabilities of the duration of human life, grounded on Mr. De Moivre's Hypotheſis, and upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ *per cent.*

For the life of a child of the age of one year.

CLXII. Let it now be required to find, by means of the foregoing artificial table of probabilities, the value of an annuity of one pound a year for the life of a child of the age of one year, when the intereſt of money is $3\frac{1}{2}$ per cent.

By Art. 87, the general expreſſion for the value of an annuity for a life of any given age is $\frac{\text{£}1}{P} \times$ the ſeries $\frac{P^{\text{I}}}{r} + \frac{P^{\text{II}}}{r^2} + \frac{P^{\text{III}}}{r^3} + \frac{P^{\text{IV}}}{r^4} + \frac{P^{\text{V}}}{r^5} + \frac{P^{\text{VI}}}{r^6} + \frac{P^{\text{VII}}}{r^7} +$ &c. in which P ſignifies the number of perſons living at the given age, P^{I} the number of perſons living at the end of the next year, P^{II} the number living at the end of the ſecond year, P^{III} the number living at the end of the third year, and ſo on to the utmoſt extent of human life.

Theſe numbers P, P^{I}, P^{II}, P^{III}, &c. are in the preſent caſe, and according to Mr. De Moivre's Hypotheſis, 94, 93, 92, 91, &c. Therefore the value of the annuity ſought is $= \frac{\text{£}1}{94} \times$ the ſeries $\frac{93}{r} + \frac{92}{r^2} + \frac{91}{r^3} + \frac{90}{r^4} + \frac{89}{r^5} + \frac{88}{r^6} + \frac{87}{r^7} +$ &c. continued to 93 terms, or to the term $\frac{1}{r^{93}}$

$\frac{1}{r^{93}} = \frac{£}{1}$ × the series $\frac{93}{94r} + \frac{92}{94r^2} + \frac{91}{94r^3} + \frac{90}{94r^4} + \frac{89}{94r^5} + \frac{88}{94r^6} + \frac{87}{94r^7}$ + &c. $+ \frac{1}{94r^{93}} = \frac{£}{1}$ × the series $\frac{94-1}{94r} + \frac{94-2}{94r^2} + \frac{94-3}{94r^3} + \frac{94-4}{94r^4} + \frac{94-5}{94r^5} + \frac{94-6}{94r^6} + \frac{94-7}{94r^7}$ + &c. $+ \frac{94-93}{94r^{93}}$. Let one term more be added to this ſeries, to wit, the term $\frac{94-94}{94r^{94}}$, in order to bring the ſeries to the ſame form as the ſeries $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7}$ + &c. $+ \frac{n-n}{nr^n}$, in Art. 144. Such addition of the term $\frac{94-94}{94r^{94}}$ will make no alteration in the value of the ſeries to which it is added, becauſe the ſaid term is $= \left(\frac{0}{94r^{94}} \text{ or}\right) 0$. Therefore the value of the annuity ſought will be $= \frac{£}{1}$ × the ſeries $\frac{94-1}{94r} + \frac{94-2}{94r^2} + \frac{94-3}{94r^3} + \frac{94-4}{94r^4} + \frac{94-5}{94r^5} + \frac{94-6}{94r^6} + \frac{94-7}{94r^7}$ + &c. $+ \frac{94-94}{94r^{94}}$. But it has been ſhewn in Art. 144, that, if n be any whole number whatſoever, the ſeries $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7}$ + &c. $+ \frac{n-n}{nr^n}$ will be equal to the fraction $\frac{1 - r\,\overline{G}^{n}}{r-1}$, in which G denotes the ſum of the terms of the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. $+ \frac{1}{r^n}$. Therefore the ſeries $\frac{94-1}{94r} + \frac{94-2}{94r^2} + \frac{94-3}{94r^3} + \frac{94-4}{94r^4} + \frac{94-5}{94r^5} + \frac{94-6}{94r^6} + \frac{94-7}{94r^7}$ + &c. $+ \frac{94-94}{94r^{94}}$ will be $= \frac{1 - r\,\overline{94}^{n}}{r-1}$; that is, (becauſe the intereſt of money is ſuppoſed to be $3\frac{1}{2}$ per

cent.

cent. and consequently r is $= 1.035$,) the said series will be $= \frac{1 - \frac{1.035}{94} G}{.035}$, in which G will denote the sum of the terms of the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. $+ \frac{1}{r^{94}}$, or $\frac{1}{1.035} + \frac{1}{1.035|^2} + \frac{1}{1.035|^3} + \frac{1}{1.035|^4}$ $+ \frac{1}{1.035|^5} + \frac{1}{1.035|^6} + \frac{1}{1.035|^7}$ + &c. $+ \frac{1}{1.035|^{94}}$, or continued to 94 terms. Now it appears from Mr. Smart's fourth table of compound interest, page 82, that G, or the sum of the terms of the said geometrical progression, is 27.445,426. Therefore $\frac{1.035}{94} G$ is $= \frac{1.035}{94} \times 27.445,426 = \frac{28.406,015,910}{94} = .302,191,658$; and $1 - \frac{1.035}{94} G$ is $= 1 - .302,191,658 = .697,808,342$; and $\frac{1 - \frac{1.035}{94} G}{1.035}$ is $= \frac{.697,808,342}{.035} = 19.908,809$. Therefore the value of an annuity of one pound a year for the life of a child of the age of one year, is, according to Mr. De Moivre's Hypothesis, $= \overset{£}{1} \times 19.908,809, = \overset{£}{19}.908,809$, or 19*l.* 18*s.* 2*d.* QEI.

CLXIII. By the like reasonings as those in the preceeding article it may be shewn, that, if n be the complement of life to any other given age whatsoever, the value of an annuity of one pound for a life of the said age, according to Mr. De Moivre's Hypothesis, will be $= \overset{£}{1} \times$ the fraction $\frac{1 - \frac{r}{n} G}{r - 1}$; as, I presume, the reader must easily perceive. I will now therefore proceed to calculate, by means of this expression, $\overset{£}{1} \times \frac{1 - \frac{r}{n} G}{r - 1}$, the values of an annuity of one pound for lives of the

several

ſeveral ages of 3 years, 5 years, 10 years, 15 years, 20 years, 25, 30, 35, 40, 45, 50, 55, 60, 65, and 70 years, according to Mr. De Moivre's Hypotheſis, to the end that we may be able to compare them with the values of the ſame annuity for lives of the ſame ages, as computed above in Art. 108, 109, &c.—114, from Monſieur de Parcieux's table.

CLXIV. When the given life is of the age of 3 years, we ſhall have For the life of a child of the age of 3 years.
n, or the complement of life, $(= 95-3) = 92$, and $\frac{r}{n} = \frac{r}{92} = \frac{1.035}{92}$, and G = the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} +$ &c. continued to $\frac{1}{r^{92}}$, or to 92 terms, $= \frac{1}{1.035} + \frac{1}{1.035|^2} + \frac{1}{1.035|^3} + \frac{1}{1.035|^4} +$ &c. continued to $\frac{1}{1.035|^{92}}$, or to 92 terms; which (by Mr. Smart's fourth table of compound intereſt, page 82,) is $= 27.365,227$. Therefore $\frac{r}{n}G$ will be $= \frac{1.035}{92} \times 27.365,227 = \frac{28.323,009,945}{92} = .307,858,803$; and $1 - \frac{r}{n}G$ will be $= 1.000,000,000 - .307,858,803 = .692,141,197$; and $\frac{1-\frac{r}{n}G}{r-1}$ will be $= \frac{.692,141,197}{r-1} = \frac{.692,141,197}{.035} = 19.775,462$. Conſequently $\overset{£}{1} \times \frac{1-\frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 3 years, according to Mr. De Moivre's Hypotheſis, will be $= \overset{£}{1} \times 19.775,462$, $= £19.775,462$, or 19*l.* 15*s.* 6*d.*

N. B. This value of the ſaid annuity is almoſt a quarter of a year's purchaſe leſs than its true value, as computed above from Monſieur de Parcieux's table of probabilities, that value being £19.987,654, or 19*l.* 19*s.* 9*d.* See Table III. Art. 118.

CLXV. When

For a life of the age of 5 years.

CLXV. When the given life is of the age of 5 years, we ſhall have n, or the complement of it to 95 years, $= 90$, and $\frac{r}{n} = \frac{r}{90} = \frac{1.035}{90}$, and $G =$ the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5}$ +&c. continued to $\frac{1}{r^{90}}$, or to 90 terms, and conſequently (by Mr. Smart's tables, page 82) $= 27.279,315$. Therefore $\frac{r}{n}G$ will be $= \frac{1.035}{90} \times 27.279,315 = \frac{28.234,091,025}{90} = .313,712,122$; and $1 - \frac{r}{n}G$ will be $= 1.000,000,000 - .313,712,122 = .686,287,878$; and $\frac{1 - \frac{r}{n}G}{r-1}$ will be $= \frac{.686,287,878}{r-1} = \frac{.686,287,878}{.035} = 19.608,225$. Therefore $\underset{1}{\pounds} \times \frac{1 - \frac{r}{n}G}{r-1}$, or the value of an annuity of one pound for a life of the age of 5 years, according to Mr. De Moivre's Hypotheſis, will be $= \underset{1}{\pounds} \times 19.608,225 = £19.608,225$, or 19*l.* 12*s.* 2*d.* Q E I.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £20.526,716, or 20*l.* 10*s.* 6*d.*$\frac{1}{2}$; which is greater than the value juſt now found for it by almoſt a year's purchaſe.

For a life of the age of 10 years.

CLXVI. When the given life is of the age of 10 years, we ſhall have n $(= 95 - 10) = 85$, and $\frac{r}{n} = \frac{1.035}{85}$, and $G =$ the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4}$ +&c. continued to $\frac{1}{r^{85}}$, or to 85 terms, which (by Mr. Smart's tables, page 82,) is $= 27.036,803$. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{85} \times 27.036,803 = \frac{27.983,091,105}{85} =$ .329,212,836;

.329,212,836; and $\frac{1-\frac{r}{n}G}{}$ is = 1.000,000,000 — .329,212,836 =

.670,787,164; and $\frac{\frac{1-\frac{r}{n}G}{}}{r-1}$ is = $\frac{.670,787,164}{.035}$ = 19.165,347.

Therefore $\overset{\pounds}{1} \times \frac{\overline{\left|\frac{1-\frac{r}{n}G}{}\right.}}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 10 years, according to Mr. De Moivre's Hypotheſis, is = $\overset{\pounds}{1} \times 19.165,347 = £19.165,347$, or 19*l.* 3*s.* 3*d.* $\frac{3}{4}$. Q. E. I.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £20 752,981, or 20*l.* 15*s.* 7*d.* $\frac{1}{2}$; which is greater than the value juſt now found for it by more than a year and a half's purchaſe.

CLXVII. When the given life is of the age of 15 years, we ſhall have n (= 95 — 15) = 80, and $\frac{r}{n} = \frac{1.035}{80}$, and G = the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5}$ + &c. continued to $\frac{1}{r^{80}}$, or to 80 terms, which (by Mr. Smart's tables, page 82) is = 26.748,775. For a life of the age of 15 years.

Therefore $\frac{r}{n}G$ is = $\frac{1.035}{80} \times 26.748,775 = \frac{27.684,982,125}{80}$ =

.346,062,276; and $\frac{1-\frac{r}{n}G}{}$ is = 1.000,000,000 — .346,062,276 =

.653,937,724; and $\frac{\frac{1-\frac{r}{n}G}{}}{r-1}$ is = $\frac{.653,937,724}{.035}$ = 18.683,934.

Therefore $\overset{\pounds}{1} \times \frac{\overline{\left|\frac{1-\frac{r}{n}G}{}\right.}}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 15 years, according to Mr. De Moivre's Hypotheſis, is = $\overset{\pounds}{1} \times 18.683,934 = £18.683,934$, or 18*l.* 13*s.* 8*d.* Q E I.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £20.137,194, or 20*l.* 2*s.* 9*d.* which is greater than the value juſt now found for it by almoſt a year and a half's purchaſe.

For a life of the age of 20 years.

CLXVIII. When the given life is of the age of 20 years, we ſhall have $n\ (=95-20)=75$; and $\frac{r}{n}=\frac{1.035}{75}$; and $G=$ the geometrical progreſſion $\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}+\frac{1}{r^5}+$ &c. continued to the term $\frac{1}{r^{75}}$, or to 75 terms, which (by Mr. Smart's tables, page 80,) is = 26.406,688. Therefore $\frac{r}{n}G$ is $=\frac{1.035}{75}\times 26.406,688=\frac{27.330,922,080}{75}$ $=.364,412,294$; and $1-\frac{r}{n}G$ is $=1.000,000,000,-.364,412,294=$ .635,587,706; and $\frac{1-\frac{r}{n}G}{r-1}$ is $=\frac{.635,587,706}{.035}=18.159,648$.

Therefore $\overset{£}{1}\times\frac{\overline{1-\frac{r}{n}G}}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 20 years, according to Mr. De Moivre's Hypotheſis, is $=\overset{£}{1}\times 18.159,648=$ £18.159,648, or 18*l.* 3*s.* 2*d.* $\frac{1}{4}$. Q E I.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £19.457,903, or 19*l.* 9*s.* 1*d.* $\frac{3}{4}$; which is greater than the value juſt now found for it by about a year and a quarter's purchaſe.

For a life of the age of 25 years.

CLXIX. When the given life is of the age of 25 years, we ſhall have $n\ (=95-25)=70$, and $\frac{r}{n}=\frac{1.035}{70}$, and $G=$ the geometrical progreſſion $\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}+$ &c. continued to the term $\frac{1}{r^{70}}$, or to 70 terms, which (by Mr. Smart's table, page 80,) is = 26.000,396.

Therefore

Therefore $\frac{r}{n}G$ is $= \frac{1.035}{70} \times 26..000,396 = \frac{26.910,411,070}{70} =$ .384,434,443; and $1 - \frac{r}{n}G$ is $= 1.000,000,000 - .384,434,443 =$ .615,565,557; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.615,565,557}{.035} = 17.587,587.$

Therefore $\overset{\pounds}{1} \times \frac{1 - \frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 25 years, according to Mr. De Moivre's Hypotheſis, is $= \overset{\pounds}{1} \times 17.587,587 = \pounds 17.587,587$, or 17*l.* 11*s.* 9*d.* QEI.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is $\pounds 18.827,070$, or 18*l.* 16*s.* 6*d.* $\frac{1}{2}$; which is greater than the value juſt now found for it by about a year and a quarter's purchaſe.

CLXX. When the given life is of the age of 30 years, we ſhall have n $(= 95 - 30) = 65$, and $\frac{r}{n} = \frac{1.035}{65}$, and $G =$ the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} +$ &c. continued to the term $\frac{1}{r^{65}}$, or to 65 terms, which (by Mr. Smart's tables, page 80,) is $=$ 25.517,849. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{65} \times 25.517,849 = \frac{26.410,974,715}{65}$ $=$ .406,322,672; and $1 - \frac{r}{n}G$ is $= 1.000,000,000 -$ .406,322,672 $=$ .593,677,328; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.593,677,328}{.035} = 16.962,209.$ For a life of the age of 30 years.

Therefore $\overset{\pounds}{1} \times \frac{1 - \frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 30 years, according to Mr. De Moivre's Hypotheſis, is $= \overset{\pounds}{1} \times 16.962,209 = \pounds 16.962,209$, or 16*l.* 19*s.* 3*d.*

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £18.095,844, or 18*l.* 1*s.* 11*d.* which is greater than the value juſt now found for it by about a year and half a quarter's purchaſe.

For a life of the age of 35 years.

CLXXI. When the given life is of the age of 35 years, we ſhall have $n\ (= 95 - 35) = 60$, and $\frac{r}{n} = \frac{1.035}{60}$, and $G =$ the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} +$ &c. $+ \frac{1}{r^{60}} =$ (by Mr. Smart's tables, page 80,) 24.944,734. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{60} \times 24.944,734 = \frac{25.817,799,690}{60} = .430,296,661$; and $1 - \frac{r}{n}G$ is $= 1.000,000,000 - .430,296,661 = .569,703,339$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.569,703,339}{.035} = 16.277,238$. Therefore $\overset{£}{1} \times \frac{1 - \frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 35 years, according to Mr. De Moivre's Hypotheſis, is $= \overset{£}{1} \times 16.277,238 =$ $= £16.277,238$, or 16*l.* 7*s.* 6*d.* $\frac{1}{2}$. QEI.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £17.240,588, or 17*l.* 4*s.* 9*d.* $\frac{3}{4}$, which is greater than the value juſt now found for it by ſomewhat leſs than a year's purchaſe.

For a life of the age of 40 years.

CLXXII. When the given life is of the age of 40 years, we ſhall have $n\ (= 95 - 40) = 55$, and conſequently $\frac{r}{n} = \frac{1.035}{55}$, and $G =$ the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} +$ &c. $+ \frac{1}{r^{55}} =$ (by Mr. Smart's tables, page 80,) 24.264,053. Therefore $\frac{r}{n}G$ is $=$

$\frac{1.035}{55}$

$\frac{1.035}{55} \times 24\,264{,}053 = \frac{25.113{,}294{,}855}{55} = .456{,}605{,}361$; and $1 - \frac{r}{n}G$ is $= 1.000{,}000{,}000 - .456{,}605{,}361 = .543{,}394{,}639$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.543{,}394{,}639}{.035} = 15.525{,}561$. Therefore $\overset{£}{1} \times \frac{1 - \frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 40 years, according to Mr. De Moivre's Hypotheſis, is $= \overset{£}{1} \times 15.525{,}561 =$ £15.525,561, or 15*l.* 10*s.* 6*d.* QEI.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £16.147,167, or 16*l.* 2*s.* 11*d.* $\frac{1}{4}$; which is greater than the value juſt now found for it by ſomething more than half a year's purchaſe.

CLXXIII. When the given life is of the age of 45 years, we ſhall have n $(= 95 - 45) = 50$, and conſequently $\frac{r}{n} = \frac{1.035}{50}$, and $G =$ the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} +$ &c. $+ \frac{1}{r^{50}} =$ (by Mr. Smart's tables, page 78,) 23.455,617. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{50} \times 23.455{,}617 = \frac{24.276{,}563{,}595}{50} = .485{,}531{,}271$; and $1 - \frac{r}{n}G$ is $= 1.000{,}000{,}000 - .485{,}531{,}271 = .514{,}468{,}729$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.514{,}468{,}729}{.035} = 14.699{,}106$. Therefore $\overset{£}{1} \times \frac{1 - \frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 45 years, according to Mr. De Moivre's Hypotheſis, is $= \overset{£}{1} \times 14.699{,}106$ = £14,699,106, or 14*l.* 14*s.* 0*d.* QED. For a life of the age of 45 years.

N. B. The

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £14.716,120, or 14*l.* 14*s.* 3*d.* $\frac{3}{4}$; which is greater than the value juſt now found for it by only the trifling ſum of 3*d.* $\frac{1}{4}$.

For a life of the age of 50 years.

CLXXIV. When the given life is of the age of 50 years, we ſhall have $n\ (= 95 - 50) = 45$, and conſequently $\frac{r}{n} = \frac{1.035}{45}$, and $G =$ the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c. + \frac{1}{r^{45}}$ = (by Mr. Smart's tables, page 78,) 22.495,450. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{45} \times 22.493,450 = \frac{23.282,790,750}{45} = .517,395,350$; and $1 - \frac{r}{n}G$ is $= 1.000,000,000, - .517,395,350, = .482,604,650$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.482,604.650}{.035} = 13.788,704$. Therefore $\frac{£}{1} \times \frac{1 - \frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 50 years, according to Mr. De Moivre's Hypotheſis, is $= \frac{£}{1} \times 13.788,704 =$ £13.788,704, or 13*l.* 15*s.* 9*d.* $\frac{1}{4}$. Q E I.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £13.183,083, or 13*l.* 3*s.* 8*d.* which, inſtead of being greater than the value juſt now found for it, (as has been the caſe in the preceeding inſtances,) is leſs than that value by more than half a year's purchaſe.

For a life of the age of 55 years.

CLXXV. When the given life is of the age of 55 years, we ſhall have $n\ (= 95 - 55) = 40$, and conſequently $\frac{r}{n} = \frac{1.035}{40}$, and $G =$ the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c. + \frac{1}{r^{40}}$ = (by Mr. Smart's tables, page 78,) 21.355,072. Therefore $\frac{r}{n}G$ is $=$

$\frac{1.035}{40}$

$\frac{1.035}{40} \times 21.355,072 = \frac{22.102,499}{40} = .552,562,487$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= 1.000,000,000, - .552,562,487, = .447,437,513$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.447,437,513}{.035} = 12.783,928$. Therefore $\overset{£}{1} \times \left|\frac{1 - \frac{r}{n}G}{r-1}\right.$, or the value of an annuity of one pound a year for a life of the age of 55 years, is $= \overset{£}{1} \times 12.783,928 = £12.783,928$, or 12*l.* 15*s.* 8*d.* Q E I.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £11.691,801, or 11*l.* 13*s.* 1*cd.* which is leſs than the value juſt now found for it by more than a year's purchaſe.

CLXXVI. When the given life is of the age of 60 years, we ſhall have n $(= 95 - 60) = 35$, and conſequently $\frac{r}{n} = \frac{1.035}{35}$, and $G =$ the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} +$ &c. $+ \frac{1}{r^{35}} =$ (by Mr. Smart's tables, page 78,) 20.000,661. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{35} \times 20.000,661 = \frac{20.700,684,135}{35} = .591,448,118$; and $1 - \frac{r}{n}G$ is $= 1.000,000,000, - .591,448,118 = 408,551,882$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.408,551,882}{.035} = 11.672,910$. Therefore $\overset{£}{1} \times \left|\frac{1 - \frac{r}{n}G}{r-1}\right.$, or the value of an annuity of one pound a year for a life of the age of 60 years, according to Mr. De Moivre's Hypotheſis, is $= \overset{£}{1} \times 11.672,910 = £11.672,910$, or 11*l.* 13*s.* 5*d.* $\frac{1}{2}$. Q E I.

For a life of the age of 60 years.

N. B. The

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £10.104,074, or 10*l.* 2*s.* 1*d.* which is leſs than the value juſt now found for it by more than a year and a half's purchaſe.

For a life of the age of 65 years.

CLXXVII. When the given life is of the age of 65 years, we ſhall have $n\ (=95-65) = 30$, and conſequently $\frac{r}{n} = \frac{1.035}{30}$, and $G =$ the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} +$ &c. $+ \frac{1}{r^{30}}$, = (by Mr. Smart's tables, page 78,) 18.392,045. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{30} \times 18.392,045 = \frac{19.035,766,575}{30} = .634,525,552$; and $1 - \frac{r}{n}G$ is $= 1.000,000,000, \ - .634,525,552 = .365,474,448$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.365,474,448}{.035} = 10.442,127$. Therefore $\frac{£}{1} \times \frac{1 - \frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 65 years, according to Mr. De Moivre's Hypotheſis, is $= \frac{£}{1} \times 10.442,127 =$ £10.442,127, or 10*l.* 8*s.* 10*d.* QEI.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £8.813,625, or 8*l.* 6*s.* 3*d.* $\frac{1}{4}$; which is leſs than the value juſt now found for it by more than two years purchaſe.

For a life of the age of 70 years.

CLXXVIII. When the given life is of the age of 70 years, we ſhall have $n\ (= 95 - 70) = 25$, and conſequently $\frac{r}{n} = \frac{1.035}{25}$, and G = the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} +$ &c. $+ \frac{1}{r^{25}}$

$\frac{1}{1-\frac{1}{r^{n-s}}}$ = (by Mr. Smart's tables, page 76,) 16.481,514. Therefore

$\frac{r}{n}G$ is = $\frac{1.035}{25}$ × 16.481,514 = $\frac{17.058,366,990}{25}$ = .682,334,679;

and $1-\frac{r}{n}G$ is = 1.000,000,000 — .682,334,679 = .317,665,321;

and $\frac{1-\frac{r}{n}G}{r-1}$ is = $\frac{.317,665,321}{.035}$ = 9.076,152. Therefore

$\mathcal{L}1 \times \frac{1-\frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 70 years, according to Mr. De Moivre's Hypotheſis, is $= \mathcal{L}1 \times 9.076,152 = \mathcal{L}9.076,152$, or 9*l*. 1*s*. 6*d*. $\frac{1}{4}$ QEI.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is $\mathcal{L}6.575,357$, or 6*l* 11*s*. 6*d*. which is leſs than the value juſt now found for it by about two years and a half's purchaſe.

CLXXIX. If the foregoing values of an annuity of one pound for the lives of perſons of the ſeveral ages of 1 year, 3 years, 5 years, 10 years, 15 years, 20 years, 25, 30, 35, 40, 45, 50, 55, 60, 65, and 70, years, which are derived from Mr. De Moivre's Hypotheſis, are ſet down in a regular ſeries, and the values of the ſame annuity for all the ſame lives, except the firſt, as computed above from Monſieur de Parcieux's table, are ſet down in an adjoining column, and the differences between the ſaid values are likewiſe ſet down in a third column, they will together form the following table.

TABLE IX.

Containing the conjectural values of an annuity of one pound a year for the lives of perſons of ſeveral different ages, derived from Mr. De Moivre's Hypotheſis, "That the decrements of human life, or the numbers of "perſons dying every year out of any given original number of perſons "living at the beginning of life, continue the ſame throughout the whole "extent of human life;" and containing likewiſe the true values of the ſame annuity of one pound a year for all the ſame lives, except the firſt, (which is a life of the age of 1 year) computed according to Monſieur de Parcieux's table of the probabilities of the duration of human life; upon a ſuppoſition, in both ways of computing the values of this annuity, that the intereſt of money is 3½ per cent. and that the utmoſt extent of human life is leſs than 95 years; and containing likewiſe, in an adjoining column, the differences of the ſaid values of the ſaid annuity from each other.

Years of Age.	*Values of a life-annuity of one pound a year, according to Mr. De Moivre's Hypotheſis.*	*Values of the ſame life-annuity according to Monſieur de Parcieux's table of probabilities.*	*Differences of the ſaid values.*
	£	£	
1	19.908,809		
3	19.775,462	19.987,654	0.212,192
5	19.608,225	20.526,716	0.918,491
10	19.165,347	20.752,981	1.587,634
15	18.683,934	20.137,194	1.453,260
20	18.159,648	19.457,903	1.298,255
25	17.587,587	18.827,070	1.239,483
30	16.962, 09	18.095,844	1.133,635
35	16.277,238	17.240,588	0.963,350
40	15.525,561	16.147,167	0.621,606
45	14.699,106	14.716,120	0.017,014
50	13.788,704	13.183,083	0.605,621
55	12.783,928	11.691,801	1.092,127
60	11 672,910	10.104,074	1.568,836
65	10.442,127	8.313,625	2.128,502
70	9.076,152	6.575,357	2.500,795

CLXXX. It appears from the foregoing table, in the 1ſt place, That, from the age of 3 years to the age of 45 years, the values of life-annuities, according to Mr. De Moivre's Hypotheſis, are leſs than their true values, as computed above from Monſieur de Parcieux's table of probabilities; Remarks on the foregoing table.

And, 2dly, That during the ſaid interval from the age of 3 years to the age of 45 years, the exceſs of the true value of a life-annuity of one pound, above its conjectural value, derived from Mr. De Moivre's Hypotheſis, increaſes from £0.212,192, or leſs than a quarter of a year's purchaſe, to £1.587,634, or more than a year and a half's purchaſe, (which is its magnitude at the age of 10 years,) and then decreaſes again to £0.017,014, or leſs than one week's purchaſe, or, we may ſay, to 0;

And, 3dly, That, from the age of 45 years to the age of 70 years, the values of life-annuities, according to Mr. De Moivre's Hypotheſis, are greater than their true values, as computed above from Monſieur de Parcieux's table of probabilities;

And, 4thly, That, during the ſaid interval from the age of 45 years to the age of 70 years, the exceſs of the conjectural value of a life-annuity of one pound, (according to Mr. De Moivre's Hypotheſis,) above its true value increaſes continually, as the age advances, or grows older, from 0.017,014, or 0, to £2.500,795, or more than two years and a half's purchaſe, which is its magnitude at the age of 70 years.

CLXXXI. Upon the whole matter, the differences of theſe values ſeem to me to be ſo conſiderable that I cannot but wonder that Mr. De Moivre ſhould have thought fit to diſregard them, and to compute his tables of life-annuities from his own Hypotheſis, inſtead of following Dr. Halley's, or Monſieur de Parcieux's, or ſome other table of the probabilities of life deduced from obſervations. And this appears the more ſurprizing when we conſider, that the labour of deriving the value of a life-annuity for a life of any given age from that of a like annuity for a life only one year older, in ſtrict conformity to a table of the probabilities of life, by means of the expreſſion $\frac{1}{r} \times \frac{P}{P \dashv d} \times \overline{1 \dashv V}\,\pounds$, is ſcarcely, if at all, greater than that of computing the ſame annuity, according to Mr. De Moivre's Hypotheſis, by means of the expreſſion $\pounds 1 \times \frac{\overline{1 - r}\,G_n}{r \quad 1}$. A general concluſion concerning Mr. De Moivre's Hypotheſis.

I would therefore recommend it to all future calculators of the values of life-annuities, or at leaſt to thoſe who undertake only to calculate annuities for ſingle lives, to lay aſide all conſideration of Mr. De Moivre's Hypotheſis, and to compute thoſe values from Monſieur de Parcieux's table of probabilities, or ſome other table of the ſame kind which they may eſteem to be of ſtill better authority, by means of the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times 1-\overline{+V}\,\mathcal{L}$, in the manner that has been above explained and exemplified in Art. 100, 101, 102, &c.—117.

End of the explanation of Mr. De Moivre's Hypotheſis.

An account of Mr. Weyman Lee's method of computing the values of Life-annuities.

CLXXXII. Some gentlemen who have undertaken to write upon the ſubject of life-annuities, but who appear to have been unacquainted with the Mathematicks and the doctrine of chances, have conſidered the method of computing the value of a life-annuity above-delivered in Problem 11, Coroll. 2, Art. 31, page 28, and exemplified in many inſtances in the ſubſequent part of this tract, and which is that which all mathematical writers, (from Dr. Halley in the year 1692, down to Dr. Price and Mr. Morgan in their late treatiſes,) have univerſally adopted;——I ſay, ſome gentlemen have conſidered this method as erroneous. One of theſe gentlemen was the late Weyman Lee, Eſq; a barriſter at law and a bencher of the Society of the Inner Temple. This gentleman publiſhed a book in one volume octavo in the year 1738, upon the doctrine of life-annuities, intitled, "*An Eſſay on the method of aſcertaining the value of annuities, and of Leaſes reduced to annuities certain, for one or more lives*;" and in the year 1751 he publiſhed a ſecond pamphlet on the ſame ſubject, intitled, "*A valuation of annuities and leaſes certain for a ſingle life.*" This latter pamphlet I have ſeen. And in it, I obſerve, he argues againſt the method of computing the values of life-annuities above-explained, in a ſtyle of great confidence, and with a high degree of contempt of his numerous mathematical adverſaries, Dr. Halley, a Mr. H. B, Mr. Richards, Mr. Hodgſon, Mr. Hayes, and Mr. De Moivre: but at the ſame time he expreſſes himſelf with great obſcurity, as might naturally be expected from a perſon who endeavoured to invalidate ſo clear and certain a propoſition. And, in lieu of the ſaid method of computing the values of life-annuities (which he conceives himſelf to have ſhewn to be erroneous,) he gives us another method of eſtimating them, which is as follows.

CLXXXIII. When

CLXXXIII. When the value of an annuity for the life of a perſon of any given age is to be determined, we muſt look into Dr. Halley's, or Monſieur de Parcieux's, or ſome other, table of the probabilities of the duration of human life, to find what number of perſons are therein repreſented as living at the ſaid age. And then we muſt divide this number by 2, and look again into the ſaid table, to find at what later age of life the number of perſons repreſented therein as living will be reduced to this latter number, or to one half of the former number. And an annuity of one pound a year for the life of a perſon of the firſt, or given, age will be equal in value, ſays Mr. Lee, to a like annuity of one pound a year for a term of years certain, equal to the difference of the ſaid two ages, or to the ſpace of time in which the number of perſons repreſented in the table as living at the given age will be reduced to half. Mr. Weyman Lee's method of valuing life-annuities.

Thus, for example, if the value of an annuity for the life of a perſon of the age of 10 years is to be determined, we ſhall find in Monſieur de Parcieux's table above-mentioned, that 880 perſons are there repreſented as living at the age of 10 years. The half of this number is 440. We muſt therefore, in the ſecond place, look out in the ſame table the age at which the number of perſons living is reduced to 440. Now we ſhall find, upon this inſpection of the ſaid table, that at the age of 61 years there will be 450 perſons living, and at the age of 62 years 437 perſons living, out of the ſaid 880 perſons who were living at the age of 10 years. Therefore at ſome intermediate age between 61 and 62, as, for inſtance, about the age of 61 years and nine months, we may ſuppoſe that the number of perſons living, (out of the ſaid original 880 perſons, who were living at the age of 10 years,) will be 440; or, in other words, the original 880 perſons, living at the age of 10 years, will be reduced to 440, or to one half, in the courſe of 51 years and 9 months. Therefore (according to Mr. Weyman Lee's method of valuing annuities,) an annuity of one pound a year for the life of a perſon of the age of 10 years is of the ſame value with an annuity of one pound a year for the ſpace of 51 years and 9 months certain, or, in round numbers, for the ſpace of 52 years. An example of the ſaid method.

CLXXXIV. According to this method of eſtimating life-annuities, an annuity of one pound a year for the life of a perſon of the age of 10 years, when the intereſt of money is $3\frac{1}{2}$ per cent. is £23.795,764, or 23*l.* 15*s.* 11*d.* that being the value of an annuity of one pound a year for a term certain of 52 years, when the intereſt of money is $3\frac{1}{2}$ per cent. as appears from Mr. Smart's tables, page 80.

CLXXXV. But

Of the difference of the values of life-annuities computed by this method from their true values.

CLXXXV. But we have ſeen above, in Art. 94, that the true value of an annuity of one pound a year for the life of a perſon of the age of 10 years, when the intereſt of money is $3\frac{1}{2}$ per cent. is only £20.73925, or 20*l.* 14*s.* 9*d.* $\frac{1}{4}$: and by the other calculation of it, in Art. 114, it appeared to be only £20.752,981, or 20*l.* 15*s.* 0*d.* $\frac{3}{4}$. Therefore the foregoing value of it, obtained by the method of Mr. Weyman Lee, to wit, 23*l.* 15*s.* 11*d.* is greater than its true value by no leſs than three years purchaſe. And in like manner it will be found upon trial, that in all other ages of life, except from the age of 74 years to the age of 83 years, the values of life-annuities obtained by this method of Mr. Lee, will be greater, and for the moſt part *very much* greater, than their true values.

CLXXXVI. But that the difference of theſe values of life-annuities, found by Mr. Lee's method, from their true values, as computed according to the directions of Problem 11, Coroll. 2, may be the more apparent, I ſhall here preſent the reader with a table containing both ſets of values of an annuity of one pound a year for the lives of perſons of all ages, differing from each other by one year, from the age of 3 years to the age of 93 years, incluſively, computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. and containing likewiſe the differences of the ſaid values. This table is as follows.

TABLE

TABLE X.

Containing the values of an annuity of one pound a year for the lives of persons of the ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively, upon a supposition that the probabilities of the duration of human life are such as they are represented to be in Monsieur de Parcieux's table of them, and that the interest of money is $3\frac{1}{2}$ per cent. computed according to two different methods, to wit, Mr. Weyman Lee's method just now mentioned in Art. 183, and the true method that has been adopted by Dr. Halley and all other mathematicians, and which has been explained above in Problem 11 and its corollaries; and containing likewise, in an adjoining column, the differences of the said values.

Years of Age.	*Values of an annuity of one pound by Mr. Weyman Lee's method.*	*Values of the same annuity by Dr. Halley's method.*	*Differences of the said values.*
	£	£	£
3	24.113,295	19.987,654	4.125,641
4	24.113,295	20.327,034	3.786,261
5	24.113,295	20.526,716	3.586,579
6	24.113,295	20.656,349	3.456,946
7	24.035,948	20.729,912	3.306,036
8	23.957,260	20.764,685	3.192,575
9	23.877,206	20.781,222	3.095,984
10	23.795,764	20.752,981	3.042,783
11	23.628,616	20.676,395	2.952,221
12	23.455,617	20.548,338	2.907,279
13	23.366,861	20.415,899	2.950,962
14	23.27 ,564	20.278,914	2.997,650
15	23 091,244	20.137,194	2 954,050
16	22.899,437	19.990,615	2.908,822
17	22.700,918	19.863,739	2.837,179
18	22.599,067	19.732,778	2 866,289
19	22.495,450	19.597,560	2.897,890
20	22.282,791	19.457,903	2.824,888
21	22.173,686	19.338,821	2.834,865
22	21.949,705	19.216,339	2.733,426
23	21.834,882	19.090,318	2.744,564
24	21.599,103	18.960,613	2.638,490

Years of Age	*Values of an annuity of one pound by Mr. Weyman Lee's method.*	*Values of the same annuity by Dr. Halley's method.*	*Differences of the said values.*
	£	£	£
25	21.478,137	18.827,070	2.651,067
26	21.355,072	18.689,528	2.665,544
27	21.102,499	18.547,817	2.554,682
28	20.972,917	18.401,759	2.571,158
29	20.706,969	18.251,167	2.455,802
30	20.570,525	18.095,844	2.474,681
31	20.290,493	17.935,582	2.354,911
32	20.146,823	17.770,162	2.376,661
33	19.851,962	17.599,354	2.252,608
34	19.700,684	17.422,915	2.277,769
35	19.390,208	17.240,588	2.149,620
36	19.068,805	17.052,103	2.016,762
37	18.904,000	16.857,174	2.046,826
38	18.565,640	16.629,189	1.936,451
39	18.392,045	16.392,655	1.999,390
40	18.035,767	16.147,167	1.888,600
41	17.667,018	15.892,297	1.774,721
42	17.285,364	15.580,939	1.704,425
43	17.089,556	15.303,763	1.785,793
44	16.890,352	15.015,669	1.874,683
45	16.481,514	14.716,120	1.765,394
46	16.058,367	14.404,549	1.653,818
47	15.841,272	14.105,200	1.736,072
48	15.395,716	13.793,859	1.601,857
49	15.167,124	13.494,425	1.672,699
50	14.697,974	13.183,083	1.514,891
51	14.212,403	12.883,449	1.328,954
52	13.963,281	12.596,296	1.366,985
53	13.709,837	12.298,386	1.411,451
54	13.189,681	11.989,093	1.200,588
55	12.922,816	11.691,801	1.231,015
56	12.651,320	11.383,427	1.267,893
57	12.094,116	11.063,485	1.030,631
58	11.808,243	10.755,123	1.053,120
59	11.517,410	10.435,566	1.081,844
60	10.920,520	10.104,074	0.816,446
61	10.302,738	9.759,829	0.542,909
62	9.985,785	9.401,925	0.583,860
63	9.663,334	9.053,059	0.610,275
64	9.335,288	8.690,648	0.644,640

Years of Age.	Values of an annuity of one pound by Mr. Weyman Lee's method.	Values of the same annuity by Dr. Halley's method.	Differences of the said values.
	£	£	£
65	8.662,023	8.313,625	0.348,398
66	8.310,605	7.944,258	0.372,347
67	7.965,194	7.583,727	0.381,467
68	7.607,686	7.233,699	0.373,987
69	7.243,976	6.896,496	0.347,480
70	6.873,955	6.575,357	0.298,598
71	6.497,515	6.249,840	0.247,675
72	6.114,543	5.945,972	0.168,571
73	5.724,928	5.644,448	0.080,480
74	5.328,553	5.347,806	0.019,253
75	4.925,300	5.059,623	0.134,323
76	4.515,052	4.754,921	0.239,869
77	4.515,052	4.461,839	0.053,213
78	4.097,686	4.187,758	0.09 ,070
79	3.673,079	3.908,000	0 234,921
80	3.673,079	3.661,781	0,011,298
81	3.241,105	3.427,856	0.186,751
82	3.241,105	3.215,658	0.025,447
83	2.801,636	2.984,473	0.182,837
84	2.801,636	2.717,188	0.08 ,4
85	2.801,636	2.456,774	0.344,862
86	2.354,544	2.211,911	0.142,63
87	2.354,544	1 999,810	0 354,734
88	1.899,694	1.728,378	0.1 1,316
89	1.899,694	1.459,699	0.439,995
90	1.436,953	1.197,512	0.239,441
91	1 436,953	0.947,669	0.480,284
92	0.966,183	0.716,468	0.249,715
93	0.966,183	0.483c9.	0.4 3,093
94	0.000,000		

Obſervations on the differences of the two ſets of values of life-annuities ſet down in the foregoing table.

CLXXXVII. From the foregoing table it appears, that at the age of 3 years the value of a life-annuity obtained by Mr. Lee's method is greater than its true value by more than 4 years purchaſe; and that, from the age of 3 years to the age of 10 years, incluſively, it exceeds the true value by more than three years purchaſe; and that, from the age of 10 years to the age of 37 years, incluſively, it exceeds the true value by more than two years purchaſe; and that, from the age of 37 years to the age of 59 years, incluſively, it exceeds the true value by more than one year's purchaſe; and that from the age of 59 years to the age of 74 years, incluſively, it exceeds the true value by leſs than one year's purchaſe, and that in that laſt interval, from the age of 59 years to the age of 74 years, its exceſs above the true value becomes gradually leſs and leſs, till, at the ſaid latter age of 74 years, it is almoſt equal to 0. But, after the ſaid age of 74 years, the value of a life-annuity obtained by the ſaid method of Mr. Lee, is, for about 9 years, or till the age of 83 years, nearly equal to, but for the moſt part ſomewhat leſs than, its true value; after which it again becomes greater than the ſaid true value at the age of 84 years, and continues to exceed the ſaid true value throughout the whole remainder of the table, or in every following year of human life.

CLXXXVIII. The differences of the foregoing values of life-annuities, obtained by Mr. Lee's method of computation, from their true values, are ſo conſiderable before the ages of 39 or 40 years, that it would be by no means adviſeable to have recourſe to thoſe values in ſettling the prices of life-annuities that are to be granted for the lives of perſons under thoſe ages. And even to the age of 64 years, incluſively, I ſhould think theſe differences too great to be neglected. But from the age of 65 years to the age of 85, or 86, years, thoſe values might do tolerably well as a guide in the purchaſe of life-annuities, if a table containing the true values of the annuities for lives of thoſe ages were not at hand. But, when tables of the true values of life-annuities, (computed ſtrictly from a table of the probabilities of human life, and not from Mr. De Moivre's Hypotheſis, or any other inaccurate ſuppoſition adopted by calculators for the ſake of abridging their trouble,) are ready-computed to our hands, it ſeems to be imprudent and unſatisfactory to have recourſe to any other values of them.

CLXXXIX. We have ſeen in the foregoing table, that Mr. Lee's method of eſtimating life-annuities is doubly erroneous. For it makes the values of them much greater than they ought to be throughout the greater part of human life; and, for a few years between the ages of 74 years and

and 84 years, it makes them rather leſs than they ought to be. Yet Mr. Lee is very confident that it is the only right method of valuing life-annuities, and ſeems to wonder that any body can doubt of its being ſo. And, to remove theſe doubts, and fully eſtabliſh the juſtneſs of his ſaid method, he has given us what he calls two diſtinct proofs of it. Theſe proofs are expreſſed in very vague and looſe terms, inſomuch that I muſt confeſs myſelf unable to underſtand them. The words *chance* and *chances* occur in them very frequently, and ſometimes the expreſſion of *the chance of, or to, a chance:* but they ſeem to be uſed without any diſtinct meanings annexed to them. However, as this is a matter which relates to the very foundation of the doctrine of life-annuities, I ſhould be glad that my readers would exerciſe their own judgements concerning it; and therefore, to enable them to do ſo, (as Mr. Lee's tracts are not now eaſily to be met with) I ſhall here preſent them with a tranſcript of the paſſage, in Mr. Lee's ſecond pamphlet above-mentioned, which contains the ſaid two ſuppoſed proofs of the juſtneſs of his method of valuing life-annuities. In this paſſage Mr. Lee has introduced ſome numbers into his argument, by way of example and illuſtration, which are grounded upon Dr. Halley's Breſlaw table of the probabilities of the duration of human life, and on a ſuppoſition that the intereſt of money is 6 per cent. Theſe numbers I have taken the liberty to alter ſo as to make them ſuit with Monſieur de Parcieux's table of probabilities and a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. in order to avoid the unneceſſary trouble and perplexity that might have ariſen from the conſideration of a different table of probabilities and a different rate of intereſt from thoſe which are become familiar to us by our frequent uſe of them in the courſe of the foregoing articles. Theſe alterations in Mr. Lee's numbers, it is evident, cannot in the leaſt affect the force of his reaſonings, (if there is any force in them,) and therefore may be made without injuring him. And, with theſe alterations in the numbers, the paſſage containing Mr. Lee's two ſuppoſed proofs of the juſtneſs of his method of valuing life-annuities, in pages 8, 9, and 10 of his ſecond pamphlet above-mentioned, intitled, *A valuation of annuities for lives*, is in the words following.

Of Mr. Weyman Lee's attempts to prove the juſtneſs of his method of valuing life-annuities.

CXC. " Since the author [Mr. H. B.] is ſo full of, and ſo learned in, " the doctrine of chances, as he calls it, I will take the liberty of offering " to his conſideration an argument, or two, in behalf of my rule, accord- " ing to his way of thinking. The value of the chances in this caſe " depends on, or is one and the ſame thing as, the value of the annuities " dependant on thoſe chances. All the chances which an annuitant has on " the life of A [a perſon of the age of 10 years] are agreed to be 880: " and of thoſe he has an even chance to enjoy a moiety, that is, 440.

Extracts from Mr. Lee's ſecond tract, publiſhed in the year 1751.

" The annuities which attend thoſe chances for the whole life are annuities " for the term of 85 years, it being ſuppoſed poſſible that he may live for " ſo many years. The total value of them for 85 years, as I compute it, " is 27.036,803; and the value of thoſe in poſſeſſion for 52 years is " 23.795,764, and of thoſe in reverſion 3.241,039. Since then the an- " nuitant on the life of A has an even chance to 440 chances on his life, " and the annuities will attend thoſe chances;——and, ſince theſe 440 " chances will, and, from the nature of the life of man, of neceſſity muſt, " ariſe in the firſt part of thoſe 85 years, if they ariſe at all; (and 'tis " ſuppoſed in the caſe to be an even chance that ſo many chances will " ariſe in his life:) he muſt then of neceſſity have the annuities attending " thoſe very chances, that is, he muſt have the annuities for the firſt 52 " years of his life. This author [Mr. H. B.], or whoever will undertake " to anſwer this argument, muſt ſhew that 'tis poſſible for an annuitant on " the life of A to have the benefit of all theſe 440 chances (to which " number of chances he has without diſpute a right, or an even chance to " enjoy,) unleſs he does enjoy thoſe which ariſe in the firſt part of life and " in immediate poſſeſſion, that is, thoſe of the greateſt value. And this, " I am very ſure, he cannot ſhew.

" Again; theſe poſitions are not conteſted:——That an annuity for a " term of 52 years is in value £23.795,764;——and that an annuitant " for the life of A has an even chance that A lives for 52 years.—'Tis a " maxim made uſe of by this author, and is certainly a true one, That, " in eſtimating the value of annuities for a life, all the poſſible chances of " life muſt be computed. From hence I argue thus. To eſtimate the " value of an annuity for the life of A, we muſt compute all the poſſible " chances on the life of A. The chance that A lives for 52 years is an " even chance, and conſequently is one of the poſſible chances on the life " of A. Therefore, to eſtimate the value of an annuity for his life, we " muſt compute this even chance. The even chance on this life is to a term " for 52 years. A term for 52 years is in value £23.795,764. Therefore " the value of an annuity for this life is £23.795,764."

CXCI. Theſe are the words of Mr. Weyman Lee's proofs of the truth of his method of valuing life-annuities. But what meaning is to be found in them, or what train of reaſoning Mr. Lee might purſue obſcurely in his own mind, when he uſed them, I will not pretend to determine. And yet I am inclined to conjecture that his meaning (though certainly it is not expreſſed

expreſſed in the foregoing words,) might be as follows; to wit, "that, if a perſon were to make 880 different grants, of one pound a year each, to as many different grantees, all of the age of 10 years, for their reſpective lives, half the ſaid grantees would be dead in the ſpace of 52 years, and the other half would live beyond that term; and conſequently 440 of the ſaid 880 annuities would become extinct in the courſe of the ſaid 52 years, and the other 440 of them would continue to be payable after the ſaid 52 years. And hence, I preſume, Mr. Lee was inclined to conclude, that therefore it would be the ſame thing, in point of advantage, to the grantor of theſe 880 annuities, whether he granted them to the ſaid 880 perſons for the life of each of them reſpectively, or whether he granted them to the ſaid perſons and their executors, or other repreſentatives, for a term certain of 52 years; becauſe that, if he had, firſt, granted them for a term certain of 52 years, and afterwards was, by the conſent of all parties, to change thoſe grants into grants of the ſame annuities of one pound each to the ſame 880 perſons of the age of 10 years for the lives of the ſeveral grantees reſpectively, *the burthen* that would fall on the ſaid grantor in conſequence of this change in the nature of the grants, by means of the payments he would thereby be obliged to make after the expiration of the ſaid term of 52 years, to the 440 annuitants who would live beyond that term, would be counter-balanced by *the profit*, or ſaving, that would accrue to him before the end of the ſaid term of 52 years by the extinction of the annuities that had been granted to the other 440 annuitants who would have died in the courſe of the ſaid term." This is the only argument that I can imagine to have been intended to be expreſſed in the foregoing paſſage of Mr. Lee's pamphlet. But, if this was the argument Mr. Lee relied on, he was greatly miſtaken in thinking it a juſt one, though it may, at firſt ſight, appear plauſible. For the truth is, that the profit accruing to the grantor of the 880 annuities of one pound by the aforeſaid change in his bargain, by means of the extinction of the annuities that had been granted to the 440 annuitants who had died before the end of the 52 years, will *much more than counter-balance* the burthen that would fall on him, in conſequence of ſuch a change of his firſt bargain, by means of the payments he would thereby be obliged to make to the other 440 annuitants who would live beyond the ſaid term of 52 years. And one very obvious reaſon why we ſhould expect that the ſaid *profit* ſhould *more than counter-balance* the ſaid *burthen*, in ſo young an age as this of 10 years, is, that the payments which conſtitute the ſaid profit, namely, the payments of the annuities ſaved by the deaths of the 440 annuitants who die before the end of the ſaid 52 years, are *much leſs remote*, and conſequently *much more valuable*, than the payments which conſtitute the ſaid burthen, or the payments incurred by the obligation of continuing the annuities, during their reſpective lives, to the other 440 annuitants who will live beyond the ſaid term of 52 years. In what preciſe proportion, or degree, the ſaid profit will

A conjecture concerning the meaning of the foregoing extracts from Mr. Lee's pamphlet.

Mr. Lee's miſtake in the ſaid argument ſuppoſed to be contained in the foregoing extracts.

will exceed the ſaid burthen, is a nice queſtion, and can only be determined by obſerving, in Monſieur de Parcieux's table of the probabilities of human life, how many of the 880 perſons who are there repreſented as living at the age of 10 years, (and to whom we ſuppoſe annuities of one pound to have been granted for their reſpective lives,) will die off in every individual year of the whole term of 85 years through which it is poſſible that their lives may be extended, and computing, firſt, the values of the ſeveral payments that will be ſaved in each of the firſt 52 years of the ſaid term of 85 years, by the deaths of thoſe 440 perſons who will die in the courſe of the ſaid 52 years, and, ſecondly, the values of the payments that will be incurred in each of the remaining 33 years of the ſaid term of 85 years after the 52d year, by the continuance of the lives of the other 440 annuitants, who will live beyond the ſaid 52 years, and then comparing the ſum of the values of the former payments with the ſum of the values of the latter payments. This would be a nice and tedious inquiry, and would be tantamount to the computation of the value of an annuity of one pound for the life of a perſon of the age of 10 years in the manner explained above in Problem 11 and its corollaries, and exemplified in Art. 94. But it is ſufficient to invalidate the foregoing argument, (which I have ſuppoſed to be that which was meant to be advanced by Mr. Lee;) that the *exact equality* of the ſaid profit to the ſaid burthen (which in the ſaid argument is taken for granted, as a thing ſelf-evident,) is by no means apparent; but that there is, on the contrary, good reaſon to ſuppoſe (without going into the nice inquiry juſt now mentioned,) that the ſaid profit in this caſe of an annuity for a life of the age of 10 years, is *much greater* than the ſaid burthen. For then it will not be the ſame thing (as it is ſuppoſed to be in the ſaid argument) in point of advantage, to the grantor of the ſaid 880 annuities of one pound to as many grantees, all of the age of 10 years, whether he grant them the ſaid annuities for the term of 52 years certain, or whether he grant them for the lives of each ſeveral grantee; but the latter bargain will be leſs burthenſome to the grantor than the former; and therefore he ought to receive a leſs price from the ſaid 880 grantees in the latter caſe than in the former. And conſequently, it will follow from Art. 21, that, when the grantor makes only one ſuch grant of an annuity of one pound to a perſon of the age of 10 years, he ought to receive a leſs price for it than he ought to receive for the grant of the ſame annuity for a term certain of 52 years; or, in other words, there is reaſon to ſuppoſe (even without going into the nice inquiry above-mentioned,) that the value of an annuity of one pound a year for the life of a perſon of the age of 10 years is leſs than the value of the ſame annuity for a term certain of 52 years; contrary to what Mr. Lee has aſſerted.

CXCII. The

CXCII. The only way of determining truly the value of any propoſed life-annuity is that which is explained above in Art 21, and made the foundation of this whole doctrine, namely, to ſuppoſe the grantor of the annuity to make many more ſuch grants to many other grantees, all of the ſame age as the firſt, or propoſed, grantee, and then to inquire what price ought to be paid by each of the ſaid grantees to the grantor of all the ſaid annuities, in order to make him, at the cloſe of the whole tranſaction, or when all the ſaid annuitants ſhall be dead, be neither a gainer nor a loſer by the ſum total of all his bargains; upon a ſuppoſition that he improves the money he receives from the ſaid grantees, as the price of their annuities, at compound intereſt according to a certain given rate, and that the grantees of the ſaid annuities die off every year in the proportions repreſented in ſome particular table of the probabilities of the duration of human life that is adopted as the ground of the calculation. That price, and no other, is the *true, or fair, value* of ſuch a life-annuity: and, whenever we attempt to eſtimate a life-annuity without thus ſuppoſing a great number of the like annuities to be granted at the ſame time to other perſons of the ſame age as the propoſed annuitant, we ſhall find ourſelves bewildered and confuſed, and without any ſolid foundation to build upon. And I have no doubt that it is only to the want of ſuch *a criterion*, or *meaſure*, whereby to judge of the true value of a life-annuity, or, perhaps I ought to ſay, of ſuch *a definition* of ſuch true value, that we ought to aſcribe the obſcurity and confuſion with which Mr. Weyman Lee treats of this ſubject, both in the paſſage above-cited from him, in which he endeavours to prove the rectitude of his own erroneous method of eſtimating life-annuities, and in other parts of the ſame tract, in which he endeavours to prove the falſhood of the method which has been uſed by Dr. Halley, and all other mathematicians, for that purpoſe, and which has been above explained and illuſtrated in Prob. 11 and its corollaries, and in ſeveral of the following articles. All this perplexity would have been avoided, if he had previouſly ſettled clearly and diſtinctly in his own mind what he meant by *the fair price, or value*, of a life-annuity.

Of the neceſſity of keeping conſtantly in view the idea of *the fair price of a life-annuity* given above in Art. XXI.

CXCIII. I have been induced to enlarge the more on the falſhood of this erroneous method of valuing life-annuities, becauſe I have obſerved, that not only Mr. Weyman Lee in the year 1751, but a later and much abler writer on the ſame ſubject, Mr. Dale, in his uſeful book on life-annuities, intitled, *Calculations deduced from firſt principles, &c.* publiſhed in the year 1772, ſeems likewiſe to be perſuaded of its truth. See the ſaid book of Mr. Dale, article 2d of the Addenda.

End of the examination of Mr. Weyman Lee's erroneous method of valuing Life-annuities.

CXCIV. From

CXCIV. From theſe digreſſions, concerning Mr. De Moivre's Hypotheſis for abridging the computations of life-annuities, and Mr. Weyman Lee's erroneous method of valuing thoſe annuities, I now return to the only true and accurate method of computing the values of the ſaid annuities, which is that which has been above explained in Problem 11 and its corollaries. From this method, (as we have ſhewn above in Art. 100, 101, and 102,) the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$ may be derived, by which the value of an annuity of one pound for any propoſed life may be computed from the value of the ſame annuity for a life that is one year older. Nothing can well be expected more conciſe and convenient for the purpoſe of computing a table of life-annuities, than this expreſſion. Nevertheleſs it is liable to this one inconvenience, that, as the values of life-annuities for different ages are deduced by it one from another, in regular ſucceſſion from the older ages to the younger, an error that ſhould happen to be made in computing the value of one of theſe life-annuities would affect the values of all the following life-annuities, or of the annuities for all the ages that were younger than that in which the error was originally made. This inconvenience we have in ſome meaſure endeavoured to guard againſt in Art. 120, by ſhewing how an uncommonly large increaſe, or decreaſe, in the differences of the values of theſe ſucceſſive life-annuities will, for the moſt part, afford a juſt ground of ſuſpicion that ſome miſtake has been made in the computation of the laſt value by means of the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|\mathcal{L}$, and thereby will induce the calculator to reviſe his laſt computation; in conſequence of which reviſion ſuch miſtake, if any has been made, will be diſcovered. But Mr. Morgan in his uſeful and learned tract upon life-annuities, intitled, *The doctrine of annuities and aſſurances on lives and ſurvivorſhips*, (pages 59, 60, 61, &c.——68,) has given us a much better and more ſatisfactory method of removing this inconvenience, and proving the truth of the ſeveral operations, as they ariſe. This method may be explained as follows.

Of Mr. Morgan's method of proving the truth of the computations of the values of Life-annuities, as faſt as they are made.

CXCV. Let A denote the number of perſons repreſented in any table of the probabilities of the duration of human life as living at the youngeſt age that is ſet down in the ſaid table; which age in Monſieur de Parcieux's table is the age of 3 years. Secondly, Let P be the number of perſons repreſented in the ſaid table as living at any ſubſequent, or older, age, conſiſting of N years; and let n be the number of years by which the ſaid

ſaid age of N years exceeds the youngeſt age in the table. And, thirdly, let $\frac{\pounds}{r}$ be the value of one pound together with its intereſt for one year.

An expreſſion of the value of a ſingle future payment of one pound, to be received by a perſon of the youngeſt age in the table, after a given number of years, if he ſhall be then alive.

Then, it is evident, that the value of a ſingle payment of one pound to be received at the end of n years by a perſon of the youngeſt age in the table, if the ſaid perſon ſhall ſo long live, that is, if he ſhall live to the age of N years, will be $\frac{\pounds 1}{r^n} \times \frac{P}{A}$, or $\pounds 1 \times \frac{P}{Ar^n}$. In Monſieur de Parcieux's table of probabilities the number of perſons repreſented as living at the age of 3 years (which is the youngeſt age in the table,) is 1000; and conſequently A is $= 1000$, and $\pounds 1 \times \frac{P}{Ar^n}$ is $= \pounds 1 \times \frac{P}{1000r^n}$.

An expreſſion of the value of a remote life-annuity of one pound a year, for the life of a perſon of the youngeſt age in the table, to commence at a given future age.

CXCVI. Further, if $P^{\prime}$ denote the number of perſons repreſented in the table of probabilities as living at the age of $N+1$ years, and $P^{\prime\prime}$ the number living at the age of $N+2$ years, and $P^{\prime\prime\prime}$ the number living at the age of $N+3$ years, and ſo on throughout the remainder of the table, the number annexed, (in Roman figures,) to the top of the letter P being every where the ſame with the number of years added to N, and the ſaid letter P, with ſuch number ſo annexed, denoting the number of perſons living at the age denoted by N with the ſame number added to it; it is evident that the value of a remote annuity of one pound a year for the life of a perſon of the youngeſt age in the table of probabilities, to commence at the end of n years, or whereof the firſt payment is to be received at the end of $n+1$ years, will be equal to $\frac{\pounds 1}{A} \times$ the ſeries

$$\frac{P^{\prime}}{r^{n+1}} + \frac{P^{\prime\prime}}{r^{n+2}} + \frac{P^{\prime\prime\prime}}{r^{n+3}} + \frac{P^{\mathrm{IV}}}{r^{n+4}} + \frac{P^{\mathrm{V}}}{r^{n+5}} + \frac{P^{\mathrm{VI}}}{r^{n+6}} + \frac{P^{\mathrm{VII}}}{r^{n+7}} + \&c.$$

continued to the end of the table, or to the utmoſt poſſible extent of human life, which in Monſieur de Parcieux's table is 95 years; or it will be equal to $\pounds 1 \times$ the ſeries $\frac{P^{\prime}}{Ar^{n+1}} + \frac{P^{\prime\prime}}{Ar^{n+2}} + \frac{P^{\prime\prime\prime}}{Ar^{n+3}} + \frac{P^{\mathrm{IV}}}{Ar^{n+4}} + \frac{P^{\mathrm{V}}}{Ar^{n+5}} + \frac{P^{\mathrm{VI}}}{Ar^{n+6}} + \frac{P^{\mathrm{VII}}}{Ar^{n+7}} +$ &c. continued to the end of the table, or to the utmoſt poſſible extent of human life.

The Propoſition that is the ground of Mr. Morgan's method above-mentioned.

CXCVII. Now this remote annuity of one pound a year for the life of a perſon of the youngeſt age in the table is equal to the product that ariſes by multiplying the value of a like annuity of one pound a year for the life of a perſon of the age of N years, into $£1 \times \frac{P}{Ar^n}$, or the preſent value of the above-mentioned ſingle payment of one pound to be made to a perſon of the youngeſt age in the table at the end of n years, or when he ſhall have attained the age of N years, if he ſhall ſo long live; or, if we put $\overset{£}{V}$ for the value of an annuity of one pound a year for the life of a perſon of the age of N years, and $\overset{£}{R}$ for the value of a remote annuity of the ſame ſum of one pound a year for the life of a perſon of the youngeſt age in the table to commence at the diſtance of n years, or whereof the firſt payment is to be received at the end of $n+1$ years, we ſhall have $\overset{£}{R} = \overset{£}{V} \times \overset{£}{1} \times \frac{P}{Ar^n}$.

DEMONSTRATION.

For $\overset{£}{V}$, or the value of an annuity of one pound a year for the life of a perſon of the age of N years, is (by Art. 86 and 87) $= £1 \times$ the ſeries $\frac{P^{I}}{Pr} + \frac{P^{II}}{Pr^2} + \frac{P^{III}}{Pr^3} + \frac{P^{IV}}{Pr^4} + \frac{P^{V}}{Pr^5} + \frac{P^{VI}}{Pr^6} + \frac{P^{VII}}{Pr^7} +$ &c. continued to the end of the table of probabilities, or to the utmoſt extent of human life. Therefore $£1 \times \frac{P}{Ar^n} \times \overset{£}{V}$ is $= £1 \times \frac{P}{Ar^n} \times £1 \times$ the ſeries $\frac{P^{I}}{Pr} + \frac{P^{II}}{Pr^2} + \frac{P^{III}}{Pr^3} + \frac{P^{IV}}{Pr^4} + \frac{P^{V}}{Pr^5} + \frac{P^{VI}}{Pr^6} + \frac{P^{VII}}{Pr^7}$ + &c. continued to the end of the table of probabilities, and conſequently is $= £1 \times$ the ſeries $\frac{P^{I}}{Ar^{n+1}} + \frac{P^{II}}{Ar^{n+2}} + \frac{P^{III}}{Ar^{n+3}} + \frac{P^{IV}}{Ar^{n+4}} + \frac{P^{V}}{Ar^{n+5}} + \frac{P^{VI}}{Ar^{n+6}} + \frac{P^{VII}}{Ar^{n+7}}$ + &c. continued to the end of the table of probabilities. But, by Art. 196, $\overset{£}{R}$, or the value of a remote annuity

annuity of one pound a year for the life of a perſon of the youngeſt age in the table of probabilities, to commence at the end of n years, or whereof the firſt payment is to be received at the end of $n+1$ years, is $= £1 \times$ the ſeries $\frac{P^{I}}{Arn+1} + \frac{P^{II}}{Arn+2} + \frac{P^{III}}{Arn+3} + \frac{P^{IV}}{Arn+4} + \frac{P^{V}}{Arn+5} + \frac{P^{VI}}{Arn+6} + \frac{P^{VII}}{Arn+7}$ + &c. continued to the end of the table of probabilities. Therefore $\overset{£}{1} \times \frac{P}{Arn} \times \overset{£}{V}$, or $\overset{£}{V} \times \overset{£}{1} \times \frac{P}{Arn}$, is $= \overset{£}{R}$. QED.

CXCVIII. From the foregoing propoſition Mr. Morgan's rule for proving the truth of a computation of a life-annuity for a life of any given age is an eaſy conſequence. This rule is as follows. Let $\overset{£}{V}$ be put for the value of an annuity of one pound a year for the life of a perſon of the given age of N years, which number of years exceeds the number of years in the youngeſt age ſet down in the table of probabilities by n years. Then, in order to try whether $\overset{£}{V}$ has been rightly computed, compute the quantity $£1 \times \frac{P}{Arn}$, or the value of a ſingle future payment of one pound to be made to a perſon of the youngeſt age ſet down in the table of probabilities at the end of n years, if ſuch perſon ſhall then be living; and likewiſe compute the ſeveral terms of the ſeries $\frac{P^{I}}{Arn+1} + \frac{P^{II}}{Arn+2} + \frac{P^{III}}{Arn+3} + \frac{P^{IV}}{Arn+4} + \frac{P^{V}}{Arn+5} + \frac{P^{VI}}{Arn+6} + \frac{P^{VII}}{Arn+7}$ + &c. continued to the end of the table of probabilities, or to the utmoſt extent of human life; and then add the terms of this ſeries up together into one ſum total, which call S: and, laſtly, multiply $\overset{£}{V}$ into $\overset{£}{1} \times \frac{P}{Arn}$. And, if the product of this multiplication is equal to $£1 \times S$, or if $V \times \frac{P}{Arn}$ is equal to S, we may conclude that $\overset{£}{V}$, or the value of the ſaid annuity of one pound a year for a life of the age of N years, has been rightly computed.

Mr. Morgan's rule for proving the truth of the computation of the value of a life-annuity for any given age.

Proof of the ſaid rule.

For it is evident from Art. 196 that $\overset{£}{1} \times S$ is the value of a remote annuity of one pound a year for the life of a perſon of the youngeſt age in the table, to commence at the diſtance of n years, ſo that the firſt payment thereof ſhall be received at the diſtance of $n+1$ years, which value is called $\overset{£}{R}$ in Art. 197; that is, $\overset{£}{1} \times S$ is $= \overset{£}{R}$. Therefore when $\overset{£}{V} \times \overset{£}{1} \times \frac{P}{Ar^n}$ is equal to $\overset{£}{1} \times S$, it will be equal likewiſe to $\overset{£}{R}$, and conſequently (by Art. 197,) the value denoted by $\overset{£}{V}$ muſt have been rightly computed. Q E D.

An account of the conſtruction of the table of numbers next enſuing, or Table XI.

CXCIX. To make the manner of applying this rule of Mr. Morgan more apparent, it will be proper to ſet down again in regular order, in a new table, the values of the life-annuities which we computed above in Art. 108, ——— 115, from Monſieur de Parcieux's table by means of the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}\,£$. And, as, in obtaining the ſaid values one from another by means of that expreſſion, we proceeded upwards, or from the older lives to the younger, it will be convenient to ſet down the ſaid values in the ſame order in the following table in a column adjoining to another which contains the numbers of years in the correſponding ages. After theſe two columns of the ages of the lives and the values of the annuities, I ſhall ſet down, in a third column, the preſent values of a ſingle payment of one pound to be received by a perſon of the age of 3 years (which is the youngeſt age in Monſieur de Parcieux's table of probabilities,) at the ends of 91 years, 90 years, 89 years, 88 years, and every following number of years reſpectively (reckoning in this backward order) to the end of one year, or at the ages of 94 years, 93 years, 92 years, 91 years, and every following younger age (proceeding by a difference of one year,) to the age of 4 years; which ſeries, it is evident, will contain in it all the terms of the ſeries $\frac{P^{\text{I}}}{Ar^{n+1}} + \frac{P^{\text{II}}}{Ar^{n+2}} + \frac{P^{\text{III}}}{Ar^{n+3}} + \frac{P^{\text{IV}}}{Ar^{n+4}} + \frac{P^{\text{V}}}{Ar^{n+5}} + \frac{P^{\text{VI}}}{Ar^{n+6}} + \frac{P^{\text{VII}}}{Ar^{n+7}} +$ &c. in whatever part of the table of probabilities the number P^{I} be taken, or whatever be the number of years denoted by $N+1$, or by N. And then, in a fourth column, I ſhall ſet down the ſums of the terms in the foregoing ſeries of values contained in the third column, as they ariſe; ſo that every term in this fourth

fourth column that is even with any given age, ſhall be equal to the ſum of all the terms in the third column that correſpond to the ages that are older than the ſaid given age. Thus, for example, the term in the fourth column that is even with the age of 50 years is equal to the ſum of all the terms in the third column that correſpond to the ages that are older than 50 years. Theſe ſums will be equal to the values of remote annuities of one pound a year for the life of a perſon of the age of 3 years, to commence at the diſtances of 90 years, 89 years, 88 years, 87 years, &c. or ſo that the firſt payments of them ſhall become due at the diſtances of 91 years, 90 years, 89 years, 88 years, &c. and will comprize all the different values of the quantity which in Art. 197 is called R, and in Art. 198 is called $\frac{£}{1} \times S$. And, laſtly, in a fifth column, I ſhall ſet down the products that ariſe by multiplying the terms of the ſecond column, or the values of the life-annuities for the ſeveral ages ſet down in the firſt column, by the correſponding terms, or terms that are placed even with them, in the third column. And ſo far as we find theſe products, ſet down in the fifth column, to coincide with the numbers in the fourth column that are placed even with them, we may conclude that the numbers in the ſecond column, or the values of the life-annuities correſponding to the ages ſet down in the firſt column, have been rightly computed. This table will be as follows.

TABLE

TABLE XI.

Conſiſting of five columns of numbers; in the firſt of which the numbers of years in the ſeveral ages of human life, that differ from each other by a year, from the age of 94 *years to the age of* 3 *years, incluſively, are ſet down in regular order; and in the ſecond column are ſet down the ſeveral values of an annuity of one pound a year for the lives of perſons of the ſeveral ages ſet down in the firſt column, reſpectively; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is* 3½ *per cent. and in the third column are ſet down the preſent values of a ſingle payment of one pound to be received by a child of the age of* 3 *years at the ends of* 91 *years,* 90 *years,* 89 *years,* 88 *years, and of every following leſſer number of years down to one year, reſpectively, if the child ſhall live to the ends of the ſaid years; and in the fourth column are ſet down the numbers that ariſe by the continual addition of the numbers ſet down in the third column, ſo that each number in the ſaid fourth column is equal to the ſum of all the numbers in the third column that are placed above it, or that correſpond to the preceeding, or older, ages; and in the fifth and laſt column are ſet down the products that ariſe by multiplying the terms of the ſecond column (or the values of the life-annuities for the ſeveral ages ſet down in the firſt column,) by the correſponding terms of the third column, reſpectively.*

Years of Age.	*Values of a life-annuity of one pound a year.*	*Preſent values of a ſingle payment of one pound, to be received by a child of the age of* 3 *years, at the ends of* 91, 90, 89, 88, *&c. years, if he ſhall be living at the ends of thoſe years reſpectively.*	*Sums of the values in the third column.*	*Products of the multiplication of the numbers in the ſecond and third columns.*
	£	£	£	£
94	0.000,000	0.000,043		
93	0.483,091	0.000,090	0.000,043	0.000,043
92	0.716,468	0.000,187	0.000,133	0.000,133
91	0.94 ,669	0.000,339	0.000,320	0.000,321
90	1.19 ,512	0.000,551	0.000,659	0.000,659
89	1.459,699	0.000,830	0.001,210	0.001,211

Years of Age.	*Values of a life-annuity of one pound a year.*	*Present values of a single payment of one pound, to be received by a child of the age of 3 years, at the ends of 91, 90, 89, 88, &c. years, if he shall be living at the ends of those years respectively.*	*Sums of the values in the third column.*	*Products of the multiplication of the numbers in the second and third columns.*
	£	£	£	£
88	1.728,378	0.001,181	0.002,040	0.002,041
87	1.999,810	0.001,612	0.003,221	0.003,223
86	2.211,911	0.002,186	0.004,833	0.004,835
85	2.456,774	0.002,858	0.007,019	0.007,021
84	2.717,188	0.003,636	0.009,877	0.009,879
83	2.984,473	0.004,529	0.013,513	0.013,516
82	3.215,658	0.005,612	0.018,042	0.018,046
81	3.427,856	0.006,901	0.023,654	0.023,655
80	3.616,781	0.008,345	0.030,555	0.030,557
79	3.908,000	0.010,955	0.038,900	0.038,904
78	4.187,758	0.011,667	0.048,855	0.048,858
77	4.461,839	0.013,566	0.060,522	0.060,529
76	4.754,921	0.015,583	0.074,488	0.074,095
75	5.059,623	0.017,724	0.089,671	0.089,676
74	5.347,806	0.020,083	0.107,395	0.107,399
73	5.644,448	0.022,586	0.127,478	0.127,485
72	5.945,972	0.025,239	0.150,064	0.150,070
71	6.249,840	0.028,050	0.175,303	0.175,308
70	6.575,357	0.030,928	0.203,353	0.203,362
69	6.896,496	0.033,972	0.234,281	0.234,287
68	7.233,699	0.037,085	0.268,253	0.268,261
67	7.583,727	0.040,263	0.305,338	0,305,343
66	7.944,258	0.043,505	0.345,601	0.345,614
65	8.313,625	0.046,805	0.389,106	0.389,119
64	8.690,648	0.050,160	0.435,911	0.435,922
63	9.053,059	0.053,693	0.486,071	0.486,085
62	9.401,925	0.057,411	0.539,764	0.539,773
61	9.759,829	0.061,188	0.597,175	0.597,184
60	10.104,074	0.065,159	0.658,363	0.658,381
59	10.435,566	0.069,334	0.723,522	0.723,539
58	10.755,123	0.073,730	0.792,856	0.792,867
57	11.063,485	0.078,329	0.866,576	0.866,591
56	11.383,427	0.083,008	0.944,905	0.944,915
55	11.691,801	0.087,919	1.027,913	1.027,931
54	11.989,093	0.093,072	1.115,832	1.115,848

Years of Age.	*Values of a life-annuity of one pound a year.*	*Present values of a single payment of one pound, to be received by a child of the age of 3 years, at the ends of 91, 90, 89, 88, &c. years, if he shall be living at the ends of those years respectively.*	*Sums of the values in the third column.*	*Products of the multiplication of the numbers in the second and third columns.*
	£	£	£	£
53	12.298,386	0.098,300	1.208,904	1.208,931
52	12.596,296	0.103,779	1.307,204	1.307,231
51	12.883,449	0.109,521	1.410,983	1.411,008
50	13.183,083	0.115,339	1.520,504	1.520,523
49	13.494,425	0.121,225	1.635,843	1.635,861
48	13.793,859	0.127,382	1.757,068	1.757.089
47	14.105,200	0.133,601	1.884,450	1.884,468
46	14.404,519	0.140,100	2.018,051	2.018,077
45	14.716,120	0.146,654	2.158,151	2.158,177
44	15.015,669	0.153,495	2.304,805	2.304,830
43	15.303,763	0.160,635	2.458,300	2.458,320
42	15.580,939	0.168,087	2.618,935	2.618,953
41	15.847,705	0.175,864	2.787,022	2.787,040
40	16.104,542	0.183,980	2.962,886	2.962,904
39	16.351,906	0.192,448	3.146,866	3.146,891
38	16.590,228	0.201,283	3.339,314	3.339,330
37	16.819,920	0.210,502	3.540,597	3.540,626
36	17.016,528	0.220,440	3.751.099	3.751,123
35	17.206,612	0.230,816	3.971,539	3.971,561
34	17.390,462	0.241,649	4.202,355	4.202,387
33	17.568,352	0.252,957	4.444,004	4.444,037
32	17.740,543	0.264,761	4.696,961	4,697,003
31	17.907,280	0.277,080	4.961,722	4.961,748
30	18.068,798	0.289,938	5.238,802	5.238,831
29	18.225,318	0.303,357	5.528,740	5.528,777
28	18.377,051	0.317,360	5.832,097	5.832,140
27	18.524,196	0.331,971	6.149,457	6.149,495
26	18.666,944	0.347,216	6.481,428	6.481,461
25	18.805,476	0.363,122	6.828,644	6.828,682
24	18.939,963	0.379,715	7.191,766	7.191,788
23	19.070.569	0.397,026	7.571,481	7.571,511
22	19.197,449	0.415,083	7.968,507	7.968,534
21	19.320,751	0.433,918	8.383,590	8.383,621
20	19.440,616	0.453,563	8.817,508	8.817,544

Years of Age.	*Values of a life-annuity of one pound a year.*	*Present values of a single payment of one pound, to be received by a child of the age of 3 years, at the ends of 91, 90, 89, 88, &c. years, if he shall be living at the ends of those years respectively.*	*Sums of the values in the third column.*	*Products of the multiplication of the numbers in the second and third columns.*
	£	£	£	£
19	19.581,000	0.473,474	9.271,071	9.271,094
18	19.716,914	0.494,224	9.744,545	9.744,572
17	19.848,540	0.515,847	10.238,769	10.238,809
16	19.976,052	0.538,378	10.754,616	10.754,666
15	20.123,320	0.561,191	11.292,994	11.293,026
14	20.265,616	0.584,943	11.854,185	11.854,230
13	20.403,141	0.609,669	12.439,128	12.439,162
12	20.536,083	0.635,410	13.048,797	13.048,835
11	20.664,641	0.662,206	13.684,207	13.684,249
10	20.741,729	0.691,671	14.346,413	14.346,452
9	20.770,473	0.724,015	15.038,084	15.038,134
8	20.754,438	0 759,459	15.762,099	15.762,144
7	20.720,153	0.797,369	16.521,558	16.521,607
6	20.647,176	0.838,806	17.318,927	17.318,975
5	20.518,022	0.884,967	18.157,733	18.157,772
4	20.318,825	0.937,197	19.042,700	19.042,741
3	19.979,961		19.979,897	

In this table the numbers in the fifth column agree with those in the fourth column in either the five or the six first places of figures; and those of the last year, or that correspond to the age of 4 years, to wit, £19.042,741 and £19.042,700, agree with each other in the six first places of figures, their difference being only £0.000,041, or 41 millionth parts of a pound. And therefore we may safely conclude that all the values of an annuity of one pound a year set down in this table have been rightly computed to the same degree of exactness, or to at least five places of figures.

Of the degree of exactness of the values of life-annuities given in the foregoing table.

An account of an error in the computation of Table XI, which makes the values of an annuity of one pound for lives under the age of 41 years a little too great.

CC. The reader will obſerve that the numbers ſet down in the foregoing table as the values of annuities of one pound a year for the lives of perſons under the age of 42 years differ a little from the values of them exhibited in Table III, page 131, et ſeq. This is owing to an error in the calculation of the value of an annuity for a life of 41 years of age in that table, which error runs through all the reſt of the table, or all the values of it that are derived from that value, that is, all the values of it for lives of younger ages than that of 41 years. The value of an annuity of one pound a year for a life of 41 years is equal to $\frac{1}{1.035} \times \frac{643}{650}$ × £16.580,939, which in page 123 (where the values afterwards ſet down in Table III, are computed,) is made to be equal to $\frac{1}{1.035}$ × £16.448,528, or £15.892,297, whereas it is in truth equal to $\frac{1}{1.035}$ × £16.402,375, or £15.847,705. But the difference between theſe two values, £15.892,297 and £15.847,705, is only £0.044,592, or about 10*d.*$\frac{3}{4}$, which is but a trifling difference upon a ſum of near 15*l.* 17*s.* And the differences between the ſubſequent values in the two tables are ſtill leſs than 10*d.*$\frac{3}{4}$. Thus, for example, the difference between £17.770,162, the value of an annuity of one pound a year for a life of the age of 32 years in Table III, page 132, and £17.740,543, the value of the ſame annuity given in the laſt table, is £0 029,619; which is leſs than 7*d.*$\frac{1}{4}$; and the difference between £20.752,981, the value of an annuity of one pound a year for a life of the age of 10 years, in Table III, page 131, and £20.741,729, the value of the ſame annuity given in the laſt table, is £0.011,252; which is leſs than 2*d.*$\frac{1}{4}$; and the difference between £19.987,654, the value of an annuity of one pound a year given in Table III, page 131, for a life of only 3 years of age (which is the youngeſt age in the table,) and £19.979,961, the value of the ſame annuity given in the laſt table, is £0.007,693; which is leſs than 2*d.* We may therefore conſider the values given in Table III as being exact to all purpoſes of practice, or as co-inciding with thoſe in the laſt table, which has been computed without any miſtake. And therefore I did not think it worth while to cauſe the errors of Table III, and of ſome articles that follow it and refer to it, to be corrected, when I had diſcovered them, which was not till after the ſheets in which that table and thoſe ſubſequent articles are contained had been printed off. I diſcovered theſe errors by means of Mr. Morgan's method, above-mentioned, of proving the truth of theſe computations as we go on. For when, in computing the fifth column in the laſt table (which conſiſts of the products of the numbers in the

the ſecond and third columns,) I came to the product belonging to the age of 41 years, I found that the ſaid product was greater than it ought to be, or exceeded by a greater difference than it ought to have done the correſponding number in the fourth column; and thereupon I computed anew the value of an annuity of one pound a year for a life of the age of 41 years from that of the like annuity for a life of the age of 42 years, by means of the expreſſion $\frac{1}{1.035} \times \frac{643}{650} \times £16.580,939$, and found that the ſaid expreſſion was not equal to $\frac{1}{1.035} \times £16.448,528$, or $£15.892,297$, (as it is ſuppoſed to be in page 123,) but to $\frac{1}{1.035} \times £16.402,375$, or $£15.847,705$. And, having thus found out and corrected this error belonging to the age of 41 years, I computed the values of an annuity of one pound a year for lives of all ages younger than 41 years over again by means of the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|\,£$, and proved them, as I went on, by Mr. Morgan's method above-mentioned, and therefore I am confident that this laſt table is correct.

CCI. Having now gone through every thing I propoſed to offer concerning the *manner* of computing the values of annuities for ſingle lives, I ſhall proceed to exhibit ſeveral different tables of thoſe values ſuited to the ſeveral following rates of the intereſt of money, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. all computed fairly and ſtrictly from Monſieur de Parcieux's table of the probabilities of the duration of human life above-mentioned by means of the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|£$. One of them, to wit, the fourth, or that which relates to the intereſt of $3\frac{1}{2}$ per cent. is the ſame with the firſt and ſecond columns of Table XI, which is given above in Art. 199. I have inſerted this part of the ſaid table a ſecond time in its proper place among the other tables, becauſe I thought it would be more convenient for the readers who ſhall want to conſult and make uſe of it, to look for it there than in the former parts of this tract. But I have there ſet down only the ſaid two firſt columns of Table XI, or the numbers of years in the ſeveral ages of human life and

An account of the following twelve tables of the values of a life-annuity of one pound.

the numbers that exhibit the values of the annuities that correſpond to them, without the additional numbers that are contained in the third, fourth, and fifth columns of Table XI. and which are only uſeful as proofs of the exactneſs of the former numbers. This fourth of the following ſet of tables, or table relating to the intereſt of $3\frac{1}{2}$ per cent. I have computed myſelf. The others have been computed by Mr. Denſham, an able arithmetician, whom Dr. Price recommended for that purpoſe. And they were proved by him, as he proceeded in the work, by Mr. Morgan's method above-mentioned, though I have not thought it neceſſary to cauſe all thoſe proof-numbers to be printed. But I have looked over thoſe proof-numbers myſelf, and found that they confirm the truth of the computations.

And, as theſe tables are not inſerted in this place to illuſtrate the manner in which they have been computed, (as was the caſe with Table XI,) but are intended to be reſorted to and made uſe of, as exhibiting the true values of life-annuities for the different ages of human life, according to the different rates of the intereſt of money, without any reference to the method of computing them, I have ſet down the values of the annuities in each table in their natural order, beginning with the youngeſt age, which is that of 3 years. Theſe tables are as follows.

TABLE

TABLE XII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years, inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 2 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	27.382,830	34	21.782,266	65	9.238,319
4	27.794,317	35	21.474,025	66	8.795,049
5	28.008,120	36	21.158,940	67	8.365,278
6	28.121,217	37	20.836,776	68	7.950,607
7	28.153,865	38	20.475,233	69	7.553,307
8	28.130,823	39	20.104,909	70	7.176,577
9	28.080,318	40	19.725,499	71	6.798,054
10	27.967,401	41	19 336,686	72	6.445,751
11	27.788,463	42	18.938,139	73	6.098,544
12	27.540,613	43	18.529,510	74	5.759,088
13	27.287,412	44	18.110,436	75	5.431,073
14	27.028,710	45	17.680,537	76	5.087,894
15	26.764,350	46	17.239,415	77	4.759,614
16	26.494,172	47	16.815,956	78	4.453,776
17	26 250,605	48	16.381,354	79	4.144,112
18	26.001,981	49	15 963,864	80	3.871,790
19	25.748,153	50	15.535,376	81	3.613,948
20	25.488,966	51	15.123,598	82	3.380,105
21	25.256,792	52	14.729,083	83	3.127,536
22	25.020,193	53	14.324,686	84	2.838,919
23	24.779.034	54	13.909,922	85	2.559,295
24	24.533,179	55	13.511,804	86	2.297,450
25	24.282,487	56	13.103,799	87	2 070,661
26	24.026,812	57	12.685,379	88	1.784,099
27	23.766,002	58	12.283,071	89	1.502,199
28	23.499,897	59	11.870,902	90	1.228,718
29	23.228,331	60	11.448,295	91	0.969,461
30	22.951,131	61	11.014,604	92	0.730,488
31	22.668,117	62	10.569,115	93	0.490,196
32	22.379,101	63	10.137,299	94	0.000,000
33	22.083,886	64	9.693,983		

TABLE XIII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 2½ per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	24.462,676	34	20.136,887	65	8.911,789
4	24.849,735	35	19.878,238	66	8.495,160
5	25.062,078	36	19.612,791	67	8.090,289
6	25.185,830	37	19.340,316	68	7.698,859
7	25.238,680	38	19.030,630	69	7.323,022
8	25.242,492	39	18.712,036	70	6.966,149
9	25.222,411	40	18.384,189	71	6.606,509
10	25.146,756	41	18.046,727	72	6.271,426
11	25.011,897	42	17.699,272	73	5.940,421
12	24.814,820	43	17.341,428	74	5.616,112
13	24.612,646	44	16.972,778	75	5.302,157
14	24.405,208	45	16.592,885	76	4.972,521
15	24.192,333	46	16.201,291	77	4.656,602
16	23.973,844	47	15.825,188	78	4.361,896
17	23.779,193	48	15.437,457	79	4.062,687
18	23.579,731	49	15.064,768	80	3.799,480
19	23.375,296	50	14.680,583	81	3.549,972
20	23.165,719	51	14.311,129	82	3.323,657
21	22.980,544	52	13.957,047	83	3.078,502
22	22.791,199	53	13.592,614	84	2.797,255
23	22.597,546	54	13.217,294	85	2.524,251
24	22.399,442	55	12.856,800	86	2.268,242
25	22.196,735	56	12.485,883	87	2.046,484
26	21.989,269	57	12.103,960	88	1.765,079
27	21.776,880	58	11.736,386	89	1.487,784
28	21.559,396	59	11.358,341	90	1.218,152
29	21.336,639	60	10.969,190	91	0.962,095
30	21.108,421	61	10.568,230	92	0.725,758
31	20.874,547	62	10.154,683	93	0.487,804
32	20.634,811	63	9.753,041	94	0.000,000
33	20.388,999	64	9.339,034		

TABLE XIV.

Containing the values of an annuity of one pound a year for the lives of perſons of the ſeveral ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years incluſively; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 3 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	22.027,85	34	18.681,86	65	8.603,99
4	22.390,40	35	18.464,13	66	8.211,93
5	22.597,31	36	18.239,84	67	7.830,08
6	22.725,72	37	18.008,72	68	7.460,10
7	22.791,23	38	17.742,48	69	7.104,30
8	22.813,30	39	17.467,42	70	6.765,92
9	22.814,53	40	17.183,14	71	6.423,92
10	22.766,00	41	16.889,24	72	6.104,99
11	22.664,17	42	16.585,30	73	5.789,19
12	22.505,84	43	16.270,90	74	5.479,13
13	22.342,75	44	15.945,54	75	5.178,44
14	22.174,71	45	15.608,74	76	4.861,62
15	22.001,56	46	15.260,00	77	4.557,43
16	21.823,10	47	14.924,96	78	4.273,29
17	21.666,23	48	14.578,02	79	3.984,05
18	21.504,89	49	14.244,41	80	3.729,54
19	21.338,90	50	13.899,02	81	3.488,01
20	21.168,08	51	13.566,71	82	3.268,91
21	21.019,54	52	13.248,20	83	3.030,89
22	20.867,17	53	12.919,06	84	2.756,77
23	20.710,84	54	12.578,70	85	2.490,19
24	20.550,40	55	12.251,64	86	2.239,87
25	20.385,70	56	11.913,80	87	2.023,06
26	20.210,57	57	11.564,55	88	1.746,59
27	20.042,84	58	11.228,16	89	1.473,61
28	19.864,33	59	10.880,86	90	1.207,74
29	19.680,86	60	10.521,96	91	0.954,82
30	19.492,23	61	10.150,71	92	0.721,08
31	19.298,24	62	9.766,26	93	0.485,43
32	19.098,66	63	9.392,18	94	0.000,00
33	18.893,29	64	9.005,09		

TABLE XV.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	19.979,961	34	17.390,462	65	8.313,625
4	20.318,825	35	17.206,612	66	7.944,258
5	20.518,022	36	17.016,528	67	7.583,727
6	20.647,176	37	16.819,920	68	7.233,699
7	20.720,153	38	16.590,228	69	6.896,496
8	20.754,438	39	16.351,906	70	6.575,357
9	20.770,473	40	16.104,542	71	6.249,840
10	20.741,729	41	15.847,705	72	5.945,972
11	20.664,641	42	15.580,939	73	5.644,448
12	20.536,088	43	15.303,763	74	5.347,806
13	20.403,141	44	15.015,669	75	5.059,623
14	20.265,616	45	14.716,120	76	4.754,921
15	20.123,320	46	14.404,549	77	4.461,839
16	19.976,052	47	14.105,200	78	4.187,758
17	19.848,540	48	13.793,859	79	3.908,000
18	19.716,914	49	13.494,425	80	3.661,781
19	19.581,000	50	13.183,083	81	3.427,856
20	19.440,616	51	12.883,449	82	3.215,658
21	19.320,751	52	12.596,296	83	2.984,473
22	19.197,449	53	12.298,386	84	2.717,188
23	19.070,569	54	11.989,093	85	2.456,774
24	18.939,963	55	11.691,801	86	2.211,911
25	18.805,476	56	11.383,427	87	1.999,810
26	18.666,944	57	11.063,485	88	1.728,378
27	18.524,196	58	10.755,123	89	1.459,699
28	18.377,051	59	10.435,566	90	1.197,512
29	18.225,318	60	10.104,074	91	0.947,669
30	18.068,798	61	9.759,829	92	0.716,468
31	17.907,280	62	9.401,925	93	0.483,091
32	17.740,543	63	9.053,059	94	0.000,000
33	17.568,352	64	8.690,648		

TABLE XVI.

Containing the values of an annuity of one pound a year for the lives of perſons of the ſeveral ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years, incluſively; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 4 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
	£		£		£
3	18.242,464	34	16 239,735	65	8.039,349
4	18.558,931	35	16.084,014	66	7.690,960
5	18.749,209	36	15.922,447	67	7.350,186
6	18.876,581	37	15.754,736	68	7.018,692
7	18.953,475	38	15.555,857	69	6.698,802
8	18.995,706	39	15.348,644	70	6.393,749
9	19.021,902	40	15.132,664	71	6.083,658
10	19.007,582	41	14.907,457	72	5.793,941
11	18.949,243	42	14.672,537	73	5.505,835
12	18.843,753	43	14.427,388	74	5.221,832
13	18.734,230	44	14.171,466	75	4.945,465
14	18.620,487	45	13.904,190	76	4.652,254
15	18.502,326	46	13.624,948	77	4.369,724
16	18.379,539	47	13.336,700	78	4.105,200
17	18.274,964	48	13 076,491	79	3.834,477
18	18.166,641	49	12.807,002	80	3.596,174
19	18 054,395	50	12.525,605	81	3.369,530
20	17.938,041	51	12.254,766	82	3.163,947
21	17.840,730	52	11.995,305	83	2.939,337
22	17.740,368	53	11.725,075	84	2 678,656
23	17.636,818	54	11.443,400	85	2.424,216
24	17.529,936	55	11.172,645	86	2.184,654
25	17.419,569	56	10.890,825	87	1.977,157
26	17.305,557	57	10.597,210	88	1.710,503
27	17.187,730	58	10.314,093	89	1.446,020
28	17.065,909	59	10.019,612	90	1.187,434
29	16.939,905	60	9.712,978	91	0.940,608
30	16.809,518	61	9.393,319	92	0.711,907
31	16.674,537	62	9.059,665	93	0.480,769
32	16.534,739	63	8.733,893	94	0.000,000
33	16.389,888	64	8.394,166		

TABLE XVII.

Containing the values of an annuity of one pound a year for the lives of perſons of the ſeveral ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 4½ per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	16.756,318	34	15.210,67	65	7.779,99
4	17.051,91	35	15.078,39	66	7.451,02
5	17.232,78	36	14.940,68	67	7.128,57
6	17.356,81	37	14.797,23	68	6.814,32
7	17.435,21	38	14.624,42	69	6.510,57
8	17.482,39	39	14.443,64	70	6.220,54
9	17.515,42	40	14.254,42	71	5.924,89
10	17.511,62	41	14.056,29	72	5.648,45
11	17.467,53	42	13.848,74	73	5.372,97
12	17.380,04	43	13.631,21	74	5.100,89
13	17.288,86	44	13.403,15	75	4.835,69
14	17.193,80	45	13.163,92	76	4.553,37
15	17.094,65	46	12.912,88	77	4.280,86
16	16.991,21	47	12.671,81	78	4.025,43
17	16.904,67	48	12.418,90	79	3.763,33
18	16.814,73	49	12.175,72	80	3.532,59
19	16.721,21	50	11.920,73	81	3.312,91
20	16.623,93	51	11.675,33	82	3.113,67
21	16.544,44	52	11.440,38	83	2.895,38
22	16 462,26	53	11.194,74	84	2.641,07
23	16.377,28	54	10.937,70	85	2.392,40
24	16.289,34	55	10.690,66	86	2.157,98
25	16.198,31	56	10.432,56	87	1.954,95
26	16.104,02	57	10.162,64	88	1.692,95
27	16.006,32	58	9.902,29	89	1.432,57
28	15.905,03	59	9.630,51	90	1.177,51
29	15.799,96	60	9.346,46	91	0.933,04
30	15.690,92	61	9.049,21	92	0.707,40
31	15.577,70	62	8.737,74	93	0.478,468
32	15.460,07	63	8.433,15	94	0.000,000
33	15.337,82	64	8.114,30		

TABLE XVIII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years, inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 5 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	15.475,38	34	14 287,24	65	7.534,57
4	15.751,70	35	14.174,54	66	7.223,59
5	15.923,11	36	14.056,84	67	6.918,17
6	16 042,87	37	13.933,83	68	6.619,96
7	16.121,17	38	13.783,16	69	6.331,26
8	16.171,20	39	13.624,89	70	6.055,28
9	16.208,71	40	13.458,56	71	5.773,18
10	16.212,55	41	13.283,68	72	5.505,20
11	16.179,35	42	13.099,71	73	5.245,60
12	16.106,03	43	12.906,09	74	4.984,76
13	16.029,32	44	12.702,21	75	4.730,12
14	15.949,04	45	12.487,42	76	4.458,12
15	15.864,99	46	12.261,04	77	4.195,13
16	15.776,95	47	12.043,77	78	3.948,35
17	15.704,68	48	11.814,85	79	3 694,48
18	15.629,33	49	11.594,84	80	3.470,95
19	15.550,71	50	11.363,18	81	3.257,93
20	15.468,67	51	11.140,30	82	3.064,75
21	15.403,32	52	10.927,09	83	2.852,53
22	15.335,63	53	10.703,34	84	2.604,35
23	15.265,47	54	10.468,30	85	2.361,25
24	15.192,73	55	10.242,48	86	2.131,77
25	15.117,25	56	10.005,69	87	1.933,03
26	15.038,89	57	9 757,12	88	1.675,04
27	14.957,50	58	9.517,33	89	1.418,35
28	14.872,90	59	9.266,13	90	1.166,21
29	14.784,92	60	9.002,61	91	0.924,25
30	14.693,37	61	8.725,83	92	0.702,94
31	14.598,05	62	8.434,68	93	0.476,19
32	14.498,74	63	8.149,54	94	0.000,00
33	14.395,22	64	7.849,93		

TABLE XIX.

Containing the values of an annuity of one pound a year for the lives of perſons of the ſeveral ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years incluſively; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 6 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	13.390,78	34	12.704,49	65	7.081,91
4	13.633,23	35	12.622,00	66	6.803,15
5	13.786,59	36	12.535,35	67	6.528,33
6	13.896,64	37	12.444,26	68	6.259,06
7	13.971,93	38	12.328,53	69	5.997,22
8	14.023,69	39	12.206,01	70	5.746,68
9	14.065,55	40	12.076,23	71	5.489,21
10	14.078,91	41	11.938,66	72	5.247,98
11	14.060,56	42	11.792,75	73	5.006,12
12	14.007,46	43	11.637,89	74	4.765,93
13	13.951,50	44	11.473,46	75	4.530,73
14	13.892,50	45	11.298,74	76	4 277,83
15	13.830,25	46	11.112,99	77	4.032,52
16	13.764,54	47	10.935,03	78	3.801,85
17	13.712,73	48	10.745,94	79	3.563,34
18	13.658,38	49	10.564,46	80	3.353,32
19	13.601,33	50	10.371,80	81	3.152,81
20	13.541,40	51	10.186,65	82	2.971,06
21	13.496,36	52	10.009,95	83	2.770,32
22	13.449,57	53	9.823,15	84	2.533,81
23	13.400,92	54	9.625,44	85	2.301,35
24	13.350,30	55	9.435,74	86	2.081,39
25	13.297,59	56	9.235,40	87	1.890,99
26	13.242,66	57	9.023,54	88	1.642,24
27	13.185,37	58	8.819,24	89	1.393,57
28	13.125,58	59	8.603,71	90	1.148,64
29	13.063,13	60	8.376,00	91	0.913,30
30	12.997,84	61	8.135,06	92	0.694,19
31	12.929,54	62	7.879,69	93	0.471,69
32	12.858,02	63	7.628,91	94	0.000,00
33	12.783,08	64	7.363,46		

TABLE XX.

Containing the values of an annuity of one pound a year for the lives of perſons of the ſeveral ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years incluſively; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 7 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	11.777,44	34	11.404,51	65	6.673,74
4	11.991,62	35	11.343,49	66	6.422,78
5	12.128,80	36	11.279,09	67	6.174,46
6	12.229,00	37	11.211,03	68	5.930,35
7	12.299,56	38	11.120,95	69	5.692,65
8	12.350,20	39	11.024,87	70	5.464,47
9	12.392,90	40	10.922,30	71	5.228,74
10	12.411,09	41	10.812,72	72	5.007,66
11	12.401,71	42	10.695,57	73	4.785,15
12	12.361,77	43	10.570,22	74	4.563,42
13	12.319,38	44	10.436,01	75	4.345,69
14	12.274,34	45	10.292,20	76	4.110,04
15	12.226,48	46	10.138,01	77	3.880,74
16	12.175,56	47	9.990,64	78	3.664,71
17	12.137,07	48	9.832,76	79	3.440,23
18	12.096,46	49	9.681,55	80	3.242,57
19	12.053,56	50	9.519,73	81	3.053,54
20	12.008,23	51	9.364,51	82	2.882,31
21	11.976,34	52	9.216,85	83	2.692,20
22	11.943,15	53	9.059,63	84	2.466,56
23	11.908,59	54	8.892,00	85	2.244,04
24	11.872,55	55	8.731,51	86	2.033,01
25	11.834,93	56	8.560,84	87	1.850,43
26	11.795,64	57	8.379,07	88	1.609,95
27	11.754,55	58	8.203,96	89	1.368,64
28	11.711,53	59	8.017,98	90	1.130,11
29	11.666,45	60	7.820,13	91	0.900,21
30	11.619,16	61	7.609,27	92	0.685,649
31	11.569,50	62	7.384,13	93	0.467,289
32	11.517,30	63	7.162,51	94	0.000,000
33	11.462,37	64	6.926,23		

TABLE XXI.

Containing the values of an annuity of one pound a year for the lives of perſons of the ſeveral ages of 3 *years,* 4 *years,* 5 *years, and every following number of years up to* 93 *years inclusively; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is* 8 *per cent.*

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	10.498,90	34	10.323,80	65	6.305,04
4	10.689,50	35	10.278,24	66	6.078,24
5	10.812,58	36	10.229,96	67	5.853,05
6	10.903,61	37	10.178,73	68	5.630,99
7	10.968,95	38	10.107,71	69	5.414,20
8	11.017,21	39	10.031,41	70	5.205,73
9	11.059,02	40	9.949,36	71	4.989,28
10	11.079,47	41	9.861,03	72	4.786,10
11	11.075,61	42	9.765,86	73	4.580,86
12	11.044,54	43	9.663,21	74	4.375,67
13	11.011,33	44	9.552,42	75	4.173,66
14	10.975,79	45	9.432,72	76	3.953,62
15	10.937,73	46	9.303,30	77	3.738,86
16	10.896,93	47	9.179,99	78	3.536,16
17	10.867,35	48	9.046,81	79	3.324,52
18	10.835,97	49	8.919,59	80	3.138,19
19	10.802,63	50	8.782,39	81	2.959,71
20	10.767.17	51	8.651,10	82	2.798,19
21	10.743,97	52	8.526,72	83	2.617,95
22	10.719,82	53	8.393,38	84	2.402,45
23	10.694,65	54	8.250,20	85	2.189,26
24	10.668,38	55	8.113,50	86	1.986,62
25	10.640,95	56	7.967,16	87	1.811,41
26	10.612,25	57	7.810,22	88	1.578,50
27	10.582,20	58	7.659,28	89	1.344,52
28	10.550,69	59	7.497,95	90	1.112,13
29	10.517,60	60	7.325,16	91	0.887,458
30	10.482,82	61	7.139,72	92	0.677,297
31	10.446,20	62	6.940,29	93	0.462,962,9
32	10.407,60	63	6.743,60	94	0.000,0.0
33	10.366,85	64	6.532,39		

TABLE XXII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 9 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
	£		£		£
3	9.464,49	34	9.415,24	65	5.970,66
4	9.635,36	35	9.380,92	66	5.764,92
5	9.746,28	36	9.344,45	67	5.559,98
6	9.829,07	37	9.305,64	68	5.357,29
7	9.889,33	38	9.248,97	69	5.158,93
8	9.934,73	39	9.187,66	70	4.967,89
9	9.974,87	40	9.121,25	71	4.768,56
10	9.996,17	41	9.049,24	72	4.581,32
11	9.995,79	42	8.971,06	73	4.391,54
12	9.970,90	43	8.886,08	74	4.201,23
13	9.944,11	44	8.793,62	75	4.013,41
14	9.915,24	45	8.692,92	76	3.807,52
15	9.884,08	46	8.583,14	77	3.606,01
16	9.850,43	47	8.478,93	78	3.415,49
17	9.826,98	48	8.365,47	79	3.215,61
18	9.801,97	49	8.257,46	80	3.039,69
19	9.775,24	50	8.140,06	81	2.870,94
20	9.746,65	51	8.028,06	82	2.718,38
21	9.729,30	52	7.922,48	83	2.547,30
22	9.711,26	53	7.808,53	84	2.341,29
23	9.692,47	54	7.685,33	85	2.136,85
24	9.672,88	55	7.568,13	86	1.942,11
25	9.652,42	56	7.441,86	87	1.773,87
26	9.631,02	57	7.305,54	88	1.548,74
27	9.608,60	58	7.174,74	89	1.321,18
28	9.585,10	59	7.034,06	90	1.094,68
29	9.560,41	60	6.882,41	91	0.875,038
30	9.534,43	61	6.718,55	92	0.669,135
31	9.507,05	62	6.541,08	93	0.458,715
32	9.478,15	63	6.365,76	94	0.000,000
33	9.447,60	64	6.176,19		

TABLE XXIII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 10 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	8.612,41	34	8.643,42	65	5.666,42
4	8.766,66	35	8.617,37	66	5.479,11
5	8.867,12	36	8 589,66	67	5.291,94
6	8.942,62	37	8.560,12	68	5.106,33
7	8.998,15	38	8.514,37	69	4.924,28
8	9.040,62	39	8.464,55	70	4.748,71
9	9.078,77	40	8.410,21	71	4.564,65
10	9.100,13	41	8.350,86	72	4.391,68
11	9.101,99	42	8.285,95	73	4.215,77
12	9.081,56	43	8.214,86	74	4.038,86
13	9.059,42	44	8.136,92	75	3.863,86
14	9.035,38	45	8.051,35	76	3.670,85
15	9.009,25	46	7.957,30	77	3.481,41
16	8.980,80	47	7.868,40	78	3.302,03
17	8.961,70	48	7.770,84	79	3.112,98
18	8.941,21	49	7.678,32	80	2.946,63
19	8.919,19	50	7.576,99	81	2.786,85
20	8.895,48	51	7.480,66	82	2.642,58
21	8.882,16	52	7.390,37	83	2.48 ,03
22	8.868,33	53	7.292,30	84	2.282,89
23	8.853,95	54	7.185,54	85	2.086,67
24	8.838,99	55	7.084,42	86	1.899,38
25	8.823,39	56	6.974,80	87	1.737,73
26	8.807,10	57	6.855,69	88	1.519,72
27	8.790,06	58	6.741,74	89	1.298,58
28	8.772,21	59	6.618,46	90	1.077,73
29	8.753,47	60	6.484,73	91	0.862,938
30	8.733,77	61	6.339,28	92	0.661,157
31	8.713,02	62	6.180,65	93	0.454,545
32	8.691,12	63	6.023,74	94	0.000,000
33	8.667,96	64	5.852,93		

Concerning life-annuities that are payable every half-year or every quarter of a year.

CC. The foregoing tables exhibit the values of life-annuities, according to the above rates of intereſt, upon a ſuppoſition that thoſe annuities are to be paid only once a year at the ends of the ſeveral years of the annuitant's life that ſhall follow the time of granting them. But it is often ſtipulated between the grantors and grantees of life-annuities that they ſhall be paid every half-year, and ſometimes that they ſhall be paid every quarter of a year, during the life of the annuitant. In theſe caſes they will be ſomewhat more valuable than if they are to be paid only once a year; and conſequently a ſmall addition ought to be made to the values of them ſet down in the foregoing tables. Mr. Simpſon (in his doctrine of annuities, pages 78, 79, 80, 81,) ſays that this addition ought, in the caſe of half-yearly payments, to be about a quarter of a year's annuity, and, in the caſe of quarterly payments, to be about three eighth parts of a year's annuity. But theſe are only approximations to the values of theſe additions: and it is impoſſible to determine them with perfect preciſion without a table of the probabilities of the duration of human life that ſhould exhibit the numbers of perſons who die off in every half-year and quarter of a year, as well as in every whole year, of the whole poſſible extent of human life, out of a given original number of perſons of the ſame age all living at the ſame time: and no ſuch table has yet been publiſhed. But we may determine the value of the firſt of theſe additions, or that which is to be made in the caſe of half-yearly payments, to a ſufficient degree of exactneſs for all uſeful purpoſes in the manner following.

Obſervations on the value of a life-annuity that is payable every half-year.

CCI. It has been ſhewn above in Art. 86, 87, that the value of an annuity of one pound a year for the life of a perſon of the age of N years is $= \frac{\pounds 1}{P} \times$ the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7} +$ &c. continued to $E - N$ terms, or, (according to Monſieur de Parcieux's table of probabilities,) to $94 - N$ terms, or to the utmoſt poſſible extent of human life.

Now let us ſuppoſe this grant of an annuity of one pound a year for the life of the perſon aged N years to be changed into a grant of an annuity, or rather *penſion*, of one pound a year for the life of the ſame perſon, to be paid by half-yearly payments of 10 ſhillings each during the ſaid perſon's life. Then it is evident that the ſaid perſon will gain by this change the three following advantages. In the firſt place, he will have a chance of receiving one half-year's payment after he has attained the age of 94 years; becauſe it is ſuppoſed, in Monſieur de Parcieux's table of the

probabilities of the duration of human life, that one perſon, out of the original 1000 perſons who are living at the age of 3 years, will live beyond the age of 94 years and die before he attains the age of 95 years: and therefore, if the annuitant ſhould be that perſon, and ſhould live to the age of 94 years and a half, he may, upon this ſecond ſuppoſition of the annuity's being paid by half-yearly payments, receive a payment of 10 ſhillings in that remote period of his life, which it is abſolutely impoſſible he ſhould receive upon the former ſuppoſition of its being paid only at the end of every year. This advantage is, however, ſo exceeding ſmall, that it is hardly worth attending to. And, in the ſecond place, the annuitant will, upon this ſecond ſuppoſition of the annuity's being paid every half-year, receive one half of all the reſt of the money which will be paid him by the grantor of the annuity, half a year ſooner than on the former ſuppoſition, namely, all thoſe payments of 10 ſhillings each which he will receive at the ends of half a year, a year and a half, two years and a half, three years and a half, and every following intermediate half-year during his life; from which circumſtance thoſe half-yearly payments will be more valuable to him, and more burthenſome to the grantor of the annuity, than they were before, when they were to be made only at the ends of a whole year, two years, three years, four years, and every following whole number of years during the ſame perſon's life. And, in the laſt place, the annuitant's chances of receiving thoſe payments on the ſecond ſuppoſition of their being made at the ends of half a year, a year and a half, two years and a half, three years and a half, &c. will be ſomewhat better than they were before, when he was to receive them half a year later, or at the ends of a whole year, two years, three years, four years, &c. becauſe fewer perſons out of the whole number P of perſons repreſented in Monſieur de Parcieux's table as living at the age of the annuitant, at the time of granting the annuity, will be dead at the ends of half a year, a year and a half, two years and a half, three years and a half, &c. than at the ends of a whole year, two years, three years, four years, &c. And for theſe reaſons, and chiefly for the two latter, (as the firſt is not worth attending to,) the value of an annuity of one pound a year payable every half-year by payments of 10 ſhillings each, during the life of a perſon of a given age, will be ſomewhat greater than that of a like annuity of one pound a year that is payable only at the end of every year.

CCII. In order to determine the value of one of theſe half-yearly annuities with exactneſs it would be neceſſary to have a table of the probabilities of the duration of human life that ſhould ſet forth the numbers of perſons who may be ſuppoſed to die in every half-year, as well as year, of human life, out of a given original number of very young perſons of the ſame age all living at the ſame time. But no ſuch tables have yet been

been published. We must therefore content ourselves with determining the values of these annuities by an approximation; which may be done to a considerable and sufficient degree of exactness in the following manner.

An investigation of the said value by approximation.

CCIII. Let $\overset{\pounds}{V}$ denote (as in the foregoing articles,) the value of an annuity of one pound a year payable at the end of every year, during the life of a person of the age of N years; and $\overset{\pounds}{W}$ the value of a like annuity of one pound a year payable at the end of every year, during the life of a person one year younger than the former, or of the age of $N-1$ years. And let P be the number of persons represented in Monsieur de Parcieux's table of probabilities as living at the age of N years, and $P+d$ be the number of persons represented therein as living at the age of $N-1$ years. And let $\overset{\pounds}{r}$ be the value of one pound together with its interest for one year, as in the foregoing articles.

Then will $\overline{r-1}$ $\times$ £1 be the interest of one pound for a year, and consequently $\frac{r-1}{2} \times$ £1 will be the interest of one pound for half a year: for, when money is lent for less than a whole year at any particular rate of interest denominated from the whole year, the parties to such short loans generally mean that the interest of the sum lent for the time the loan is to last shall bear the same proportion to its interest for a whole year as the time for which it is lent bears to a whole year. Thus, if the interest of money is 4 per cent. per annum, and 100 pounds are lent for half a year, it is generally understood that the borrower, when he repays the 100 pounds at the end of the half-year, is to pay with it an additional sum of 40 shillings by way of interest for it. And, in like manner in general, if the interest upon one pound for a whole year is $\overline{r-1} \times$ £1, its interest for half a year will be $\frac{r-1}{2} \times$ £1, and its interest for the m^{th} part of a year will be $\frac{r-1}{m} \times$ £1.

Therefore the value of one pound together with its interest for half a year will be £1 $+ \frac{r-1}{2} \times$ £1, $\left(\text{or } \frac{2}{2} \times \text{£1} + \frac{r-1}{2} \times \text{£1}, \text{ or } \frac{2+r-1}{2} \times \text{£1},\right)$ or $\frac{1+r}{2} \times$ £1. Let this value be denoted by $\overset{\pounds}{b}$.

CCIV. Now this quantity h, or $\frac{1+r}{2}$, will be greater than $r^{\frac{1}{2}}$. For, since 1 is to $r^{\frac{1}{2}}$ as $r^{\frac{1}{2}}$ is to r, it follows from Euclid's Elements, book 5, prop. 25, that the sum of the two extreme terms 1 and r will be greater than twice the middle term $r^{\frac{1}{2}}$, and consequently that half that sum, or $\frac{1+r}{2}$, will be greater than the said middle term $r^{\frac{1}{2}}$. Therefore h^2, h^3, h^4, h^5, h^6, h^7, h^8, h^9, &c. will be greater than $(\overline{r^{\frac{1}{2}}}|^2$, $\overline{r^{\frac{1}{2}}}|^3$, $\overline{r^{\frac{1}{2}}}|^4$, $\overline{r^{\frac{1}{2}}}|^5$, $\overline{r^{\frac{1}{2}}}|^6$, $\overline{r^{\frac{1}{2}}}|^7$, $\overline{r^{\frac{1}{2}}}|^8$, $\overline{r^{\frac{1}{2}}}|^9$, &c. or $r^{\frac{2}{2}}$, $r^{\frac{3}{2}}$, $r^{\frac{4}{2}}$, $r^{\frac{5}{2}}$, $r^{\frac{6}{2}}$, $r^{\frac{7}{2}}$, $r^{\frac{8}{2}}$, $r^{\frac{9}{2}}$, &c. or) r, $r^{\frac{3}{2}}$, r^2, $r^{\frac{5}{2}}$, r^3, $r^{\frac{7}{2}}$, r^4, $r^{\frac{9}{2}}$, &c. respectively.

CCV. Further, let us suppose that a table of the probabilities of the duration of human life had been prepared from observations on births and burials to every half-year of human life, and that in such table the number of persons represented as living at the age of N years and a half had been Π', and those living at the age of $N+1$ years and a half had been Π'', and those living at the age of $N+2$ years and a half had been Π''', and in like manner that those living at the ends of $N+3$ years and a half, $N+4$ years and a half, $N+5$ years and a half, and so on, had been denoted by Π^{IV}, Π^{V}, Π^{VI}, &c. out of the number P who are living at the age of N years.

Then it is evident that the numbers Π', Π'', Π''', Π^{IV}, Π^{V}, &c. will be respectively greater than the numbers P', P'', P''', P^{IV}, P^{V}, &c. which represent the numbers of persons living at the ages of $N+1$ years, $N+2$ years, $N+3$ years, $N+4$ years, $N+5$ years, &c. but less than the numbers P, P', P'', P''', P^{IV}, &c. which represent the numbers of persons living at the ages of N years, $N+1$ years, $N+2$ years, $N+3$ years, $N+4$ years, &c.

CCVI. These things being premised, we may now find a quantity which shall always be somewhat, though very little, greater than the true value of an annuity of one pound a year to be paid half-yearly by payments of 10 shillings each, during the life of a person of the age of N years, by proceeding in the manner following.

CCVII. We

CCVII. We have already seen in Art. 78,——87, that, when the interest of money is supposed to be paid once a year, and then added to the principal, and the whole lent out again immediately at the same rate of interest, the present value of an annuity, or yearly payment, of one pound for a certain number of years denoted by the letter n will be equal to $\frac{\pounds 1}{1}$ × the series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} +$ &c. $+ \frac{1}{r^n}$; and that the value of the same annuity for the same term of years, but depending on the life of a person of the age of N years, so as to cease whenever he shall die, is $= \frac{\pounds 1}{P}$ × the series $\frac{P^{\text{I}}}{r} + \frac{P^{\text{II}}}{r^2} + \frac{P^{\text{III}}}{r^3} + \frac{P^{\text{IV}}}{r^4} + \frac{P^{\text{V}}}{r^5} + \frac{P^{\text{VI}}}{r^6} +$ &c. $+ \frac{P^n}{r^n}$; and that the value of the same annuity for the whole life of the said person of the age of N years is $= \frac{\pounds 1}{P}$ × the series $\frac{P^{\text{I}}}{r} + \frac{P^{\text{II}}}{r^2} + \frac{P^{\text{III}}}{r^3} + \frac{P^{\text{IV}}}{r^4} + \frac{P^{\text{V}}}{r^5} + \frac{P^{\text{VI}}}{r^6} +$ &c. $+ \frac{P^{E-N}}{r^{E-N}}$, the letter E denoting 94, or, in general, the greatest number of years to which it is supposed, in the table of probabilities which is made the ground of the calculation, that human life can be extended.

CCVIII. Now, by reasonings similar to those by which these conclusions were obtained, it will follow that, if we suppose the interest of money to be paid every half-year, instead of every year, and then to be added to the principal, and the sum of both to be lent out again at the same rate of interest, the present value of a half-yearly set of payments of one pound each, for the life of a person of the age of N years, will be equal to $\frac{\pounds 1}{P}$ × the series $\frac{\Pi^{\text{I}}}{h} + \frac{P^{\text{I}}}{h^2} + \frac{\Pi^{\text{II}}}{h^3} + \frac{P^{\text{II}}}{h^4} + \frac{\Pi^{\text{III}}}{h^5} + \frac{P^{\text{III}}}{h^6} + \frac{\Pi^{\text{IV}}}{h^7} + \frac{P^{\text{IV}}}{h^8} + \frac{\Pi^{\text{V}}}{h^9} +$ &c. continued to the utmost extent of human life, or to the term $\frac{\Pi^{E-N+1}}{h^{2E-2N+1}}$, and therefore (since h has been shewn to be greater than $r^{\frac{1}{2}}$,) will be less than $\frac{\pounds 1}{P}$ × the series $\frac{\Pi^{\text{I}}}{r^{\frac{1}{2}}} + \frac{P^{\text{I}}}{r} + \frac{\Pi^{\text{II}}}{r^{\frac{1}{2}}} + \frac{P^{\text{II}}}{r^2}$

$+\frac{P^{II}}{r^2}+\frac{\Pi^{I\frac{1}{2}}}{r^{\frac{1}{2}}}+\frac{P^{III}}{r^3}+\frac{\Pi^{II\frac{1}{2}}}{r^{\frac{3}{2}}}+\frac{P^{IV}}{r^4}+\frac{\Pi^{V}}{r^{\frac{9}{2}}}+$ &c. $+\frac{\Pi^{E-N+1}}{r^{E-N+\frac{1}{2}}}$, or than $\frac{\pounds 1}{P}\times$ the series $\frac{\Pi^{I}}{r^{\frac{1}{2}}}+\frac{\Pi^{II}}{r^{\frac{3}{2}}}+\frac{\Pi^{III}}{r^{\frac{5}{2}}}+\frac{\Pi^{IV}}{r^{\frac{7}{2}}}+\frac{\Pi^{V}}{r^{\frac{9}{2}}}+$ &c. $+\frac{\Pi^{E-N+1}}{r^{E-N+\frac{1}{2}}}$ together with $\frac{\pounds 1}{P}\times$ the series $\frac{P^{I}}{r}+\frac{P^{II}}{r^2}+\frac{P^{III}}{r^3}+\frac{P^{IV}}{r^4}+\frac{P^{V}}{r^5}+$ &c. $+\frac{P^{E-N}}{r^{E-N}}$. Therefore the preſent value of a ſet of half-yearly payments of only 10 ſhillings, or half a pound, each, for the life of the ſame perſon will be leſs than $\frac{\pounds 1}{2P}\times$ the ſeries $\frac{\Pi^{I}}{r^{\frac{1}{2}}}+\frac{\Pi^{II}}{r^{\frac{3}{2}}}+\frac{\Pi^{III}}{r^{\frac{5}{2}}}+\frac{\Pi^{IV}}{r^{\frac{7}{2}}}+\frac{\Pi^{V}}{r^{\frac{9}{2}}}+$ &c. $+\frac{\Pi^{E-N+1}}{r^{E-N+\frac{1}{2}}}$ together with $\frac{\pounds 1}{2P}\times$ the ſeries $\frac{P^{I}}{r}+\frac{P^{II}}{r^2}+\frac{P^{III}}{r^3}+\frac{P^{IV}}{r^4}+\frac{P^{V}}{r^5}+$ &c. $+\frac{P^{E-N}}{r^{E-N}}$.

CCIX. Now $\frac{\pounds 1}{P}\times$ the ſeries $\frac{P^{I}}{r}+\frac{P^{II}}{r^2}+\frac{P^{III}}{r^3}+\frac{P^{IV}}{r^4}+\frac{P^{V}}{r^5}+$ &c. $+\frac{P^{E-N}}{r^{E-N}}$ is equal to $\pounds V$, or the value of an annuity of one pound a year, paid at the end of every year, for the life of a perſon of the age of N years. Therefore $\frac{\pounds 1}{2P}\times$ the ſeries $\frac{P^{I}}{r}+\frac{P^{II}}{r^2}+\frac{P^{III}}{r^3}+\frac{P^{IV}}{r^4}+\frac{P^{V}}{r^5}+$ &c. $+\frac{P^{E-N}}{r^{E-N}}$ is $=\frac{\pounds V}{2}$. Therefore the preſent value of a ſet of half-yearly payments of 10 ſhillings each for the life of a perſon of the age of N years is leſs than $\frac{\pounds 1}{2P}\times$ the ſeries $\frac{\Pi^{I}}{r^{\frac{1}{2}}}+\frac{\Pi^{II}}{r^{\frac{3}{2}}}+\frac{\Pi^{III}}{r^{\frac{5}{2}}}+\frac{\Pi^{IV}}{r^{\frac{7}{2}}}+\frac{\Pi^{V}}{r^{\frac{9}{2}}}+$ &c. $+\frac{\Pi^{E-N+1}}{r^{E-N+\frac{1}{2}}}$ together with $\frac{\pounds V}{2}$.

Therefore

Therefore it is leſs than $r^{\frac{1}{2}} \times \frac{\text{£}1}{2P} \times$ the ſeries $\frac{\Pi'}{r} + \frac{\Pi''}{r^2} + \frac{\Pi'''}{r^3} + \frac{\Pi^{IV}}{r^4} + \frac{\Pi^{V}}{r^5} +$ &c. $+ \frac{\Pi^{E-N+1}}{rE-N+1}$ together with $\frac{\text{£}V}{2}$.

CCX. Now it is probable that the number of perſons who die in the firſt half of every year of human life is ſomewhat leſs than the number of thoſe who die in the ſecond half of the ſame year; and conſequently the number of perſons living at the end of the firſt half-year will be ſomewhat greater than an arithmetical mean proportional between the numbers of perſons living at the beginning of the firſt half-year and at the end of the ſecond half-year. Therefore Π' will be ſomewhat greater than an arithmetical mean between P and P', and Π'' than an arithmetical mean between P' and P'', and Π''' than an arithmetical mean between P'' and P''', and Π^{IV}, Π^{V}, &c. than arithmetical means between P''' and P^{IV}, and between P^{IV} and P^{V}, &c. Conſequently Π', Π'', Π''', Π^{IV}, Π^{V}, &c. will be ſomewhat greater than $\frac{P+P'}{2}$, $\frac{P'+P''}{2}$, $\frac{P''+P'''}{2}$, $\frac{P'''+P^{IV}}{2}$, and $\frac{P^{IV}+P^{V}}{2}$, &c. But the difference will probably be ſo extremely ſmall that we may very well conſider them as equal. And upon this ſuppoſition the ſeries $\frac{\Pi'}{r} + \frac{\Pi''}{r^2} + \frac{\Pi'''}{r^3} + \frac{\Pi^{IV}}{r^4} + \frac{\Pi^{V}}{r^5} +$ &c. $+ \frac{\Pi^{E-N+1}}{rE-N+1}$ will be equal to the ſeries $\frac{P+P'}{2r} + \frac{P'+P''}{2r^2} + \frac{P''+P'''}{2r^3} + \frac{P'''+P^{IV}}{2r^4} + \frac{P^{IV}+P^{V}}{2r^5} +$ &c. $+ \frac{\Pi^{E-N+1}}{rE-N+1}$, and conſequently to one half of the ſeries $\frac{P+P'}{r} + \frac{P'+P''}{r^2} + \frac{P''+P'''}{r^3} + \frac{P'''+P^{IV}}{r^4} + \frac{P^{IV}+P^{V}}{r^5} +$ &c. together with the term $\frac{\Pi^{E-N+1}}{rE-N+1}$, and conſequently to one half of the ſeries $\frac{P}{r} + \frac{P'}{r^2} + \frac{P''}{r^3} + \frac{P'''}{r^4} + \frac{P^{IV}}{r^5} +$ &c. together with one half of the ſeries $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} +$ &c. and with the term $\frac{\Pi^{E-N+1}}{rE-N+1}$.

Therefore

Therefore $r^{\frac{1}{2}} \times \frac{\mathcal{L}}{2P} \times$ the ſeries $\frac{\Pi^{I}}{r} + \frac{\Pi^{II}}{r^2} + \frac{\Pi^{III}}{r^3} + \frac{\Pi^{IV}}{r^4} + \frac{\Pi^{V}}{r^5} +$ &c. $+ \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ is equal $r^{\frac{1}{2}} \times \frac{\mathcal{L}}{2P} \times \frac{1}{2} \times$ the ſeries $\frac{P}{r} + \frac{P^{I}}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} +$ &c. together with $r^{\frac{1}{2}} \times \frac{\mathcal{L}}{2P} \times \frac{1}{2}$ $\times$ the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} +$ &c. $+ \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ $= \frac{r^{\frac{1}{2}}}{4} \times \frac{\mathcal{L}}{2P}$ the ſeries $\frac{P}{r} + \frac{P^{I}}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} +$ &c. $+ \frac{r^{\frac{1}{2}}}{4} \times \frac{\mathcal{L}}{P} \times$ the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} +$ &c. $+ \frac{\Pi^{E-N+1}}{r^{E-N+1}} = \frac{r^{\frac{1}{2}}}{4} \times \frac{\mathcal{L}}{P} \times$ the ſeries $\frac{P}{r} + \frac{P^{I}}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} +$ &c. $+ \frac{r^{\frac{1}{2}}}{4} \times \mathcal{L}V + \frac{r^{\frac{1}{2}}}{4} \times \frac{\mathcal{L}}{P} \times$ the term $\frac{\Pi^{E-N+1}}{r^{E-N+1}} =$ (if we neglect this laſt quantity $\frac{r^{\frac{1}{2}}}{4} \times \frac{\mathcal{L}}{P} \times \frac{\Pi^{E-N+1}}{r^{E-N+1}}$, on account of its extreme ſmallneſs,) $\frac{r^{\frac{1}{2}}}{4} \times \frac{\mathcal{L}}{P} \times$ the ſeries $\frac{P}{r} + \frac{P^{I}}{r^2} + \frac{P^{II}}{r^3} + \frac{P^{III}}{r^4} + \frac{P^{IV}}{r^5} +$ &c. $+ \frac{r^{\frac{1}{2}} \times \mathcal{L}V}{4}$.

CCXI. Therefore $r^{\frac{1}{2}} \times \frac{\mathcal{L}}{2P} \times$ the ſeries $\frac{\Pi^{I}}{r} + \frac{\Pi^{II}}{r^2} + \frac{\Pi^{III}}{r^3} + \frac{\Pi^{IV}}{r^4} + \frac{\Pi^{V}}{r^5} +$ &c. $+ \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ together with $\frac{\mathcal{L}V}{2}$ is $= \frac{r^{\frac{1}{2}}}{4} \times \frac{\mathcal{L}}{P}$

$\times \frac{\text{£}1}{P}$ × the series $\frac{P}{r} + \frac{P^{\text{I}}}{r^2} + \frac{P^{\text{II}}}{r^3} + \frac{P^{\text{III}}}{r^4} + \frac{P^{\text{IV}}}{r^5}$ + &c. $+ \frac{r^{\frac{1}{2}} \times \text{£}V}{4} + \frac{\text{£}V}{2} = \frac{r^{\frac{1}{2}}}{4} \times \frac{\text{£}1}{P}$ × the series $\frac{P}{r} + \frac{P^{\text{I}}}{r^2} + \frac{P^{\text{II}}}{r^3} + \frac{P^{\text{III}}}{r^4} + \frac{P^{\text{IV}}}{r^5}$ + &c. $+ \frac{r^{\frac{1}{2}}+2}{4} \times \text{£}V$.

CCXII. But it has been shewn, in Art. 209, that the present value of a set of half-yearly payments of 10 shillings each, during the life of a person of the age of N years, is somewhat less than $r^{\frac{1}{2}} \times \frac{\text{£}1}{2P}$ × the series $\frac{\Pi^{\text{I}}}{r} + \frac{\Pi^{\text{II}}}{r^2} + \frac{\Pi^{\text{III}}}{r^3} + \frac{\Pi^{\text{IV}}}{r^4} + \frac{\Pi^{\text{V}}}{r^5}$ + &c. $+ \frac{\Pi^{E-N+1}}{rE-N+1}$ together with $\frac{\text{£}V}{2}$. Therefore the said value of the said half-yearly payments is somewhat less than $\frac{r^{\frac{1}{2}}}{4} \times \frac{\text{£}1}{P}$ × the series $\frac{P}{r} + \frac{P^{\text{I}}}{r^2} + \frac{P^{\text{II}}}{r^3} + \frac{P^{\text{III}}}{r^4} + \frac{P^{\text{IV}}}{r^5}$ + &c. $+ \frac{r^{\frac{1}{2}}+2}{4} \times \text{£}V$.

CCXIII. Now $\text{£}W$, or the value of an annuity of one pound a year, paid at the end of every year, for the life of a person of the age of $N-1$ years, is $= \frac{\text{£}1}{P+d}$ × the series $\frac{P}{r} + \frac{P^{\text{I}}}{r^2} + \frac{P^{\text{II}}}{r^3} + \frac{P^{\text{III}}}{r^4} + \frac{P^{\text{IV}}}{r^5}$ + &c. Therefore $\frac{P+d}{P} \times \text{£}W$ is $= \frac{\text{£}1}{P}$ × the series $\frac{P}{r} + \frac{P^{\text{I}}}{r^2} + \frac{P^{\text{II}}}{r^3} + \frac{P^{\text{III}}}{r^4} + \frac{P^{\text{IV}}}{r^5}$ + &c. and $\frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \text{£}W$ is $= \frac{r^{\frac{1}{2}}}{4} \times \frac{\text{£}1}{P}$ × the series $\frac{P}{r} + \frac{P^{\text{I}}}{r^2} + \frac{P^{\text{II}}}{r^3} + \frac{P^{\text{III}}}{r^4} + \frac{P^{\text{IV}}}{r^5}$ + &c.

Therefore $\frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \overset{\text{£}}{W} + \frac{r^{\frac{1}{2}}+2}{4} \times \overset{\text{£}}{V}$ is $= \frac{r^{\frac{1}{2}}}{4} \times \frac{\overset{\text{£}}{1}}{P}$ × the series $\frac{P}{r} + \frac{P^{\text{I}}}{r^2} + \frac{P^{\text{II}}}{r^3} + \frac{P^{\text{III}}}{r^4} + \frac{P^{\text{IV}}}{r^5}$ + &c. $+ \frac{r^{\frac{1}{2}}+2}{4} \times \overset{\text{£}}{V}$; and consequently the preſent value of a ſet of half-yearly payments of 10 ſhillings each, for the life of a perſon of the age of N years, is leſs than $\frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \overset{\text{£}}{W} + \frac{r^{\frac{1}{2}}+2}{4} \times \overset{\text{£}}{V}$.

CCXIV. But $\overset{\text{£}}{W}$ is $= \frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|\text{£}$. Therefore $\frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \overset{\text{£}}{W}$ is $= \frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|\text{£} = \frac{r^{\frac{1}{2}}}{4r} \times \overline{1+V}|\text{£} = \frac{1}{4r^{\frac{1}{2}}} \times \overline{1+V}|\text{£} = \frac{\overset{\text{£}}{1}}{4r^{\frac{1}{2}}} + \frac{\overset{\text{£}}{V}}{4r^{\frac{1}{2}}}$. Therefore $\frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \overset{\text{£}}{W} + \frac{r^{\frac{1}{2}}+2}{4} \times \overset{\text{£}}{V}$ is $= \frac{\overset{\text{£}}{1}}{4r^{\frac{1}{2}}} + \frac{\overset{\text{£}}{V}}{4r^{\frac{1}{2}}} + \frac{r^{\frac{1}{2}}+2}{4} \times \overset{\text{£}}{V} = \frac{\overset{\text{£}}{1}}{4r^{\frac{1}{2}}} + \frac{\overset{\text{£}}{V}}{4r^{\frac{1}{2}}} + \frac{r+2r^{\frac{1}{2}}}{4r^{\frac{1}{2}}} \times \overset{\text{£}}{V} = \frac{\overset{\text{£}}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\text{£}}{V}$.

Therefore the preſent value of a ſet of half-yearly payments of 10 ſhillings each, or of an annuity of one pound a year payable half-yearly, for the life of a perſon of the age of N years, is ſomewhat leſs than $\frac{\overset{\text{£}}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\text{£}}{V}$, or $\frac{\overset{\text{£}}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\text{£}}{V}$ is a quantity ſomewhat greater than, but very nearly equal to, the value of the ſaid half-yearly annuity. Q E I.

CCXV. Since

CCXV. Since 1 is to $r^{\frac{1}{2}}$ as $r^{\frac{1}{2}}$ is to r, $1+r$ will be greater than $2r^{\frac{1}{2}}$, and consequently $1+r+2r^{\frac{1}{2}}$ will be greater than $2r^{\frac{1}{2}}+2r^{\frac{1}{2}}$, or $1+2r^{\frac{1}{2}}+r$ will be greater than $4r^{\frac{1}{2}}$, and therefore $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V}$ will be greater than $\overset{\pounds}{V}$. But the difference between them will be extremely small. Thus, for example, if the interest of money is 4 per cent. we shall have $r = 1.04$, and $r^{\frac{1}{2}} = \sqrt{1.04} = 1.0198$, and consequently $2r^{\frac{1}{2}} = 2 \times 1.0198 = 2.0396$, and $1+2r^{\frac{1}{2}}+r = 1+2.0396+1.04 = 4.0796$, and $4r^{\frac{1}{2}} = 4.0792$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V} = \frac{4.0796}{4.0792} \times \overset{\pounds}{V} = 1.000,09 \times \overset{\pounds}{V}$, or less than $\overset{\pounds}{V} + .0001 \times \overset{\pounds}{V}$, or $\overset{\pounds}{V} + \frac{1}{10000}$ part of $\overset{\pounds}{V}$. And, when the interest of money is 10 per cent. (which is the highest interest supposed in the foregoing tables,) and r is consequently equal to 1.1, we shall have $r^{\frac{1}{2}} = 1.0488$, and $2r^{\frac{1}{2}} = 2.0976$, and $1+2r^{\frac{1}{2}}+r = 1+2.0976+1.1 = 4.1976$, and $4r^{\frac{1}{2}} = 4 \times 1.0488 = 4.1952$, and consequently $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V} = \frac{4.1976}{4.1952} \times \overset{\pounds}{V} = 1.00057 \times \overset{\pounds}{V}$, or $\overset{\pounds}{V} + \frac{57}{100,000} \times \overset{\pounds}{V}$, or $\overset{\pounds}{V} + \frac{1}{1754}$ part of $\overset{\pounds}{V}$. Therefore, if the interest of money be either 10 per cent. or any lower interest, the excess of $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V}$ above $\overset{\pounds}{V}$ will be either $\frac{1}{1754}$ part, or some lesser part, of $\overset{\pounds}{V}$; and consequently the excess of $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V}$ above $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}} + \overset{\pounds}{V}$ will be only the 1754th, or some lesser, part of $\overset{\pounds}{V}$. And therefore, if we suppose the said quantity $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V}$,

The excess of the quantity $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V}$, or the second member of the foregoing near value of an annuity of one pound a year, paid half-yearly, above $\overset{\pounds}{V}$, or the value of the same annuity, when paid at the end of every year, is always extremely small.

A conjectural conclusion concerning the magnitude of the true value of the ſaid half-yearly annuity.

which has been ſhewn to be always ſomewhat, and but a very little, greater than the true value of an annuity of one pound a year paid half-yearly for the life of a perſon of the age of N years, to be greater than the ſaid true value by the ſame exceſs by which it exceeds the quantity $\frac{\mathcal{L}1}{4r^{\frac{1}{2}}} + \frac{\mathcal{L}}{V}$, the ſaid true value will be equal to $\frac{\mathcal{L}1}{4r^{\frac{1}{2}}} + \frac{\mathcal{L}}{V}$. QEI.

Agreement of the ſaid conjectural concluſion with the value of the ſaid half-yearly annuity aſſigned by Mr. Thomas Simpſon.

CCXVI. If this laſt ſuppoſition be a true one, the value of an annuity of one pound a year for the life of a perſon of the age of N years, when paid half-yearly, will be obtained by adding to $\frac{\mathcal{L}}{V}$, or the value of the like annuity, when paid at the end of every year, (which is given in the foregoing tables,) the ſum of $\frac{\mathcal{L}1}{4r^{\frac{1}{2}}}$, or ſomewhat leſs than $\frac{\mathcal{L}1}{4}$, or a quarter of a year's annuity; which agrees with the direction given by Mr. Thomas Simpſon for this purpoſe in his *Doctrine of Annuities, &c.* page 81.

A computation of the ſeveral different values of the expreſſion $\frac{\mathcal{L}1}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\mathcal{L}}{V}$, *(found in Art.* 203—214,) *according to the twelve different values of* r, *or the twelve different rates of intereſt, above-mentioned.*

CCXVII. But, if this laſt ſuppoſition ſhould not be true, or if the truth of it ſhould not be thought ſufficiently evident or probable, it will be neceſſary to have recourſe to the full expreſſion $\frac{\mathcal{L}1}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\mathcal{L}}{V}$, which has been ſhewn in the foregoing articles to be always ſomewhat greater than, but very nearly equal to, the true value of an annuity of one pound a year that is paid half-yearly, during the life of a perſon of the

the age of N years. Now, if we compute the different values of this expreſſion according to the twelve different rates of intereſt above-mentioned, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. thoſe values will be as follows.

In the firſt place, when the intereſt of money is 2 per cent. we ſhall have $r = 1.02$, and $r^{\frac{1}{2}} = \sqrt{1.02} = 1.009,950$, and $2r^{\frac{1}{2}} = 2.019,900$, and $4r^{\frac{1}{2}} = 4.039,800$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.019,900 + 1.02)$ $= 4.039,900$; and conſequently $\frac{\text{£}\,1}{4r^{\frac{1}{2}}} = \frac{\text{£}\,1}{4.039,800} = .2475\,\text{£}$, and $\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V} = \frac{4.039,900}{4.039,800} \times \frac{\text{£}}{V} = 1.000,024 \times \frac{\text{£}}{V}$. Therefore, when the intereſt of money is 2 per cent. the value of an annuity of one pound a year, for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2475\,\text{£} + 1.000,024 \times \frac{\text{£}}{V}$.

Secondly, when the intereſt of money is $2\frac{1}{2}$ per cent. we ſhall have $r = 1.025$, and $r^{\frac{1}{2}} = \sqrt{1.025} = 1.012,422$, and $2r^{\frac{1}{2}} = 2.024,844$, and $4r^{\frac{1}{2}} = 4.049,688$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.024,844 + 1.025)$ $= 4.049,844$; and conſequently $\frac{\text{£}\,1}{4r^{\frac{1}{2}}} = \frac{\text{£}\,1}{4.049,688} = .2469\,\text{£}$, and $\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V} = \frac{4.049,844}{4.049,688} \frac{\text{£}}{V} = 1.000,038 \times \frac{\text{£}}{V}$. Therefore, when the intereſt of money is $2\frac{1}{2}$ per cent. the value of an annuity of one pound a year, for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2469\,\text{£} + 1.000,038 \times \frac{\text{£}}{V}$.

Thirdly, when the intereſt of money is 3 per cent. we ſhall have $r = 1.03$, and $r^{\frac{1}{2}} = \sqrt{1.03} = 1.014,889$, and $2r^{\frac{1}{2}} = 2.029,778$, and $4r^{\frac{1}{2}} = 4.059,556$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.029,778 + 1.03)$ $= 4.059,778$;

$= 4.059,778$; and conſequently $\frac{\text{£}\ 1}{4r^{\frac{1}{2}}} = \frac{\text{£}\ 1}{4.059,556} = .2463\,\text{£}$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V} = \frac{4.059,778}{4.059,556} \times \frac{\text{£}}{V} = 1.000,054 \times \frac{\text{£}}{V}$. Therefore, when the intereſt of money is 3 per cent. the value of an annuity of one pound a year, for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2463\,\text{£} + 1.000,054 \times \frac{\text{£}}{V}$.

Fourthly, when the intereſt of money is $3\frac{1}{2}$ per cent. we ſhall have $r = 1.035$, and $r^{\frac{1}{2}} = \sqrt{1.035} = 1.017,349$, and $2r^{\frac{1}{2}} = 2.034,698$, and $4r^{\frac{1}{2}} = 4.069,396$, and $1+2r^{\frac{1}{2}}+r$ $(= 1+2.034,698+1.035)$ $= 4.069,698$; and conſequently $\frac{\text{£}\ 1}{4r^{\frac{1}{2}}} = \frac{\text{£}\ 1}{4.069,396} = .2457\,\text{£}$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V} = \frac{4.069,698}{4.069,396} \times \frac{\text{£}}{V} = 1.000,074 \times \frac{\text{£}}{V}$. Therefore, when the intereſt of money is $3\frac{1}{2}$ per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2457\,\text{£} + 1.000,074 \times \frac{\text{£}}{V}$.

Fifthly, when the intereſt of money is 4 per cent. we ſhall have $r = 1.04$, and $r^{\frac{1}{2}} = \sqrt{1.04} = 1.019,804$, and $2r^{\frac{1}{2}} = 2.039,608$, and $4r^{\frac{1}{2}} = 4.079,216$, and $1+2r^{\frac{1}{2}}+r$ $(= 1+2.039,608+1.04)$ $= 4.079,608$; and conſequently $\frac{\text{£}\ 1}{4r^{\frac{1}{2}}} = \frac{\text{£}\ 1}{4.079,216} = .2451\,\text{£}$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V} = \frac{4.079,608}{4.079,216} \times \frac{\text{£}}{V} = 1.000,096 \times \frac{\text{£}}{V}$. Therefore, when the intereſt of money is 4 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2451\,\text{£} + 1.000,096 \times \frac{\text{£}}{V}$.

Sixthly,

Sixthly, when the intereſt of money is $4\frac{1}{2}$ per cent. we ſhall have $r = 1.045$, and $r^{\frac{1}{2}} = \sqrt{1.045} = 1.022,252$, and $2r^{\frac{1}{2}} = 2.044,504$, and $4r^{\frac{1}{2}} = 4.089,008$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.044,504 + 1.045)$ $= 4.089,504$; and conſequently $\frac{\text{£}\ 1}{4r^{\frac{1}{2}}} = \frac{\text{£}\ 1}{4.089,008} = .2445\,\text{£}$, and $\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V} = \frac{4.089,504}{4.089,008} \times \frac{\text{£}}{V} = 1.000,121 \times \frac{\text{£}}{V}$. Therefore, when the intereſt of money is $4\frac{1}{2}$ per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2445\,\text{£} + 1.000,121 \times \frac{\text{£}}{V}$.

Seventhly, when the intereſt of money is 5 per cent. we ſhall have $r = 1.05$, and $r^{\frac{1}{2}} = \sqrt{1.05} = 1.024,695$, and $2r^{\frac{1}{2}} = 2.049,390$, and $4r^{\frac{1}{2}} = 4.098,780$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.049,390 + 1.05)$ $= 4.099,390$; and conſequently $\frac{\text{£}\ 1}{4r^{\frac{1}{2}}} = \frac{\text{£}\ 1}{4.098,780} = .2439\,\text{£}$, and $\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V} = \frac{4.099,390}{4.098,780} \times \frac{\text{£}}{V} = 1.000,148 \times \frac{\text{£}}{V}$. Therefore, when the intereſt of money is 5 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2439\,\text{£} + 1.000,148 \times \frac{\text{£}}{V}$.

Eighthly, when the intereſt of money is 6 per cent. we ſhall have $r = 1.06$, and $r^{\frac{1}{2}} = \sqrt{1.06} = 1.029,563$, and $2r^{\frac{1}{2}} = 2.059,126$, and $4r^{\frac{1}{2}} = 4.118,252$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.059,126 + 1.06)$ $= 4.119,126$; and conſequently $\frac{\text{£}\ 1}{4r^{\frac{1}{2}}} = \frac{\text{£}\ 1}{4.118,252} = .2428\,\text{£}$, and $\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V} = \frac{4.119,126}{4.118,252} \times \frac{\text{£}}{V} = 1.000,212 \times \frac{\text{£}}{V}$. Therefore,

when

when the intereſt of money is 6 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2428\,£ + 1.000{,}212 \times \frac{£}{V}$.

Ninthly, when the intereſt of money is 7 per cent. we ſhall have $r = 1.07$, and $r^{\frac{1}{2}} = \sqrt{1.07} = 1.034{,}408$, and $2r^{\frac{1}{2}} = 2.068{,}816$, and $4r^{\frac{1}{2}} = 4.137{,}632$, and $1 + 2r^{\frac{1}{2}} + r$ $(1 + 2.068{,}816 + 1.07)$ $= 4.138{,}816$; and conſequently $\frac{\frac{£}{1}}{4r^{\frac{1}{2}}} = \frac{\frac{£}{1}}{4.137{,}632} = .2416\,£$, and $\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{£}{V} = \frac{4.138{,}816}{4.137{,}632} \times \frac{£}{V} = 1.000{,}286 \times \frac{£}{V}$. Therefore, when the intereſt of money is 7 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2416\,£ + 1.000{,}286 \times \frac{£}{V}$.

Tenthly, when the intereſt of money is 8 per cent. we ſhall have $r = 1.08$, and $r^{\frac{1}{2}} = \sqrt{1.08} = 1.039{,}230$, and $2r^{\frac{1}{2}} = 2.078{,}460$, and $4r^{\frac{1}{2}} = 4.156{,}920$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.078{,}460 + 1.08)$ $= 4.158{,}460$; and conſequently $\frac{\frac{£}{1}}{4r^{\frac{1}{2}}} = \frac{\frac{£}{1}}{4.156{,}920} = .2405\,£$, and $\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{£}{V} = \frac{4.158{,}460}{4.156{,}920} \times \frac{£}{V} = 1.000{,}370 \times \frac{£}{V}$. Therefore, when the intereſt of money is 8 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2405\,£ + 1.000{,}370 \times \frac{£}{V}$.

Eleventhly,

Eleventhly, when the intereſt of money is 9 per cent. we ſhall have $r = 1.09$, and $r^{\frac{1}{2}} = \sqrt{1.09} = 1.044,031$, and $2r^{\frac{1}{2}} = 2.088,062$, and $4r^{\frac{1}{2}} = 4.176,124$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.088,062 + 1.09)$ $= 4.178,062$; and conſequently $\frac{\pounds\ 1}{4r^{\frac{1}{2}}} = \frac{\pounds\ 1}{4.176,124} = .2394\,\pounds$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V} = \frac{4.178,062}{4.176,124} \times \overset{\pounds}{V} = 1.000,464 \times \overset{\pounds}{V}$. Therefore, when the intereſt of money is 9 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2394\,\pounds + 1.000,464 \times \overset{\pounds}{V}$.

And, laſtly, when the intereſt of money is 10 per cent. we ſhall have $r = 1.1$, and $r^{\frac{1}{2}} = \sqrt{1.1} = 1.048,809$, and $2r^{\frac{1}{2}} = 2.097,618$, and $4r^{\frac{1}{2}} = 4.195,236$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.097,618 + 1.1)$ $= 4.197,618$; and conſequently $\frac{\pounds\ 1}{4r^{\frac{1}{2}}} = \frac{\pounds\ 1}{4.195,236} = .2383\,\pounds$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V} = \frac{4.197,618}{4.195,236} \times \overset{\pounds}{V} = 1.000,567 \times \overset{\pounds}{V}$. Therefore, when the intereſt of money is 10 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2383\,\pounds + 1.000,567 \times \overset{\pounds}{V}$.

CCXVIII. The ſubſtance of the concluſions obtained in the laſt article may be expreſſed more conciſely in the manner following.

The concluſions obtained in the laſt article, expreſs'd more conciſely.

Let $\overset{\pounds}{V}$ denote, as before, the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid at the end of every year; and let $\overset{\pounds}{H}$ denote the value of the ſame annuity for the

ſame life, when it is to be paid at the end of every half-year by payments of 10 ſhillings each. Then, if the intereſt of money be

2 per cent. $\overset{£}{H}$ will be (nearly) $= .2475£ + 1.000{,}024 \times \overset{£}{V}$;

If $2\frac{1}{2}$ per cent. $\overset{£}{H}$ will be (nearly) $= .2469£ + 1.000{,}038 \times \overset{£}{V}$;

If 3 per cent. $\overset{£}{H}$ will be (nearly) $= .2463£ + 1.000{,}054 \times \overset{£}{V}$;

If $3\frac{1}{2}$ per cent. $\overset{£}{H}$ will be (nearly) $= .2457£ + 1.000{,}074 \times \overset{£}{V}$;

If 4 per cent. $\overset{£}{H}$ will be (nearly) $= .2451£ + 1.000{,}096 \times \overset{£}{V}$;

If $4\frac{1}{2}$ per cent. $\overset{£}{H}$ will be (nearly) $= .2445£ + 1.000{,}121 \times \overset{£}{V}$;

If 5 per cent. $\overset{£}{H}$ will be (nearly) $= .2439£ + 1.000{,}148 \times \overset{£}{V}$;

If 6 per cent. $\overset{£}{H}$ will be (nearly) $= .2428£ + 1.000{,}212 \times \overset{£}{V}$;

If 7 per cent. $\overset{£}{H}$ will be (nearly) $= .2416£ + 1.000{,}286 \times \overset{£}{V}$;

If 8 per cent. $\overset{£}{H}$ will be (nearly) $= .2405£ + 1.000{,}370 \times \overset{£}{V}$;

If 9 per cent. $\overset{£}{H}$ will be (nearly) $= .2394£ + 1.000{,}464 \times \overset{£}{V}$;

And, if 10 per cent. $\overset{£}{H}$ will be (nearly) $= .2383£ + 1.000{,}567 \times \overset{£}{V}$.

Expreſſions of the differences of the values of an annuity of one pound a year, when paid yearly and when paid half-yearly, according to the aforeſaid rates of intereſt.

CCXIX. Therefore the values of $\overset{£}{H} - \overset{£}{V}$, or the differences of the values of the ſaid yearly and half-yearly annuities of one pound a year for the life of a perſon of the age of N years, according to the aforeſaid different rates of the intereſt of money, will be as follows.

If the intereſt of money is

2 per cent. $\overset{£}{H} - \overset{£}{V}$ will be (nearly) $= .2475£ + 0.000{,}024 \times \overset{£}{V}$;

If $2\frac{1}{2}$ per cent. $\overset{£}{H} - \overset{£}{V}$ will be (nearly) $= .2469£ + 0.000{,}038 \times \overset{£}{V}$;

If 3 per cent. $\overset{£}{H} - \overset{£}{V}$ will be (nearly) $= .2463£ + 0.000{,}054 \times \overset{£}{V}$;

If

If $3\frac{1}{2}$ per cent. $\overset{\pounds.}{H}-\overset{\pounds.}{V}$ will be (nearly) $= .2457\pounds + 0.000,074 \times \overset{\pounds.}{V}$;

If 4 per cent. $\overset{\pounds.}{H}-\overset{\pounds.}{V}$ will be (nearly) $= .2451\pounds + 0.000,096 \times \overset{\pounds.}{V}$;

If $4\frac{1}{2}$ per cent. $\overset{\pounds.}{H}-\overset{\pounds.}{V}$ will be (nearly) $= .2445\pounds + 0.000,121 \times \overset{\pounds.}{V}$;

If 5 per cent. $\overset{\pounds.}{H}-\overset{\pounds.}{V}$ will be (nearly) $= .2439\pounds + 0.000,148 \times \overset{\pounds.}{V}$;

If 6 per cent. $\overset{\pounds.}{H}-\overset{\pounds.}{V}$ will be (nearly) $= .2428\pounds + 0.000,212 \times \overset{\pounds.}{V}$;

If 7 per cent. $\overset{\pounds.}{H}-\overset{\pounds.}{V}$ will be (nearly) $= .2416\pounds + 0.000,286 \times \overset{\pounds.}{V}$;

If 8 per cent. $\overset{\pounds.}{H}-\overset{\pounds.}{V}$ will be (nearly) $= .2405\pounds + 0.000,370 \times \overset{\pounds.}{V}$;

If 9 per cent. $\overset{\pounds.}{H}-\overset{\pounds.}{V}$ will be (nearly) $= .2394\pounds + 0.000,464 \times \overset{\pounds.}{V}$;

And, if 10 per cent. $\overset{\pounds.}{H}-\overset{\pounds.}{V}$ will be (nearly) $= .2383\pounds + 0.000,567 \times \overset{\pounds.}{V}$.

CCXX. The ſecond terms of the foregoing values of $\overset{\pounds.}{H}-\overset{\pounds.}{V}$, to wit, $0.000,024 \times \overset{\pounds.}{V}$, $0.000,038 \times \overset{\pounds.}{V}$, $0.000,054 \times \overset{\pounds.}{V}$, &c. are always very ſmall in compariſon of the firſt terms, $.2475\pounds$, $.2469\pounds$, $.2463\pounds$, &c. as will be evident from the following computation of their magnitudes when they are greateſt, or when the values of $\overset{\pounds.}{V}$ are greateſt.

Of the ſmallneſs of the ſecond terms of the foregoing expreſſions in compariſon of their firſt terms.

CCXXI. When the intereſt of money is 2 per cent. it appears from the foregoing tables, (Table XII, page 221,) that $\overset{\pounds.}{V}$, or the value of an annuity of one pound a year, paid at the end of every year, for the life of a perſon of the age of N years, is greateſt when N is $= 7$, or the annuitant is 7 years of age: and then the ſaid annuity is worth $\overset{\pounds.}{28.153,865}$. Therefore the greateſt magnitude of $.000,024 \times \overset{\pounds.}{V}$, or of the ſecond term of the expreſſion $.2475\pounds + 0.000,024 \times \overset{\pounds.}{V}$, (which, at this rate

The magnitudes of the ſaid ſecond terms when they are greateſt.

of intereſt, is equal to $\overset{£}{H} - \overset{£}{V},$) is $= 0.000{,}024 \times \overset{£}{28.153{,}865} = \overset{£}{0.000{,}675}$; which is leſs than the 366th part of .2475£, or the firſt term of the ſaid expreſſion.

When the intereſt of money is $2\frac{1}{2}$ per cent. it appears from the foregoing tables (Table XIII, page 222,) that $\overset{£}{V}$, or the value of a life-annuity of one pound a year, paid yearly, is greateſt when the annuitant is of the age of 8 years, and that it is then $= \overset{£}{25.242{,}492}$. Therefore the greateſt magnitude of $0.000{,}038 \times \overset{£}{V}$, or of the ſecond term of the expreſſion $.2469£ + .000{,}038 \times \overset{£}{V}$, (which, at this rate of intereſt, is equal to $\overset{£}{H} - \overset{£}{V},$) is $= 0.000{,}038 \times \overset{£}{25.242{,}492} = \overset{£}{0.000{,}959}$; which is leſs than $\overset{£}{0.001}$, or $\frac{1}{1000}$ part of a pound, and conſequently than the 246th part of .2469£, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is 3 per cent. it appears from Table XIV, page 223, that the greateſt magnitude of $\overset{£}{V}$ is at the age of 9 years, and that it then is $= \overset{£}{22.814{,}53}$. Therefore the greateſt magnitude of $0.000{,}054 \times \overset{£}{V}$, or of the ſecond term of the expreſſion $.2463£ + 0.000{,}054 \times \overset{£}{V}$, (which, at this rate of intereſt, is equal to $\overset{£}{H} - \overset{£}{V},$) is $= 0.000{,}054 \times \overset{£}{22.814{,}53} = \overset{£}{0.001{,}221}$; which is leſs than the 201ſt part of 2463£, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is $3\frac{1}{2}$ per cent. it appears from Table XV, page 224, that the greateſt magnitude of $\overset{£}{V}$ is at the age of 9 years, and that it is then $= \overset{£}{20.770{,}473}$. Therefore the greateſt magnitude of

of 0.000,074 × $\overset{£}{V}$, or of the ſecond term of the expreſſion .2457£ + 0.000,074 × $\overset{£}{V}$ (which, at this rate of intereſt, is equal to $\overset{£}{H}-\overset{£}{V},$) is = 0.000,074 × $\overset{£}{20.770,473}$ = $\overset{£}{0.001,537}$; which is leſs than the 159th part of .2457£, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is 4 per cent. it appears from Table XVI, page 225, that the greateſt magnitude of $\overset{£}{V}$ is at the age of 9 years, and that it is then = $\overset{£}{19.021,902}$. Therefore the greateſt magnitude of 0.000,096 × $\overset{£}{V}$, or of the ſecond term of the expreſſion .2451£ + 0.000,096 × $\overset{£}{V}$, (which, at this rate of intereſt, is equal to $\overset{£}{H}-\overset{£}{V},$) is = 0.000,096 × $\overset{£}{19.021,902}$ = $\overset{£}{0.001,826}$; which is leſs than the 134th part of .2451£, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is 4½ per cent. it appears from Table XVII, page 226, that the greateſt magnitude of $\overset{£}{V}$ is at the age of 9 years, and that it is then = $\overset{£}{17.51542}$. Therefore the greateſt magnitude of 0.000,121 × $\overset{£}{V}$, or of the ſecond term of the expreſſion .2445£ + 0.000,121 × $\overset{£}{V}$, (which, at this rate of intereſt, is = $\overset{£}{H}-\overset{£}{V},$) is = 0.000,121 × 17.51542 = 0.002,129; which is leſs than the 114th part of .2445£, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is 5 per cent. it appears from Table XVIII, page 227, that the greateſt magnitude of $\overset{£}{V}$ is at the age of 10 years, and that it is then = $\overset{£}{16.212,55}$. Therefore the greateſt magnitude of 0.000,148 × $\overset{£}{V}$, or of the ſecond term of the expreſſion .2439£ + 0.000,148 × $\overset{£}{V}$,

$+ 0.000,148 \times \overset{£}{V}$, (which, at this rate of intereſt, is $= \overset{£}{H} - \overset{£}{V}$,) is $= 0.000,148 \times 16.212,55 = 0.002,499$; which is leſs than the 97th part of .2439£, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is 6 per cent. it appears from Table XIX, page 228, that the greateſt magnitude of $\overset{£}{V}$ is at the age of 10 years, and that it is then $= \overset{£}{14.078,91}$. Therefore the greateſt magnitude of $0.000,212 \times \overset{£}{V}$, or of the ſecond term of the expreſſion .2428£ $+ 0.000,212 \times \overset{£}{V}$, (which, at this rate of intereſt, is $= \overset{£}{H} - \overset{£}{V}$,) is $= 0.000,212 \times \overset{£}{14.07891} = \overset{£}{0.002,984}$; which is leſs than the 81ſt part of .2428£, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is 7 per cent. it appears from Table XX, page 229, that the greateſt magnitude of $\overset{£}{V}$ is at the age of 10 years, and that it is then $= \overset{£}{12.411,09}$. Therefore the greateſt magnitude of $0.000,286 \times \overset{£}{V}$, or of the ſecond term of the expreſſion .2416£ $+ 0.000,286 \times \overset{£}{V}$, (which, at this rate of intereſt, is $= \overset{£}{H} - \overset{£}{V}$,) is $= 0.000,286 \times \overset{£}{12.411,09} = \overset{£}{0.003,549}$; which is leſs than the 68th part of .2416£, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is 8 per cent. it appears from Table XXI, page 230, that the greateſt magnitude of $\overset{£}{V}$ is at the age of 10 years, and that it is then $= \overset{£}{11.079,47}$. Therefore the greateſt magnitude of $\overset{£}{0.000,370} \times \overset{£}{V}$, or of the ſecond term of the expreſſion .2405£ $+ 0.000,370 \times \overset{£}{V}$, (which, at this rate of intereſt, is $= \overset{£}{H} - \overset{£}{V}$,) is $= 0.000,370 \times \overset{£}{11.079,47} = \overset{£}{0.004,099}$; which is leſs than the 58th part of .2405£, or of the firſt term of the ſaid expreſſion.

When

When the intereſt of money is 9 per cent. it appears from Table XXII, page 231, that the greateſt magnitude of $\overset{£}{V}$ is at the age of 10 years, and that it is then $= \overset{£}{9.996,17}$. Therefore the greateſt magnitude of $0.000,464 \times \overset{£}{V}$, or of the ſecond term of the expreſſion $.2394£ + 0.000,464 \times \overset{£}{V}$, (which, at this rate of intereſt, is $= \overset{£}{H} - \overset{£}{V}$,) is $= 0.000,464 \times \overset{£}{9.996,17} = \overset{£}{0.004,638}$; which is leſs than the 51ſt part of $.2394£$, or of the firſt term of the ſaid expreſſion.

And, when the intereſt of money is 10 per cent. it appears from Table XXIII, page 232, that the greateſt magnitude of $\overset{£}{V}$ is at the age of 11 years, and that it is then $= \overset{£}{9.101,99}$. Therefore the greateſt magnitude of $0.000,567 \times \overset{£}{V}$, or of the ſecond term of the expreſſion $.2383£ + 0.000,567 \times \overset{£}{V}$, (which, at this rate of intereſt, is $= \overset{£}{H} - \overset{£}{V}$,) is $= 0.000,567 \times \overset{£}{9.101,99} = \overset{£}{0.005,160}$; which is leſs than the 46th part of $.2383£$, or of the firſt term of the ſaid expreſſion.

CCXXII. It follows from the foregoing article that the greateſt poſſible magnitudes of the ſeveral expreſſions $.2475£ + 0.000,024 \times \overset{£}{V}$, $.2469£ + 0.000,038 \times \overset{£}{V}$, $.2463£ + 0.000,054 \times \overset{£}{V}$, $.2457£ + 0.000,074 \times \overset{£}{V}$, $.2451£ + 0.000,096 \times \overset{£}{V}$, $.2445£ + 0.000,121 \times \overset{£}{V}$, $.2439£ + 0.000,148 \times \overset{£}{V}$, $.2428£ + 0.000,212 \times \overset{£}{V}$, $.2416£ + 0,000,286 \times \overset{£}{V}$, $.2405£ + 0.000,370 \times \overset{£}{V}$, $.2394£ + 0.000,464 \times \overset{£}{V}$, and $.2383£ + 0.000,567 \times \overset{£}{V}$, (to which the general expreſſion $\frac{\overset{£}{1}}{4r^{\frac{1}{2}}}$ The magnitudes of the whole expreſſions mentioned in Art 219, when they are greateſt.

$\frac{\text{£}}{1} \div \frac{1 \div 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V}$ becomes equal when r is equal to 1.02, 1.025, 1.03, 1.35, 1.04, 1.045, 1.05, 1.06, 1.07, 1.08, 1.09, and 1.10, or when the intereſt of money is 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent.) are as follows.

When the intereſt of money is 2 per cent. the greateſt poſſible magnitude of the expreſſion $.2475\text{£} + 0.000{,}024 \times \frac{\text{£}}{V}$ is $(= .2475\text{£} + 0.000{,}675\text{£}) = .248{,}175\text{£}$.

When the intereſt of money is $2\frac{1}{2}$ per cent. the greateſt poſſible magnitude of the expreſſion $.2469\text{£} + 0.000{,}038 \times \frac{\text{£}}{V}$ is $(= .2469\text{£} + 0.000{,}959\text{£}) = .247{,}859\text{£}$.

When the intereſt of money is 3 per cent. the greateſt poſſible magnitude of the expreſſion $.2463\text{£} + 0.000{,}054 \times \frac{\text{£}}{V}$ is $(= .2463\text{£} + 0.001{,}221) = .247{,}521\text{£}$.

When the intereſt of money is $3\frac{1}{2}$ per cent. the greateſt poſſible magnitude of the expreſſion $.2457\text{£} + 0.000{,}074 \times \frac{\text{£}}{V}$ is $(= .2457\text{£} + 0.001{,}537\text{£}) = .247{,}237\text{£}$.

When the intereſt of money is 4 per cent. the greateſt poſſible magnitude of the expreſſion $.2451\text{£} + 0.000{,}096 \times \frac{\text{£}}{V}$ is $(= .2451\text{£} + 0.001{,}826\text{£}) = .246{,}926\text{£}$.

When the intereſt of money is $4\frac{1}{2}$ per cent. the greateſt poſſible magnitude of the expreſſion $.2445\text{£} + 0.000{,}121 \times \frac{\text{£}}{V}$ is $(= .2445\text{£} + 0.002{,}129\text{£}) = .246{,}629\text{£}$.

When

When the intereſt of money is 5 per cent. the greateſt poſſible magnitude of the expreſſion $.2439\mathcal{L} + 0.000{,}148 \times \overset{\mathcal{L}}{V}$ is $(= .2439\mathcal{L} + 0.002{,}499\mathcal{L}) = .246{,}399\mathcal{L}$.

When the intereſt of money is 6 per cent. the greateſt poſſible magnitude of the expreſſion $.2428\mathcal{L} + 0.000{,}212 \times \overset{\mathcal{L}}{V}$ is $(= .2428\mathcal{L} + 0.002{,}984\mathcal{L}) = .245{,}784\mathcal{L}$.

When the intereſt of money is 7 per cent. the greateſt poſſible magnitude of the expreſſion $.2416\mathcal{L} + 0.000{,}286 \times \overset{\mathcal{L}}{V}$ is $(= .2416\mathcal{L} + 0.003{,}549\mathcal{L}) = .245{,}149\mathcal{L}$.

When the intereſt of money is 8 per cent. the greateſt poſſible magnitude of the expreſſion $.2405\mathcal{L} + 0.000{,}370 \times \overset{\mathcal{L}}{V}$ is $(= .2405\mathcal{L} + 0.004{,}099\mathcal{L}) = .244{,}599\mathcal{L}$.

When the intereſt of money is 9 per cent. the greateſt poſſible magnitude of the expreſſion $.2394\mathcal{L} + 0.000{,}464 \times \overset{\mathcal{L}}{V}$ is $(= .2394\mathcal{L} + 0.004{,}638\mathcal{L}) = .244{,}038\mathcal{L}$.

And, when the intereſt of money is 10 per cent. the greateſt poſſible magnitude of the expreſſion $.2383\mathcal{L} + 0.000{,}567 \times \overset{\mathcal{L}}{V}$ is $(= .2383\mathcal{L} + 0.005{,}160\mathcal{L}) = .243{,}460\mathcal{L}$.

The ſaid greateſt magnitudes of the whole expreſſions mentioned in Art. 217 are, all of them, leſs than a quarter of a year's annuity.

CCXXIII. Theſe ſeveral quantities, .248,175£, .247,859£, .247,521£, .247,237£, .246,926£, .246,629£, .246,399£, .245,784£, .245,149£, .244,599£, .244,038£, and .243,460£, or (neglecting the two laſt figures, as inconſiderable,) .2482£, .2478£, .2475£, .2472£, .2469£, .2466£, .2464£, .2458£, .2451£, .2446£, .2440£, and .2434£, (which are the greateſt poſſible magnitudes of the ſeveral values of the expreſſion $\frac{\text{£}\ 1}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V}$, or $\frac{\text{£}}{H} - \frac{\text{£}}{V}$, according to the twelve different rates of intereſt above-mentioned,) are all of them leſs than .2500£, or $\frac{1}{4}$ of a pound, or a quarter of a year's annuity. Therefore the greateſt poſſible magnitude of the exceſs of the value of an annuity of one pound a year, paid half-yearly by payments of 10 ſhillings each, above the value of a like annuity of one pound a year, paid at the end of every year, when the intereſt of money is at any of the rates before-mentioned, is leſs than $\frac{1}{4}$ part of a pound, or a quarter of a year's annuity.

The ſecond terms of the ſaid expreſſions, mentioned in Art. 217, are ſo ſmall in compariſon of the firſt terms, that they may juſtly be neglected.

CCXXIV. Since the ſeveral values of $\frac{\text{£}\ 1}{4r^{\frac{1}{2}}}$, or of the firſt term of the expreſſion $\frac{\text{£}\ 1}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V}$, at the ſeveral rates of intereſt before-mentioned, have been ſhewn, in Art. 217, to be .2475£, and .2469£, and .2463£, and .2457£, and .2451£, and .2445£, and .2439£, and .2428£, and .2416£, and .2405£, and .2394£, and .2383£; and (by Art. 221) the greateſt poſſible magnitudes of the ſecond term, $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\text{£}}{V}$, of the ſame expreſſion, at the ſame rates of intereſt, are only equal to 0.000,675£, and 0.000,959£, and 0.001,221£, and 0.001,537£, and 0.001,826£, and 0.002,129£, and 0.002,499£, and 0.002,984£, and 0.003,549£, and 0.004,099£, and 0.004,638£, and 0.005,160£; of which latter quantities the laſt and greateſt, to wit, 0.005,160£, is only about 5 thouſandth parts, or one two hundredth part, of a pound, or one two hundredth part of a year's

year's annuity, or leſs than two days annuity; we may ſafely neglect the ſecond term, $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V}$, of the expreſſion $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V}$, as being inconſiderable in compariſon of the firſt term, $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}}$, and may therefore eſteem the firſt term $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}}$ to be equal to the whole expreſſion, and conſequently to $\overset{\pounds}{H}-\overset{\pounds}{V}$, or to the difference of the values of an annuity of one pound a year paid yearly and paid half-yearly. And, upon this ſuppoſition, we ſhall have $\overset{\pounds}{H}-\overset{\pounds}{V}$,

The values of the ſaid expreſſions, when their ſecond terms are neglected.

When the intereſt of money is 2 per cent. = .2475£;
And, when it is $2\frac{1}{2}$ per cent. = .2469£;
And, when it is 3 per cent. = .2463£;
And, when it is $3\frac{1}{2}$ per cent. = .2457£;
And, when it is 4 per cent. = .2451£;
And, when it is $4\frac{1}{2}$ per cent. = .2445£;
And, when it is 5 per cent. = .2439£;
And, when it is 6 per cent. = .2428£;
And, when it is 7 per cent. = .2416£;
And, when it is 8 per cent. = .2405£;
And, when it is 9 per cent. = .2394£;
And, when it is 10 per cent. = .2383£.

CCXXV. And, if theſe laſt-mentioned quantities, .2475£, .2469£, .2463£, &c. (which, it is evident, differ very little from each other,) be conſidered as equal to each other, and alſo as equal to .25£, (from which they differ by only about the 100th part of a pound, or about four days annuity,) we ſhall then have $\overset{\pounds}{H}-\overset{\pounds}{V}$, or the difference of the values of an annuity of one pound a year, paid half-yearly, and paid yearly, in all the different rates of intereſt above-mentioned, equal to .25£, or $\frac{1}{4}$ part of a pound, or a quarter of a year's annuity; agreeably to Mr. Thomas Simpſon's determination above-mentioned.

Agreement of theſe values with Mr. Thomas Simpſon's determination.

Of the value of a life-annuity that is paid at the end of every quarter of a year.

CCXXVI. When the annuity is to be paid at the end of every quarter of a year, it will be worth ſomewhat more than when it is to be paid at the end of every half-year; and Mr. Simpſon ſays that it will be worth an eighth part of a year's annuity more than in the other caſe, or $\frac{3}{8}$ of a year's annuity more than when it is to be paid at the end of every year. And this, I think, ſeems very probable. But after the foregoing very tedious inveſtigation of the difference of the values of a yearly and a half-yearly annuity, (which yet I knew not how to make ſhorter or eaſier,) I ſhall not trouble either myſelf or my reader with any attempt to determine it exactly. Indeed I thought the difference of the values of a yearly and a half-yearly annuity hardly worth attending to, and ſhould not therefore have gone into the foregoing inveſtigation of it, (eſpecially when I found into what length it led me) if I had not obſerved that ſeveral other writers upon this ſubject, and particularly Mr. De Moivre, Mr. Simpſon, and Dr. Price, have thought fit to mention it as an object of ſome importance. But, having now found that the ſaid difference is worth only about .24$\mathcal{L}$, or leſs than a quarter of a year's annuity, we may be the better ſatisfied that the further addition that ought to be made to $\frac{\mathcal{L}}{V}$, or to the value of the annuity, when paid yearly, in order to make it equal to the value of the ſame annuity paid quarterly, can be but a very trifling quantity. And therefore we may ſafely decline any further conſideration of it.

[*End of the conſideration of the values of life-annuities paid half-yearly and paid quarterly.*].

Of the limit of the increaſe of a ſum of money at compound intereſt.

CCXXVII. Nevertheleſs, as it is evident that the advantage ariſing from a given ſum of money put out at compound intereſt continually increaſes in *ſome* degree when the term for which the money is lent is diminiſhed, it will, I doubt not, be entertaining to the reader to ſee a determination of *the limit* of this increaſe, or the ſum to which the given ſum of money may, (by increaſing the number, and diminiſhing the lengths, of the terms for which the money is lent,) be made to approach as near as we pleaſe, in the ſpace of a year, or in any other given time, but which it can never exceed, or even abſolutely become equal to, into whatever number of parts the year be divided. Now this limit may be determined without much difficulty by the ſolution of the following Problem.

A Problem

A Problem in the Doctrine of Compound Intereſt.

CCXXVIII. Let a be any given ſum of money, and b its intereſt for one year. Let m be any whole number whatſoever; and let it be ſuppoſed that the money a is lent out for the m^{th} part of a year at an intereſt that bears the ſame proportion to the intereſt b, (which would be due for a whole year,) as the term for which it is lent, to wit, the m^{th} part of a year, bears to a whole year; and that, when the intereſt and principal are both repaid at the end of the m^{th} part of a year, they are added together and immediately lent out again at the ſame rate of intereſt for another m^{th} part of a year; and ſo on continually, at the end of every m^{th} part of a year, till the whole year is exhauſted. It is required to find the ſum to which the original ſum a will have increaſed, by theſe repeated loans of principal and intereſt, at the end of the whole year.

SOLUTION.

Since b is the intereſt of the ſum a for a whole year, it is evident that $\frac{b}{m}$ will be the intereſt of the ſame ſum a for the m^{th} part of a year. Therefore the ſum to be lent at the end of the firſt, and the beginning of the ſecond, m^{th} part of a year will be $a+\frac{b}{m}$. This ſum is to be lent for the m^{th} part of a year, as the ſum a was at firſt; and it is to be lent at the ſame intereſt. Therefore the ſaid ſum $a+\frac{b}{m}$ together with its intereſt for the m^{th} part of a year will bear the ſame proportion to the ſaid ſum $a+\frac{b}{m}$ alone as the ſaid ſum $a+\frac{b}{m}$ (which is equal to the original ſum a together with its intereſt for the m^{th} part of a year,) bears to the original ſum a. Therefore the ſaid ſum $a+\frac{b}{m}$, together with its intereſt for the m^{th} part of a year, is equal to $\frac{\overline{a+\frac{b}{m}}\Big|^2}{a}$; or the quantity to which the original ſum a will have increaſed by theſe loans at the end of the

ſecond

second m^{th} part of a year will be $\frac{\overline{a+\frac{b}{m}}|^2}{a}$. In the same manner it may be shewn that the quantity to which the original sum a will have increased in the next, or third, m^{th} part of the year will be $= \frac{\overline{a+\frac{b}{m}}|^2}{a} \times \frac{a+\frac{b}{m}}{a}$, or $\frac{\overline{a+\frac{b}{m}}|^3}{aa}$; and the quantity to which it will have increased in the fourth m^{th} part of the year will be $= \frac{\overline{a+\frac{b}{m}}|^3}{aa} \times \frac{a+\frac{b}{m}}{a}$, or $\frac{\overline{a+\frac{b}{m}}|^4}{a^3}$; and consequently that the quantity to which it will have increased in the last m^{th} part of the year, or in the course of the whole year, will be $= \frac{\overline{a+\frac{b}{m}}|^m}{a^{m-1}}$. QEI.

Examples of the foregoing Solution.

Case of interest paid every half-year.

CCXXIX. COROLL. 1. Let m be $= 2$; or let the sum a be lent for only half a year, and then paid in with the interest due upon it, and immediately lent out again, together with the interest, at the same rate of interest, for the other half-year. Then will $\frac{\overline{a+\frac{b}{m}}|^m}{a^{m-1}}$ be $=$

$$\frac{\overline{a+\frac{b}{2}}|^2}{a^{2-1}} = \frac{\overline{a+\frac{b}{2}}|^2}{a} = \frac{aa+2a\times\frac{b}{2}+\frac{bb}{4}}{a} = a+b+\frac{bb}{4a}.$$

Therefore by lending the sum a in this manner for two successive half-years, instead of lending it at once for a whole year, the said sum will, at the end of the year, be increased to $a+b+\frac{bb}{4a}$, instead of being increased only to $a+b$.

Thus,

Thus, for example, if we ſuppoſe the ſum a to be £100, and b, the intereſt of it for a year, to be 5 pounds, we ſhall have $bb = \overset{£}{25}$, and $\frac{bb}{4a}$ $\left(= \frac{\overset{£}{25}}{4 \times 100} = \frac{\overset{£}{25}}{4 \times 4 \times 25} = \frac{\overset{£}{1}}{16} = \frac{\overset{s}{20}}{16} = \frac{\overset{s}{5}}{4}\right)$ $= \overset{s}{1}.\ \overset{d}{3}.$

Therefore, if a man were to lend a hundred pounds at the intereſt of 5 per cent. per annum, for two ſucceſſive half-years, ſo that the principal together with its intereſt for the firſt half-year, ſhould be made a new principal and lent out at the ſame intereſt for the ſecond half-year, he would thereby increaſe it in the courſe of the whole year to only 1s. 3d. more than it would have been increaſed to in the ſame time by lending it at once for a whole year at the ſame intereſt.

CCXXX. COROLL. 2. If m is $= 4$, we ſhall have $\overline{a + \frac{b}{m}}\Big|^m$ $= \overline{a + \frac{b}{4}}\Big|^4$ $\left(a^4 + \frac{4a^3b}{4} + \frac{6aabb}{16} + \frac{4a \times b^3}{64} + \frac{b^4}{256}\right) = a^4 + a^3b + \frac{3aabb}{8} + \frac{ab^3}{16} + \frac{b^4}{256}$, and $a^{m-1} = a^{4-1} = a^3$; and conſequently $\frac{\overline{a + \frac{b}{m}}\Big|^m}{a^{m-1}} = a + b + \frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^3}$. Therefore, if the ſum a be lent out for only a quarter of a year, or three months, at the intereſt of b per annum, or $\frac{b}{4}$ for the ſaid three months, and it be then paid in with the intereſt due upon it; and then the ſaid principal and intereſt be added together, ſo as to make a new principal, and the ſaid new principal be lent out again immediately, at the ſame rate of intereſt as before, for three months more; and ſo on for the two remaining quarters of the year; the quantity to which the original ſum a will have increaſed at the end of the year by means of theſe four ſucceſſive loans, will be $a + b + \frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^3}$, which is greater than $a + b$, (or the quantity to which the ſaid ſum a would have increaſed in the ſame time by means of a ſingle loan at the ſame intereſt for the whole year,) by $\frac{3bb}{8a} + \frac{b^3}{10aa} + \frac{b^4}{256a^3}$.

Caſe of intereſt paid every quarter of a year.

If

If a is 100 pounds, and b is 5 pounds, we shall have $\frac{3bb}{8a}$ $\left(= \frac{3 \times 25}{8 \times 100}\right.$ $= \frac{3 \times 1}{8 \times 4} = \frac{3 \times 1}{32} = \frac{3 \times 20}{32} = \frac{60}{32} = \frac{720}{32} = 22.5\left.\right) = 1, 10\frac{1}{2}$; and $\frac{b^3}{16aa}$ $\left(= \frac{125}{16 \times 10{,}000} = \frac{25}{16 \times 2000} = \frac{5}{16 \times 400} = \frac{1}{16 \times 80} = \frac{1}{1280} = \frac{240}{1280} = \frac{24}{128} = \frac{3}{16}\right) = 0.1875$; and $\frac{b^4}{256a^3}$ $\left(= \frac{625}{256 \times 1000{,}000} = \frac{125}{256 \times 200{,}000} = \frac{25}{256 \times 40{,}000} = \frac{5}{256 \times 8000} = \frac{1}{256 \times 1600} = \frac{240}{256 \times 1600} = \frac{24}{256 \times 160} = \frac{3}{256 \times 20} = \frac{3}{5120}\right)$ $= 0.000{,}581$. Therefore $\frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^3}$ will be $= 1, 10.5 + 0.18750 + 0.000{,}58 = 1, 10.68808$, or less than $1, 10\frac{3}{4}$; that is, the excess of the increase of the original sum of a hundred pounds in the space of a year by means of these four quarterly loans above the increase of it in the same time by means of a single loan of it for the whole year at the interest of 5 pounds, is a little more than $1, 10\frac{1}{2}$, but less than $1, 10\frac{3}{4}$.

Case of interest paid every month.

CCXXXI. COROLL. 3. If m is $= 12$, or the sum a is lent out for only the 12^{th} part of a year, or a calendar month, and then is paid in, and the principal and interest added together so as to form a new principal, and the said new principal lent out again immediately for another calendar month at the same interest as before; and the like loans of the principal with its monthly interest, be repeated for the remaining ten months of the year, we shall have $\overline{\left|a + \frac{b}{m}\right|}^m = \overline{\left|a + \frac{b}{12}\right|}^{12}$, and $a^m - 1 = a^{11}$, and consequently $\frac{\overline{\left|a + \frac{b}{m}\right|}^m}{a^m - 1} = \frac{\overline{\left|a + \frac{b}{12}\right|}^{12}}{a^{11}}$.

Now

Now, by Sir Iſaac Newton's binomial theorem, $\overline{\frac{a+b}{12}}\Big|^{12}$ is $\left(= a^{12} + \frac{12}{1} \times a^{12-1}\frac{b}{12} + \frac{12}{1} \times \frac{12-1}{2} \times \frac{a^{12-2} \times bb}{12\times12} + \frac{12}{1} \times \frac{12-1}{2} \times \frac{12-2}{3} \times a^{12-3} \times \frac{b^3}{12\times12\times12} + \text{&c. continued to the term } \frac{b^{12}}{\overline{12}|^{12}} = a^{12} + \frac{12\,a^{11}b}{12} + 12 \times \frac{11}{2} \times \frac{a^{10}bb}{12\times12} + 12 \times \frac{11}{2} \times \frac{10}{3} \times \frac{a^9 b^3}{12\times12\times12} + \text{&c. continued to the term } \frac{b^{12}}{\overline{12}|^{12}}\right) = a^{12} + a^{11}b + \frac{11\,a^{10}bb}{24} + \frac{55a^9b^3}{432} + \text{&c. continued to the term } \frac{b^{12}}{\overline{12}|^{12}}$. Therefore $\frac{\overline{\frac{a+b}{12}}\Big|^{12}}{a^{11}}$ is $=$ $a + b + \frac{11\,bb}{24a} + \frac{55b^3}{432\,aa} + \text{&c. continued to the term } \frac{b^{12}}{\overline{12}|^{12}a^{11}}$.

Therefore the ſum a will, in the courſe of the whole year, have increaſed, by means of theſe twelve repeated monthly loans, to $a + b + \frac{11\,bb}{24a} + \frac{55b^3}{432\,aa} + \text{&c. continued to the term } \frac{b^{12}}{\overline{12}|^{12}a^{11}}$; the exceſs of which above $a + b$, (or above the quantity to which it would have increaſed in the ſame time by a ſingle loan for the intereſt b,) is $\frac{11\,bb}{24a} + \frac{55b^3}{432\,aa} + \text{&c. continued to the term } \frac{b^{12}}{\overline{12}|^{12}a^{11}}$; in which ſeries all the terms after $\frac{55b^3}{432\,aa}$ are omitted on account of their extreme ſmallneſs in compariſon of $\frac{11\,bb}{24a}$ and $\frac{55b^3}{432\,aa}$.

If a is $\overset{£}{100}$, and b is $\overset{£}{5}$, we ſhall have $bb = \overset{£}{25}$, and $\frac{11\,bb}{24\,a}$ $\left(= \frac{11 \times 25}{24 \times 100} = \frac{11 \times 25}{24 \times 4 \times 25} = \frac{11 \times 1}{24 \times 4} = \frac{\overset{£}{11}}{96} = \frac{11 \times 240}{96} = \frac{\overset{D}{2640}}{96} = \overset{D}{27.5}\right) = \overset{S\ D}{2,\ 3.5}$, or $\overset{S\ D}{2,\ 3\frac{1}{2}}$; and $\frac{55\,b^3}{432\,aa}$ $\left(= \frac{55 \times 125}{432 \times 100 \times 100} = \frac{55 \times 25 \times 5}{432 \times 4 \times 25 \times 5 \times 20} = \frac{55}{432 \times 4 \times 20} = \frac{55}{432 \times 4 \times 4 \times 5} = \frac{11}{432 \times 4 \times 4} = \frac{11}{432 \times 16} = \frac{11}{6912} = \frac{11 \times 240}{6912} = \frac{2640}{6912}\right) = \overset{D}{0.38}$, or leſs than one half-penny. Therefore $\frac{11\,bb}{24\,a} + \frac{55\,b^3}{432\,aa}$ will be equal to $\overset{S\ D}{2,\ 3.5} + \overset{D}{0.38}$, or $\overset{S\ D}{2,\ 3.88}$, or leſs than $\overset{S\ D}{2,\ 4}$. So that the exceſs of the intereſt of 100 pounds for a year, ariſing from theſe 12 monthly loans, at this rate of intereſt, above 5 pounds, or the intereſt of the ſame ſum of money for the ſame time, ariſing from a ſingle loan of it for the whole year, is only $\overset{S\ D}{2,\ 3.88}$, or leſs than $\overset{S\ D}{2,\ 4}$.

Caſe of intereſt paid every week.

CCXXXII. COROLL. 4. If m is $= 52$, or the ſum a is lent out for only the 52d part of a year, or for one week, and then paid in, and, with its intereſt, immediately lent out again for another week, and ſo on for every week throughout the whole year, we ſhall have $\overline{a + \frac{b}{m}}\Big|^m = \overline{a + \frac{b}{52}}\Big|^{52}$, and $a^{m-1} = a^{51}$, and conſequently $\frac{\overline{a + \frac{b}{m}}\Big|^m}{a^{m-1}} = \frac{\overline{a + \frac{b}{52}}\Big|^{52}}{a^{51}}$. Now, by Sir Iſaac Newton's binomial theorem, $\overline{a + \frac{b}{52}}\Big|^{52}$ is $\left(= a^{52} + \frac{52}{1} \times a^{51} \times \frac{b}{52} + \frac{52}{1} \times \frac{52-1}{2} \times \frac{a^{50} \times bb}{52 \times 52} + \frac{52}{1}\right.$

$+\frac{52}{1}\times\frac{52-1}{2}\times\frac{52-2}{3}\times\frac{a^{49}\times b^3}{52\times 52\times 52}+\&c. = a^{52}+a^{51}b$

$+\frac{51\,a^{50}\,bb}{104}+\frac{51}{2}\times\frac{50}{3}\times\frac{a^{49}b^3}{52\times 52}+\&c. = a^{52}+a^{51}b+\frac{51a^{50}bb}{104}$

$+\frac{17\times 25\times a^{49}b^3}{52\times 52}+\&c.\Big) = a^{52}+a^{51}b+\frac{51a^{50}bb}{104}+\frac{425a^{49}b^3}{2704}$

$+\&c.$ Therefore $\frac{\overline{a+b}\,\Big|^{52}}{a^{51}}$ is $= a+b+\frac{51\,bb}{104\,a}+\frac{425\,b^3}{2704\,aa}+\&c.$

Therefore the ſum a will, in the courſe of the whole year, have increaſed, by means of theſe 52 repeated weekly loans, to $a+b+\frac{51\,bb}{104\,a}+\frac{425\,b^3}{2704\,aa}+\&c.$ which exceeds $a+b$ (or the quantity to which the ſaid ſum a would have increaſed in the ſame time by a ſingle loan for the intereſt b,) by the difference $\frac{51\,bb}{104\,a}+\frac{425\,b^3}{2704\,aa}+\&c.$ in which difference all the terms after the ſecond term, $\frac{425\,b^3}{2704\,aa}$, are omitted on account of their extreme ſmallneſs in compariſon of $\frac{51\,bb}{104\,a}$ and $\frac{425\,b^3}{2704\,aa}$.

Let a be $= \overset{£}{100}$, as before, and $b = \overset{£}{5}$. Then will bb be $= \overset{£}{25}$, and $\frac{51\,bb}{104\,a}$ will be $\Big(= \frac{51\times 25}{104\times 100}^{£} = \frac{51\times 25}{104\times 4\times 25}^{£} = \frac{51\times 1}{104\times 4}^{£} = \frac{51\times 240}{104\times 4}^{D}$

$= \frac{51\times 60}{104}^{D} = \frac{51\times 15}{26}^{D} = \frac{766}{26}^{D}\Big) = 29.46^{D}$, or $2, 5.46^{S\ D}$; and $\frac{425\,b^3}{2704\,aa}$ will be $\Big(= \frac{425\times 5\times 5\times 5}{2704\times 100\times 100}^{£} = \frac{425\times 5\times 5\times 5}{2704\times 5\times 5\times 4\times 5\times 20}^{£} = \frac{425\times 1}{2704\times 4\times 20}^{£}$

$= \frac{425\times 1}{2704\times 4\times 4\times 5}^{£} = \frac{85\times 1}{2704\times 4\times 4}^{£} = \frac{85\times 240}{2704\times 4\times 4}^{D} = \frac{85\times 60}{2704\times 4}^{D}$

$= \frac{85\times 15}{2704}^{D}$

$= \frac{85 \times 15}{2704} = \frac{1275}{2704}\Big) = \overset{D}{0.47}$; and conſequently $\frac{51\,bb}{104\,a} + \frac{425\,b^3}{2704\,aa}$ will be $= \overset{S\ D}{2,\ 5.46} + \overset{D}{0.47} = \overset{S\ D}{2,\ 5.93}$, or leſs than 2, 6. Conſequently the exceſs of the compound intereſt of 100 pounds for a year ariſing from theſe 52 weekly loans of it, at this rate of intereſt, above 5 pounds, or the ſimple intereſt of the ſame ſum of money for the ſame time, or the intereſt ariſing from a ſingle loan of it for the whole year, is only $\overset{S.\ D}{2,\ 5.93}$, or leſs than $\overset{S\ D}{2,\ 6}$.

Recapitulation of the concluſions obtained in the four preceeding articles.

CCXXXIII. Thus it appears that, when the intereſt of money is 5 per cent. per annum, the exceſs of the compound intereſt of 100 pounds for a year, ariſing from two ſucceſſive half-yearly loans of it at that intereſt, above the ſimple intereſt of it, is only $\overset{S\ D}{1,\ 3}$; and the exceſs of the compound intereſt above the ſimple intereſt when the loans of it are made for a quarter of a year at a time, is about $\overset{S\ D}{1,\ 10\frac{1}{2}}$; and, when the ſaid loans are made for a month at a time, it is $\overset{S\ D}{2,\ 3\frac{1}{2}}$; and, when they are made for only a week at a time, it is about 2, 6. So that very little advantage ariſes to the lender of a ſum of money by lending it for terms leſs than a year, or by receiving the intereſt of it at the end of every half-year, or quarter of year, or month, or week, inſtead of lending it for a whole year at once, or receiving his intereſt only at the end of the year.

Caſe of intereſt paid at the end of every minute, or ſecond, of time, or rather, of every infinitely ſmall portion of time.

CCXXXIV. COROLL. 5. The *limit* of the magnitude to which the ſum a will increaſe in the courſe of a year by ſuppoſing the terms for which it is lent to be continually diminiſhed, and the number of them to be increaſed in the ſame proportion, may be found by ſuppoſing the number m (which denotes the number of thoſe terms contained in the whole year,) to become infinite.

Now, when m is infinite, the ſeveral fractions $\frac{m-1}{2}$, $\frac{m-2}{3}$, $\frac{m-3}{4}$, $\frac{m-4}{5}$, $\frac{m-5}{6}$, &c. will become equal to $\frac{m}{2}$, $\frac{m}{3}$, $\frac{m}{4}$, $\frac{m}{5}$, &c. be-

cauſe

cauſe the ſubtracted numbers 1, 2, 3, 4, 5, &c. in the ſeveral numerators of thoſe fractions will be infinitely ſmaller than the number m, from which they are ſubtracted. Therefore $\overline{a+\frac{b}{m}}\Big|^m$, (which, by Sir Iſaac Newton's binomial theorem, is in all caſes $= a^m + \frac{m}{1}\, a^{m-1}\frac{b}{m} + \frac{m}{1}\times\frac{m-1}{2}\times a^{m-2}\times\frac{bb}{mm} + \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times a^{m-3}\times\frac{b^3}{m^3} + \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times a^{m-4}\times\frac{b^4}{m^4} + \frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\times a^{m-5}\times\frac{b^5}{m^5} +$ &c. continued to the $m+1^{\text{th}}$ term,) will in this caſe be $= a^m + \frac{m}{1}\times a^{m-1}\times\frac{b}{m} + \frac{m}{1}\times\frac{m}{2}\times a^{m-2}\times\frac{bb}{mm} + \frac{m}{1}\times\frac{m}{2}\times\frac{m}{3}\times a^{m-3}\times\frac{b^3}{m^3} + \frac{m}{1}\times\frac{m}{2}\times\frac{m}{3}\times\frac{m}{4}\times a^{m-4}\times\frac{b^4}{m^4} + \frac{m}{1}\times\frac{m}{2}\times\frac{m}{3}\times\frac{m}{4}\times\frac{m}{5}\times a^{m-5}\times\frac{b^5}{m^5} +$ &c. *ad infinitum* $= a^m + a^{m-1}b + \frac{a^{m-2}bb}{1.2} + \frac{a^{m-3}b^3}{1.2.3} + \frac{a^{m-4}b^4}{1.2.3.4} + \frac{a^{m-5}b^5}{1.2.3.4.5} +$ &c. *ad infinitum*; and conſequently $\frac{\overline{a+\frac{b}{m}}\Big|^m}{a^{m-1}}$ will in this caſe be $= a + b + \frac{bb}{1.2.a} + \frac{b^3}{1.2.3.aa} + \frac{b^4}{1.2.3.4.a^3} + \frac{b^5}{1.2.3.4.5.a^4} +$ &c. *ad infinitum*. Therefore the limit of the quantity to which the original ſum a may be made to increaſe in the ſpace of a year by continually diminiſhing the magnitude, and increaſing the number, of the parts into which the year is divided, or the periods for which the money is to be lent, is the ſaid ſeries $a + b + \frac{bb}{1.2.a} + \frac{b^3}{1.2.3.aa} + \frac{b^4}{1.2.3.4.a^3} + \frac{b^5}{1.2.3.4.5.a^4} +$ &c. *ad infinitum*, or $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} +$ &c. *ad infinitum*;

and

and the excefs of the faid limit above $a + b$, or the quantity to which the faid fum a would have increafed in the fame time by a fingle loan of it at the intereft b for the whole year, is = the feries

$$\frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5\,a^4} + \text{&c. } ad\ infinitum.\ \text{QEI.}$$

Now let a (as before) be $= \overset{\pounds}{100}$, and b be $= \overset{\pounds}{5}$. Then will $\frac{bb}{2a}$ be $\left(= \frac{b \times b}{2a} = \overset{\pounds}{5} \times \frac{5}{2\times 100} = \overset{\pounds}{5} \times \frac{1}{2} \times \frac{1}{20} = \overset{\pounds}{5} \times \frac{1}{40} = \overset{S}{\frac{100}{40}} = \overset{D}{\frac{1200}{40}}\right) = \overset{D}{30} = \overset{S}{2},\ \overset{D}{6}$; and $\frac{b^3}{2.3.aa}$ will be $\left(= \frac{bb}{2a} \times \frac{b}{3a} = \frac{bb}{2a} \times \frac{1}{3\times 20} = \frac{bb}{2a} \times \frac{1}{60} = \overset{D}{30} \times \frac{1}{60}\right) = \overset{D}{\frac{1}{2}}$; and $\frac{b^4}{2.3.4.a^3}$ will be $\left(= \frac{b^3}{2.3.aa} \times \frac{b}{4a} = \frac{b^3}{2.3.aa} \times \frac{1}{4\times 20} = \frac{b^3}{2.3.aa} \times \overset{D}{\frac{1}{80}} = \overset{D}{\frac{1}{2}} \times \overset{D}{\frac{1}{80}}\right) = \overset{D}{\frac{1}{160}}$; and $\frac{b^5}{2.3.4.5.a^4}$ will be $\left(= \frac{b^4}{2.3.4.a^3} \times \frac{b}{5a} = \frac{b^4}{2.3.4.a^4} \times \frac{1}{5\times 20} = \frac{b^4}{2.3.4.a^3} \times \frac{1}{100} = \overset{D}{\frac{1}{160}} \times \overset{D}{\frac{1}{100}}\right) = \overset{D}{\frac{1}{16000}}$; and consequently the feries $\frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} + \text{&c.}$ will be $\left(= \overset{D}{30} + \overset{D}{\frac{1}{2}} + \overset{D}{\frac{1}{160}} + \overset{D}{\frac{1}{16000}} + \text{&c.} = \overset{D}{30} + \overset{D}{0.500{,}000} + \overset{D}{0.006{,}250} + 0.000{,}062 + \text{&c.} = \overset{D}{30.506{,}312}\right) = \overset{S}{2},\ \overset{D}{6.506{,}312}$, or $\overset{S}{2},\ \overset{D}{6\frac{1}{2}} + \overset{D}{0.006{,}312}$, or $\overset{S}{2},\ \overset{D}{6\frac{1}{2}}$ together with $\frac{6312}{1000{,}000}$ parts, or one 158^{th} part, of a penny. Therefore the limit of the excefs of the fum to which it is poffible to increafe the fum of 100 pounds in the fpace of a year by lending it at the intereft of 5 per cent. per annum for very fmall portions of a year, above 105 pounds, or the fum to which it will increafe in

in the ſame time by a ſingle loan of it for the whole year at the ſame rate of intereſt, is only two ſhillings, and ſixpence, half-penny, together with the 158th part of a penny. QEI.

From this and the preceeding articles we may conclude that the advantages that may be made by lending money for very ſmall portions of a year, inſtead of lending it for a year, or half a year, at a time, are ſo very ſmall as not to be worth attending to.

A remarkable analogy between the foregoing limit of the increaſe of a ſum of money at compound intereſt, in the courſe of a year, and a certain ordinate of a logarithmick curve.

CCXXXV. Let the ſum a be repreſented by the ſubtangent of any logarithmick curve; and let an ordinate to the axis of the ſaid curve be drawn that ſhall be equal to the ſubtangent of it; which, from the nature of the ſaid curve, (which admits of ordinates of all poſſible magnitudes,) is always poſſible: and from the bottom of the ſaid ordinate, (or the point in which it touches the axis of the curve,) and on that ſide of the curve on which the ordinates increaſe, let a portion of the ſaid axis be taken that ſhall bear the ſame proportion to the ſaid ordinate, (or to the ſubtangent of the curve,) as b, (the intereſt of the ſum a for a year, when lent at once for the whole year,) bears to the ſaid ſum a itſelf; and at the extremity of the ſaid portion, or abſciſs, of the axis, let another ordinate be erected; which, it is evident, will be greater than the former ordinate, or than the ſubtangent. Then will the ſaid ſecond, or greater, ordinate repreſent, or be proportional to, the ſeries $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} +$ &c. *ad infinitum*; or the ſaid firſt, or leſſer ordinate, (which is equal to the ſubtangent,) the ſaid abſciſs of the axis intercepted between the two ordinates, and the ſaid greater ordinate, will bear to each other the ſame proportions, reſpectively, as the original ſum a, the ſum b (which is the intereſt of a for one year, when lent out at once for the whole year,) and the ſeries $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} +$ &c. *ad infinitum*, or the *limit* of the quantity to which the ſum a may be made to increaſe in the ſame ſpace of one year by lending it out at compound intereſt for very ſhort terms.

For

For it is ſhewn in my Elements of Plane Trigonometry, Art. 354, page 470, that, if a be the ſubtangent of a logarithmick curve, and a and $a + x$ be two ordinates to the axis of the ſaid curve, and z the abſciſs of the axis intercepted between the ſaid ordinates, the difference, x, of the ſaid ordinates will be equal to the ſeries $z + \frac{zz}{2a} + \frac{z^3}{2.3.aa} + \frac{z^4}{2.3.4.a^3} + \frac{z^5}{2.3.4.5.a^4} +$ &c. *ad infinitum*. Conſequently $a + x$, or the greater of the ſaid two ordinates, will be equal to the ſeries $a + z + \frac{zz}{2a} + \frac{z^3}{2.3.aa} + \frac{z^4}{2.3.4.a^3} + \frac{z^5}{2.3.4.5.a^4} +$ &c. *ad infinitum*. Therefore, if we denote the ſaid abſciſs of the axis by the letter b inſtead of the letter z, we ſhall have $a + x =$ the ſeries $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} +$ &c. *ad infinitum*. Q E D.

N. B. This analogy between the foregoing limit of the increaſe of the ſum a in the courſe of a year, by being lent out for ſhort periods at compound intereſt, and the ordinate of a logarithmick curve, has ſomething in it that is pleaſing and amuſing to the imagination: and therefore I thought the reader might be glad to ſee it ſtated in this place, though I do not know of any practical uſe in the doctrine of annuities to which it can be applied. See upon this ſubject Dr. Keil's tract on Logarithms, at the end of Commandine's edition of Euclid's Elements, printed at Oxford in the year 1747, page 71.

Of the foregoing infinite ſeries, obtained for the limit of the increaſe of the ſum a *at compound intereſt, when* b *denotes the ſimple intereſt of it for a greater, or leſſer, term than one year.*

CCXXXVI. In Art. 227 and all the following articles we have ſuppoſed b to be the intereſt of the ſum a for a ſingle year, becauſe that is the moſt common way of eſtimating, or denominating, the intereſt of money. But the reaſonings uſed in thoſe articles will be equally juſt, and the concluſions obtained in them will be equally true, if b be made to denote the intereſt of a for any other given time, either greater or leſs than a year; as, for example, if b be made to denote the intereſt of a for

for fifteen or twenty years. And therefore it will always be true, (whatever be the length of the term for which the money is lent, or whatever be the magnitude of the intereſt b,) that the limit of the quantity to which the original ſum a may be made to increaſe, (in the ſame time in which the intereſt b becomes due,) by lending it at compound intereſt for a great number of very ſhort terms, is equal to the foregoing ſeries

$$a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} + \text{&c. } \textit{ad infinitum}.$$

The foregoing ſeries is true in all caſes, whatever be the proportion of the intereſt b to the original ſum a.

One of theſe caſes ſeems more curious than the reſt, namely, that in which the intereſt b, which is to be paid for the original ſum a at the end of the term for which it is lent, is equal to the ſaid ſum a itſelf; as, for example, if the ſum a were to be lent for a term of 20 years, and were then to be repaid with an intereſt equal to itſelf; which, it is evident, would be by no means a hard or unreaſonable agreement. Now in this caſe the foregoing ſeries $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4}$ + &c. *ad infinitum* will become equal to $a + a + \frac{aa}{2a} + \frac{a^3}{2.3.aa}$ $+ \frac{a^4}{2.3.4.a^3} + \frac{a^5}{2.3.4.5.a^4}$ + &c. *ad infinitum* $= a + a + \frac{a}{2} + \frac{a}{2.3}$ $+ \frac{a}{2.3.4} + \frac{a}{2.3.4.5}$ + &c. *ad infinitum* $= a \times$ the ſeries $1 + 1 + \frac{1}{2}$ $+ \frac{1}{2.3} + \frac{1}{2.3.4} + \frac{1}{2.3.4.5}$ + &c. *ad infinitum* $= a \times 2.718,281,828,$ 459,036, &c. Therefore, if the intereſt due for the ſum a for any given time were equal to the ſaid ſum itſelf, inſtead of being only a 20th, or 25th, part of it, (as is moſt commonly the caſe, when money is lent only for a year,) the limit of the quantity to which the ſaid ſum might be made to increaſe in the ſame ſpace of time by improving it at compound intereſt by means of very frequent payments of the intereſt and immediate renewals of the loan of both principal and intereſt, would be equal to 2.718, 281, 828, 459, 036, &c. times, or leſs than three times, the original ſum a.

Caſe of the equality of b to a.

The value of the ſaid ſeries in that caſe.

CCXXXVII. The computation of the terms of the ſeries $1 + 1 + \frac{1}{2} + \frac{1}{2.3}$ $\frac{1}{2.3.4} + \frac{1}{2.3.4.5}$ + &c. ſo far as to obtain the foregoing number 2.718,281,828,459,036, may be performed as follows.

A computation of the value of the ſaid ſeries in that caſe to ſixteen places of figures.

Let the capital letters *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *K*, *L*, *M*, &c. be put for the ſeveral terms of the ſaid ſeries, as they ariſe; ſo that *A* ſhall be = 1, and *B* ſhall be = 1, and *C* ſhall be = $\frac{1}{2}$, and *D* ſhall be = $\frac{1}{2 \cdot 3}$, and *E* ſhall be = $\frac{1}{2 \cdot 3 \cdot 4}$, and *F* ſhall be = $\frac{1}{2 \cdot 3 \cdot 4 \cdot 5}$, and ſo on of the following terms. Then, from the law of the continuation of the terms of this ſeries, it is evident that *D* will be = $\frac{C}{3}$, and *E* will be = $\frac{D}{4}$, and *F* will be = $\frac{E}{5}$, and *G* will be = $\frac{F}{6}$, and *H* will be = $\frac{G}{7}$, and *I* will be = $\frac{H}{8}$, and *K* will be = $\frac{I}{9}$, and *L* will be = $\frac{K}{10}$, and *M* will be = $\frac{L}{11}$, and *N* will be = $\frac{M}{12}$, and *O* will be = $\frac{N}{13}$, and *P* will be = $\frac{O}{14}$, and *Q* will be = $\frac{P}{15}$, and *R* will be = $\frac{Q}{16}$, and *S* will be = $\frac{R}{17}$. We ſhall therefore have

$$A = 1 = 1.000,000,000,000,000;$$

$$\text{and } B = 1 = 1.000,000,000,000,000;$$

$$\text{and } C = \frac{1}{2} = .500,000,000,000,000;$$

$$\text{and } D = \frac{C}{3} = .166,666,666,666,666;$$

$$\text{and } E = \frac{D}{4} = .041,666,666,666,666;$$

$$\text{and } F = \frac{E}{5} = .008,333,333,333,333;$$

$$\text{and } G = \frac{F}{6} = .001,388,888,888,888;$$

$$\text{and } H = \frac{G}{7} = .000,198,412,698,412;$$

and

and $I = \frac{H}{8} =$ .000,024,801,587,301;

and $K = \frac{I}{9} =$ .000,002,755,731,922;

and $L = \frac{K}{10} =$ .000,000,275,573,192;

and $M = \frac{L}{11} =$ .000,000,025,052,108;

and $N = \frac{M}{12} =$ .000,000,002,087,075;

and $O = \frac{N}{13} =$ .000,000,000,160,590;

and $P = \frac{O}{14} =$ .000,000,000,011,470;

and $Q = \frac{P}{15} =$ .000,000,000,000,764;

and $R = \frac{Q}{16} =$ .000,000,000,000,047;

and $S = \frac{R}{17} =$ .000,000,000,000,002.

2.718,281,828,459,036.

N. B. The proportion of this number, 2.718,281,828,459,036, &c. to 1, is what Mr. Cotes calls the *ratio modularis*, being the ratio, or proportion, of two ordinates to the axis of a logarithmick curve drawn through the extremities of an abſciſs of the axis equal to the *ſubtangent* of the curve, or that ratio of which the ſubtangent of a logarithmick curve, and, in general, in any other ſyſtem of the meaſures of ratios, the *modulus* of the ſyſtem, is the meaſure.

CCXXXVIII. If the ſum a be ſuppoſed to be lent for ſo long a term of years before its intereſt b is to be paid, that the ſaid intereſt ſhall be greater than the principal, it is evident that the literal parts of the terms of the foregoing ſeries $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4}$ Of the value of the foregoing ſeries when the intereſt b is greater than the original ſum a.

$+\frac{b^5}{2.3.4.5.a^4}$ + &c. *ad infinitum*, to wit, $a, b, \frac{bb}{a}, \frac{b^3}{aa}, \frac{b^4}{a^3}, \frac{b^5}{a^4}$, &c. will continually increaſe. And hence it may ſeem, perhaps, at firſt ſight, that the terms of the ſeries will in theſe caſes continually diverge, and that conſequently their ſum, or the ſeries itſelf, will become infinitely great, and therefore will not exhibit truely the value of the aforeſaid limit. But, if we conſider the matter with a little attention, we ſhall perceive that the ſeries muſt in all caſes be a converging one, whatever be the proportion in which the intereſt b may exceed the original ſum a, though the convergency will not, in theſe caſes, begin from the firſt term a, as it does when the intereſt b is leſs than a. For, as, upon this ſuppoſition, the literal parts of the terms, to wit, $a, b, \frac{bb}{a}, \frac{b^3}{aa}, \frac{b^4}{a^3}, \frac{b^5}{a^4}$, &c. will diverge, or increaſe, continually in the ſame given proportion of b to a, and the numeral parts of the third, fourth, fifth, ſixth, and other following terms, to wit, $\frac{1}{2}, \frac{1}{2.3}, \frac{1}{2.3.4}, \frac{1}{2.3.4.5}$, &c. do, in all caſes, decreaſe in proportions which are continually and indefinitely increaſing, to wit, in the proportions of 2 to 1, 3 to 1, 4 to 1, and ſo on, it is evident that the proportion of the numeral parts of two contiguous terms in the ſeries (in which the latter of the ſaid numeral parts will be leſs than the former of them,) muſt always, in ſome part or other of the ſeries, become greater than the proportion of the literal parts of the ſame terms, (in which the latter of the ſaid literal parts will be greater than the former of them,) and conſequently that the latter of the ſaid two contiguous terms will, upon the whole, be leſs than the former of them. And after the terms in which this happens all the following terms of the ſeries will continually decreaſe in a greater and greater proportion. Thus, for example, if b is $= 10a$, the firſt ten terms of the ſeries $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4}$ + &c. *ad infinitum*, or (if we denote the numeral parts of theſe terms, to wit, $1, 1, \frac{1}{2}, \frac{1}{2.3}, \frac{1}{2.3.4}, \frac{1}{2.3.4.5}$, &c. by the capital letters A, B, C, D, E, F, &c. as in Art. 237,) of the ſeries $Aa + Bb + \frac{Cbb}{a} + \frac{Db^3}{aa} + \frac{Eb^4}{a^3} + \frac{Fb^5}{a^4}$ + &c.

The ſaid ſeries will in this caſe diverge in its firſt terms, but afterwards converge in all the remaining terms.

$+$ &c. *ad infinitum*, or of the series $Aa + Bb + \frac{Bbb}{2a} + \frac{Cb^3}{3aa} + \frac{Db^4}{4a^3} + \frac{Eb^5}{5a^4} + \frac{Fb^6}{6a^5} + \frac{Gb^7}{7a^6} + \frac{Hb^8}{8a^7} + \frac{Ib^9}{9a^8} + \frac{Kb^{10}}{10a^9} + \frac{Lb^{11}}{11a^{10}} + \frac{Mb^{12}}{12a^{11}} + \frac{Nb^{13}}{13a^{12}}$ + &c. will diverge, or increaſe; becauſe the proportion of b to a, or of 10 to 1, (in which the literal part of every term exceeds the literal part of the term that immediately preceeds it,) is greater than the proportions of 2 to 1, 3 to 1, 4 to 1, 5 to 1, 6 to 1, 7 to 1, 8 to 1, and 9 to 1, in which the numeral parts of third, fourth, fifth, ſixth, ſeventh, eighth, ninth, and tenth terms are exceeded by the numeral parts of the terms that immediately preceed them: but the eleventh term $\frac{Kb^{10}}{10a^9}$ will not be greater than the tenth term $\frac{Ib^9}{9a^8}$, but exactly equal to it, being equal to $\left(K \times \frac{b^{10}}{10a^9} = \frac{I}{9} \times \frac{b^{10}}{10a^9} = \frac{I}{9} \times \frac{b^9}{a^8} \times \frac{b}{10a} = \frac{Ib^9}{9a^8} \times \frac{b}{10a} = \frac{Ib^9}{9a^8} \times \frac{1}{10} \times \frac{b}{a} = \frac{Ib^9}{9a^8} \times \frac{1}{10} \times \frac{10}{1} =\right) \frac{Ib^9}{9a^8} \times \frac{10}{10}$, which is $= \frac{Ib^9}{9a^8}$: and the twelfth term $\frac{Lb^{11}}{11a^{10}}$ will be leſs than the eleventh term $\frac{Kb^{10}}{10a^9}$, being $\left(= L \times \frac{b^{11}}{11a^{10}} = \frac{K}{10} \times \frac{b^{11}}{11a^{10}} = \frac{K}{10} \times \frac{b^{10}}{a^9} \times \frac{b}{11a} = \frac{Kb^{10}}{10a^9} \times \frac{b}{11a} = \frac{Kb^{10}}{10a^9} \times \frac{1}{11} \times \frac{b}{a} = \frac{Kb^{10}}{10a^9} \times \frac{1}{11} \times \frac{10}{1}\right) = \frac{Kb^{10}}{10a^9} \times \frac{10}{11}$, which is leſs than $\frac{Kb^{10}}{10a^9}$ in the proportion of 10 to 11. And in like manner it will be found, that the thirteenth term $\frac{Mb^{12}}{12a^{11}}$ is leſs than the twelfth term $\frac{Lb^{11}}{11a^{10}}$ in the proportion of 10 to 12, and that the fourteenth term $\frac{Nb^{13}}{13a^{12}}$ is leſs than the thirteenth term $\frac{Mb^{12}}{12a^{11}}$ in the proportion of 10 to 13, and that every following term is leſs than the term that immediately

diately preceeds it in the increaſing proportions of 10 to 14, 10 to 15, 10 to 16, and 10 to every following higher number whatſoever. So that from the eleventh term $\frac{Kb^{10}}{10a^9}$ the foregoing ſeries will be a converging ſeries. And the ſame convergency muſt take place, (though farther from the beginning of the ſeries,) if we ſhould ſuppoſe the intereſt b to exceed the original ſum a in the proportion of ten thouſand to one, or any other and greater proportion, how great ſoever, inſtead of exceeding it only in the proportion of 10 to 1. We may therefore conclude that the ſaid ſeries

$$a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2\ 3.4.5.a^4} + \text{&c. } \textit{ad infinitum}$$

will always exhibit the true value of the aforeſaid limit of the quantity to which the original ſum a may be made to increaſe by compound intereſt, in the ſame time as at ſimple intereſt it increaſes to $a + b$, whatever be the proportion of the intereſt b to the ſaid original ſum a.

But it is now time to return from this digreſſion, concerning the increaſe of money at compound intereſt, to the conſideration of the values of annuities depending upon lives, which is the principal ſubject of this tract.

[*End of the inquiry concerning the increaſe of a ſum of money by means of compound intereſt.*]

Concerning remote life-annuities, that are to commence at the diſtance of 30 *years.*

CCXXXIX. Now the next thing I propoſe to enter upon, concerning this ſubject of Life-annuities, is the computation of the values of certain remote life-annuities, which ſeem to deſerve particular conſideration: I mean life-annuities that are to commence at the diſtance of 30 years, or whereof the firſt payments are to be made at the end of 31 years;—I ſay, at the end of 31 years, and not at the end of 30 years and a half, becauſe I ſhall now return to the ſuppoſition that has been made throughout all the former part of this book, as far as Art. 200, "that the payments of annuities are to be made only at the end of every year," and ſhall not trouble either my reader or myſelf any more with the perplexing, but not very important, conſideration of the difference between the values of annuities paid yearly, and annuities paid half-yearly. Now theſe particular remote life-annuities, which are to take place at the diſtance of 30 years, ſeem to me to be more intereſting than

than any others (as, for inſtance, than life-annuities that are to commence at the diſtance of 20, or 25, or 35, or 40, years,) becauſe I conceive that it would be a very uſeful and convenient meaſure, both for the Publick, and the individuals whom it would concern, if the Parliament were to eſtabliſh ſuch annuities, which the people ſhould be at liberty to purchaſe at their full and proper values according to the ſeveral ages of the purchaſers. For, as the Parliament has, within theſe few years paſt, thought fit to eſtabliſh annuities for a term of 30 years certain, it ſeems reaſonable to ſuppoſe that it would be a great ſatisfaction to the younger part of the proprietors of thoſe annuities to be able, for a moderate ſum of money, (ſuch as about two years annuity,) to purchaſe an additional intereſt in them for their own lives, and thereby to rid themſelves of the uneaſy apprehenſion, that, without living to an uncommonly great old age, they might poſſibly outlive the income that ſupports them. If this objection to theſe annuities for 30 years were removed, by eſtabliſhing ſuch additional life-annuities as have been juſt now mentioned, it would probably induce more people to become purchaſers of them, and conſequently would tend to enhance their value, and enable the Government to get a better price for them. And thus both the Government and the individuals who ſhould purchaſe theſe 30 years annuities, or any other annuities for the ſame term that may be hereafter created, would find their account in the eſtabliſhment of the ſaid additional remote life-annuities.

It would probably be an uſeful publick meaſure for the Parliament to eſtabliſh ſuch remote life-annuities.

Nor would the eſtabliſhment of ſuch remote life-annuities retard, in any *conſiderable* degree, the diminution of the national debt ariſing from the expiration of the annuities for 30 years; becauſe it is almoſt certain that much the greater part of the proprietors of thoſe annuities will be dead before the expiration of the ſaid term of 30 years; and of the few that will live beyond it, and who conſequently might, if they had become purchaſers of theſe additional life-annuities, continue to enjoy their reſpective annuities beyond it, much the greater part will die in the courſe of the next ten or fifteen years: and it muſt alſo be remembered that they will have paid to the Government a full and fair price for the ſaid prolongation of their ſaid annuities during the time they do enjoy them.

For theſe reaſons I am inclined to think that the meaſure of eſtabliſhing ſuch a ſet of remote life-annuities is not liable to any objection but that of the difficulty (as it may be ſuppoſed) of diſtinctly aſcertaining their ſeveral values according to the ſeveral ages of the perſons who ſhall purchaſe them. But this difficulty is a ſmall one, and may be eaſily removed by computing a few tables of thoſe values according to the moſt received and cuſtomary rates of intereſt at which money is at this time

time (July 1781,) borrowed by the Government, and will, probably, be borrowed by them when the bleſſing of peace ſhall be reſtored to us. And with this view I have procured the four following tables of the values of ſuch remote life-annuities to be computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon ſuppoſitions that the intereſt of money is 5 per cent. $4\frac{1}{2}$ per cent. 4 per cent. and $3\frac{1}{2}$ per cent. It does not ſeem likely that Government ſhould be able to borrow money at a lower intereſt than $3\frac{1}{2}$ per cent. for many years to come, even though peace ſhould be ſpeedily re-eſtabliſhed.

Of the manner of computing the values of theſe remote life-annuities.

CCXL. The values of remote life-annuities may be either computed in the manner explained above in Problem II, Coroll. 3, Art. 36, page 33, without any reference to the values of immediate life-annuities for the lives of perſons of the ages at which the remote annuities are to take place, or they may be derived from the values of thoſe immediate life-annuities by an eaſy operation or two of multiplication and diviſion. And, as the values of thoſe immediate life-annuities have been already computed, and are contained in the foregoing tables, I conceive the latter method of obtaining the values of the ſaid remote annuities to be upon the whole the moſt convenient. This method may be explained in the manner following.

A method of deriving them from the values of immediate life-annuities contained in the foregoing tables in Art. 201, pages 221, 222, &c. —232.

CCXLI. Let $\overset{\pounds}{R}$ be the value ſought of an annuity of one pound a year, that is to commence at the end of m years, (or ſo that the firſt payment of it ſhall become due at the end of $m+1$ years,) in caſe a perſon who is now of the age of N years ſhall be then alive, and which is to continue during the whole remainder of the ſaid perſon's life. And let $\overset{\pounds}{V}$ be the value of an immediate annuity of one pound a year for the life of a perſon of the age of $N+m$ years. Let P be the number of perſons repreſented in Monſieur de Parcieux's table of the probabilities of the duration of human life, as being alive at the age of N years, and F the number of perſons repreſented there as living at the age of $N+m$ years, and F' the number living at the age of $N+m+1$ years, and F'' the number living at the age of $N+m+2$ years, and F''', F^{IV}, F^{V}, F^{VI}, F^{VII}, &c. the numbers living at the ages of $N+m+3$ years, $N+m+4$ years, $N+m+5$ years, $N+m+6$ years,

years, $N+m+7$ years, &c. respectively. Then, by Art. 86, we shall have $\overset{\pounds}{V} = \frac{\overset{\pounds}{1}}{F} \times$ the series $\frac{F^{I}}{r} + \frac{F^{II}}{r^2} + \frac{F^{III}}{r^3} + \frac{F^{IV}}{r^4} + \frac{F^{V}}{r^5} + \frac{F^{VI}}{r^6} + \frac{F^{VII}}{r^7}$ + &c. continued to the end of the table of probabilities. And, because F, F^{I}, F^{II}, F^{III}, F^{IV}, F^{V}, F^{VI}, F^{VII}, &c. in this notation are respectively equal to P^{m}, P^{m+I}, P^{m+II}, P^{m+III}, P^{m+IV}, P^{m+V}, P^{m+VI}, P^{m+VII}, &c. in the notation used in Art. 90, page 99, we shall have $\overset{\pounds}{R}$ (which, by that article, is $= \frac{\overset{\pounds}{1}}{P} \times$ the series $\frac{P^{m+I}}{r^{m+1}} + \frac{P^{m+II}}{r^{m+2}} + \frac{P^{m+III}}{r^{m+3}} + \frac{P^{m+IV}}{r^{m+4}} + \frac{P^{m+V}}{r^{m+5}} + \frac{P^{m+VI}}{r^{m+6}} + \frac{P^{m+VII}}{r^{m+7}}$ + &c. continued to the end of the table of probabilities,) $= \frac{\overset{\pounds}{1}}{P} \times$ the series $\frac{F^{I}}{r^{m+1}} + \frac{F^{II}}{r^{m+2}} + \frac{F^{III}}{r^{m+3}} + \frac{F^{IV}}{r^{m+4}} + \frac{F^{V}}{r^{m+5}} + \frac{F^{VI}}{r^{m+6}} + \frac{F^{VII}}{r^{m+7}}$ + &c. continued to the end of the table of probabilities, $= \frac{\overset{\pounds}{1}}{P} \times \frac{1}{r^m} \times$ the series $\frac{F^{I}}{r} + \frac{F^{II}}{r^2} + \frac{F^{III}}{r^3} + \frac{F^{IV}}{r^4} + \frac{F^{V}}{r^5} + \frac{F^{VI}}{r^6} + \frac{F^{VII}}{r^7}$ + &c. continued to the end of the table of probabilities, $= \frac{\overset{\pounds}{1}}{P} \times \frac{1}{r^m} \times F \times \frac{\overset{\pounds}{1}}{F} \times$ the series $\frac{F^{I}}{r} + \frac{F^{II}}{r^2} + \frac{F^{III}}{r^3} + \frac{F^{IV}}{r^4} + \frac{F^{V}}{r^5} + \frac{F^{VI}}{r^6} + \frac{F^{VII}}{r^7}$ + &c. continued to the end of the table of probabilities, $= \frac{\overset{\pounds}{1}}{P} \times \frac{1}{r^m} \times F \times \overset{\pounds}{V} = \frac{F}{P} \times \frac{1}{r^m} \times \overset{\pounds}{V}$; that is, the value of the remote annuity of one pound, that is to take place at the end of m years, in case a person that is now of the age of N years shall be then alive, and which is to continue during the whole remainder of the said person's life, is equal

to the product that arises by multiplying $\overset{£}{V}$, (or the value of an immediate annuity of one pound a year for the life of a person of the age of $N+m$ years,) first into the fraction $\frac{1}{r^m}$, and then into the fraction $\frac{F}{P}$. Q E F.

CCXLII. Thus, for example, if it is required to find the value of a remote annuity of one pound a year for the life of a person of the age of 10 years, to commence at the distance of 30 years, or when the said annuitant shall be 40 years of age, or so that the first payment of it shall become due at the end of 31 years, or when the said annuitant shall be 41 years old, the interest of money being $3\frac{1}{2}$ per cent. we must proceed in the manner following.

In the first place we must look out in the foregoing tables, (Table XV, page 224,) the value, $\overset{£}{V}$, of an immediate annuity of one pound a year for the life of a person of the age of ($N+m$, or $10+30$, or) 40 years; which we shall find to be $\overset{£}{16.104,542}$.

Secondly, since the interest of money is supposed to be $3\frac{1}{2}$ per cent. we shall have $r = 1.035$, and $\frac{1}{r} = \frac{1}{1.035} = 0.966,183$, and $\frac{1}{r^m} = \frac{1}{r^{30}} = 0.356,278$; as appears by Mr. Smart's second table of compound interest, pages 60 and 61.

And, lastly, it appears from Monsieur de Parcieux's table of the probabilities of the duration of human life, that P, or the number of persons living at the age of (N, or) 10 years, out of an original number of 1000 living at the age of 3 years, is 880, and that F, or the number of persons living at the age of ($N+m$, or $10+30$, or) 40 years, out of the same original number, is 657.

Therefore $\frac{F}{P} \times \frac{1}{r^m} \times \overset{£}{V}$, or $\frac{F}{P} \times \frac{1}{r^{30}} \times \overset{£}{V}$, is $= \frac{657}{880} \times 0.356,278 \times \overset{£}{16.104,542}$ $\left(= \frac{234.074,646}{880} \times \overset{£}{16.104,542} = 0.265,993 \times \overset{£}{16.104,542}\right)$

$\times$ £16.104,542) = £4.283,695; or the preſent value of a remote annuity of one pound a year for the life of a perſon of the age of 10 years, that is to commence at the end of 30 years, or when the annuitant ſhall be 40 years of age, or ſo that the firſt payment of it ſhall become due at the end of 31 years, or when the ſaid annuitant ſhall be 41 years old, upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. is £4.283,695, or 4*l.* 5*s.* 8*d.* or ſomewhat more than four years and a quarter's annuity. QE I.

And in this manner all the values of remote life-annuities contained in the four following tables have been computed.

TABLE XXIV.

Containing the values of a remote annuity of one pound a year for the lives of perſons of the ſeveral ages of 3 *years,* 4 *years,* 5 *years, and every following number of years up to* 63 *years inclusively, that is to commence only at the end of* 30 *years, or whereof the firſt payment is to be received at the end of* 31 *years, after the time of purchaſing it; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is* $3\frac{1}{2}$ *per cent.*

Years of Age.	*Values of a remote life-annuity of one pound a year, that is to commence at the end of* 30 *years.*	*Years of Age.*	*Values of a remote life-annuity of one pound a year, that is to commence at the end of* 30 *years.*
	£		£
3	4.444,037	16	3.748,438
4	4.483,991	17	3.653,162
5	4.487,811	18	3.555,256
6	4.471,977	19	3.455,031
7	4.440,391	20	3.352,405
8	4.397,007	21	3.251,937
9	4.346,451	22	3.149,313
10	4.283,695	23	3.044,957
11	4.208,738	24	2.938,658
12	4.121,688	25	2.830,830
13	4.032,235	26	2.721,424
14	3.940,261	27	2.610,440
15	3.845,704	28	2.498,339

Years of Age.	*Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.*	*Years of Age.*	*Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.*
	£		£
29	2.385,100	47	0.453,064
30	2.270,749	48	0.383,586
31	2.155,438	49	0.320,944
32	2.038,874	50	0.264,962
33	1.921,611	51	0.216,020
34	1.803,961	52	0.173,893
35	1.685,836	53	0.137,512
36	1.567,839	54	0.106,103
37	1.450,584	55	0.079,874
38	1.332,772	56	0.058,249
39	1.217,431	57	0.041,158
40	1.105,337	58	0.027,702
41	0.996,861	59	0.017,798
42	0.892,829	60	0.010,135
43	0.793,643	61	0.005,251
44	0.699,717	62	0.002,352
45	0.611,500	63	0.000,813
46	0.528,105	64	0.000,000

TABLE

TABLE XXV.

Containing the values of a remote annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 63 years inclusively, that is to commence only at the end of 30 years, or whereof the first payment is to be received at the end of 31 years, after the time of purchasing it: computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 4 per cent.

Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.
	£		£		£
3	3.587,85	24	2.427,34	45	0.517,24
4	3.623,63	25	2.341,00	46	0.447,80
5	3.630,32	26	2.253,22	47	0.383,98
6	3.621,19	27	2.163,84	48	0.325,40
7	3.599,31	28	2.073,38	49	0.272,51
8	3.567,87	29	1.981,77	50	0.225,18
9	3.530,60	30	1.889,02	51	0.183,76
10	3.483,36	31	1.795,12	52	0.148,06
11	3.426,10	32	1,700,08	53	0.117,20
12	3.358,91	33	1.604,31	54	0.090,57
13	3.289,62	34	1.507,87	55	0.068,20
14	3.218,16	35	1.410,77	56	0.049,79
15	3.144,42	36	1.313,53	57	0.035,21
16	3.068,30	37	1.216,66	58	0.023,72
17	2.993,65	38	1.119,08	59	0.014,98
18	2.916,67	39	1.023,35	60	0.008,69
19	2.837,63	40	0.930,14	61	0.004,51
20	2.756,45	41	0.839,74	62	0.002,009
21	2.676,12	42	0.752,89	63	0.00 ,700
22	2.595,36	43	0.669,94	64	0.000,000
23	2.512,24	44	0.591,26		

TABLE XXVI.

Containing the values of a remote annuity of one pound a year for the lives of persons of the several ages of 3 *years,* 4 *years,* 5 *years, and every following number of years up to* 63 *years, inclusively, that is to commence only at the end of* 30 *years, or whereof the first payment is to be received at the end of* 31 *years, after the time of purchasing it: computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is* $4\frac{1}{2}$ *per cent.*

Years of Age.	*Values of a remote life-annuity of one pound a year, that is to commence at the end of* 30 *years.*	*Years of Age.*	*Values of a remote life-annuity of one pound a year, that is to commence at the end of* 30 *years.*	*Years of Age.*	*Values of a remote life-annuity of one pound a year, that is to commence at the end of* 30 *years.*
	£		£		£
3	2.907,57	24	2.009,15	45	0.437,99
4	2.939,17	25	1.939,82	46	0.379,55
5	2.947,25	26	1.869,12	47	0.325,76
6	2.942,54	27	1.797,02	48	0.276,32
7	2.927,52	28	1.723,83	49	0.231,62
8	2.904,73	29	1.649,54	50	0.191,56
9	2.877,17	30	1.574,14	51	0.156,46
10	2.841,47	31	1.497,61	52	0.126,19
11	2.797,56	32	1.419,93	53	0.099,98
12	2.745,46	33	1.341,48	54	0.077,33
13	2.691,56	34	1.262,26	55	0.058,29
14	2.635,79	35	1.182,30	56	0.042,60
15	2.578,05	36	1.102,01	57	0.030,15
16	2.518,24	37	1.021,84	58	0.020,34
17	2.459,53	38	0.940,89	59	0.012,86
18	2.398,78	39	0.861,31	60	0.007,47
19	2.336,23	40	0.783,67	61	0.003,88
20	2.271,78	41	0.708,23	62	0.001,73
21	2.208,42	42	0.635,62	63	0.000,604
22	2.143,56	43	0.566,16	64	0.000,000
23	2.077,16	44	0.500,17		

TABLE XXVII.

Containing the values of a remote annuity of one pound a year for the lives of perſons of the ſeveral ages of 3 years, 4 years, 5 years, and every following number of years up to 63 years, inclusively, that is to commence at the end of 30 years, or whereof the firſt payment is to be received at the end of 31 years, after the time of purchaſing it: computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 5 per cent.

Years of Age.	*Values of a remote life annuity of one pound a year, that is to commence at the end of 30 years.*	*Years of Age.*	*Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.*	*Years of Age.*	*Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.*
	£		£		£
3	2.364,81	24	1.666,37	45	0.371,26
4	2.393,50	25	1.610,16	46	0.322,03
5	2.400,38	26	1.553,46	47	0.276,64
6	2.399,10	27	1.495,12	48	0.234,87
7	2.388,90	28	1.435,76	49	0.197,04
8	2.372,38	29	1.375,37	50	0.163,10
9	2.351,96	30	1.313,93	51	0.133,33
10	2.324,88	31	1.251,42	52	0.107,63
11	2.291,05	32	1.187,80	53	0.085,35
12	2.250,48	33	1.123,40	54	0.066,08
13	2.208,88	34	1.058,21	55	0.049,85
14	2.164,67	35	0.992,23	56	0.036,46
15	2.119,27	36	0.925,83	57	0.025,83
16	2.072,10	37	0.859,37	58	0.017,43
17	2.025,74	38	0.792,10	59	0.011,03
18	1.977,62	39	0.725,83	60	0.006,410
19	1.927,94	40	0.661,07	61	0.003,326
20	1.876,60	41	0.598,01	62	0.001,488
21	1.826,07	42	0.536,84	63	0 000,520
22	1.774,23	43	0.478,99	64	0.000,000
23	1.721,01	44	0.423,57		

OBSER-

OBSERVATIONS

ON THE

NATIONAL DEBT,

AND

The moſt likely Methods of paying off a Part of it.

ART. CCXLIII.

WHEN the bleſſing of peace ſhall be reſtored to us (which, in our preſent friendleſs and declining condition, we ought ſurely to wiſh for upon almoſt any terms;) it is to be hoped that our rulers will ſet themſelves ſeriouſly to work to reduce to a more moderate quantity the immenſe load of debt under which the nation now labours, and which will then be at leaſt two hundred and twenty millions of pounds ſterling, even if a peace ſhould be made as ſoon as poſſible. For at this preſent time, November, 1781, it amounts to more than a hundred and ninety-eight millions, two hundred thouſand pounds; or at leaſt it will do ſo on the firſt of next January, 1782; as will appear from an accurate ſtatement of it which has been printed in the Publick Advertiſer of the ſecond of October laſt, and which the reader may ſee in the note below.* And to

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* An account of the taxes laid on ſince the beginning of the American War.

	Computed produce per annum.	
1776.	£	
Stamps on deeds,	30,000	
Ditto on news-papers,	18,000	
Ditto on cards,	6,000	
Additional duty on coaches, &c.	19,000	
		73,000
1777.		
Tax on ſervants,	105,000	
Stamps,	55,000	
Additional duty on glaſs,	45,000	
Duty on ſales by auction,	37,000	
		242,000

Tax

to this ſum we muſt add at leaſt 25 millions more for the expence attending the concluſion of the war, before every thing ſhall be compleatly ſettled on a peace eſtabliſhment: ſo that at the final re-eſtabliſhment of peace the whole debt will be at leaſt two hundred and twenty millions. Now, when the important buſineſs of reducing this enormous debt to a leſs ſum ſhall be undertaken, it ſeems probable that one, or more, of the following methods will be adopted for that purpoſe.

The

1778.		
Tax upon houſe rents,	£.264,000	
Additional duty on wines,	72,000	
		336,000
1779.		
A tax upon taxes, viz. an additional ſurcharge of 5 per cent. on cuſtoms and exciſe,	314,000	
A tax upon poſt horſes,	164,000	
		478,000
1780.		
An additional tax upon malt,	310,000	
Additional duty on Britiſh low wines,	20,617	
Ditto on Britiſh ſpirits,	34,557	
Ditto on brandy,	35,310	
Ditto on rum,	70,958	
Second additional duty on wines,	72,000	
Additional duty on coals exported,	12,899	
Additional 5 per cent. on the above laid taxes,	46,193	
Additional duty upon ſalt,	69,000	
Additional ſtamp duties,	21,000	
Duty on licences to ſell tea,	9,082	
		701,616
1781.		
Five per cent. on exciſe, except malt, ſoap, candles, and hides,	150,000	
Diſcount of the cuſtoms,	167,000	
Tobacco one penny three farthings per pound,	61,000	
Sugar halfpenny per pound,	326,000	
Since laid,		704,000
Duty on paper,	100,000	
Ditto on almanacks,	10,000	
		110,000
		2,644,616

The exact national debt up to July 5, 1781, is £.177,206,000.
The annual intereſt raiſed on the publick is £.6,889,000.

So far the funded debt, and the taxes laid, in order to diſcharge the intereſt to the publick creditors. The debt unfunded may be computed as here under:

The firſt method of employing a given ſum of money every year in the reduction of the national debt.

The firſt method is to employ the annual Surplus of the Publick Revenue, which can be ſpared for this good work, in repaying to ſome of the national creditors a part of the capital which is due to them, and alſo to employ the annual ſavings of the intereſt upon the capital ſo diſcharged (which, it is evident, will increaſe every year,) in the ſame manner, that is, in diſcharging, or repaying, ſome more of the ſaid capital due to the publick creditors. Thus, for example, if the ſum that can be ſpared out of the publick revenue for this purpoſe ſhould be a million of pounds ſterling a year, and ſome of the publick funds, as, for example, the 4 per cent. annuities, ſhould riſe to par, or above par, ſo that 100 pounds in thoſe annuities ſhall ſell for a hundred pounds, or more, in hard money, the ſaid ſurplus million of the publick revenue may be employed

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Navy debt on the 1ſt of January, 1782, about —	£.9,000,000	
Army extraordinaries,	3,000,000	
Vote of credit of laſt ſeſſion,	1,000,000	
Ordnance debt,	1,000,000	
Money to be voted for navy extras,	1,000,000	
Exchequer bills in circulation, about	4,000,000	
Borrowed from the Bank of England,	2,000,000	
		21,000,000
Suppoſe, when this ſum comes to be funded, that the loan or bargain with the publick may, as it has for the two or three laſt years, be negociated at 5 and a half per cent. the annual intereſt to be paid on 21 millions will be	1,155,000	

RECAPITULATION.

Principal funded on the 5th of July, 1781, — —	177,206,000	
Principal which will remain unfunded on the 1ſt of January, 1782,	21,000,000	
Total of the national debt on the ſaid laſt mentioned day,		198,206,000
Intereſt paid, for which proviſions have been made by taxes, 5th July, 1781,	6,889,000	
Intereſt to be paid for the debts not yet funded, which will ſtand due on the 1ſt of January, 1782, — —	1,155,000	
		8,044,000

So that on the 1ſt of January, 1782, the national debt, funded and unfunded, will amount to *one hundred ninety-eight millions*, and a conſiderable fraction, and the intereſt to *eight millions*, which is nearly double to what was paid by the people, in taxes, previous to the breaking out of the preſent war, the annual intereſt, on the 1ſt of January, 1776, being in or about *four millions three hundred thouſand pounds*.

in

in paying off a million of the capital of the ſaid annuities: and thus the national debt will be diminiſhed by the ſum of one million by this firſt payment. But by this firſt diminution of the debt it is evident the publick will have leſs intereſt to pay to the publick creditors at the beginning of the next year than it had before by the intereſt of the million that has been ſo paid off, that is, (if the annuities are 4 per cents,) by 40,000 pounds. Therefore this ſum of 40,000 pounds, which, before the diſcharge of the ſaid million, uſed to be paid away every year to the publick creditors as the intereſt of it, may then be employed, together with the annual million of this year originally deſtined to this purpoſe, in diſcharging, or repaying, ſome more of the capital of the ſaid 4 per cent. annuities. And in like manner there will be in every following year a greater and greater ſum of money, by means of theſe ſavings of the intereſt of the diſcharged debt, that may be ſo applied. And in the courſe of fifty, or ſixty, years the amount of theſe annual payments to the publick creditors, or the portion of the national debt extinguiſhed by them, will be exceeding great, being evidently equal to the amount of an annual ſum of one million of pounds for the ſame period of time, improved at compound intereſt. In fifty years time it would be more than a hundred and fifty-two millions; and in ſixty years it would be nearly two hundred and thirty-eight millions: as appears from Mr. Smart's third table of compound intereſt, pages 70 and 72. And conſequently, as the annual intereſt of a hundred and fifty-two millions, at 4 per cent. is 152 times 40,000 pounds, or 6,080,000 pounds, and the annual intereſt of two hundred and thirty-eight millions, at the ſame rate, is 238 times 40,000 pounds, or 9,520,000 pounds; the quantity of the annual intereſt of the national debt, or of the perpetual annuities now due to the publick creditors, that would be redeemed by the operation of this annual ſurplus million in the courſe of fifty years would be £.6,080,000 *per annum*; and the quantity of thoſe perpetual annuities that would be redeemed by the operation of this annual million in the courſe of ſixty years would be £.9,520,00 *per annum*; which is more than the whole intereſt of the preſent national debt, though increaſed to ſo alarming a quantity. So great would be the effect of the faithful application of only the moderate ſum of one million of pounds *per annum* for the ſpace of only 60 years to this important purpoſe!

CCLXV. This firſt method of employing the ſuppoſed ſurplus part of the publick revenue is only fit to be applied to ſuch of the public funds as ſhall riſe to *par* or above *par*, as I have ſuppoſed may be the caſe with the 4 per cent. annuities. But the greateſt part of the national debt conſiſts of 3 per cent. annuities; which will not, in all probability, riſe at the peace to any thing like their *par*, or nominal value, they being at this time, November, 1781, at the very low price of 56 per cent. But, perhaps,

A remark on the ſaid method.

perhaps, they may riſe, upon a peace, to £75 per cent. Now, if they ſhould then riſe to this value, or to any other value leſs than a hundred pounds per cent. it is not to be expected that the Government will redeem them at the full price of 100 pounds per cent. or pay the publick creditors a real hundred pounds for every nominal hundred pounds of their ſtock in theſe annuities; becauſe the nation is not under any obligation to redeem them at that, or any other, price, provided it continues to pay the intereſt of them with punctuality. The Government will therefore chuſe to continue to pay the intereſt due upon theſe funds, unleſs the owners of them will conſent to be paid off at a leſs price than the *par*, or nominal value, of their ſtocks, and will accept a price equal to, or a little exceeding, the price at which thoſe ſtocks ſhall then be found to ſell at the publick market. And many of the owners of theſe ſtocks will, doubtleſs, be glad to accept of ſuch a price for them. This brings me to the conſideration of the ſecond method in which it is probable that a given annual ſum of money, that can be ſpared from the publick revenue for that purpoſe, may be employed in diminiſhing the national debt, and redeeming the perpetual annuities that are now due to the publick creditors as the intereſt of it.

A ſecond method of employing a given ſum of money every year in the reduction of the national debt.

CCXLVI. This ſecond method of employing a given annual ſum of money in the reduction, or diminution, of the national debt conſiſts in buying up with it, from the owners of ſome of the publick ſtocks that ſhall be ſold at the publick market for leſs than their *par*, or nominal value, a part of their ſaid ſtocks, with the full and free conſent of the ſaid owners, at the market-price, or a price a little exceeding the market-price, of the ſaid ſtocks; and in employing the intereſt that would thereby be every year redeemed to the nation, in the ſame manner.

Thus, for example, if we ſuppoſe, as before, that the annual ſurplus of the publick revenue that can be ſpared for this purpoſe, is a million of pounds ſterling, and that the value of the 3 per cent. annuities ſhould riſe, after the concluſion of a peace, to 74 or 75 pounds per cent. the ſaid million of pounds ſterling *per annum* may be employed in buying up a part of the capital of the ſaid 3 per cent. annuities, with the conſent of the reſpective owners of it, at the price of £75 per cent. and the intereſt that would thereby be redeemed to the nation at the end of every year may alſo be employed in the ſame manner.

CCXLVII. If

CCXLVII. If a million of pounds ſterling were to be ſo employed, it is evident that 750,000 pounds of it would buy up, or extinguiſh, a million of the ſaid capital of the 3 per cent. annuities, and conſequently that the whole million ſo employed would extinguiſh a million and the third part of a million, or £1,333,333, of the ſaid capital; of which capital of £1,333,333, the annual intereſt is 40,000 pounds. Of the effect of this ſecond method of diminiſhing the national debt.

This would be the effect of employing a ſingle million of pounds ſterling in this manner, or of the firſt year's execution of the ſuppoſed project of employing a million of pounds in this way for ſeveral years together. But at the end of the ſaid firſt year it is evident that the Government would have the 40,000 pounds, which uſed before to pay the intereſt of the capital of £1,333,333 ſo extinguiſhed, to employ in the ſame manner, over and above the annual million belonging to this year, which was originally deſtined to this purpoſe; ſo that the quantity of the ſaid capital of the 3 per cent. annuities that would be extinguiſhed by the ſecond operation of this method would be ſo much as could be purchaſed at the price of £75 per cent. by the ſum of 1,040,000 pounds, that is, $\left(\frac{100}{75} \times £1.040{,}000,\text{ or } \frac{4}{3} \times £1{,}040{,}000,\text{ or } \frac{£\ 4{,}160{,}000}{3},\text{ or}\right)$ £1.386,666; of which ſum the annual intereſt is 41,600 pounds.

Therefore at the beginning of the third year the Government would have the three following ſums to employ in this manner; to wit, firſt, the original million belonging to the ſaid third year; 2dly, the 40,000 pounds of the intereſt of the national debt which was redeemed by the firſt operation; and, 3dly, the 41,600 pounds of the intereſt of the ſame debt which was redeemed by the ſecond operation; which ſums together amount to 1,081,600 pounds. And this ſum would be ſufficient to extinguiſh $\left(\frac{4}{3} \times £1{,}081{,}600,\text{ or}\right)$ £1,442,133 of the ſaid capital of the 3 per cent. annuities; the intereſt of which capital is 43,264 pounds *per annum*.

Therefore the ſums that the Government will have to employ in this manner at the beginnings of the firſt, ſecond, third, and fourth years during the time that this method ſhall be purſued, will be as follows; to wit,

1ſt, £1,000,000,
2dly, £1,000,000 + £40,000, or £1,040,000,
3dly, £1,000,000 + £40,000 + £41,600, or £1,081,600, and
4thly, £1,000,000 + £40,000 + £41,600 + £43,264, or £1,124,864.

Theſe

Theſe ſums, £1,000,000, £1,040,000, £1,081,600, and £1,124,864, are in continual geometrical proportion, the common ratio of the terms being that of 100 to 104; and the ſecond, third and fourth terms are the values of a million of pounds at the ends of one, two, and three years, when improved at compound intereſt at the rate of 4 per cent. And in like manner it will be found that the ſums which the Government will have to employ in the ſame manner at the beginnings of the fifth, ſixth, ſeventh, and other following years during which this method ſhall be purſued, will the values of a million of pounds at the ends of four, five, ſix, and every following number of years, when the ſaid million is improved during thoſe years at compound intereſt at the rate of 4 per cent. Therefore the total amount of all the annual ſums which the Government would have to employ in this manner during any given number of years, in conſequence of having ſet apart an original annual ſum of a million of pounds for this purpoſe, would be equal to the total amount of an annual ſum of a million of pounds at the end of the ſame number of years, if the ſaid annual ſum were, as faſt as it was received by the perſon to whom it was due, to be immediately lent out and improved at compound intereſt at the rate of 4 per cent. But we have already obſerved that it appears from Mr. Smart's tables of compound intereſt, pages 70 and 72, that the total amount of an annual payment of a million of pounds, ſo improved at compound intereſt at the rate of 4 per cent. for a period of fifty years, is more than a hundred and fifty-two millions; and that the amount of it for a period of ſixty years is very nearly two hundred and thirty-eight millions. Therefore the total amount of all the ſums which the Government would have to employ in thus buying up the ſtock of the 3 per cent. annuities at the price of £75 per cent. if an annual ſum of a million of pounds ſterling *per annum*, together with the intereſt that would be continually redeemed by it, were to be faithfully applied to this purpoſe for a period of fifty years, would be more than a hundred and fifty-two millions; and the amount of all the ſums which the Government would have to employ in this manner in the courſe of ſixty years would be very nearly two hundred and thirty-eight millions. And, as every million of this money would be ſufficient to extinguiſh £1,333,333 of the capital of the ſaid 3 per cent. annuities, it follows that the amount of all the ſaid annual ſums that would, according to this ſecond method of employing the ſaid annual ſurplus million, be applied to the extinguiſhment of the capital of the ſaid 3 per cent. annuities, in the courſe of fifty years, would be ſufficient to have extinguiſhed in that time more than 152 times £1,333,333, or than £202,666,666 of that capital, and conſequently to have redeemed for the nation more than 152 times 40,000 pounds *per annum*, or 6,080,000 pounds *per annum* of the perpetual annuities which pay the intereſt of that capital: and the amount of the ſaid annual ſums, that might be ſo employed, in the courſe of ſixty years would be ſufficient to have extinguiſhed,

very

very nearly, 238 times £1,333,333, or, very nearly, £317,33,254, of the ſaid capital of the 3 per cent. annuities, and conſequently to have redeemed for the nation very nearly 238 times 40,000 pounds *per annum*, or very nearly 9,520,000 pounds *per annum* of the perpetual annuities which pay the intereſt of the ſaid capital: which latter ſum, £9,520,000, is greater than the whole intereſt of the preſent national debt, though increaſed of late to ſo alarming a quantity.

The two foregoing methods of diminiſhing the national debt would be equally efficacious and beneficial to the nation.

CCXLVIII. It is evident that the two foregoing methods of applying an annual ſum of a million of pounds, together with the intereſt continually redeemed by it, for a given number of years, to the diminution of the national debt, to wit, by paying off at *par* thoſe ſtocks which carry an intereſt of 4 per cent. when the price of thoſe ſtocks ſhall riſe to *par*, and by buying up at the price of £75 per cent. (with the conſent of the owners of them,) thoſe ſtocks which ſhall carry an intereſt of 3 per cent. would be equally beneficial to the publick, or that the quantity of the intereſt of the national debt that would be redeemed by either of them in a given time would be exactly the ſame as the quantity of the ſaid intereſt that would be redeemed by the other in the ſame time.

Expediency of appropriating by act of parliament, in the ſtricteſt manner poſſible, the ſum of one million of pounds ſterling per annum *out of the ſinking fund, during a term of fifty, or ſixty, years, to the diſcharge of the national debt.*

CCXLIX. The foregoing very great operation of only one million of pounds ſterling a year, with the intereſt continually redeemed by it, in the courſe of ſo moderate a period of time as fifty, or ſixty, years, when ſtrictly applied to this purpoſe of diminiſhing the national debt without any interruption, in either of the two foregoing methods, ought, one would think, to induce the Parliament to appropriate that ſum out of the ſinking fund to this important purpoſe in the ſtricteſt manner that can be deviſed, for the ſpace of fifty, or ſixty, years, and to forbear to interrupt its operation during that period upon any account, or occaſion, whatſoever, however urgent. And it ſeems the more reaſonable to expect that ſuch a meaſure will ſoon be adopted, becauſe the ſinking fund has of late years produced no leſs a ſum than three millions of pounds ſterling *per annum:* and our miniſters of ſtate, as well as the owners of property in the publick funds, ought to recollect that the whole of the ſaid fund, as its name imports, was once appropriated by Parliament to this very purpoſe, of *ſinking*, or diminiſhing, the national debt, in the manner now recommended for one third of it.

A third

A third method of employing a given annual ſum of money in the reduction of the national debt.

CCL. A third method of employing the ſaid annual ſurplus million, or other part of the publick revenue that can be ſpared for this purpoſe, to the diminution of the national debt, would be to convert ſome of the *perpetual* annuities now due to the publick creditors as the intereſt of the ſaid debt, into *temporary* annuities, or annuities that ſhould continue only for a limited number of years, as, for example, for 100 years, or 80 years, or 60 years, or 30 years, but which ſhould be greater than the perpetual annuities in the room of which they ſhould be ſubſtituted, in a certain juſt and reaſonable proportion that was adapted to the change made in their duration: the ſaid change being made with the conſent of the proprietors of the perpetual annuities that ſhould be ſo converted. For without this conſent a meaſure of this kind would be a breach of the national faith: but with ſuch conſent of the proprietors concerned it would be a perfectly juſt meaſure, and might, I imagine, be ſometimes an expedient one. And, if it were adopted, the ſaid annual ſurplus million, or other part of the publick revenue that could be ſpared for this purpoſe, might be employed in paying the additions that would be neceſſary to be made to the ſaid perpetual annuities in compenſation for the abridgement of their duration.

A remark on the ſaid third method of diminiſhing the national debt.

CCLI. By this third method of proceeding the national debt would not be diminiſhed gradually, or every year a little, as in the two former methods; but, on the contrary, the intereſt annually paid to the publick creditors would be ſomewhat increaſed during the continuance of the new temporary annuities that had been ſubſtituted in the room of ſome of the former perpetual ones. But at the end of that time the whole of the ſaid temporary annuities would ceaſe at once, and conſequently the perpetual annuities, in lieu of which they had been ſubſtituted, would be redeemed to the nation.

The ſaid third method would be equally efficacious with either of the two former in diminiſhing the national debt.

CCLII. This third method of diminiſhing the national debt will have preciſely the ſame effect at the end of the term during which the new temporary annuities which are ſubſtituted in lieu of the former perpetual annuities, are to continue, as either of the two former methods; that is, the quantity of the intereſt of the national debt which will be redeemed by this third method at the end of the ſaid limited number of years, by employing a million of pounds ſterling, or any other given ſum of money, every year in this manner, will be the ſame as the quantity of the

the ſaid intereſt that would be redeemed in the ſame courſe of time by employing the ſame million of pounds, or other annual ſum, together with the ſavings of intereſt annually produced by it, in either of the two former methods.

CCLIII. Thus, for example, if the intereſt of money is ſuppoſed, as before, to be 4 per cent. and the term during which the new temporary annuities, (which are ſubſtituted in lieu of the former perpetual annuities,) are to continue, is ſixty years, we ſhall find that the quantity of the intereſt of the national debt, or of the perpetual annuities, that would be redeemed by the application of a million of pounds every year during the ſaid ſixty years according to this third method, or by employing the ſaid million in paying the additions which would have been made to the perpetual annuities which had been converted into temporary ones, would be exactly the ſame as the quantity of the intereſt of the national debt, or of the perpetual annuities, that would be redeemed in the ſame period of ſixty years by employing the ſame ſum of a million every year, together with the annual ſavings of intereſt produced by it, in either of the two former methods; which ſum we have ſeen above to be £9,520,000 *per annum*. This may be ſhewn in the manner following. An example of its effect in a period of ſixty years.

When the intereſt of money is 4 per cent. or a perpetual annuity of four pounds a year is worth a hundred pounds, and conſequently a perpetual annuity of one pound a year is worth twenty-five pounds, a temporary annuity of one pound a year, that is to continue only for ſixty years, will be worth only £22.6234; as appears from Mr. Smart's fourth table of compound intereſt, page 80. Therefore a temporary annuity for 60 years that ſhall be worth 25 pounds, or ſhall be equal in value to a perpetual annuity of one pound a year, muſt be greater than a like temporary annuity of one pound a year in the proportion of £25 to £22.6234, and therefore will be $= £1 \times \frac{25}{22.6234}$, or £1.1050, or ſomething leſs than 1*l.* 2*s.* 1*d.*$\frac{1}{4}$ a year. And conſequently, if the Government were to convert ſome of the perpetual annuities, now due to the publick creditors, into temporary annuities for 60 years, (with the conſent of the proprietors of them,) and were to allow to each proprietor the ſum of 1*l.* 2*s.* 1*d.*$\frac{1}{4}$ *per annum* during the ſaid term of 60 years, inſtead of every ſum of 1*l.* *per annum* that had formerly been due to him for ever, ſuch a bargain ought to be conſidered as fair and equal on both ſides.

Now, ſince for every perpetual annuity of one pound a year that ſhould be thus changed into a temporary annuity for 60 years, it would be neceſſary

ceſſary to allow the proprietor of it an annuity of 1*l.* 2*s.* 1*d.*$\frac{1}{2}$, or £1.1050, a year during the ſaid term of 60 years: it follows that for every million of pounds *per annum* of the ſaid perpetual annuities that ſhould be thus changed into temporary annuities for a term of 60 years, it would be neceſſary to allow to the proprietors of the ſaid million of perpetual annuities a million times £1.1050 *per annum*, or £1,105,000, or a million and a hundred and five thouſand pounds *per annum*, to be continued during the ſaid term of 60 years; or, in other words, an annual additional ſum of £105,000 would be required to enable the Government to convert £1000,000 *per annum* of the perpetual annuities into temporary annuities for 60 years. Therefore an annual additional ſum of £1000,000 would be ſufficient to enable the Government to convert into annuities for 60 years a quantity of the perpetual annuities that is as much greater than £1,000,000 a year as £1,000,000 is greater than £105,000; which greater quantity is (£1,000,000 × $\frac{1,000,000}{105,000}$, or £1000,000 × $\frac{1000}{105}$, or £1,000,000 × 9.52. or) £9,520,000. Therefore the quantity of the intereſt of the national debt, or of the perpetual annuities, that would be redeemed to the nation in the courſe of 60 years by employing the ſum of a million of pounds *per annum* during that time in this third manner would be £9,520,000 *per annum*. Q E D.

In one reſpect this third method of applying a given ſum of money every year to the diminution of the national debt ſeems to be preferable to either of the two former methods of applying it to the ſame purpoſe.

CCLIV. It appears therefore that theſe three different methods of applying a given annual ſum of money to the diminution of the national debt, or to the redemption of the publick revenue that is mortgaged for the payment of the intereſt of it, are equally efficacious, if they are purſued with equal ſteadineſs. But in this laſt reſpect, I mean the probability of their being purſued with ſteadineſs for a conſiderable length of time, they do not ſeem to be quite equal; but the third method ſeems rather to have the advantage of the former two. For, if either the firſt method of applying a given annual ſum of money to this purpoſe, to wit, "that of employing it every year in repaying, at their *par*, or nominal value, ſome of thoſe ſtocks, or branches of the national debt, which ſhall have riſen at the publick market either to, or above, their ſaid *par*, or nominal value;" or the ſecond method of applying it to the ſame purpoſe, to wit, "by buying up at the market price, or at a price very little exceeding the market price, ſome of the publick ſtocks that were under their *par*, or nominal value, with the conſent of the reſpective owners of them;" I ſay, if either of theſe methods were to be adopted, there would be reaſon to apprehend that, whenever any particular emergence ſhould ariſe that required a greater expenditure of publick money than ordinary, the miniſters of ſtate might be tempted to propoſe to the Parliament, and the Parliament

Parliament to adopt the propoſal, that the operation of diminiſhing the national debt ſhould be ſuſpended for a year, and the ſaid annual ſum that had been allotted to this purpoſe, ſhould be applied to the diſcharge of the extra-ordinary expence that was made neceſſary by the ſuppoſed emergency: becauſe by ſo doing they would avoid the odium and difficulty of raiſing the ſaid extra-ordinary money by laying freſh taxes upon the people. The experience of the nation affords ſufficient grounds for ſuch an apprehenſion. For we have had but too many inſtances of ſuch a diſpoſition both in our miniſters of ſtate and in our parliaments ever ſince the year 1733, when Sir Robert Walpole firſt began to divert the ſinking fund from its original deſtination of *ſinking*, or diminiſhing, the national debt, and employed it in defraying the current ſervices of the year: without which change in the application of it, the continual operation of that large and uſeful fund from the year 1733 to the preſent year, 1781, would have extinguiſhed almoſt all our publick debts ſoon after they had been contracted, though they have now increaſed to ſuch an enormous quantity. But, if the aforeſaid annual ſurplus million were to be applied to the diminution of the national debt in the third method above-mentioned, that is, "by paying with it the additions that ſhould have been made to ſome of the perpetual annuities now due to the publick creditors, upon a converſion of the ſaid annuities (with the conſent of the creditors to whom they were due,) into greater annuities for a limited number of years," it could never afterwards be diverted from this deſtination, and applied to any other purpoſe, without an abſolute breach of the national faith; which, I preſume, we may conſider as a kind of moral impoſſibility. For, as ſuch a meaſure would not only be exorbitantly unjuſt and cruel towards the numerous unhappy individuals who would be the immediate ſufferers by it, but likewiſe moſt ruinous to the general credit and commerce of the kingdom, and likely to produce the moſt dreadful ſcenes of internal miſery and confuſion; we may reaſonably hope that our miniſters of ſtate, and our parliaments, will always look upon it with the utmoſt horror and deteſtation, and will conſequently think themſelves bound in duty to prevent it by the moſt vigorous and extenſive exertions of their right of impoſing new taxes that may become neceſſary for that purpoſe, however diſagreeable and unpopular ſuch exertions may appear. And therefore, I think, we may conſider ſuch an event as morally impoſſible. And, if we are right in ſo conſidering it, we may conclude that the aforeſaid third method of applying any given annual ſum to the diminution of the national debt will deſerve to be preferred to both the former methods of applying it to the ſame purpoſe, becauſe of the poſſibility, and, perhaps, even probability, that thoſe former methods of applying it may occaſionally be ſuſpended and interrupted.

A fourth method of employing a given ſum of money every year in the reduction of the national debt.

CCLV. A fourth method of applying a given annual ſum of money to the diminution of the national debt, or the redemption of the intereſt of it, might be as follows. Some of the perpetual annuities now due to the publick creditors might, with the conſent of the owners of them, be converted into life-annuities for the lives of their reſpective owners, or for the lives of other perſons whom they ſhould name, and who may be called their *nominees:* which life-annuities, it is evident, muſt be greater than the correſponding perpetual annuities, in the room of which they would have been ſubſtituted, in various proportions ſuited to the ſeveral ages of the perſons for whoſe lives they would be to continue. And the ſaid annual ſum of money, that could be ſpared out of the publick revenue for the purpoſe of diminiſhing the national debt, might be employed in the payment of the additions neceſſary to be made to the ſaid perpetual annuities upon their being ſo converted into life-annuities.

This fourth method of diminiſhing the national debt, if properly purſued, would be equally effectual for that purpoſe with either of the three preceeding methods.

CCLVI. In this method of diminiſhing the national debt, it is evident, there will be continual ſavings of intereſt ariſing to the nation every year by the deaths of ſome of the life-annuitants. Now, if theſe ſavings are continually employed, for the ſame purpoſe of diminiſhing the national debt, in either the firſt or the ſecond method above-mentioned, that is to ſay, either in paying off at *par* ſome part of thoſe ſtocks which ſhall have riſen to *par*, or above *par*, or in buying up at the market price, or at a price very little greater than the market price, (with the conſent of the reſpective owners of it) ſome part of thoſe ſtocks which ſhall continue under *par*; the quantity of annual intereſt that would be redeemed to the nation by applying a given ſum of money every year in this fourth method during the life of the longeſt liver of all the ſaid life-annuitants, would be the very ſame that would be redeemed by applying the ſame ſum of money to the diminution of the national debt during the ſame length of time in either of the three preceeding methods. Of which it may not be amiſs to give an example: which may be done as follows.

An example of this third method of applying a given ſum of money every year to the diminution of the national debt.

CCLVII. It has been ſhewn above that a million of pounds ſterling *per annum*, applied conſtantly to the diminution of the national debt, or the redemption of the intereſt of it, for the ſpace of 60 years, in either of the

the three foregoing methods, will redeem an annual intereſt of 9,520,000 pounds. We will therefore now inquire what quantity of annual intereſt the ſame annual ſum of a million of pounds ſterling will redeem in the ſame courſe of time, if applied in this fourth method; making the ſame ſuppoſitions as before, to wit, that the ſtocks which carry an intereſt of 4 per cent. will riſe to *par*, and that thoſe which carry an intereſt of 3 per cent. will riſe to £75 per cent. and uſing the table of the values of life-annuities when the intereſt of money is 4 per cent. which is given above in page 225, for the valuation of the life-annuities which we are now going to conſider, and Monſieur de Parcieux's table of the probabilities of the duration of human life (upon which the ſaid table in page 225 is founded,) for the purpoſe of aſcertaining the numbers of the life-annuitants that will die off in every year of the whole term, and the conſequent ſavings of intereſt that will accrue every year to the publick by their deaths.

CCLVIII. Now, that this calculation (which will unavoidably run into conſiderable length,) may not be too tedious and intricate, we will ſuppoſe that all the perſons for whoſe lives the new annuities, that are to be ſubſtituted inſtead of the former perpetual annuities, ſhall be granted, are of the ſame age. And further,—as it is ſuppoſed that the whole operation of this fourth method is to continue during the ſpace of 60 years;——and in Monſieur de Parcieux's table of the probabilities of the duration of human life, only one perſon, out of the whole original number of 1000 perſons living at the age of 3 years, is ſuppoſed to live to the age of 94 years, and that one perſon is ſuppoſed to die before he attains the age of 95 years;——we will ſuppoſe the ſeveral perſons, for whoſe lives the ſaid new annuities ſhall be granted, to be all of the age of 35 years: for then it may be ſuppoſed that the longeſt liver of them will live almoſt to the very end of the ſaid term of 60 years during which this operation is to continue.

A computation of the quantity of the intereſt of the national debt that would be redeemed to the publick by this method of life-annuities at the end of 60 years without the help of the annual ſavings

CCLIX. Now it appears above by Table XVI, page 225, that, when the intereſt of money is 4 per cent. or the value of a perpetual annuity of four pounds a year is a hundred pounds, and conſequently the value of a perpetual annuity of one pound a year is twenty-five pounds, the value of an annuity of one pound a year for the life of a perſon of the age of 35 years is £16.084,014. Therefore an annuity for the life of a perſon of the age of 35 years that ſhall be worth 25 pounds, or ſhall be equal in value to a perpetual annuity of one pound a year, will be greater than an annuity of one pound a year for the life of the ſame perſon in the proportion of £25 to £16.084,014, and therefore will be equal

that would accrue to the publick by the deaths of ſome of the life-annuitants in the courſe of the ſaid term.

equal to £1 × $\frac{25.000,000}{16.084,014}$, or £1.554,338. This therefore is the life-annuity which each of the perſons aged 35 years for whoſe lives we ſuppoſe the ſaid life-annuities to be granted, ought to receive in lieu of the perpetual annuity of one pound a year in the room of which it is ſubſtituted. Therefore for every million of pounds *per annum* of the perpetual annuities, now due to the publick creditors, which ſhould be thus converted into life-annuities for the lives of perſons of the age of 35 years, it would be neceſſary that Government ſhould eſtabliſh life-annuities to the amount of a million times £1.554,338 *per annum*, or to the amount of £1,554,338, *per annum*; which would require an additional annual intereſt of £554,338: ſo that an additional annual intereſt of 554,338 pounds would be ſufficient to enable the Government to convert a million of pounds *per annum* of the perpetual annuities into life-annuities for the lives of perſons of the age of 35 years. Therefore an additional annual intereſt of £1000,000 will be ſufficient to enable the Government to convert into life-annuities for the lives of perſons of the age of 35 years a quantity of the perpetual annuities that is greater than £1000,000 *per annum* in the proportion of £1000,000 to £554,338, that is, a quantity of the ſaid perpetual annuities that is equal to (£1000,000 × $\frac{1000,0000}{554,338}$, or) £1,803,953 *per annum*. Therefore at the end of the ſaid term of 60 years, when all the ſaid life-annuitants (who were of the age of 35 years at the beginning of the ſaid term,) will be dead, the ſaid ſum of £1,803,953 *per annum* of the perpetual annuities will be intirely redeemed to the nation by means of this converſion of them into life-annuities.

Of the ſavings that would accrue every year to the publick by the deaths of ſome of the life-annuitants.

CCLX. This would be the quantity of the perpetual annuities that would be redeemed by this operation in the courſe of 60 years, if all the aforeſaid life-annuitants had lived to within a day of the end of the whole 60 years, or if, when they had died off in the different years of the ſaid term according to the courſe of nature, the ſavings which would have accrued every year to the publick by their deaths had not been applied to the ſame purpoſe of diminiſhing the national debt, but had been ſpent upon ſome other ſervice. But it was ſuppoſed above that theſe ſavings were to be employed continually to the ſame purpoſe of diminiſhing the national debt in either the firſt or the ſecond of the methods above-deſcribed. We muſt now therefore inquire what theſe ſavings would amount to in every year of this whole term, and what quantity of the perpetual annuities they would reſpectively be ſufficient to redeem in the courſe of the ſaid term, if they were applied to that uſe in either the firſt or the ſecond method above-mentioned. Theſe inquiries may be made in the manner following.

CCLXI. The

CCLXI. The number of perſons repreſented in Monſieur de Parcieux's rable as living at the age of 35 years is 694. The money to be paid annually by the Government in life-annuities, ſo long as the life-annuitants ſhall be all alive, in lieu of the £1,803,953 *per annum* of perpetual annuities which are to be redeemed, is the ſaid ſum of £1,803,953 *per annum* together with the annual million of pounds which is deſtined to this purpoſe, and conſequently is £2,803,953 *per annum*. Therefore, if we ſuppoſe theſe life-annuities to be diſtributed equally amongſt 694 perſons, all of the age of 35 years, each of the ſaid perſons will be poſſeſſed of a life-annuity of $\left(\frac{£\ 2,803,953}{694}, \text{ or}\right)$ £4040.27 *per annum*. And conſequently, if we ſuppoſe them to be equally diſtributed amongſt 6940 perſons, all of the ſame age of 35 years, each of the ſaid perſons will be poſſeſſed of a life-annuity of £404.027 *per annum:* and, if we ſuppoſe them to be equally diſtributed amongſt 69400 perſons, all of the ſame age of 35 years, each of the ſaid perſons will be poſſeſſed of a life-annuity of £40.4027, or 40*l.* 8*s.* ½*d.* *per annum*. Let us make this laſt ſuppoſition.

Of the number of perſons ſuppoſed to be life-annuitants, and the value of their annuities.

CCLXII. Then, ſince it appears by Monſieur de Parcieux's table of the probabilities of the duration of human life, that out of 694 perſons of the age of 35 years, all living at the ſame time, only 686 perſons will be living at the end of a year, or at the age of 36 years, it follows that out of the ſaid 69400 perſons above-mentioned, of the age of 35 years, all living at the ſame time, for whoſe lives theſe annuities of £40.4027, or 40*l.* 8*s.* ½*d.* each, are ſuppoſed to be granted, only 68600 will be living at the end of a year, or at the age of 36 years. And conſequently the annuities that would have been payable to the other 800 life-annuitants at the end of the year, if the ſaid annuitants had been then living, will be ſaved to the publick, and will be ready in the publick treaſury to be employed, in any manner that may be thought fit, towards the diminution of the national debt. The amount of theſe annuities is 800 times £40.4027, or £32,322.1600. Now, if this ſum £32,322.1000 is immediately laid out either in paying off ſome of the capital of the 4 per cent. annuities at *par*, according to the firſt method above-deſcribed, or in buying up ſome of the capital of the 3 per cent. annuities at the price of £75 per cent. according to the ſecond method above-deſcribed; and the intereſt of the capital thereby extinguiſhed, or the portion of the perpetual annuities that will be redeemed at the end of the next, or ſecond, year, by thus employing the ſaid ſum of £32,322.1600, be likewiſe employed in the ſame manner; and the intereſt of the capital extinguiſhed by this ſecond operation, or the portion of the perpetual annuities that will thereby be redeemed at

Of the ſaving made at the end of the firſt year by the deaths of ſome of the life-annuitants.

Of the manner in which the ſaid ſaving is to be employ'd during the remaining 59 years of the ſaid term of 60 years.

at the end of the third year, be likewiſe employed in the ſame manner; and all the following portions of the perpetual annuities that will be thus redeemed by means of this firſt ſum of £32,322.1600, that is ſaved to the nation at the end of the firſt year by the death of the aforeſaid 800 life-annuitants, be likewiſe employed in the ſame manner during the whole remainder of the ſaid term of 60 years;—I ſay, if theſe ſucceſſive ſums of money be ſo employed without interruption during the remainder of the ſaid term of 60 years, the quantity of the capital of the national debt which will be thereby extinguiſhed at the end of the ſaid term of 60 years, or at the end of 59 years from the time when this firſt ſaving of £32,322.1600 will have accrued, will be equal in value to the ſum to which the ſaid firſt ſaving of £32,322.1600 would have increaſed in the ſame period of 59 years, if it had been improved all the time at compound intereſt at the rate of 4 per cent. and therefore it will, (according to Mr. Smart's firſt table of compound intereſt, page 56,) be equal in value to the ſum of £32,322.1600 × 10.115,026, or £326,939.4887; the intereſt of which at 4 per cent. is $\left(\frac{4}{100} \times £326,939.4887, \text{ or}\right)$ £13077.5795 *per annum*. Therefore the quantity of the intereſt of the national debt, or of the perpetual annuities now due to the public creditors, which would be redeemed at the end of the whole term of 60 years by means of this firſt ſaving of £32,322.1600, which will accrue to the publick at the end of the firſt year of the ſaid term by the deaths of the aforeſaid 800 life-annuitants, would be £13,077.5795 *per annum*.

Of the quantity of the capital of the national debt that will have been extinguiſhed by means of the ſaid ſaving at the end of the ſaid term of 60 years.

Of the ſaving made at the end of the ſecond year by the deaths of life-annuitants.

CCLXIII. In the courſe of the ſecond year of this term the 68600 life-annuitants of the age of 36 years, who are living at the end of the firſt year, will be reduced to 67800. Therefore 800 more of the ſaid life-annuitants will have died in the courſe of this ſecond year. And conſequently at the end of this ſecond year there will be a ſaving to the Government of twice 800 payments of £40.4027 each, which would have then become due to the 1600 life-annuitants who will have died in the two preceeding years, if they had lived till that time. There will therefore be in the hands of the Government at the end of this ſecond year the ſum of 1600 × £40.4027, or £64,644.3200, to be employed during the remainder of the ſaid term of 60 years, that is, during 58 years, in diminiſhing the national debt; which will be ſufficient to extinguiſh as much of the capital of the ſaid debt as is equal in value to the ſum to which this ſecond ſaving of £64,644.3200 will increaſe in the courſe of 58 years, if conſtantly improved during that time at compound intereſt at the rate of 4 per cent. Now the amount of £64,644.3200 at the end of 58 years, when improved during that time at compound intereſt at 4 per cent. appears

appears by Mr. Smart's firſt table of compound intereſt, page 56, to be = £64,644.3200 x 9.725,986, or £628,729.7513; the intereſt of which at 4 per cent. is $\left(\frac{4}{100} \times £628,729\ 7513,\ \text{or}\ \frac{£2,514,919.0052}{100},\ \text{or}\right)$ £25,149.1900. Therefore the quantity of the capital of the national debt that would be extinguiſhed at the end of the ſaid term of 60 years by means of this ſecond ſaving of £64,644.3200 (which would have accrued to the publick at the end of the ſecond year of the ſaid term by the deaths of the ſaid 1600 life-annuitants who would have died in the two preceeding years,) will be equal in value to £628,729.7513; and the quantity of the intereſt of the ſaid debt, or of the perpetual annuities now due to the publick creditors, that would then be redeemed to the nation by means of this ſecond ſaving, would be the ſum of £25,149.1900 *per annum*.

Of the quantity of the capital of the national debt that will have been extinguiſhed by means of this ſecond ſaving at the end of the ſaid term of 60 years.

CCLXIV. In the ſame manner we muſt proceed to inveſtigate the ſavings that will accrue to the publick at the ends of the third, fourth, fifth, ſixth, and every following year of the whole term of 60 years, and the quantities of the capital of the national debt that will be extinguiſhed by each of the ſaid ſavings at the end of the ſaid term of 60 years, if they are conſtantly employed for that purpoſe in either the firſt or the ſecond method above-mentioned, during the ſeveral remaining years of the ſaid term after the times when they will have reſpectively accrued. And, in order to theſe inveſtigations, it will be neceſſary in the firſt place to aſcertain, by the help of Monſieur de Parcieux's table of the probabilities of the duration of human life, the numbers of life-annuitants who will be alive at the ends of the ſeveral remaining years of this term, except the laſt year of it, (at the end of which they will be all dead,) and thence to determine the numbers of thoſe who will have died in each of the ſaid years, and from whoſe deaths the ſeveral ſavings to the publick will ariſe.

Of the ſavings that will be made at the ends of the third, and fourth, and other following years of the ſaid term of 60 years, by the deaths of life-annuitants.

CCLXV. Now of the aforeſaid original number of 6940 life-annuitants, all of the age of 35 years, who were ſuppoſed to be living at the beginning of the ſaid term of 60 years, and for whoſe lives the ſaid annuities, of £40.4027 to each, were ſuppoſed to be granted, it appears from Monſieur de Parcieux's table of probabilities that the following numbers will be living at the ſubſequent ages of 38 years, 39 years, 40 years, 41 years, &c. to the age of 94 years, incluſive, or at the ends of the third, fourth, fifth, and ſixth, and every following, year of the ſaid term of 60 years; to wit,

The numbers of life-annuitants who will be living at the ends of all the ſaid years, except the laſt year.

67100,	59900,	48900,	34700,	15400,	2200,
66400,	59000,	47600,	32900,	13600,	1600,
65700,	58100,	46300,	31000,	11800,	1100,
65000,	57100,	45000,	29100,	10100,	700,
64300,	56000,	43700,	27100,	8500,	400,
63600,	54900,	42300,	25100,	7100,	200,
62900,	53800,	40900,	23100,	5900,	100.
62200,	52600,	39500,	21100,	4800,	
61500,	51400,	38000,	19200,	3800,	
60700,	50200,	36400,	17300,	2900,	

The numbers of life-annuitants who will have died in each of the faid years, and in the laft year.

Therefore the numbers who will have died in each of thoſe years, and in the laſt year of the ſaid period of 60 years, or from the age of 94 years to the age of 95 years, will be as follows; to wit,

700,	800,	1300,	1700,	1900,	700,
700,	900,	1300,	1800,	1800,	600,
700,	900,	1300,	1900,	1800,	500,
700,	1000,	1300,	1900,	1700,	400,
700,	1100,	1300,	2000,	1600,	300,
700,	1100,	1400,	2000,	1400,	200,
700,	1100,	1400,	2000,	1200,	100,
700,	1200,	1400,	2000,	1100,	100.
700,	1200,	1500,	1900,	1000,	
800,	1200,	1600,	1900,	900,	

CCLXVI. Conſequently the ſavings of intereſt that will be made by the publick at the ends of thoſe years in conſequence of the deaths of life-annuitants that will have happened in the preceeding years of the ſaid term, will be as follows; to wit,

At the end of the third year

$$\overline{1600 + 700} \times £40.4027,$$

or $2300 \times £40.4027 = £92,926.2100.$

At the end of the fourth year

$$\overline{2300 + 700} \times £40.4027,$$

or $3000 \times £40.4027, = £121,208.1000.$

At

At the end of the fifth year,

$\overline{3000 + 700} \times £40.4027,$

or $3700 \times £40.4027, = £149,489.9900.$

At the end of the ſixth year,

$\overline{3700 + 700} \times £40.4027,$

or $4400 \times £40.4027, = £177,771.8800.$

At the end of the ſeventh year,

$\overline{4400 + 700} \times £40.4027,$

or $5100 \times £40.4027, = £206,053.7700.$

At the end of the eighth year,

$\overline{5100 + 700} \times £40.4027,$

or $5800 \times £40.4027, = £234,335.6600.$

At the end of the ninth year,

$\overline{5800 + 700} \times £40.4027,$

or $6500 \times £40.4027, = £262,617.5500.$

At the end of the tenth year,

$\overline{6500 + 700} \times £40.4027,$

or $7200 \times £40.4027, = £290,899.4400.$

At the end of the eleventh year,

$\overline{7200 + 700} \times £40.4027,$

or $7900 \times £40.4027, = £319,181.3300.$

At the end of the twelfth year,

$\overline{7900 + 800} \times £40.4027,$

or $8700 \times £40.4027, = £351,503.4900.$

At the end of the thirteenth year,

$\overline{8700 + 800} \times £40.4027,$

or $9500 \times £40.4027, = £383,825.6500.$

At the end of the fourteenth year,

$\overline{9500 + 900} \times £40.4027,$

or $10,400 \times £40.4027, = £420,188.0800.$

At the end of the fifteenth year,

$\overline{10,400 + 900} \times £40.4027,$

or $11,300 \times £40.4027, = £456,550.5100.$

At the end of the ſixteenth year,

$\overline{11,300 + 1000} \times £40.4027,$

or $12,300 \times £40.4027, = £496,953.2100.$

At the end of the ſeventeenth year,

$\overline{12,300 + 1100} \times £40.4027,$

or $13,400 \times £40.4027, = £541,396.1800.$

At the end of the eighteenth year,

$\overline{13,400 + 1100} \times £40.4027,$

or $14,500 \times £40.4027, = £585,839.1500.$

At the end of the nineteenth year,

$\overline{14,500 + 1100|} \times £40\ 4027,$

or $15,600 \times £40.4027, = £630,282.1200.$

At the end of the twentieth year,

$\overline{15,600 + 1200|} \times 40.4207,$

or $16,800 \times £40.4027, = £678,765.3600.$

At the end of the twenty-firſt year,

$\overline{16,800 + 1200|} \times £40.4027,$

or $18,000 \times £40.4027, = £727,248.6000.$

At the end of the twenty-ſecond year,

$\overline{18,000 + 1200|} \times £40\ 4027,$

or $19,200 \times £40.4027, = £775,731.8400.$

At the end of the twenty-third year,

$\overline{19,200 + 1300|} \times £40.4027,$

or $20,500 \times £40.4027, = £828,255.3500.$

At the end of the twenty-fourth year,

$\overline{20,500 + 1300|} \times £40.4027,$

or $21,800 \times £40.4027, = £880,778.8600.$

At the end of the twenty-fifth year,

$\overline{21,800 + 1300|} \times £40.4027,$

or $23,100 \times £40.4027, = £933,302.3700.$

At

At the end of the twenty-ſixth year,

$\overline{|23{,}100 + 1300|} \times £40.4027,$

or $24{,}400 \times £40.4027, = £985{,}825.8800.$

At the end of the twenty-ſeventh year,

$\overline{|24{,}400 + 1300|} \times £40.4027,$

or $25{,}700 \times £40.4027, = £1{,}038{,}349.3900.$

At the end of the twenty-eighth year,

$\overline{|25{,}700 + 1400|} \times £40.4027,$

or $27{,}100 \times £40.4027, = £1{,}094{,}913.1700.$

At the end of the twenty-ninth year,

$\overline{|27{,}100 + 1400|} \times £40.4027,$

or $28{,}500 \times £40.4027, = £1{,}151{,}476.9500.$

At the end of the thirtieth year,

$\overline{|28{,}500 + 1400|} \times £40.4027,$

or $29{,}900 \times £40.4027, = £1{,}208{,}040.7300.$

At the end of the thirty-firſt year,

$\overline{|29{,}900 + 1500|} \times £40.4027,$

or $31{,}400 \times £40.4027, = £1{,}268{,}644.7800.$

At the end of the thirty-ſecond year,

$\overline{|31{,}400 + 1600|} \times £40.4027,$

or $33{,}600 \times £40.4027, = £1{,}333{,}289.1000.$

At the end of the thirty-third year,

$\overline{33{,}000 + 1700|} \times £40.4027,$

or $34{,}700 \times £40.4027, = £1{,}401{,}973.6900.$

At the end of the thirty-fourth year,

$\overline{34{,}700 + 1800|} \times £40.4027,$

or $36{,}500 \times £40.4027, = £1{,}474{,}698.5500.$

At the end of the thirty-fifth year,

$\overline{36{,}500 + 1900|} \times £40.4027,$

or $38{,}400 \times £40.4027, = £1{,}551{,}463.6800.$

At the end of the thirty-fixth year,

$\overline{38{,}400 + 1900|} \times £40.4027,$

or $40{,}300 \times £40.4027. = £1{,}628{,}228.8100.$

At the end of the thirty-feventh year,

$\overline{40{,}300 + 2000|} \times £40.4027,$

or $42{,}300 \times £40.4027, = £1{,}709{,}034.2100.$

At the end of the thirty-eighth year,

$\overline{42{,}300 + 2000|} \times £40.4027,$

or $44{,}300 \times £40.4027, = £1{,}789{,}839.6100.$

At the end of the thirty-ninth year,

$\overline{44{,}300 + 2000|} \times £40.4027,$

or $46{,}300 \times £40.4027, = £1{,}870{,}645.0100.$

At

At the end of the fortieth year,

$\overline{|46{,}300 + 2000|} \times £40.4027,$

or $48{,}300 \times £40.4027, = £1{,}951{,}450.4100.$

At the end of the forty-firſt year,

$\overline{|48{,}300 + 1900|} \times £40.4027,$

or $50{,}200 \times £40.4027, = £2{,}028{,}215.5400.$

At the end of the forty-ſecond year,

$\overline{|50{,}200 + 1900|} \times £40.4027,$

or $52{,}100 \times £40.4027, = £2{,}104{,}980.6700.$

At the end of the forty-third year,

$\overline{|52{,}100 + 1900|} \times £40.4027,$

or $54{,}000 \times £40.4027, = £2{,}181{,}745.8000.$

At the end of the forty-fourth year,

$\overline{|54{,}000 + 1800|} \times £40.4027,$

or $55{,}800 \times £40.4027, = £2{,}254{,}470.6600.$

At the end of the forty-fifth year,

$\overline{|55{,}800 + 1800|} \times £40.4027,$

or $57{,}600 \times £40.4027, = £2{,}327{,}195.5200.$

At the end of the forty-ſixth year,

$\overline{|57{,}600 + 1700|} \times £40.4027,$

or $59{,}300 \times £40.4027, = £2{,}395{,}880.1100.$

At

At the end of the forty-ſeventh year,

$\overline{59{,}300 + 1600|} \times £40.4027,$

or $60{,}900 \times £40.4027, = £2{,}460{,}524.4300.$

At the end of the forty-eighth year,

$\overline{60{,}900 + 1400|} \times £40.4027,$

or $62{,}300 \times £40.4027, = £2{,}517{,}088.2100.$

At the end of the forty-ninth year,

$\overline{62{,}300 + 1200|} \times £40.4027,$

or $63{,}500 \times £40.4027, = £2{,}565{,}571.4500.$

At the end of the fiftieth year,

$\overline{63{,}500 + 1100|} \times £40.4027,$

or $64{,}600 \times £40.4027. = £2{,}610{,}014.4200.$

At the end of the fifty-firſt year,

$\overline{64{,}600 + 1000|} \times £40.4027,$

or $65{,}600 \times £40.4027, = £2{,}650{,}417.1200.$

At the end of the fifty-ſecond year,

$\overline{65{,}600 + 900|} \times £40.4027,$

or $66{,}500 \times £40.4027, = £2{,}686{,}779.5500.$

At the end of the fifty-third year,

$\overline{66{,}500 + 700|} \times £40.4027,$

or $67{,}200 \times £40.4027, = £2{,}715{,}061.4400.$

At the end of the fifty-fourth year,

$\overline{|67,200 + 600|} \times £40.4027,$

or $67,800 \times £40.4027, = £2,739,303.0600.$

At the end of the fifty-fifth year,

$\overline{|67,800 + 500|} \times £40.4027,$

or $68,300 \times £40.4027, = £2,759,504.4100.$

At the end of the fifty-sixth year,

$\overline{|68,300 + 400|} \times £40.4027,$

or $68,700 \times £40.4027, = £2,775,665.4900.$

At the end of the fifty-seventh year,

$\overline{|68,700 + 300|} \times £40.4027,$

or $69,000 \times £40.4027, = £2,787,786.3000.$

At the end of the fifty-eighth year,

$\overline{|69,000 + 200|} \times £40.4027,$

or $69,200 \times £40.4027, = £2,795,866.8400.$

At the end of the fifty-ninth year,

$\overline{|69,200 + 100|} \times £40.4027,$

or $69,300 \times £40.4027, = £2,799,907.1100.$

And at the end of the sixtieth and last year,

$\overline{|69,300 + 100|} \times £40.4027,$

or $69,400 \times £40.4027, = £2,803,947.3800.$

Note,

Note. This laſt ſaving £2,803,947.3800, ought to be equal to the whole ſum allotted to the payment of the life-annuities; ſince at the time of its accruing to the publick all the life-annuitants are ſuppoſed to be dead. It ought therefore to be equal to £1000,0 0 + £1,803,953, or £2,803,953, which was the ſum allotted to that purpoſe. And ſo we find it very nearly is, ſince it differs from it by leſs than £6, which, upon ſo large a ſum as £2,803,953, is not worth attending to. And it would have been more nearly equal to £2,803,953, if in dividing £2,803,953 by 694 (in Art. CCLXI) we had carried the quotient to more than ſix places of figures. For, if we had carried it only to one figure more, we ſhould have found it to be 4040.278. And, in conſequence of this increaſe of this quotient, the annuity which would have belonged to each of the aforeſaid 69400 life-annuitants for his life, would have been £40.40278, inſtead of £40.4027; and 69400 × £40.40278 is = £2,803,952.93200, which differs from £2,803,953 by leſs than $\frac{1}{10}$ of a pound, or than two ſhillings. But the numbers £40.4027 and £2,803,947.3800 are near enough to the truth to anſwer the purpoſe of this computation.

A remark on the laſt ſaving, £2,803,947.3000.

CCLXVII. Now all theſe ſavings are to be employed, as faſt as they ariſe, for the purpoſe of diminiſhing the national debt, in either the firſt or the ſecond method above-mentioned. By this means the laſt ſaving, £2,803,947.3800, will either extinguiſh £2,803,947.3800 of the capital of the 4 per cent. annuities, (which are ſuppoſed to ſell at their *par*, or nominal value,) or, if employed in the ſecond method above-deſcribed, it will extinguiſh a proportionally greater capital of the 3 per cent. annuities, which are ſuppoſed to ſell at the price of £75 per cent. and in either caſe it will redeem to the publick the intereſt of 4 per cent. upon it, or the annual ſum of $\left(\frac{4}{100} \times £2,803,947.3800, \text{ or } \frac{£11,215,789.5200}{100}, \text{ or}\right)$ £112,157.8952. But all the other ſavings will have time to perform more than one operation of this kind towards the diminution of the national debt, to wit, a new operation at the end of every year of the term that is remaining after the time when it accrued: and, in conſequence of theſe repeated operations, the quantity of the national debt which each of theſe ſavings will have extinguiſhed at the end of the ſaid term of 60 years, will be equal in value to the amount of ſuch ſaving at the end of the ſaid term, if improved in the mean time at compound intereſt at the rate of four per cent.

Of the effect of the ſaid ſavings in diminiſhing the national debt.

We muſt therefore now compute the amounts of theſe ſavings at the end of the ſaid term, if ſo improved at compound intereſt in the mean time, or during the remaining years of the ſaid term after the times at which they will have reſpectively accrued, that is, during 57 years, 56 years, 55 years, 54 years, &c. to the laſt year of the ſaid term. Now theſe amounts will be as follows.

A computation of the amounts of all the foregoing ſavings, except the laſt, at the end of the ſaid term of 60 years, if improved in the mean time at compound intereſt at the rate of 4 per cent.

CCLXVIII. The amount of £92,926.21, improved at compound intereſt at 4 per cent. during 57 years, is = £92,926.21 × 9.351,910 = £869,037.5525.

The amount of £121,208.1000, improved in the ſame manner during 56 years, is = £121,208.1000 × 8.992,221 = £1,089,930.0221.

The amount of £149,489.9900, improved in the ſame manner during 55 years, is = £149,489.9900 × 8.646,367 = £1,292,545.3163.

And the amounts of all the other ſavings, improved in the ſame manner during the remaining years of the ſaid term of 60 years after they ſhall have reſpectively accrued to the publick, will be as follows.

£177,771.8800	×	8.313,814	=	£1,477,962.3447.
£206,053.7700	×	7.994,052	=	£1,647,204.5521.
£234,335.6600	×	7.686,588	=	£1,801,241.6721.
£262,617.5500	×	7.390,950	=	£1,940,993.1811.
£290,899.4400	×	7.106,683	=	£2,067,330.1049.
£319,181.3300	×	6.833,349	=	£2,181,077.4221.
£351,503.4900	×	6.570,528	=	£2,309,563.5231.
£383,825.6500	×	6.317,815	=	£2,424,939.4489.
£420,188.0800	×	6.074,822	=	£2,552,567.7925.
£456,550.5100	×	5.841,175	=	£2,666,791.4252.
£496,953.2100	×	5.616,515	=	£2,791,145.1576.
£541,396.1800	×	5.400,495	=	£2,923,807.3631.
£585,839.1500	×	5.192,783	=	£3,042,135.5788.
£630,282.1200	×	4.993,061	=	£3,147,037.0723.
£678,765.3600	×	4.801,020	=	£3,258,766.0686.
£727,248.6000	×	4.616,366	=	£3,357,245.7105.
				£775,731.8400

£775,731.8400	×	4.438,813	=	£3,443,328.5759.
£828,255.3500	×	4.268,089	=	£3,535,067.5285.
£880,778.8600	×	4.103,932	=	£3,614,656.5484.
£933,302.3700	×	3.946,089	=	£3,682,894.2159.
£985,825.8800	×	3.794,316	=	£3,740,534.9097.
£1,038,349.3900	×	3.648,381	=	£3,788,294.1858.
£1,094,913.1700	×	3.508,058	=	£3,841,018.9053.
£1,151,476.9500	×	3.373,133	=	£3,884,084.8987.
£1,208,040.7300	×	3.243,397	=	£3,918,155.6795.
£1,268,644.7800	×	3.118,651	=	£3,956,460.3117.
£1,333,289.1000	×	2.998,703	=	£3,998,138.0240.
£1,401,973.6900	×	2.883,368	=	£4,042,406.0745.
£1,474,698.5500	×	2.772,469	=	£4,088,556.0142.
£1,551,463.6800	×	2.665,836	=	£4,135,947.7308.
£1,628,228.8100	×	2.563,304	=	£4,173,645.4215.
£1,709,034.2100	×	2.464,715	=	£4,212,282.2529.
£1,789,839.6100	×	2.369,918	=	£4,241,773.0758.
£1,870,645.0100	×	2.278,768	=	£4,262,765.9881.
£1,951,450.4100	×	2.191,123	=	£4,275,867.8767.
£2,028,215.5400	×	2.106,849	=	£4,273,143.8822.
£2,104,980.6700	×	2.025,816	=	£4,264,203.5209.
£2,181,745.8000	×	1.947,900	=	£4,249,822.6438.
£2,254,470.6600	×	1.872,981	=	£4,222,580.7112.
£2,327,195.5200	×	1.800,943	=	£4,191,146.4813.
£2,395,880.1100	×	1.731,676	=	£4,148,588.0853.
£2,460,524.4300	×	1.665,073	=	£4,096,952.7942.
£2,517,088.2100	×	1.601,032	=	£4,029,938.7710.
£2,565,571.4500	×	1.539,454	=	£3,949,579.2309.
£2,610,014.4200	×	1.480,244	=	£3,863,458.1851.
£2,650,417.1200	×	1.423,311	=	£3,772,367.8414.
				£2,686,779.5500

£2,686,779.5500	×	1.368,569	=	£3,677,043.2019.
£2,715,061.4400	×	1.315,931	=	£3,572,833.5158.
£2,739,303.0600	×	1.265,319	=	£3,466,092.2085.
£2,759,504.4100	×	1.216,652	=	£3,357,356.5594.
£2,775,665.4900	×	1.169,858	=	£3,247,134.4788.
£2,787,786.3000	×	1.124,864	=	£3,135,880.4485.
£2,795,866.8400	×	1.081,600	=	£3,024,009.5741.
£2,799,907.1100	×	1.040,000	=	£2,911,903.3944.

The addition of the amounts of all the yearly savings at the end of the said term of 60 years, into one sum.

CCLXIX. Having thus found the amounts of the several yearly savings at the end of the said term of 60 years, we must now add these several amounts together; which may be done as follows.

The amounts of the savings made at the ends of the first ten years, the second ten years, and the third ten years, of the said term, are as follows.

£	£	£
326,939.4887	2,181,077.4221	3,357,245.7105
628,729.7513	2,309,563.5231	3,443,328.5759
869,037.5525	2,424,939.4489	3,535,067.5285
1,089,930.0221	2,552,567.7925	3,614,656.5484
1,292,545.1668	2,666,791.4252	3,682,894.2159
1,477,962.3447	2,791,145.1576	3,740,534.9097
1,647,204.5521	2,923,807.3631	3,788,294.1858
1,801,241.6721	3,042,135.5788	3,841,018.9053
1,940,993.1811	3,147,037.0723	3,884,084.8987
2,067,330.1049	3,258,766.0686	3,918,155.6795
13,141,913.8363	27,297,830.8522	36,805,281.1582

And

And the amounts of the ſavings made at the ends of the fourth ten years, the fifth ten years, and the ſixth ten years, are as follows.

£	£	£
3,956,460.3117	4,273,143.8822	3,772,367.8414
3,998,138.0240	4,264,203.5209	3,677,043.2019
4,042,406.0745	4,249,822.6438	3,572,833.5158
4,088,556.0142	4,222,580.7112	3,466,092.2085
4,135,947.7308	4,191,146.4813	3,357,356.5594
4,173,645.4215	4,148,888.0853	3,247,134.4788
4,212,282.2529	4,096,952.7942	3,135,880.4485
4,241,773.0758	4,029,938.7710	3,024,009.5741
4,262,765.9881	3,949,579.2309	2,911,903.3944
4,275,867.8767	3,863,458.1851	2,803,947.3800
41,387,842.7702	41,289,714.3059	32,968,568.6028

And the ſum total of theſe ſix ſeveral ſums, or the ſum total of the amounts that will ariſe from all the ſavings made at the ends of all the 60 years of the ſaid term, at the end of the ſaid term, is £192,891,151.5256.

£
13,141,913.8363
27,297,830.8522
36,805,281.1582
41,387,842.7702
41,289,714.3059
32,968,568.6028
192,891,151.5256

CCLXX. Therefore the quantity of the capital of the national debt that would have been extinguiſhed by means of all theſe ſavings at the end of the ſaid term of 60 years, will be equal in value to the ſaid ſum of £192,891,151.5256; of which the annual intereſt at 4 per cent. is

£ £ £
$\left(\frac{4}{100} \times 192,891,151.5256,\text{ or } \frac{771,564,606.1024}{100},\text{ or}\right)$ 7,715,646.0610.

Therefore

The quantity of the national debt that will be extinguiſhed at the end of the ſaid term of 60 years by means of all the ſaid yearly ſavings.

The quantity of the intereſt of the ſaid debt that will thereby be redeemed to the publick at the end of the ſaid term of 60 years.

Therefore the quantity of the intereſt of the national debt, or of the perpetual annuities employed in the payment of the ſaid intereſt, that would be redeemed to the publick at the end of the ſaid term of 60 years by means of all theſe ſavings, (ſo that at the end of the next, or 61ſt, year it would be free to be diſpoſed of by the Parliament in any other way,) is £7,715,646.0610 *per annum*. QEI.

The whole quantity of the intereſt of the national debt that will be redeemed to the publick at the end of the ſaid term of 60 years by the foregoing ſcheme of life-annuities.

CCLXXI. This ſum of £7,715,646.0610 *per annum* muſt therefore be added to £1,803,953 *per annum*, which was the quantity of the perpetual annuities that was ſuppoſed to be converted into life-annuities for the lives of 69400 perſons, all of the age of 35 years, and which will therefore be now wholly redeemed to the publick by the deaths of all the ſaid life-annuitants: and the ſum of both, or the ſum of £9,519,599.0610, or (neglecting the fraction .0610,) £9,519,599 *per annum* will be the whole quantity of the intereſt of the national debt, or of the perpetual annuities employed in the payment of it, which will be redeemed to the publick at the end of the ſaid term of 60 years (ſo as to be free at the end of the next, or 61ſt, year to be diſpoſed of by the Parliament in any other way,) by the faithful application of a million of pounds ſterling a year to that ſalutary purpoſe during that period according to the fourth method above-deſcribed. QEI.

Agreement between the effect of the foregoing ſcheme of life-annuities, and the effects of the three preceeding methods.

CCLXXII. This ſum of £9,519,599 *per annum*, which is the quantity of the intereſt of the national debt that would be redeemed to the publick in the courſe of 60 years by the application of a million of pounds ſterling *per annum* in this fourth method, or in the way of life-annuities, is the ſame with the quantity of the intereſt of the ſaid debt that would have been redeemed to the publick in the ſame time by applying a million of pounds *per annum* to the ſame purpoſe in either of the three former methods. For that was ſhewn above, (in Art. CCXLIV, CCXLVII, CCLIII,) to be £9,520,000 *per annum*; which differs from £9,519,599 *per annum*, by only £401 *per annum*, which is a mere trifle upon ſo large a ſum. And, if we had computed the ſaid former quantity more accurately, we ſhould have found it to approach ſtill nearer to £9,519,599. For it is not exactly equal to £9,520,000 *per annum*, or 238 × £40,000 *per annum*, (as it is there, for the ſake of round numbers, ſuppoſed to be;) but more nearly to 237.9906 × £40,000 *per annum*, that is, to £9,519,624.0000, or £9,519,624, *per annum*; which differs from £9,519,599 *per annum* by only £25 *per annum*. They may therefore be conſidered as equal; agreeably to what is aſſerted in Art. CCLVI.

CCLXXIII. We

CCLXXIII. We may therefore conclude that this fourth method of applying a million of pounds ſterling, or any other given ſum of money, every year, during a given number of years, to the purpoſe of diminiſhing the national debt, by converting ſome of the perpetual annuities, now due to the publick creditors, into life-annuities, would be juſt as efficacious in redeeming the intereſt of the ſaid debt, and conſequently juſt as beneficial to the nation, as either of the three former methods of applying the ſame ſum of money to the ſame purpoſe during the ſame period of time; to prove which was the principal object of the foregoing computation. But this, it muſt be remembered, is true only upon a ſuppoſition that the ſavings of the intereſt of the national debt, that will accrue to the publick every year by the deaths of ſome of the life-annuitants, are immediately employed in diſcharging a part of the ſaid debt in either the firſt or ſecond method above-deſcribed, or in ſome other method that is equivalent to them. For, if thoſe ſavings are not ſo employed, but are diverted to other uſes, the converſion of the perpetual annuities, now due to the publick creditors, into life-annuities would be evidently a *much leſs* efficacious method of diminiſhing the national debt than either of the three former methods above-mentioned. Thus, for example, we have ſeen that, if the ſaid ſavings are employed in the manner above-deſcribed, the ſum of one million of pounds, ſterling, a year will be ſufficient to redeem to the publick, in the courſe of 60 years, the ſum of £9,519,599 *per annum* of the intereſt of the national debt; but without the help of the ſaid ſavings, it will redeem in that time only £1,803,953 *per annum*. Therefore, whenever any propoſal is made to pay off any part of the national debt by means of life-annuities, great care muſt be taken that the ſavings of the intereſt of the ſaid debt, that will continually accrue to the publick by the deaths of ſome of the life-annuitants, ſhall be conſtantly applied to the ſame purpoſe; or otherwiſe the nation muſt (as Dr. Price has juſtly obſerved in his Eſſay on Publick Credit and the National Debt in the third chapter of his Treatiſe on Reverſionary Payments, firſt edition, page 149,) loſe greatly by all ſchemes of this kind.

General concluſion concerning the efficacy of this fourth method of applying a given ſum of money every year to the diminution of the national debt.

CCLXXIV. This fourth method of employing a given ſum of money every year in the diminution of the national debt, "by converting ſome of the perpetual annuities, that now pay the intereſt of it, into life-annuities, and allotting the ſaid annual ſum to the payment of the additions thereupon made to the perpetual annuities ſo converted," partakes in ſome degree of the advantage mentioned in Art. CCLIV as belonging to the third method, to wit, that of ſecuring to the publick the faithful application of ſuch annual ſum to the purpoſes of its deſtination, without ſuſpenſion or interruption. For it would then be impoſſible to with-hold any

A compariſon between this fourth method and the third method, with reſpect to the advantage of ſecuring to the publick the faithful application of the ſaid annual ſum of money to the purpoſes of its deſtination.

part of the life-annuities ſo created from the perſons to whom they would be due, without an abſolute breach of the publick faith; which, (as we before obſerved,) may be conſidered as a moral impoſſibility. But this advantage will relate only to ſo much of the publick revenue as will be due at any time to the life-annuitants that are ſtill alive; and not to that other part of it which was allotted to the payment of the annuities of the other life-annuitants who will have died ſince the eſtabliſhment of them, and which will be every year increaſing by the deaths of more of the ſaid life-annuitants. For this part of the revenue, which will be thus ſaved every year to the publick, will be as liable to be diverted, by the miniſters of ſtate and the Parliament, from its original deſtination of diminiſhing the national debt, in order to defray the expence of ſome temporary meaſure of Government, as the money that ſhould be allotted to the diminution of the national debt in either the firſt or the ſecond method above-deſcribed: whereas in the third method of applying a given ſum of money every year to the diminution of the national debt, "by converting ſome of the perpetual annuities, that now pay the intereſt of it, into greater temporary annuities that ſhould continue for a certain limited number of years, and allotting the ſaid annual ſum to the payment of the additions thereupon made to the perpetual annuities ſo converted," the whole of the ſaid annual ſum is effectually ſecured from being diverted from its original and proper deſtination, and applied to any other purpoſe whatſoever, during the whole of the ſaid term.

A fifth method of employing a given ſum of money every year in the reduction of the national debt.

CCLXXV. A fifth method of applying a given ſum of money every year to the diminution of the national debt, would be to combine the foregoing third and fourth methods together, by converting ſome of the perpetual annuities now due to the publick creditors, into greater temporary annuities of a mixt nature, or that ſhould continue both for a certain moderate number of years at all events, or whether the proprietors of them lived to the end of the ſaid term, or not, and ſhould, in caſe of their dying before the end of the ſaid term, be paid to their executors or other perſonal repreſentatives, and ſhould likewiſe continue during the lives of ſuch of the ſaid proprietors, or of other perſons to be named by the ſaid proprietors, as ſhould live beyond the end of the ſaid term, and by employing the ſaid annual ſum of money in paying the additions that it would be neceſſary to make to the perpetual annuities ſo converted, in compenſation for the abridgement of their duration: ſuch converſion being made (as in the ſecond, third, and fourth methods above-mentioned,) with the conſent of the proprietors of the annuities ſo converted.

CCLXXVI. In

CCLXXVI. In this fifth method of employing a given ſum of money every year for the purpoſe of diminiſhing the national debt, it is evident there would be no ſavings accruing to the publick till after the expiration of the limited term of years during which the new annuities, into which the perpetual annuities had been converted, were to continue at all events, whether the proprietors of the ſaid perpetual annuities, or their *nominees*, were living or dead. But after the expiration of the ſaid term there would accrue to the publick very large ſavings every year by the deaths of ſome of the perſons for whoſe lives the ſaid annuities ſhould have been granted. Now theſe ſavings ought to be employed, as faſt as they aroſe, in diminiſhing the national debt in either the firſt or the ſecond method above-deſcribed, or in ſome other method equivalent to them.

Of the ſavings that would accrue every year to the publick, after the expiration of a certain term, by the deaths of life-annuitants.

CCLXXVII. Thus, for example, it might be expedient to convert a million, or two millions, or, perhaps, three millions, of pounds, ſterling, *per annum* of the perpetual annuities, now due to the publick creditors, (with the conſent of the proprietors of ſuch annuities,) into annuities which ſhould continue for 30 years at all events, and likewiſe, after the expiration of the ſaid 30 years, during the lives of their reſpective proprietors, or of other perſons that ſhould be named by the ſaid proprietors, and who may be called their *nominees*. And then the million of pounds, ſterling, a year, or other given annual ſum that could be ſpared out of the publick revenue for this purpoſe, or a part of the ſaid ſum, might be employed in paying the additions which it would be neceſſary to make to the perpetual annuities ſo converted, in order to make amends to the proprietors of them for this abridgement of the time of their continuance. And, as there would accrue to the publick at the end of the ſaid term of 30 years, during which the ſaid annuities were to continue at all events, a great ſaving of intereſt by the deaths of the many life-annuitants who would have died in the courſe of the ſaid term;—and other and ſtill greater ſavings of intereſt would accrue in like manner to the publick at the end of every following year after the expiration of the ſaid term by the deaths of life-annuitants, until all the ſaid life-annuitants were dead;—theſe ſeveral ſavings ſhould, as faſt as they accrued, be employed, for the ſame purpoſe of diminiſhing the national debt, in either the firſt or the ſecond method above-mentioned.

CCLXXVIII. This fifth method of paying off a part of the national debt, by converting ſome of the perpetual annuities, now due to the publick creditors, into theſe compound, temporary, annuities, (which are to continue at all events during a certain given number of years, and further for the lives of certain life-annuitants,) would have exactly the ſame

This fifth method of diminiſhing the national debt, if properly purſued, would be equally effectual for that purpoſe with either of the four preceeding methods.

ſame effect towards diminiſhing the national debt, in any given period of time, as either of the four preceeding methods; that is, if a given ſum of money was every year to be faithfully applied to the diminution of the national debt in this fifth method during a given number of years;—and the ſavings of intereſt accruing to the publick, by the deaths of the ſaid life-annuitants, at the expiration of the ſaid term certain (during which the annuities were to continue at all events,) and at the ends of all the following years, after the expiration of the ſaid term, during the lives of any of the ſaid life-annuitants, were, as ſoon as they accrued, to be faithfully employed, for the ſame purpoſe of diminiſhing the national debt, in either the firſt or the ſecond method above-deſcribed;—the quantity of the intereſt of the national debt, or of the perpetual annuities now allotted to the payment of it, that would be redeemed to the publick at the end of any ſuppoſed number of years, by employing the ſaid given ſum of money in this fifth method, would be the ſame as the quantity of the ſaid intereſt that would have been redeemed to the publick by the ſame annual ſum of money, if it had been applied to the ſame uſeful purpoſe during the ſame number of years in either of the four preceeding methods. Of this it may not be amiſs to give the following example.

An example of the ſaid fifth method of applying a given ſum of money every year to the diminution of the national debt.

Preliminary ſuppoſitions.

CCLXXIX. Let us ſuppoſe (as in the foregoing articles,) that the ſum of money that is to be applied every year to the diminution of the national debt in this fifth method, is a million of pounds, ſterling, a year; and that the period during which it is to be ſo applied, is 60 years. And let us ſuppoſe further, that the perſons upon whoſe lives the new annuities, (which are, according to this fifth method, to be eſtabliſhed in lieu of ſome of the perpetual annuities now due to the publick creditors,) are to depend, are all, at the time of eſtabliſhing theſe annuities, of the age of 35 years; and that they will die off in the courſe of the following years in the proportions expreſſed in Monſieur de Parcieux's table of the probabilities of the duration of human life; and conſequently that the longeſt liver of them will live almoſt through the whole of the ſaid period of 60 years, and die a few days before the end of it. And let the number of years during which the ſaid annuities are to continue at all events, or whether the ſaid life-annuitants are alive or dead, (and during which they are to be paid to the ſaid life-annuitants, if they are alive, and, if they are dead, to their executors or other perſonal repreſentatives,) be 30 years. And let the intereſt of money be ſuppoſed (as in the foregoing articles) to be 4 per cent. and the price at which the 3 per cent. annuities ſell at the publick market, to be £75 per cent.

CCLXXX. Theſe

CCLXXX. Theſe things being premiſed, we muſt inquire, in the next place, what quantity of the perpetual annuities, which now are employed in paying the intereſt of the national debt, might be converted into theſe compound, temporary, annuities (which are at the ſame life-annuities and annuities for a term certain of 30 years,) by the help of the ſaid ſum of a million of pounds, ſterling, a year. Now this quantity may be determined in the manner following.

Of the quantity of the perpetual annuities, now due to the publick creditors, which might be converted into theſe compound life annuities by the help of a million of pounds *per annum*.

CCLXXXI. It appears from Mr. Smart's fourth table of compound intereſt, page 78, that the value of an annuity of one pound a year for a term certain of 30 years, is £17.292,03. And it appears above in Table XXV, page 285, that the value of a remote life-annuity of one pound a year for the life of a perſon of the age of 35 years, that is to commence at the diſtance of 30 years, or whereof the firſt payment is to be received at the end of 31 years, is £1.410,77. It follows therefore that the value of a compound, temporary, annuity of one pound a year, that is to continue for 30 years certain, and alſo for the life of a perſon of the age of 35 years, muſt be £17.292,03 + £1.410,77; or £18.702,80. Now, when the intereſt of money is 4 per cent. (as it is here ſuppoſed to be,) the value of a perpetual annuity of one pound a year is £25. Therefore, in order to make an annuity of one pound a year of the compound and temporary kind here deſcribed, or that is to continue only for 30 years certain and during the life of a perſon of the age of 35 years, equal in value to a perpetual annuity of one pound a year, we muſt increaſe it in the proportion of £25 to £18.702,80; by which means it will become equal to $\left(\frac{25.00000}{18.70280} \times £1, \text{ or}\right)$ £1.336,698. Therefore a compound, temporary, annuity of £1.336,698 a year, that is to continue for 30 years certain and alſo for the life of a perſon of the age of 35 years, will be equal in value to a perpetual annuity of one pound a year: and conſequently, if the Government were to convert ſome of the perpetual annuities, now due to the publick creditors as the intereſt of their debts, into compound, temporary, annuities of the kind here deſcribed, or that ſhould continue for 30 years certain and further for the lives of perſons of the age of 35 years, they ought to allow to the ſaid publick creditors, whoſe perpetual annuities ſhould be ſo converted into theſe temporary ones, an annuity of £1.336,698 a year of the kind here deſcribed, for every perpetual annuity of £1 a year of which they had been before poſſeſſed. Therefore for every million of pounds *per annum* of the perpetual annuities that ſhould be ſo converted, it would be neceſſary to allow to the proprietors of them the ſum of a million times £1.336,698 *per annum*, or of £1,336,698 *per annum* of theſe compound and temporary annuities: and conſequently

An inveſtigation of the ſaid quantity.

conſequently an additional annual ſum of £336,698 would be required to enable the Government to convert £1,000,000 *per annum* of the perpetual annuities, now due to the publick creditors as the intereſt of their debts, into theſe compound, temporary, annuities for 30 years certain and for the lives of perſons of the age of 35 years. Therefore an additional annual ſum of £1,000,000 would be ſufficient to enable the Government to convert into theſe compound, temporary, annuities for 30 years certain and for the lives of perſons of the age of 35 years, a quantity of the perpetual annuities that is greater than £1,000,000 *per annum* in the ſame proportion as £1,000,000 is greater than £336,698, that is, a quantity of the ſaid perpetual annuities that is equal to $\left(£1{,}000{,}000 \times \frac{1{,}000{,}000}{336{,}698}\right.$, or $\frac{£\ 1{,}000{,}000{,}000{,}000}{336{,}698}$, or $\left.\right)$ £2,970,020 *per annum*. QEI.

CCLXXXII. It is evident therefore that at the end of the ſaid term of 60 years. when all the ſaid life-annuitants (who were of the age of 35 years at the beginning of the ſaid term,) will be dead, the ſaid ſum of £2,970,020 *per annum* of the perpetual annuities, now due to the publick creditors, will be intirely redeemed to the publick by means of this converſion of them into theſe compound, temporary, annuities.

Of the ſavings that would accrue to the publick at the end of every year, after the firſt 30 years, by the deaths of life-annuitants.

CCLXXXIII. This would be the quantity of the ſaid perpetual annuities that would be redeemed to the nation by this operation in the courſe of 60 years, if all the aforeſaid life-annuitants had lived to within a day of the end of the whole 60 years, or if, when they had died off in the different years of the ſaid term of 60 years according to the courſe of nature, the ſavings which would have accrued every year to the publick by their deaths during the laſt 30 years of the ſaid term, had not been applied to the ſame purpoſe of diminiſhing the national debt, but had been ſpent upon ſome other ſervice. But it was ſuppoſed above that theſe ſavings were to be employed continually, as faſt as they aroſe, for the ſame purpoſe of diminiſhing the national debt, in either the firſt, or the ſecond, of the methods above-deſcribed. We muſt therefore now proceed to inquire, what theſe ſavings would amount to in every year of the ſaid term of 60 years after the firſt 30 years of it, and what quantity of the perpetual annuities, now due to the publick creditors, they would be ſufficient to redeem in the courſe of the latter half of the ſaid term, if they were conſtantly employed for that purpoſe, until the end of the ſaid term, in either the firſt or the ſecond method above-mentioned. Theſe inquiries may be made in the manner following.

CCLXXXIV. The

CCLXXXIV. The number of perſons repreſented in Monſieur de Parcieux's table of probabilities as living at the age of 35 years, is 694. The money that is to be paid annually by the Government in the aforeſaid compound annuities (for the lives of ſeveral perſons of the age of 35 years, and, in caſe of their dying within the ſpace of 30 years, to the executors of ſuch deceaſed life-annuitants for the remainder of the ſaid term of 30 years,) in lieu of the £2,970,020 *per annum* of perpetual annuities which are to be thereby redeemed, is the ſaid ſum of £2,970,020 *per annum* together with the annual million of pounds which is deſtined to this purpoſe, and conſequently is £3,970,020 *per annum*. Therefore, if we ſuppoſe theſe compound annuities to be diſtributed equally amongſt 694 perſons, all of the age of 35 years, each of the ſaid perſons will be poſſeſſed of an annuity, for 30 years certain and alſo for his life, of $\left(\frac{£3,970,020}{694}\right.$, or £5720.4899, or, very nearly,$\left.\right)$ £5720.4900 *per annum*. And conſequently, if we ſuppoſe them to be equally diſtributed amongſt 6940 perſons, all of the ſame age of 35 years, each of the ſaid perſons will be poſſeſſed of an annuity, of the ſame kind, or £572.0490 *per annum*: and, if we ſuppoſe them to be equally diſtributed amongſt 69400 perſons, all of the ſame age of 35 years, each of the ſaid perſons will be poſſeſſed of an annuity, of the ſame kind, of £57.2049, or 57*l*. 4*s*. 1*d*.$\frac{1}{4}$, *per annum*. Let us make this laſt ſuppoſition.

Of the number of perſons ſuppoſed to be life-annuitants, and the value of their annuities.

CCLXXXV. Then, ſince it appears by Monſieur de Parcieux's table of the probabilities of the duration of human life, that out of 694 perſons of the age of 35 years, all living at the ſame time, only 380 perſons will be living at the end of 31 years, or at the age of 66 years, it follows that out of the ſaid 69400 perſons above-mentioned, of the age of 35 years, all living at the ſame time, (for whoſe lives, and likewiſe for a term certain of 30 years, theſe annuities of £57.2049, or 57*l*. 4*s*. 1*d*.$\frac{1}{4}$ each, are ſuppoſed to be granted,) only 38000 will be living at the end of 31 years, or at the age of 66 years. And conſequently the annuities of £57.2049, or 57*l*. 4*s*. 1*d*.$\frac{1}{4}$ each, which would have been payable to the other (69400 — 38000, or) 31400 life-annuitants at the end of the ſaid 31 years, if the ſaid annuitants had been then living, will be ſaved to the publick, and will be ready in the publick treaſury to be employed, in any manner that may be thought fit, towards the diminution of the national debt. The amount of theſe annuities is 31,400 times £57.2049, or £1,796,233.8600. Now, if this ſum of £1,796,233.8600 is immediately laid out either in paying off ſome of the capital of the 4 per cent. annuities at *par*, according to the firſt method above-deſcribed, or in buying up ſome of the capital of the 3 per cent. annuities at the price of £75 per cent.

Of the ſaving made at the end of the 31ſt year by the deaths of ſome of the life-annuitants.

Of the manner in which the ſaid ſaving is to be employ'd during the remaining 29 years of the ſaid term of 60 years.

cent. according to the ſecond method above-deſcribed; and the intereſt of the capital thereby extinguiſhed, or the portion of the perpetual annuities that will be redeemed at the end of the next, or 32d, year, by thus employing the ſaid ſum of £1,796,233.8600, be likewiſe employed in the ſame manner; and the intereſt of the capital extinguiſhed by this ſecond operation, or the portion of the perpetual annuities that will thereby be redeemed at the end of the 33d year, be likewiſe employed in the ſame manner; and all the following portions of the perpetual annuities that will be thus redeemed by means of this firſt ſum of £1,796,233.8600, that is ſaved to the nation at the end of the 31ſt year by the deaths of the aforeſaid 31400 life-annuitants, be likewiſe employed in the ſame manner during the whole remainder of the ſaid term of 60 years; ——I ſay, if theſe ſucceſſive ſums of money be ſo employed without interruption during the remainder of the ſaid term of 60 years, the quantity of the capital of the national debt that will be thereby extinguiſhed at the end of the ſaid term of 60 years, or at the end of 29 years from the time when this firſt ſaving of £1,796,233.8600 will have accrued, will be equal in value to the ſum to which the ſaid firſt ſaving of £1,796,233.8600 would have increaſed in the ſame period of 29 years, if it had been improved all the time at compound intereſt at the rate of 4 per cent. and therefore it will (according to Mr. Smart's firſt table of compound intereſt, page 54,) be equal in value to the ſum of £1,796,233.8600 × 3.118,651, or £5,601,826.5237; the intereſt of which at 4 per cent. is $\left(\frac{4}{100} \times £5,601,826.5237, \text{ or } \frac{£22,407,306.0948}{100}, \text{ or}\right)$ £224,073.0609 *per annum*. Therefore the quantity of the intereſt of the national debt, or of the perpetual annuities now due to the publick creditors, which would be redeemed at the end of the whole term of 60 years by means of this firſt ſaving of £1,796,233.8600, which would have accrued to the publick at the end of the 31ſt year of the ſaid term by the deaths of the aforeſaid 31,400 life-annuitants, would be £224,073.0609 *per annum*.

The quantity of the capital of the national debt that will have been extinguiſhed by means of the ſaid ſaving at the end of the ſaid term of 60 years.

The quantity of the intereſt of the ſaid debt that will have been thereby redeemed at the end of the ſaid term of 60 years.

Of the ſaving made at the end of the thirty-ſecond year by the deaths of life-annuitants.

CCLXXXVI. In the courſe of the thirty-ſecond year of this term the 38,000 life-annuitants of the age of 66 years who were living at the end of the thirty-firſt year, or at the age of 66 years, will be reduced to 36,400. Therefore 1600 more of the ſaid life-annuitants will have died in the courſe of this thirty-ſecond year. And conſequently at the end of this thirty-ſecond year there will be a ſaving to the Government of (31,400 + 1,600, or) 33,000 payments of £57.2049 each, which would have then become due to the 33,000 life annuitants who will have died in the thirty-

two

two preceeding years, if they had lived till that time. There will therefore be in the hands of Government at the end of this thirty-ſecond year the ſum of 33,000 × £57.2049, or £1,887,761.7000, to be employed during the remainder of the ſaid term of 60 years, that is, during 28 years, in diminiſhing the national debt; which will be ſufficient to extinguiſh as much of the capital of the ſaid debt as is equal in value to the ſum to which this ſecond ſaving of £1,887,761.7000 will increaſe in the courſe of 28 years, if it be conſtantly improved during that time at compound intereſt at the rate of 4 per cent. Now the amount of £1,887,761.7000 at the end of 28 years, when improved during that time at compound intereſt at the rate of 4 per cent. appears by Mr. Smart's firſt table of compound intereſt, page 54, to be £1,887,761.7000 × 2.998,703, or £5,660,836.6730; the intereſt of which at 4 per cent. is $\left(\frac{4}{100} \times £5,660,836.6730, \text{ or } \frac{£22,643,346.6920}{100}, \text{ or}\right)$ £226,433.4669 *per annum*.

Therefore the quantity of the capital of the national debt which would be extinguiſhed at the end of the whole term of 60 years by means of this ſecond ſaving of £1,887,761.7000, (which would have accrued to the publick at the end of the thirty ſecond year of the ſaid term by the deaths of the aforeſaid 33,000 life-annuitants who would have died in the preceeding 32 years,) would be equal in value to the ſum of £5,660,836.6730; and the quantity of the intereſt of the ſaid debt, or of the perpetual annuities now due to the publick creditors, which would be redeemed at the end of the ſaid term of 60 years by means of the ſaid ſecond ſaving, would be the ſum of £226,433.4669 *per annum*. Q E I.

Of the ſavings that will be made at the ends of the 33d, 34th, 35th, and other following years of the ſaid term of 60 years, by the deaths of life-annuitants.

CCLXXXVII. In the ſame manner we muſt proceed to inveſtigate the ſavings that will accrue to the publick at the ends of the thirty-third, the thirty-fourth, the thirty-fifth, the thirty-ſixth, and every following year of the whole term of 60 years, and the quantities of the capital of the national debt that will be extinguiſhed by each of the ſaid ſavings at the end of the ſaid term of 60 years, if they are conſtantly employed for that purpoſe, in either the firſt or the ſecond method above deſcribed, during the ſeveral remaining years of the ſaid term after the times when they will have reſpectively accrued. And, in order to theſe inveſtigations, it will be neceſſary, in the firſt place, to aſcertain, by the help of Monſieur de Parcieux's table of the probabilities of the duration of human life, the numbers of life-annuitants who will be alive at the ends of the ſeveral remaining years of this term, and thence to determine the numbers of thoſe who will have died in each of the ſaid years, and from whoſe deaths the ſeveral new and ſucceſſive ſavings to the publick will ariſe.

The numbers of life-annuitants who will be living at the ends of all the ſaid years, except the laſt year.

CCLXXXVIII. Now of the aforeſaid original number of 69,400 life-annuitants, all of the age of 35 years, who were ſuppoſed to be living at the beginning of the ſaid term of 60 years, and for whoſe lives (as well as for a term certain of 30 years,) the ſaid annuities, of £57.2049 each, were ſuppoſed to be granted, it appears from Monſieur de Parcieux's table of probabilities that the following numbers will be living at the ſubſequent ages of 68 years, 69 years, 70 years, 71 years, &c. as far as the age of 94 years incluſive, or at the ends of the thirty-third, the thirty-fourth, the thirty-fifth, the thirty-ſixth, and every following year of the ſaid term of 60 years, except the laſt; to wit,

34,700,	21,100,	8,500,	1,600,
32,900,	19,200,	7,100,	1,100,
31,000,	17,300,	5,900,	700,
29,100,	15,400,	4,800,	400,
27,100,	13,600,	3,800,	200,
25,100,	11,800,	2,900,	100.
23,100,	10,100,	2,200,	

The numbers of life-annuitants who will have died in each of the ſaid years, and in the laſt year.

Therefore the numbers of the ſaid life-annuitants who will have died in each of thoſe years, to wit, the 33d, the 34th, the 35th, the 36th, &c. and in the laſt year of the ſaid period of 60 years, or from the age of 94 years to the age of 95 years, are as follows; to wit,

1700,	2000,	1600,		600,
1800,	1900,	1400,		500,
1900,	1900,	1200,		400,
1900,	1900,	1100,		300,
2000,	1800,	1000,		200,
2000,	1800,	900,		100,
2000,	1700,	700,	and	100.

The ſavings that will be made at the ends of the ſaid years in conſequence of the deaths of the ſaid life-annuitants.

CCLXXXIX. Therefore the ſavings of intereſt that will be made by the publick at the ends of thoſe years in conſequence of the deaths of life-annuitants that will have happened in the courſe of them reſpectively, will be as follows; to wit,

At

At the end of the thirty-third year,

$\overline{33{,}000 + 1{,}700} \times £57.2049,$

or $34{,}700 \times £57.2049, = £1{,}985{,}010.0300.$

At the end of the thirty-fourth year,

$\overline{34{,}700 + 1800} \times £57.2049,$

or $36{,}500 \times £57.2049, = £2{,}087{,}978.8500.$

At the end of the thirty-fifth year,

$\overline{36{,}500 + 1900} \times £57.2049,$

or $38{,}400 \times £57.2049, = £2{,}196{,}668.1600.$

At the end of the thirty-fixth year,

$\overline{38{,}400 + 1900} \times £57.2049,$

or $40{,}300 \times £57.2049, = £2{,}305{,}357.4700.$

At the end of the thirty-feventh year,

$\overline{40{,}300 + 2000} \times £57.2049,$

or $42{,}300 \times £57.2049, = £2{,}419{,}767.2700.$

At the end of the thirty-eighth year,

$\overline{42{,}300 + 2000} \times £57.2049,$

or $44{,}300 \times £57.2049, = £2{,}534{,}177.0700.$

At the end of the thirty-ninth year,

$\overline{44{,}300 + 2000} \times £57.2049,$

or $46{,}300 \times £57.2049, = £2{,}648{,}586.8700.$

At the end of the fortieth year,

$\overline{\overline{46,300 + 2000}} \times £57.2049,$

or $48,300 \times £57.2049, = £2,762,996.6700.$

At the end of the forty-firſt year,

$\overline{\overline{48,300 + 1900}} \times £57.2049,$

or $50,200 \times £57.2049, = £2,871,685.9800.$

At the end of the forty-ſecond year,

$\overline{\overline{50,200 + 1900}} \times £57.2049,$

or $52,100 \times £57.2049, = £2,980,375.2900.$

At the end of the forty-third year,

$\overline{\overline{52,100 + 1900}} \times £57.2049,$

or $54,000 \times £57.2049, = £3,089,064.6000.$

At the end of the forty-fourth year,

$\overline{\overline{54,000 + 1800}} \times £57.2049,$

or $55,800 \times £57.2049, = £3,192,033.4200.$

At the end of the forty-fifth year,

$\overline{\overline{55,800 + 1800}} \times £57.2049,$

or $57,600 \times £57.2049, = £3,295,002.2400.$

At the end of the forty-ſixth year,

$\overline{\overline{57,600 + 1700}} \times £57.2049,$

or $59,300 \times £57.2049, = £3,392,250.5700.$

At

At the end of the forty-ſeventh year,

$\overline{59{,}300 + 1600|} \times £57.2049,$

or $60{,}900 \times £57.2049, = £3{,}483{,}778.4100.$

At the end of the forty-eighth year,

$\overline{60{,}900 + 1400|} \times £57.2049,$

or $62{,}300 \times £57.2049, = £3{,}563{,}865.2700.$

At the end of the forty-ninth year,

$\overline{62{,}300 + 1200|} \times £57.2049,$

or $63{,}500 \times £57.2049, = £3{,}632{,}511.1500.$

At the end of the fiftieth year,

$\overline{63{,}500 + 1100|} \times £57.2049,$

or $64{,}600 \times £57.2049, = £3{,}695{,}436.5400.$

At the end of the fifty-firſt year,

$\overline{64{,}600 + 1000|} \times £57.2049,$

or $65{,}600 \times £57.2049, = £3{,}752{,}641.4400.$

At the end of the fifty-ſecond year,

$\overline{65{,}600 + 900|} \times £57.2049,$

or $66{,}500 \times £57.2049, = £3{,}804{,}125.8500.$

At the end of the fifty-third year,

$\overline{66{,}500 + 700|} \times £57.2049,$

or $67{,}200 \times £57.2049, = £3{,}844{,}169.2800.$

At the end of the fifty-fourth year,

$$\overline{67{,}200 + 600} \times £57.2049,$$

or $67{,}800 \times £57.2049, = £3{,}878{,}492.2200.$

At the end of the fifty-fifth year,

$$\overline{67{,}800 + 500} \times £57.2049,$$

or $68{,}300 \times £57.2049, = £3{,}907{,}094.6700.$

At the end of the fifty-sixth year,

$$\overline{68{,}300 + 400} \times £57.2049,$$

or $68{,}700 \times £57.2049, = £3{,}929{,}976.6300.$

At the end of the fifty-seventh year,

$$\overline{68{,}700 + 300} \times £57.2049,$$

or $69{,}000 \times £57.2049, = £3{,}947{,}138.1000.$

At the end of the fifty-eighth year,

$$\overline{69{,}000 + 200} \times £57.2049,$$

or $69{,}200 \times £57.2049, = £3{,}958{,}579.0800.$

At the end of the fifty-ninth year,

$$\overline{69{,}200 + 100} \times £57.2049,$$

or $69{,}300 \times £57.2049, = £3{,}964{,}299.5700.$

And at the end of the sixtieth and last year,

$$\overline{69{,}300 + 100} \times £57.2049,$$

or $69{,}400 \times £57.2049, = £3{,}970{,}020.0600.$

CCXC. All

CCXC. All theſe ſavings are to be laid out in diminiſhing the national debt in either the firſt or the ſecond method above-deſcribed; by which means the portion of the national debt that will have been extinguiſhed at the end of the ſaid term of 60 years in conſequence of each of theſe ſavings, except the laſt, to wit, £3,970,020.0600, or (neglecting the fraction .0600,) £3,970,020, will be equal in value to a ſum that is greater than ſuch ſaving, namely, to the ſum which is the amount to which ſuch ſaving will have increaſed at the end of the ſaid term, if we ſuppoſe it to be improved in the mean time at compound intereſt at the rate of 4 per cent. We muſt therefore, in the next place, compute the amounts of theſe ſavings at the end of the ſaid term, if ſo improved at compound intereſt in the mean time, or during the remaining years of the ſaid term after the times at which they will have reſpectively accrued, that is, during 27 years, 26 years, 25 years, 24 years, &c. to the laſt year of the ſaid term. Now theſe amounts will be as follows.

The ſaid ſavings are to be employed in diminiſhing the national debt, in either the firſt or the ſecond method above-deſcribed.

CCXCI. The amount of £1,985,010.0300, improved at compound intereſt at the rate of 4 per cent. during 27 years, is = £1,985,010.0300 × 2.883,368 = £5,723,514.4001.

The amount of £2,087,978.8500, improved in the ſame manner during 26 years, is £2,087,978.8500 × 2.772,469 = £5,788,856.6342.

The amount of £2,196,668.1600, improved in the ſame manner during 25 years, is £2,196.668.1600 × 2.665,836 = £5,855,957.0609.

And the amounts of all the following ſavings, improved in the ſame manner during the remaining years of the ſaid term of 60 years after the times at which they will have reſpectively accrued to the publick, will be as follows.

A computation of the amounts of all the ſaid ſavings, except the laſt, at the end of the ſaid term of 60 years, if improved in the mean time at compound intereſt at the rate of 4 per cent.

£2,305,357.4700	×	2.563,304	=	£5,909,332.0242.
£2,419,767.27	×	2.464,715	=	£5,964,036.6868.
£2,534,177.0700	×	2.369,918	=	£6,005,791.8533.
£2,648,586.8700	×	2.278,768	=	£6,035,515.0045.
£2,762,996.6700	×	2.191,123	=	£6,054,065.5525.
£2,871,685.9800	×	2.106,849	=	£6,050,208.7352.
£2,980,375.2900	×	2.025,816	=	£6,037,691.9484.
£3,089,064.6000	×	1.947,900	=	£6,017,188.9343.
				£3,192,033.4200

	£3,192,033.4200	×	1.872,981	= £5,978,617.9647.
	£3,295,002.2400	×	1.800,943	= £5,934,111.2191.
	£3,392,250.5700	×	1.731,676	= £5,874,278.8980.
	£3,483,778.4100	×	1.665,073	= £5,800,745.3684.
	£3,563,865.2700	×	1.601,032	= £5,705,862.3409.
	£3,632,511.1500	×	1.539,454	= £5,592,083.8199.
	£3,695,436.5400	×	1.480,244	= £5,476,147.7657.
	£3,752,641.4400	×	1.423,311	= £5,341,175.8406.
	£3,804,125.8500	×	1.368,569	= £5,206,208.7104.
	£3,844,169.2800	×	1.315,931	= £5,058,661.5248.
	£3,878,492.2200	×	1.265,319	= £4,907,529.8973.
	£3,907,094.6700	×	1.216,652	= £4,753,574.5444.
	£3,929,976.6300	×	1.169,858	= £4,597,514.6004.
	£3,947,138.1000	×	1.124,864	= £4,439,993.5517.
	£3,958,579.0800	×	1.081,600	= £4,281,599.1329.
And	£3,964,299.5700	×	1.040,000	= £4,122,871.5528.

The addition of the amounts of all the yearly savings at the end of the said term of 60 years, into one sum.

CCXCII. Having thus found the amounts to which the several yearly savings made at the ends of the thirty-first year and every following year of the said term of 60 years, except the last, will have increased at the end of the said term by being improved in the mean time at compound interest at the rate of 4 per cent. we must now add all these amounts, together with the saving of £3,970,020.0600, which will accrue at the end of the last year, into one sum: which may be done as follows.

The amounts of the savings made in the first period of six years after the expiration of the term certain of 30 years, and in the second period of six years after the expiration of the said term, and in the third period of six years after the expiration of the said term, will be as follows.

£	£	£
5,601,826.5237	5,964,036.6868	6,017,138.9343
5,660,836.6730	6,005,791.8533	5,978,617.9647
5,723,514.4001	6,035,515.0045	5,934,111.2191
5,788,856.6342	6,054,065.5525	5,874,278.8980
5,855,957.0609	6,050,208.7352	5,800,745.3684
5,909,332.0242	6,037,691.9484	5,705,862.3409
34,540,323.3161	36,147,309.7807	35,310,804.7254

And

And the amounts of the ſavings made in the fourth period of ſix years and the fifth period of ſix years after the expiration of the ſaid term certain of 30 years, will be as follows.

£	£
5,592,083.8199	4,753,574.5444
5,476,147.7657	4,597,514.6004
5,341,175.8406	4,439,993.5517
5,206,208.7104	4,281,599.1329
5,058,661.5248	4,122,871.5528
4,907,529.8973	3,970,020.0600
31,581,807.5587	26,165,573.4422

Therefore the ſum total of the amounts of all theſe ſavings at the end of the ſaid term of 60 years, is £163,745,818 8231.

£
34,540,323.3161
36,147,309.7807
35,310,804.7254
31,581,807.5587
26,165,573.4422
163,745,818.8231

CCXCIII. Therefore the quantity of the capital of the national debt that would have been extinguiſhed, by means of all theſe ſavings, at the end of the ſaid term of 60 years, will be equal in value to the ſaid ſum of £163,745,818.8231; the annual intereſt of which at 4 per cent. is $\left(\frac{4}{100} \times £163,745,818.8231, \text{ or } \frac{£\ 654,983,275.2924}{100}, \text{ or}\right)$ £6,549,832.7529. Therefore the quantity of the intereſt of the national debt, or of the perpetual annuities now due to the publick creditors, that would be redeemed to the publick at the end of the ſaid term of 60 years by means of all theſe ſavings, (ſo that at the end of the next, or 61ſt, year it would be free to be diſpoſed of by the Parliament in any other way,) would be £6,549,832.7529 *per annum*, or (neglecting the fractional part .7529,) £6,549,832 *per annum*. Q E I.

The quantity of the national debt that will have been extinguiſhed by means of all the foregoing ſavings, at the end of the ſaid term of 60 years.

The quantity of the intereſt of the national debt that will be then redeemed to the publick by means of the ſaid ſavings.

The whole quantity of the intereſt of the national debt that will be redeemed to the publick at the end of the ſaid term of 60 years by the foregoing operation.

CCXCIV. This ſum of £6,549,832 *per annum* muſt therefore be added to £2,970,020 *per annum*; which was the quantity of the perpetual annuities, now due to the publick creditors, which was ſuppoſed to be converted into theſe compound, temporary, annuities for 30 years certain and for the lives of 69400 perſons, all of the age of 35 years; and which will therefore be now wholly redeemed to the publick by the expiration of the ſaid term of 30 years, and the deaths of all the ſaid life-annuitants: and the ſum of both, or the ſum of £9,519,852 *per annum*, will be the whole quantity of the intereſt of the national debt, or of the perpetual annuities employed in the payment of it, which will be redeemed to the publick at the end of the ſaid term of 60 years, (ſo as to be free at the end of the next, or 61ſt, year to be diſpoſed of by the Parliament in any other way,) by the faithful application of a million of pounds, ſterling, a year to that ſalutary purpoſe during that period according to this fifth method. Q E I.

Agreement between the effect of the foregoing ſcheme, "of life-annuities with a term certain annexed to them," and the effects of the four preceeding methods.

CCXCV. This ſum of £9,519,852 *per annum* may be conſidered as equal to the quantity of the perpetual annuities, now due to the publick creditors, which would have been redeemed to the publick in the ſame ſpace of 60 years by employing the ſame annual ſum of a million of pounds, ſterling, for the ſame purpoſe in either of the four preceeding methods; which quantity we have ſeen in the foregoing pages to be £9,520,000 *per annum*, or, more accurately, $\left(\frac{4}{100} \times £237,990,685.24,\right.$ or $\frac{£951,962,740.96}{100}$, or $\left.\right)$ £9,519,627.4096, or (neglecting the fraction .4096,) £9,519,627 *per annum*. For the difference between them is only £225 *per annum*, which, upon ſo large a ſum as £9,519,627, is a mere triſle.

General concluſion concerning the efficacy of this fifth method of applying a given ſum of money every year to the diminution of the national debt.

CCXCVI. We may therefore conclude that this fifth method of employing a million of pounds, ſterling, or any other given ſum of money, every year to the purpoſe of diminiſhing the national debt, "by converting ſome of the perpetual annuities, now due to the publick creditors, into theſe compound, temporary, annuities, which are to continue at all events during a certain term of years, and are alſo to be extended contingently beyond the ſaid term and during the lives of certain life-annuitants," would be juſt as efficacious in redeeming the perpetual annuities that are now due to the publick creditors as the intereſt of the ſaid debt, and therefore would be juſt as beneficial to the nation as either of the four preceeding methods of applying the ſame ſum of money to the ſame purpoſe

purpoſe during the ſame period of time: which was the principal object of the foregoing computation. But this, it muſt be remembered, is true only upon a ſuppoſition that the annual ſavings which will accrue to the publick at the end of every year after the expiration of the ſaid term certain (during which the ſaid annuities are to continue at all events,) by the deaths of the ſaid life-annuitants, are immediately employed in diſcharging a part of that debt in either the firſt or the ſecond method above-deſcribed, or in ſome other method that is equivalent to them. For, if thoſe ſavings are not ſo employed, but are diverted to other uſes, the converſion of the perpetual annuities, now due to the publick creditors, into compound, temporary, annuities of the kind deſcribed in this fifth method, would be evidently a much *leſs* efficacious method of diminiſhing the national debt than either of the four preceeding methods. Thus, for example, we have ſeen that, by employing the ſaid ſavings in the manner above-deſcribed, the ſum of one million of pounds, ſterling, a year, employed in this fifth method, will be ſufficient to redeem to the publick, in the courſe of 60 years, the ſum of £9,519,852 *per annum* of the intereſt of the national debt; but without the help of the ſaid ſavings, it will redeem in that time only £2,970,020 *per annum* of the ſaid intereſt. Therefore, whenever any propoſal is made to pay off any part of the national debt by means of theſe compound, temporary, annuities, (which are to continue for a certain number of years at all events, and afterwards during the lives of certain life-annuitants,) great care muſt be taken that the ſavings of the publick revenue, which will accrue every year to the publick, (after the expiration of the term certain, during which ſuch annuities are to continue at all events,) by the deaths of the ſaid life-annuitants, ſhall be conſtantly applied to the ſame purpoſe; or otherwiſe the nation muſt (as Dr. Price has obſerved concerning the converſion of the perpetual annuities, now due to the publick creditors, into life-annuities,) loſe greatly by all ſchemes of this kind.

A ſixth method of employing a given ſum of money every year in the reduction of the national debt, by means of life-annuities with benefit of ſurvivorſhip during a certain term of years.

CCXCVII. A ſixth method of applying a given ſum of money every year to the diminution of the national debt, might be grafted on the fourth method, in which ſome of the perpetual annuities, now due to the publick creditors, are ſuppoſed to be converted into life-annuities. For, inſtead of ſuppoſing (as we did in the ſaid fourth method,) that, upon the deaths of the ſaid life-annuitants, their annuities ſhall revert to the publick, and be immediately employed, for the purpoſe of diminiſhing

the national debt, in either the firſt, or the ſecond, method above-mentioned, we may ſuppoſe them to accrue to the ſurviving life annuitants, and to be equally divided amongſt them, untill their annuities (which, it is evident, will, upon this plan, be continually increaſing,) ſhall have increaſed to ten times their original quantity, or to ſome other very great ſum, or during a certain number of years leſs than the whole number of years during which it is probable that ſome few of the ſaid life-annuitants may live: and then the annuities of thoſe life-annuitants who ſhall die after the expiration of the ſaid term of years, or after the annuities of the ſurviving life-annuitants ſhall have been increaſed to their greateſt allowed magnitude, may be ſuppoſed to accrue to the publick, (as in the aforeſaid fourth method,) and to be employed, for the purpoſe of diminiſhing the national debt, in either the firſt, or the ſecond, method above-mentioned. And, if this plan were adopted, the annual ſum of money, which could be ſpared from the publick revenue for this purpoſe, might be employed in paying to the proprietors of the perpetual annuities ſo converted into temporary ones, the additions which it would be neceſſary to make to the ſaid perpetual annuities in compenſation for the ſaid abridgement of their duration.

In this method, as well as in the four preceeding methods, it is ſuppoſed that the proprietors of the perpetual annuities that are ſo to be converted into temporary ones, have previouſly conſented to ſuch converſion.

CCXCVIII. Thus, for example, if a million of pounds, ſterling, *per annum* could be ſpared from the publick revenue for this purpoſe, the Government might convert a proportionable quantity of the perpetual annuities, now due to the publick creditors, with the conſent of the proprietors of ſuch annuities, into annuities for the lives of the ſaid proprietors, or of ſuch perſons as they ſhould name, with the benefit of ſucceeding to each other's annuities by ſurvivorſhip during the ſpace of 30 years; after which the annuities of the then ſurviving life-annuitants ſhould no longer accrue, upon the deaths of the ſaid life-annuitants, to their ſurviving companions, but ſhould belong to the publick, and be employed every year in the diminution of the national debt in either the firſt, or the ſecond, method above-mentioned. And the ſaid million of pounds *per annum* would, in ſuch caſe, be employed in paying the additions which it would be neceſſary to make to the perpetual annuities that ſhould be ſo converted, as a compenſation to the proprietors of them for the abridgement of the time of their continuance.

CCXCIX. If

CCXCIX. If a given ſum of money were to be employed every year for the purpoſe of diminiſhing the national debt in this ſixth method, its effect would be exactly the ſame in a given period of time as it would be, if employed for the ſame purpoſe during the ſame period in either of the five preceeding methods; that is, the quantity of the intereſt of the national debt, or of the perpetual annuities now due to the publick creditors, that would be redeemed to the publick by the application of a given annual ſum of money to the diminution of the national debt during a given number of years in this ſixth method would be exactly the ſame as that which would be redeemed to the publick by applying the ſame ſum of money every year to the ſame purpoſe during the ſame number of years in either of the five preceeding methods. Of this I will now proceed to give an example.

This ſixth method of diminiſhing the national debt, if properly purſued, would be equally effectual for that purpoſe with either of the five preceeding methods.

An example of the ſaid ſixth method of employing a given ſum of money every year in the reduction of the national debt.

CCC. Let the ſum of money to be employed in the reduction of the national debt according to this ſixth method be (as in the former examples,) a million of pounds, ſterling, a year; and the period during which it is to be ſo employed, be alſo the ſame as was choſen in the ſaid former examples, to wit, a period of 60 years. And let the proprietors of the perpetual annuities which are to be converted into life-annuities with benefit of ſurvivorſhip according to this ſixth method, or the *nominees* of the ſaid proprietors, during whoſe lives the new life-annuities are to continue, be all of the age of 35 years; ſo that it may reaſonably be expected, (according to Monſieur de Parcieux's table of the probabilities of the duration of human life,) that the laſt ſurvivor of them will live almoſt through the whole period of 60 years, and die a few days before the end of it. And let the number of the life-annuitants for whoſe lives the new annuities are to continue, be ſuppoſed to be the ſame as in the examples above given of the foregoing fourth and fifth methods, to wit, 69,400. And let the ſaid 69,400 life-annuitants, (who are all of the age of 35 years at the time of granting the ſaid life-annuities,) be ſuppoſed to die off every year in the proportions ſet forth in Monſieur de Parcieux's table of the probabilities of the duration of human life. And, laſtly, let the intereſt of money be ſuppoſed to be 4 per cent. and the price at which the 3 per cent. annuities ſell at the publick market, to be £75 per cent.

Preliminary ſuppoſitions.

CCCI. Theſe

An inveſtigation of the quantity of the perpetual annuities, now due to the publick creditors, which might be converted into theſe compound life-annuities, with benefit of ſurvivorſhip, by the help of a million of pounds, ſterling, *per annum*.

CCCI. Theſe things being premiſed, we muſt, in the next place, inquire what quantity of the perpetual annuities, now due to the publick creditors, might, by the help of a million of pounds, ſterling, a year, be converted into temporary annuities of the kind above-deſcribed. Now this may be determined in the manner following.

Let us firſt ſuppoſe that theſe 69,400 life-annuitants (who are all of the age of 35 years,) receive from the Government, for an adequate conſideration, grants of annuities of £10 a piece, of the kind juſt now deſcribed, or that ſhall increaſe by ſurvivorſhip during the ſpace of 30 years, and then ſhall continue at their laſt magnitude, (to which they will have increaſed in the courſe of the ſaid 30 years,) during the lives of each of the then ſurviving life-annuitants reſpectively. The ſum of all theſe annuities will be £694,000 *per annum*; which, it is evident, the Government will be obliged to pay for 30 years together to ſuch of the ſaid life-annuitants as ſhall be living to receive it. But at the end of the ſaid term of 30 years the Government will have leſs than £694,000 to pay every year to the ſaid ſurviving life-annuitants; becauſe then the right of ſucceeding to each other's annuities by ſurvivorſhip is ſuppoſed to ceaſe, and the annuities of the life-annuitants who ſhall die after the end of the ſaid 30 years are ſuppoſed to accrue to the publick. The Government will therefore have a ſaving at the end of every year after the expiration of the ſaid 30 years, which may be employed in diminiſhing the national debt in either the firſt or the ſecond method above-mentioned: and theſe ſavings will be greater and greater every year untill all the ſaid life-annuitants are dead, or till the end of the whole term of 60 years. The number of life-annuitants will be reduced in the courſe of 30 years from 69,400 to 39,500; ſo that at the end of the 30th year the ſum of £694,000 will be to be divided between 39,500 perſons, inſtead of 69,400 perſons, and conſequently each of the ſaid 39,500 perſons will receive, for his ſhare of the ſaid £694,000, the ſum of $\left(\frac{£694,000}{39,500}\right.$, or$\left.\right)$ £17.569,620. And this ſum of £17.569,620, which he will be intitled to receive at the end of the ſaid 30th year, he will continue to receive at the end of every following year of his whole life. We may therefore divide the whole annuity which each of the ſaid 69,400 life-annuitants will be intitled to according to the foregoing ſuppoſitions, into the two following parts; to wit, 1ſt, an annuity of £10 a year for the firſt 30 years of his life, and no longer, but accompanied with the benefit of ſucceeding by ſurvivorſhip to the annuities of ſuch of his fellow life-annuitants as ſhould die in the courſe of the ſaid 30 years; by which means the ſaid annuity of £10 a year will gradually increaſe from £10 a year to £17.569,620 a year; and ſecondly, a remote life-annuity of £17.569,620, that is to commence at the end of 30 years, (or whereof the firſt payment is

An original life-annuity of £10 a year for the life of a perſon of the age of 35 years, will increaſe in the courſe of 30 years to £17.569,620 *per annum*.

Diviſion of a compound life-annuity of £10 a year, for the life of a perſon of the age of 35 years, with benefit of ſurvivorſhip during 30 years, into two parts.

is to be received at the end of 31 years,) and that is to continue during the remainder of his life. Now the values of theſe two annuities may be found in the manner following.

A computation of the value of the firſt part of ſuch compound life-annuity.

CCCII. As the ſeveral annuities of thoſe life-annuitants who ſhall die every year during a term of 30 years, are ſuppoſed to be divided amongſt their ſurviving companions, and not to accrue to the publick, it is evident that the price which ought to be paid to the publick by the whole body of the ſaid 69,400 life-annuitants for the enjoyment of the ſaid life-annuities during the ſaid term of 30 years, in caſe they ſhall live ſo long, accompanied with the aforeſaid benefit of ſurvivorſhip during the ſaid term, will be the ſame that ought to be paid by them for the ſame annuities, if the ſaid annuities were to be granted to them for a term certain of 30 years, ſo as to become payable to their executors, or other perſonal repreſentatives, in caſe the ſaid life-annuitants themſelves ſhould happen to die in the courſe of the ſaid term: becauſe it would be a matter of indifference to the publick, whether the annuities of the life-annuitants that died in the courſe of the ſaid term of 30 years, were made payable, during the ſeveral remaining years of the ſaid term, to the executors, or other perſonal repreſentatives, of the ſaid deceaſed life-annuitants, or to their ſurviving companions, who would, in ſuch caſe, ſupply the place of their executors. And conſequently, as all the ſaid 69,400 life-annuitants are ſuppoſed to be of the ſame age of 35 years, and there is no reaſon that any one of them ſhould pay more for his annuity than any other; the price which each of them ought to pay for the enjoyment of his life-annuity during the ſaid term of 30 years, or ſuch part of the ſaid term as he ſhall happen to live through, when accompanied with this benefit of ſucceeding, by ſurvivorſhip, to his proportional ſhare of the annuities of thoſe of his companions who ſhall die in the courſe of the ſaid term, will be the ſame as he ought to pay for the purchaſe of the ſame annuity for the ſame term of 30 years, when it is to be enjoyed at all events during the whole of the ſaid term either by himſelf or his executor, or other perſonal repreſentative, but without any chance of augmentation. Now it appears by Mr. Smart's fourth table of compound intereſt, page 78, that when the intereſt of money is 4 per cent. the value of an annuity of one pound a year for a term of 30 years certain (or not depending on the continuance of any life, or any other contingency,) is £17.292,033. Therefore the value of an annuity of £10 a year for the ſame term of 30 years is £172.920,33. And conſequently the value of a life-annuity of £10 a year for the life of one of the aforeſaid 69,400 life-annuitants of the age of 35 years, during the ſame term of 30 years, if he ſhall ſo long live, but not to continue after the ſaid term, accompanied with the above-mentioned benefit of ſucceeding, by ſurvivorſhip, to the annuities of ſuch of his fellow life-annuitants as ſhall die in the courſe of the ſaid term, will alſo be £172.920,33. Q.E.I.

CCCIII. The

Accomputation of the value of the fecond part of fuch compound life-annuity.

CCCIII. The other annuity, of which we were to determine the value, is a remote life-annuity of £17.569,620 *per annum*, for the life of a perſon of the age of 35 years, that is to commence at the end of 30 years, or whereof the firſt payment is to be received at the end of 31 years, the intereſt of money being 4 per cent. Now the value of a life-annuity of only one pound a year of this remote kind for the life of a perſon of the age of 35 years, when the intereſt of money is 4 per cent. appears above (in Table XXV, page 285,) to be £1.410,77. Therefore the value of a life-annuity of £17.569,620 *per annum* of this remote kind for the life of a perſon of the age of 35 years, is = 17.569,620 × £1.410,77, or £24.786,692. Q E I.

The value of the whole of the ſaid compound life-annuity of 1*o*l. *per annum*.

CCCIV. Therefore the value of both the foregoing annuities, or of the whole annuity which is ſuppoſed to be granted by the Government to each of the aforeſaid 69,400 life-annuitants of the age of 35 years, is £172.920,33 + £24.786,69, or £197.707,02. Therefore £197.707,02 is the price which each of the ſaid 69,400 life-annuitants ought to pay to the Government for a grant of an annuity of £10 a year of the kind above-deſcribed. Q E I.

The ſaid compound life-annuity of 10*l*. *per annum* is equal in value to a perpetual annuity of £7.908,28 *per annum*.

CCCV. Now, when the intereſt of money is 4 per cent. the intereſt of £197.707,02 will be $\left(\frac{4}{100} \times £197.707,02, \text{ or } \frac{790.828,08}{100}, \text{ or}\right)$ £7.908,28 *per annum*; and conſequently a perpetual annuity of £7 908,28 *per annum* is worth the ſum of £197.707,02. Therefore a perpetual annuity of £7.908,28 *per annum* is equal in value to a life-annuity of the kind above-deſcribed of £10 *per annum* for the life of a perſon of the age of 35 years. Therefore the 69,400 life-annuities of £10 each which are above ſuppoſed to be granted by the Government to 69,400 perſons, all of the age of 35 years, will be equal in value to 69,400 perpetual annuities of £7.908,28 *per annum*, or to a perpetual annuity of 69,400 × £7.908,28, or £548,834.63200 *per annum*. Therefore, if the yearly ſum of £548,834.63200 of the perpetual annuities, now due to the publick creditors, were the property of the ſaid 69,400 perſons of the age of 35 years, to whom we ſuppoſed above that the Government had granted life-annuities of £10 each of the kind above-deſcribed, and the ſaid 69,400 life-annuitants were to reſign to the Government their right to the ſaid annual ſum of £548,834.63200 of the perpetual annuities, in exchange for, and as the price of, their ſaid life-annuities of £10 each, or the annual ſum of £694,000 amongſt them, ſuch a bargain would be fair and equal

on

on both ſides. Therefore the difference between £694,000 *per annum* and £548,834.63200 *per annum*, that is, the ſum of £145,165.368 0 *per annum*, would be the addition which it would be neceſſary for the Government to make to the ſaid ſum of £548,834.63200 of the perpetual annuities, upon their being converted into life-annuities of the kind above-deſcribed, as a compenſation to the proprietors of them for the abridgement of the time of their continuance. And conſequently, ſince the additional ſum of £145,165.36800 *per annum* would be ſufficient to enable the Government to convert the ſum of £548,834.63200 *per annum* of the perpetual annuities into life-annuities of the kind above-deſcribed, the additional ſum of a million of pounds *per annum* would be ſufficient to enable the Government to convert into life-annuities of the ſame kind a quantity of the perpetual annuities that is greater than £548,834.63200 *per annum* in the ſame proportion in which £1000,000 *per annum* is greater than £145,165.36800 *per annum*, or a quantity of the perpetual annuities that is equal to $\left(\frac{1000,000}{145,165.36800} \times £548,834.63200\right.$, or $\frac{548,834,632,000}{145,165.368,00}$, or) £3,780,754 *per annum*. Therefore the quantity which we propoſed in Art. cccı as neceſſary to be determined, to wit, the quantity of the perpetual annuities, now due to the publick creditors, which might be converted into life-annuities of the kind above-deſcribed by the help of a fund of a million of pounds *per annum*, is the ſum of £3,780,754 *per annum*. Q E I.

The quantity of the perpetual annuities mentioned in Art. 301 as neceſſary to be determined, is the ſum of £3,780,754 *per annum*.

CCCVI. Let us now therefore ſuppoſe that the Government were to employ the ſaid fund of a million of pounds *per annum* in converting £3,780,754 *per annum* of the perpetual annuities into life-annuities of the kind above-deſcribed; and that the number of perſons of the age of 35 years, to whom, or for whoſe lives, the ſaid life-annuities were to be granted, was 69,400. Then it is evident that the whole ſum of money which would be paid every year to theſe 69,400 life-annuitants, or the ſurvivors of them, for the ſpace of 30 years, would be £3,780,754 *per annum* together with the ſaid additional ſum of £1,000,000 *per annum*, which would be neceſſary to enable the Government to make this converſion; that is, it would be £4,780,754 *per annum*. And conſequently the life-annuity which would be due to each of the ſaid 69,400 life-annuitants at firſt, or before any of his companions had died, would be equal to $\frac{£4,780,754}{69,400}$, or £68.886,94 *per annum*.

The original quantity of each man's life annuity, upon a ſuppoſition that the whole ſum of £3,780,754 *per annum* of the perpetual annuities is converted into theſe life-annuities, and that the number of the life-annuitants continues as before) to be 69,400, is £68.88694 *per annum*.

Another manner of determining the quantity of the ſaid original life-annuity.

CCCVII. The quantity of each of theſe life-annuities may likewiſe be determined in the following manner.

Since, when each of the 69,400 perſons of the age of 35 years was ſuppoſed to have a life-annuity, of the kind above-deſcribed, of £10 a year granted to him by Government, the additional annual ſum of money which was required to enable the Government to convert a portion of the perpetual annuities due to the publick creditors, that was equivalent to the annual ſum of £694,000 (which was thus ſuppoſed to be granted, in life-annuities of £10 a piece, to the ſaid 69,400 life-annuitants,) into the ſaid life-annuities, was £145,165.36800 *per annum*, it follows that, when an additional ſum of £1000,000 *per annum* is employed for the ſame purpoſe, and the number of the life-annuitants, who are to receive theſe life-annuities from Government, ſtill continues the ſame as before, to wit, 69,400, (as is the caſe on the preſent ſuppoſition,) the life-annuity which each of the ſaid perſons will be intitled to upon this ſecond ſuppoſition will be greater than the life-annuity of £10, (to which he was intitled upon the former ſuppoſition,) in the proportion £1000,000 to £145,165.36800, and therefore will be equal to $£10 \times \frac{1,000,000}{145,165.36800}$, or $\frac{£10,000,000}{145,165.36800}$, or £68.88695, which agrees with the number found before, to wit, £68.88694, to ſix places of figures, and differs from it only by an unit in the ſeventh figure. We may therefore be confident that the ſaid firſt ſix figures are true, or that the quantity of each of the original life-annuities belonging to the ſaid 69,400 life-annuitants, or that which he would be intitled to at the end of the firſt year, if none of his companions had died in the courſe of it, is £68.8869 *per annum*.

Of the increaſe of the ſaid original life-annuity of £68 88694 *per annum*, by the benefit of ſurvivorſhip, in the courſe of 30 years.

CCCVIII. We muſt now proceed to inquire to what quantity this original life-annuity, belonging to each of the ſaid 69,400 life-annuitants, will increaſe by the deaths of ſome of the ſaid life-annuitants, in the courſe of the ſaid term of 30 years. Now, ſince, when each of the original life-annuities was ſuppoſed to be £10 a year, it was found (in Art. cccı,) to increaſe, in the courſe of the ſaid 30 years, to £17.569,620 a year, it follows that the original life-annuity of £68.8869, or, more accurately, £68.88694 *per annum*, to which each of the ſaid 69,400 life-annuitants of the age of 35 years will be intitled upon this ſecond ſuppoſition, will increaſe in the courſe of the ſaid 30 years to a quantity that will exceed £68.886,94 *per annum* in the ſame proportion as £17.569,620 exceeds £10, that

that is, to the quantity $\left(\frac{17.569,620}{10} \times £68.88694\right.$, or $\frac{£ 1210.317,368,762,8}{10}$,

or) £121.031,736,876,28, or (neglecting the ſeven laſt figures,) £121.0317. Therefore at the end of the 30th year of the ſaid term of 60 years, each of the ſurvivors, out of the original 69,400 life-annuitants, will be intitled to receive from the Government the ſum of £121.0317; and he will alſo be intitled to receive the ſame ſum at the end of every following year during his whole life; but without any further increaſe, becauſe it is ſuppoſed that from this time the annuities of the ſeveral life-annuitants, who ſhall die in the following years of the ſaid term of 60 years, are to accrue to the publick.

Its laſt and greateſt magnitude will be £121.0317 *per annum.*

CCCIX. That this ſum of £121.0317 is the true quantity of the increaſed annuity, to which each of the ſurviving life-annuitants will be intitled at the end of the 30th year of the ſaid term of 60 years, will likewiſe appear from the following conſideration. The number of the ſaid life-annuitants will, in the courſe of the ſaid 30 years, be reduced from 69,400 to 39,500: and conſequently the ſum total of all the money paid at the end of every year, during the ſaid 30 years, to all the ſurviving life-annuitants together, to wit, the ſum £4,780,754, will be divided at the end of the 30th year amongſt 39,500 perſons; and conſequently the annuity belonging to each of the ſaid 39,500 perſons will be $\frac{£4,780,754}{39,500}$, or £121.0317 *per annum.* QEI.

Another way of determining the ſaid greateſt magnitude.

CCCX. We muſt therefore, in the next place, inquire into the ſavings which will now accrue to the publick at the end of every following year after the 30th, by the deaths of the remaining life-annuitants: which ſavings are to be employed, for the purpoſe of diminiſhing the national debt, in either the firſt, or the ſecond, method above-mentioned.

Of the ſavings that will accrue to the publick after the expiration of the ſaid term of 30 years.

CCCXI. Now it appears from Monſieur de Parcieux's table of the probabilities of the duration of human life, that, out of the 39,500 life-annuitants who will be living at the end of the 30th year of the ſaid term of 60 years, and who will be then of the age of 65 years, only 38,000 will be living at the age of 66 years, or at the end of the 31ſt year of the ſaid term, and that the numbers living at the ſubſequent ages of 67 years, 68 years, 69 years, 70 years, &c. or at the ends of the thirty ſecond year, the thirty-third year, the thirty-fourth year, the thirty-fifth year, and every following year of the ſaid term of 60 years, will be as follows; to wit,

The numbers of the ſaid 69,400 life-annuitants that will be living at the ends of the 31ſt year, and every following year, of the ſaid period of 60 years.

36,400,	21,100,	7,100,	700,
34,700,	19,200,	5,900,	400,
32,900,	17,300,	4,800,	200,
31,000,	15,400,	3,800,	100,
29,100,	13,600,	2,900,	000.
27,100,	11,800,	2,200,	
25,100,	10,100,	1,600,	
23,100,	8,500,	1,100,	

The numbers of the said life-annuitants that will have died in each of the said years.

CCCXII. Therefore the numbers of the said life-annuitants who will have died in the course of the 31st year, the 32d year, the 33d year, the 34th year, and every following year of the whole period of 60 years, in which they will all have died, will be as follows; to wit,

1,500,	2,000,	1,600,	500,
1,600,	2,000,	1,400,	400,
1,700,	1,900,	1,200,	300,
1,800,	1,900,	1,100,	200,
1,900,	1,900,	1000,	100,
1,900,	1,800,	900, and	100.
2,000,	1,800,	700,	
2,000,	1,700,	600,	

The savings that will accrue to the publick at the ends of each of those years by the deaths of the said life-annuitants.

CCCXIII. Therefore at the end of the 31st year there will accrue to the Government a saving of the annuities of the 1500 life-annuitants who will have died in the course of the said year, or which would have become due to them at the end of the said year, if they had been then alive; that is, a saving of 1500 times £121.0317, or of the sum of £181,547.5500. And at the end of the thirty-second year there will accrue to the Government a still greater saving, to wit, a saving of the annuities of the 1500 life-annuitants who will have died in the course of the 31st year, and likewise of the annuities of the 1600 life-annuitants who will have died in the course of the 32d year, that is, a saving of 1500 + 1600 times £121.0317, or 3100 times £121.0317, or of the sum of £375,198.2700. And in like manner we shall find the savings at the ends of the 33d year, the 34th year, the 35th year, the 36th year, and every following year, of the whole term of 60 years, to be as follows; to wit,

At

At the end of the thirty-third year,

$\overline{3{,}100 + 1700} \times £121.0317,$

or $4{,}800 \times £121.0317, = £580{,}952.1600.$

At the end of the thirty-fourth year,

$\overline{4{,}800 + 1800} \times £121.0317,$

or $6{,}600 \times £121.0317, = £798{,}809.2200.$

At the end of the thirty-fifth year,

$\overline{6{,}600 + 1900} \times £121.0317,$

or $8{,}500 \times £121.0317, = £1{,}028{,}769.4500.$

At the end of the thirty-fixth year,

$\overline{8{,}500 + 1900} \times £121.0317,$

or $10{,}400 \times £121.0317, = £1{,}258{,}729.6800.$

At the end of the thirty-feventh year,

$\overline{10{,}400 + 2000} \times £121.0317,$

or $12{,}400 \times £121.0317, = £1{,}500{,}793.0800.$

At the end of the thirty-eighth year,

$\overline{12{,}400 + 2000} \times £121.0317,$

or $14{,}400 \times £121.0317, = £1{,}742{,}856.4800.$

At the end of the thirty-ninth year,

$\overline{14{,}400 + 2000} \times £121.0317,$

or $16{,}400 \times £121.0317, = £1{,}984{,}919.8800.$

At

At the end of the fortieth year,

$\overline{16{,}400 + 2000}$ × £121.0317,

or 18,400 × £121.0317, = £2,226,983.2800.

At the end of the forty-firſt year,

$\overline{18{,}400 + 1900}$ × £121.0317,

or 20,300 × £121.0317, = £2,456,943.5100.

At the end of the forty-ſecond year,

$\overline{20{,}300 + 1900}$ × £121.0317,

or 22,200 × £121.0317, = £2,686,903.7400.

At the end of the forty-third year,

$\overline{22{,}200 + 1900}$ × £121.0317,

or 24,100 × £121.0317, = £2,916,863.9700.

At the end of the forty-fourth year,

$\overline{24{,}100 + 1800}$ × £121.0317,

or 25,900 × £121.0317, = £3,134,721.0300.

At the end of the forty-fifth year,

$\overline{25{,}900 + 1800}$ × £121.0317,

or 27,700 × £121.0317, = £3,352,578.0900.

At the end of the forty-ſixth year,

$\overline{27{,}700 + 1700}$ × £121.0317,

or 29,400 × £121.0317, = £3,558,331.9800.

At

At the end of the forty-ſeventh year,

$\overline{29{,}400 + 1600|} \times £121.0317,$

or $31{,}000 \times £121.0317, = £3{,}751{,}982.7000.$

At the end of the forty-eighth year,

$\overline{31{,}000 + 1400|} \times £121.0317,$

or $32{,}400 \times £121.0317, = £3{,}921{,}427.0800.$

At the end of the forty-ninth year,

$\overline{32{,}400 + 1200|} \times £121.0317,$

or $33{,}600 \times £121.0317, = £4{,}066{,}665.1200.$

At the end of the fiftieth year,

$\overline{33{,}600 + 1100|} \times £121.0317,$

or $34{,}700 \times £121.0317, = £4{,}199{,}799.9900.$

At the end of the fifty-firſt year,

$\overline{34{,}700 + 1000|} \times £121.0317,$

or $35{,}700 \times £121.0317, = £4{,}320{,}831.6900.$

At the end of the fifty-ſecond year,

$\overline{35{,}700 + 900|} \times £121.0317,$

or $36{,}600 \times £121.0317, = £4{,}429{,}760.2200.$

At the end of the fifty-third year,

$\overline{36{,}600 + 700|} \times £121.0317,$

or $37{,}300 \times £121.0317, = £4{,}514{,}482.4100.$

At the end of the fifty-fourth year,

$$\overline{37{,}300 + 600} \times £121.0317,$$

or $37{,}900 \times £121.0317, = £4{,}587{,}101.4300.$

At the end of the fifty-fifth year,

$$\overline{37{,}900 + 500} \times £121.0317,$$

or $38{,}400 \times £121.0317, = £4{,}647{,}617.2800.$

At the end of the fifty-sixth year,

$$\overline{38{,}400 + 400} \times £121.0317,$$

or $38{,}800 \times £121.0317, = £4{,}696{,}029.9600.$

At the end of the fifty-seventh year,

$$\overline{38{,}800 + 300} \times £121.0317,$$

or $39{,}100 \times £121.0317, = £4{,}732{,}339.4700.$

At the end of the fifty-eighth year,

$$\overline{39{,}100 + 200} \times £121.0317,$$

or $39{,}300 \times £121.0317, = £4{,}756{,}545.8100.$

At the end of the fifty-ninth year,

$$\overline{39{,}300 + 100} \times £121.0317,$$

or $39{,}400 \times £121.0317, = £4{,}768{,}648.9800.$

And at the end of the sixtieth and last year,

$$\overline{39{,}400 + 100} \times £121.0317,$$

or $39{,}500 \times £121.0317, = £4{,}780{,}752.1500.$

CCCXIV. Now

CCCXIV. Now all theſe ſavings are to be employed, as faſt as they ariſe, for the purpoſe of diminiſhing the national debt, in either the firſt or the ſecond method above-mentioned. By this means the laſt ſaving, £4,780,752.1500, (which accrues to the publick at the end of the whole period of 60 years,) will either extinguiſh £4,780,752.1500 of the capital of the 4 per cent. annuities (which are ſuppoſed to ſell at their *par*, or nominal value,) or, if employed in the ſecond method above-deſcribed, it will extinguiſh a proportionably greater capital of the 3 per cent. annuities, which are ſuppoſed to ſell at the price of £75 per cent. and in either caſe it will redeem to the publick the intereſt of 4 per cent. upon it, or the annual ſum of $\left(\frac{4}{100} \times £4,780,752.1500,\right.$ or $\left.\frac{£19,123,008.6000}{100},\right.$ or $\left.£191,230.0860\right)$. But all the other ſavings will have time to perform more than one operation of this kind towards the diminution of the national debt, to wit, a new operation at the end of every year of the term that is remaining after the time when it accrued: and, in conſequence of theſe repeated operations, the quantity of the national debt which each of theſe ſavings will have extinguiſhed at the end of the ſaid term of 60 years, will be equal in value to the amount of ſuch ſaving at the end of the ſaid term, if improved in the mean time at compound intereſt at the rate of 4 per cent.

The ſaid ſavings are to be employed in diminiſhing the national debt, in either the firſt or the ſecond method above-deſcribed.

We muſt therefore now compute the amounts of theſe ſavings at the end of the ſaid term, if ſo improved at compound intereſt in the mean time, or during the remaining years of the ſaid term after the times at which they will have reſpectively accrued, that is, during 29 years, 28 years, 27 years, 26 years, &c. to the laſt year of the ſaid term. Now theſe amounts will be as follows.

CCCXV. The amount of £181,547.5500, improved at compound intereſt at 4 per cent. during 29 years, is = £181,547.5500 × 3.118,651 = £566,183.4483.

The amount of £375,198.2700, improved in the ſame manner during 28 years, is = £375,198,2700 × 2.998,703 = £1,125,108.1778.

And the amounts of all the other ſavings, improved in the ſame manner during the remaining years of the ſaid term of 60 years after they ſhall have reſpectively accrued to the publick, will be as follows.

A computation of the amounts of all the ſaid ſavings, except the laſt, at the end of the ſaid term of 60 years, if improved in the mean time at compound intereſt at the rate of 4 per cent.

£580,952.1600 × 2.883,368 will be = £1,675,098.8676.
£798,809.2200 × 2.772,469 = £2,214,673.7993.
£1,028,769.4500 × 2.665,836 = £2,742,530.6375.
£1,258,729.6800 × 2.563,304 = £3,226,506.8236.
£1,500,793.0800 × 2.464,715 = £3,699,027.2161.
£1,742,856.4800 × 2.369,918 = £4,130,426.9433.
£1,984,919.8800 × 2.278,768 = £4,523,171.9051.
£2,226,983.2800 × 2.191,123 = £4,879,594.2854.
£2,456,943.5100 × 2.106,849 = £5,176,408.9871.
£2,686,903.7400 × 2.025,816 = £5,443,172.5869.
£2,916,863.9700 × 1.947,900 = £5,681,759.3271.
£3,134,721.0300 × 1.872,981 = £5,871,272.9294.
£3,352,578.0900 × 1.800,943 = £6,037,802.0431.
£3,558,331.9800 × 1.731,676 = £6,161,878.0897.
£3,751,982.7000 × 1.665,073 = £6,247,325.0902.
£3,921,427.0800 × 1.601,032 = £6,278,330.2407.
£4,066,665.1200 × 1.539,454 = £6,260,443.8856.
£4,199,799.9900 × 1.480,244 = £6,216,728.7363.
£4,320,831.6900 × 1.423,311 = £6,149,887.2735.
£4,429,760.2200 × 1.368,569 = £6,062,432.5145.
£4,514,482.4100 × 1.315,931 = £5,940,747.3522.
£4,587,101.4300 × 1.265,319 = £5,804,146.5943.
£4,647,617.2800 × 1.216,652 = £5,654,532.8589.
£4,696,029.9600 × 1.169,858 = £5,493,688.2149.
£4,732,339.4700 × 1.124,864 = £5,323,238.3055.
£4,756,545.8100 × 1.081,600 = £5,144,679.9480.
£4,768,648.9800 × 1.040,000 = £4,959,394.9392.

CCCXVI. Having

CCCXVI. Having thus found the amounts to which the ſeveral yearly ſavings made at the ends of the thirty-firſt year and every following year of the ſaid term of 60 years, except the laſt, will have increaſed at the end of the ſaid term by being improved in the mean time at compound intereſt at the rate of 4 per cent. we muſt now add up all theſe amounts, together with the ſaving of £4,780,752.1500, (which will accrue at the end of the 60th, or laſt, year,) into one ſum: which may be done as follows.

The addition of the amounts of all the aforeſaid yearly ſavings, at the end of the ſaid term of 60 years, into one ſum.

The amounts of the ſavings made in the firſt period of ſix years after the expiration of the above-mentioned term of 30 years, and in the ſecond period of ſix years after the expiration of the ſaid term, and in the third period of ſix years after the expiration of the ſaid term, will be as follows.

£	£	£
566,183.4483,	3,699,027.2161,	5,681,759.3271,
1,125,108.1778,	4,130,426.9433,	5,871,272.9294,
1,675,098.8676,	4,523,171.9051,	6,037,802.0431,
2,214,673.7993,	4,879,594.2854,	6,161,878.0897,
2,742,530.6375,	5,176,408.9871,	6,247,325.0902,
3,226,506.8236.	5,443,172.5869.	6,278,330.2407,
11,550,101.7541	27,851,801.9239	36,278,367.7202

And the amounts of the ſavings made in the fourth period of ſix years after the expiration of the ſaid term of 30 years, and in the fifth and laſt period of ſix years after the expiration of the ſaid term, will be as follows.

£	£
6,260,443.8856,	5,654,532.8589,
6,216,728.7363,	5,493,688.2149,
6,149,887.2735,	5,323,238.3055,
6,062,432.5145,	5,144,679.9480,
5,940,747.3522,	4,959,394.9392,
5,804,146.5943.	4,780,752.1500.
36,434,386.3564	31,356,286.4165

And the ſum total of theſe five ſeveral ſums, or the ſum total of the amounts that will ariſe from all the ſavings made at the ends of all the years of the ſaid term of 60 years after the 30th year, at the end of the ſaid term of 60 years, is £143,470,944.1711.

£
11,550,101.7541,
27,851,801.9239,
36,278,367.7202,
36,434,386.3564,
31,356,286.4165.
143,470,944.1711.

The quantity of the capital of the national debt that will have been extinguiſhed, by means of all the foregoing ſavings, at the end of the ſaid term of 60 years.

CCCXVII. Therefore the quantity of the capital of the national debt that would have been extinguiſhed, by means of all theſe ſavings, at the end of the ſaid term of 60 years, will be equal in value to the ſaid ſum of £143,470,944.1711; of which the annual intereſt at 4 per cent. is

$$\left(\frac{4}{100} \times £143{,}470{,}944.1711, \text{ or } \frac{£\ 573{,}883{,}776.6844}{100}, \text{ or}\right) £5{,}738{,}837.7668.$$

The quantity of the intereſt of the national debt that will be then redeemed to the publick by means of the ſaid ſavings.

Therefore the quantity of the intereſt of the national debt, or of the perpetual annuities employed in the payment of the ſaid intereſt, that would be redeemed to the publick at the end of the ſaid term of 60 years by means of all theſe ſavings, (ſo that at the end of the next, or 61ſt, year it would be free to be diſpoſed of by the Parliament in any other way,) is £5,738,837.7668 *per annum*. QE I.

The whole quantity of the intereſt of the national debt that will be redeemed to the publick, at the end of the ſaid term of 60 years, by the foregoing operation.

CCCXVIII. This ſum of £5,738,837.7668 *per annum* muſt therefore be added to £3,780,753 *per annum*, which was the quantity of the perpetual annuities, now due to the publick creditors, which was ſuppoſed to be converted into life annuities of the kind above-deſcribed for the lives of 69400 perſons, all of the age of 35 years, and which will therefore be now wholly redeemed to the publick by the deaths of all the ſaid life-annuitants: and the ſum of both, or the ſum of £9,519,590.7668, or (neglecting the fraction .7668) £9,519,590 *per annum* will be the whole quantity of the intereſt of the national debt, or of the perpetual annuities employed in the payment of it, which will be redeemed to the publick at the end of the ſaid term of 60 years, (ſo as to be free at the end of the next, or 61ſt, year to be diſpoſed of by the Parliament in any other

other way,) by the faithful application of a million of pounds, ſterling, *per annum* to that ſalutary purpoſe, during that period, according to the ſixth method above-deſcribed. Q E I.

CCCXIX. This ſum of £9,519,590 *per annum*, which is the quantity of the intereſt of the national debt that would be redeemed to the publick in the courſe of 60 years by the application of a million of pounds, ſterling, *per annum* to that purpoſe in this ſixth method, or in the way of life-annuities with a right of ſucceſſion by ſurvivorſhip to the annuities of the deceaſed life-annuitants for a term of 30 years, is the ſame with the quantity of the intereſt of the ſaid debt that would have been redeemed to the publick in the ſame time by applying the ſame ſum of a million of pounds *per annum* to the ſame purpoſe in either of the five former methods. For the ſum that would have been ſo redeemed in either the firſt or the ſecond method is £9,519,624 *per annum*; which differs from the preſent ſum of £9,519,590 *per annum* by only £34 *per annum*, which, upon ſo large a ſum as £9,519,624, is perfectly inconſiderable. And the quantities of the ſaid intereſt that would be redeemed by the application of the ſame ſum of a million of pounds *per annum* to the ſame purpoſe during the ſame period of ſixty years, in either the third, or the fourth, or the fifth, method above-mentioned, have been before ſhewn to be very nearly equal to the ſame ſum of £9,519,624 *per annum*. Therefore the quantity of the ſaid intereſt which would be redeemed by this ſixth method may be conſidered as equal to that which would be redeemed in the ſame time by the ſame annual ſum of money in either of the five preceeding methods; agreeably to what is aſſerted above in Art. CCXCIX. Q E D.

Agreement between the effect of the foregoing ſcheme, "of life annuities with the benefit of ſurvivorſhip during a certain number of years," in redeeming to the publick the intereſt of the national debt, and the effects of the five preceeding methods.

CCCXX. This ſixth method of employing a given ſum of money every year in the diminution of the national debt, "by means of the above-mentioned life-annuities with the benefit of ſurvivorſhip during a certain number of years," partakes in a greater degree than the fourth method, (which conſiſted in eſtabliſhing life-annuities without the benefit of ſurvivorſhip,) of the advantage mentioned in Art. CCLIV as belonging to the third method, to wit, that of ſecuring to the publick the faithful application of ſuch annual ſum to the purpoſes of its deſtination, without ſuſpenſion or interruption. For it would then be impoſſible to with-hold not only any part of the original life-annuities ſo created, but alſo any part of the additions that would accrue to the ſurviving life-annuitants every year by the deaths of ſome of their companions in the courſe of the given term of years during which the ſaid benefit of ſurvivorſhip had been granted

A compariſon between this ſixth method and the third and fourth methods above-mentioned, with reſpect to the advantage of ſecuring to the publick the faithful application of the ſaid annual ſum of money to the purpoſes of its deſtination.

granted to them, without an abſolute breach of the publick faith; which (as we obſerved above in Art. CCLIV) may be conſidered as a moral impoſſibility. And conſequently during the whole of the ſaid term throughout which ſuch benefit of ſurvivorſhip was to continue, the annual ſum of money allotted to the purpoſe of diminiſhing the national debt would be tied down to the purpoſe of its deſtination. But, when the ſaid term was at an end, the ſubſequent ſavings that would ariſe from the deaths of thoſe life-annuitants who would die after the ſaid term, would be as liable to be diverted, by the miniſters of ſtate and the parliament, from its original deſtination of diminiſhing the national debt, and to be employed in defraying the expence of ſome temporary meaſure of Government, as the money that ſhould be allotted to the ſame purpoſe, of diminiſhing the national debt, in either the firſt or the ſecond method above-deſcribed: whereas in the *third* method of applying a given ſum of money every year to the diminution of the national debt, "by converting ſome of the perpetual annuities that now pay the intereſt of it, into greater temporary annuities that ſhould continue for a certain limited number of years, and allotting the ſaid annual ſum to the payment of the additions thereupon made to the perpetual annuities ſo converted," *the whole* of the ſaid annual ſum is effectually ſecured from being diverted from its original and proper deſtination, and applied to any other purpoſe whatſoever, during the *whole* of the ſaid term.

A ſeventh method of employing a given ſum of money every year in the reduction of the national debt, by means of life-annuities, which ſhould continue at all events for a certain term of years, and, in caſe of the deaths of the life-annuitants, ſhould become payable to their executors, or other perſonal repreſentatives, during the remainder of the ſaid term, and which ſhould afterwards be accompanied with the benefit of ſurvivorſhip during a ſecond term of years.

CCCXXI. A ſeventh method of applying a given ſum of money every year to the diminution of the national debt would be to combine the foregoing fifth and ſixth methods together in the following manner.

A portion of the perpetual annuities, now due to the publick creditors, might (with the conſent of the proprietors of ſuch annuities) be converted into equivalent life-annuities of the following kind. The ſaid annuities ſhould continue for a certain moderate term of years at all events, ſo that, in caſe the proprietors themſelves ſhould die before the expiration of the ſaid term, the annuities ſhould be payable to the executors, or other perſonal

ſonal repreſentatives, of the ſaid proprietors during the remaining years of it. At the end of the ſaid term of years the annuities payable to the executors of the deceaſed proprietors ſhould ceaſe; but thoſe that were payable to the ſurviving proprietors ſhould continue during their lives, (as in the fifth method above-deſcribed,) and ſhould alſo, during a ſecond term of years, increaſe continually (as in the laſt, or ſixth, method above-deſcribed) by a diviſion of the annuities of the life-annuitants that died in every year after the end of the firſt term, amongſt their ſurviving companions. And after the end of this ſecond term of years there ſhould be no further increaſe of the ſaid life-annuities by ſurvivorſhip; but the then ſurviving life-annuitants ſhould receive during the remainder of their lives the ſame annuities as they had received at the end of the laſt year of the ſaid ſecond term. And the ſavings that would accrue to the publick after the end of the ſaid ſecond term by the deaths of the few remaining life-annuitants, ſhould be applied, as faſt as they aroſe, to the diminution of the national debt in either the firſt or the ſecond method above-deſcribed.

Thus, for example, if the life-annuitants for whoſe lives theſe annuities were to be granted, were all of the age of 35 years, and conſequently a few of the longeſt livers of them would (according to Monſieur de Parcieux's table of the probabilities of the duration of human life,) live to almoſt the end of a period of 60 years, the ſaid period of 60 years might be divided into three parts, to wit, 1ſt, a term of 30 years, 2dly, a term of 20 years, and, 3dly, a term of 10 years. And during the firſt of theſe terms the annuities might be made payable not only to the ſaid life-annuitants themſelves, if they were living, but, in caſe of their deceaſe, to their executors, or other perſonal repreſentatives; but at the end of the ſaid firſt term of 30 years the payments of theſe annuities to the repreſentatives of the ſaid deceaſed life-annuitants ſhould intirely ceaſe, and only thoſe due to the ſurviving life-annuitants ſhould be continued. And during the ſecond period of 20 years the ſurviving life-annuitants ſhould not only continue to receive their original life-annuities, but ſhould ſucceed, by ſurvivorſhip, to the life-annuities of ſuch of their companions as ſhould die in the courſe of the ſaid 20 years; by which means the annuities of the few perſons who ſhould live to the end of the ſaid 20 years, or to the age of 85 years, would be increaſed in the proportion of the number of the ſaid life-annuitants who would be living at the ſaid age of 85 years to the number of them that had been living at the beginning of the ſaid term of 20 years, or at the age of 65 years, that is (according to Monſieur de Parcieux's table,) in the proportion of 48 to 395, or ſomething more than in the proportion of 1 to 8. At the end of the ſaid ſecond term of 20 years this benefit of ſurvivorſhip ſhould ceaſe, and the few life-annuitants that ſhould live beyond that term ſhould continue to receive the ſame annuities during the remaining years of their lives as they had received at the

the end of the laſt year of the ſaid term of 20 years, or when they were of the age of 85 years: and the ſavings that would then ariſe every year by the deaths of ſome of the ſaid life-annuitants, ſhould accrue to the publick, and be employed in the diminution of the national debt in either the firſt or the ſecond method above-deſcribed.

And, if this method of diminiſhing the national debt were adopted, the annual ſum of money that could be ſpared out of the publick revenue for this purpoſe, or a part of that ſum, muſt be employed in paying the additions which it would be neceſſary for the Government to make to the perpetual annuities, now due to the publick creditors, which ſhould be converted into life-annuities of the kind juſt now deſcribed, in compenſation of the abridgement of the time of their continuance.

This ſeventh method of diminiſhing the national debt, if properly purſued, would be equally effectual for that purpoſe with either of the ſix preceeding methods.

CCCXXII. This ſeventh method of applying a given ſum of money every year to the diminution of the national debt would have exactly the ſame effect in the courſe of any given period of time as either of the ſix preceeding methods; or the quantity of the intereſt of the national debt, or of the perpetual annuities now employed in paying it, which would be redeemed to the publick in any given number of years by employing a given ſum of money every year for that purpoſe according to this ſeventh method, would be the very ſame that would be redeemed to the publick in the ſame period of time by employing the ſame ſum of money every year for the ſame purpoſe according to either of the preceeding methods. Of this I ſhall now proceed to give an example.

An example of the ſaid ſeventh method of employing a given ſum of money every year in the reduction of the national debt.

Preliminary ſuppoſitions.

CCCXXIII. Let the ſum of money that is to be employed every year for the purpoſe of diminiſhing the national debt according to this ſeventh method, be, (as in the foregoing examples,) a million of pounds, ſterling, *per annum*; and the period during which it is to be ſo employed, be alſo (as before,) a term of 60 years. Let the life-annuitants for whoſe lives the new life-annuities, of the compound nature above-deſcribed, are to be granted, in lieu of certain portions of the perpetual annuities, now due to the publick creditors, (which are thereupon, with the conſent of the proprietors of them, to ceaſe,) be all of the age of 35 years: and let them be ſuppoſed to die off every year in the proportions ſet forth in Monſieur de Parcieux's table of the probabilities of the duration of human life; in conſequence of which it muſt be ſuppoſed that ſome few of them will

will live almoſt throughout the whole of the ſaid period of 60 years, and die a few days before the end of it. Alſo let the number of life-annuitants for whoſe lives theſe life-annuities are granted, be (as before,) 69,400.

Further, let the firſt term, during which the life-annuities of the deceaſed life-annuitants are to be paid to their executors, or other perſonal repreſentatives, be a term of 30 years; and the ſecond term, during which the life-annuitants, who ſhall outlive the firſt term, ſhall be intitled to ſucceed by ſurvivorſhip to the annuities of ſuch of their companions as ſhall die in the courſe of the ſaid ſecond term, be 20 years. And, laſtly, let the intereſt of money be ſuppoſed to be 4 per cent. and the perpetual annuities of 4 per cent. now due to the publick creditors, be ſuppoſed to ſell at their *par*, or nominal value; and the perpetual annuities of 3 per cent. now due to the publick creditors, be ſuppoſed to ſell at the price of £75 per cent.

An inveſtigation of the quantity of the perpetual annuities, now due to the publick creditors, which might be converted into theſe compound life-annuities, with a term of 30 years certain, and the benefit of ſurvivorſhip during a ſecond term of 20 years, by the help of a million of pounds, ſterling, *per annum* allotted to that purpoſe during a period of 60 years.

Theſe things being premiſed, we muſt now proceed to inquire what quantity of the perpetual annuities, now due to the publick creditors, the Government would be able to convert into life-annuities of the compound kind juſt now deſcribed, by means of the ſaid ſum of a million of pounds, ſterling, *per annum*, allotted to that purpoſe during the ſpace of 60 years. Now this quantity may be determined by proceeding in the manner following.

CCCXXIV. The number of life-annuitants of the age of 35 years, to whom theſe life-annuities are ſuppoſed to be granted, is 69,400. If therefore we ſuppoſe the annuities granted to them to be £10 a year a piece, the annual expence to the Government ariſing from theſe grants would be £694,000. And this expence would continue without any diminution during the whole of the ſaid firſt term of 30 years, notwithſtanding the numbers of the ſaid life-annuitants who would be continually dying in every year of the ſaid term; becauſe during the whole of the ſaid term the annuities of the deceaſed life-annuitants are to be paid to their reſpective executors, or other perſonal repreſentatives.

CCCXXV. But at the end of the ſaid term of 30 years a conſiderable ſaving would accrue every year to the publick from the ſaid annuities of the life-annuitants who had died in the courſe of the ſaid term; becauſe then the ſaid annuities are no longer to be paid to the repreſentatives of the deceaſed life-annuitants. This ſaving will be equal to £10 multiplied into the number of life-annuitants who will have died in the courſe of the ſaid

30 years. Now it appears from Monſieur de Parcieux's table of the probabilities of the duration of human life, that the ſaid original 69,400 life-annuitants, of the age of 35 years, for whoſe lives the ſaid annuities of £10 each were ſuppoſed to be granted, will, in the courſe of 30 years, be reduced to 39,500. Therefore the number of the ſaid life-annuitants who will have died in the courſe of the ſaid 30 years is (69,400 — 39,500, or) 29,900: and conſequently the ſaving which will accrue every year to the publick, after the expiration of the ſaid term of 30 years, in conſequence of the deaths of the ſaid life-annuitants, who will have died in the courſe of the ſaid term, will be (29,900 x £10, or) £299,000. And therefore the ſum which the Government will be obliged to pay, in conſequence of the grants of theſe annuities, after the expiration of the ſaid term of 30 years, will not, as before, be £694,000 a year, but only (£694,000 — £299,000, or) £395,000 a year. And this expence of £395,000 *per annum* will continue the ſame during the whole ſecond term of 20 years, notwithſtanding the deaths of ſeveral of the ſaid life-annuitants in every year of the ſaid term; becauſe it is ſuppoſed that the annuities of the ſeveral life-annuitants who ſhall die in the courſe of the ſaid ſecond term are to be divided amongſt their ſurviving companions.

CCCXXVI. But after the expiration of this ſecond term of 20 years, the annual expence of the Government, in conſequence of the grants of theſe annuities, will grow leſs and leſs every year; becauſe then the annuities of the life-annuitants who ſhall die in the remaining 10 years of the ſaid period of 60 years are no longer to be enjoyed, by right of ſurvivorſhip, by their few ſurviving companions, but are to accrue to the publick.

An original life-annuity of 10 pounds a year, for the life of a perſon of the age of 35 years, will increaſe in the courſe of the aforeſaid ſecond term, (which conſiſts of 20 years,) to £82.291,666 *per annum.*

CCCXXVII. In the courſe of the ſaid ſecond term of 20 years the annuities of the life-annuitants who ſhall have lived to the end of it, will have increaſed from £10 *per annum* to £82.291,666 *per annum.* For the number of the ſaid life-annuitants will have decreaſed in the ſaid term from 39,500 to 4,800: and conſequently the ſum of £395,000, which is every year to be divided among them, will, at the end of the laſt year of the ſaid term, be divided amongſt only 4,800 perſons; and therefore the annuity then paid to each of them will be $= \frac{£395,000}{4,800} = \frac{£3950}{48} =$ £82.291,666. And this annuity, of £82.291,666 *per annum*, each of the ſaid 4,800 ſurviving life-annuitants will be intitled to receive during the remainder of his life, but without any further increaſe of it by ſurvivorſhip.

CCCXXVIII. In

CCCXXVIII. In order to diſcover the value of one of theſe compound life-annuities of £10 *per annum*, it will be convenient to divide it into the three following parts, to wit, 1ſt, an annuity of £10 a year for a term of 30 years, to be certainly paid either to the life-annuitant, or, in caſe of his deceaſe, to his perſonal repreſentative; and, 2dly, A remote annuity of £10 a year, depending upon the life of the life-annuitant, ſo as not to take place unleſs the ſaid life-annuitant ſhall be then alive, and afterwards to ceaſe immediately in caſe of his death; and which, even in the other event, of his living, ſhall continue only during 20 years; but which during the ſaid 20 years ſhall be accompanied with the advantage of a right in the life-annuitant to ſucceed, by ſurvivorſhip, to a ſhare of the annuities of ſuch of his fellow life-annuitants as ſhall die in the courſe of the ſaid 20 years; and 3dly, A remote life-annuity of £82.291,666 *per annum*, that is to commence at the diſtance of 50 years, or whereof the firſt payment is to be received at the end of 51 years. The values of theſe three annuities may be determined in the manner following.

Diviſion of a compound life-annuity of 10 pounds a year, for the life of a perſon of the age of 35 years, with a term certain of 30 years and the benefit of ſurvivorſhip during another term conſiſting of 20 years, into three parts.

CCCXXIX. In the firſt place, it appears from Mr. Smart's fourth table of compound intereſt, page 78, that the value of an annuity of one pound a year for 30 years certain, when the intereſt of money is 4 per cent. is £17.292,033,30. And conſequently the value of an annuity of £10 a year for 30 years certain is £172.920,333. Q E I.

A computation of the value of the firſt part of the ſaid compound life-annuity of 10*l*. *per annum*.

CCCXXX. Secondly, ſince during the whole ſecond term of 20 years the Government is every year to pay to the ſurviving life-annuitants the ſum of £395,000 *per annum*, it is evident that the price which the Government ought to receive from all the 69,400 original life-annuitants together for this annual payment of £395,000 during the ſaid ſecond term of 20 years, muſt be the ſame as if the ſame annual payment were to be made to them during the ſame term of 20 years in the more ſimple form of a remote annuity for a term certain of 20 years, that was to be paid either to them or their executors, or other perſonal repreſentatives, during the ſaid term, and was to commence at the end of 30 years, or whereof the firſt payment was to be received at the end of 31 years. Now the value of a remote annuity of £395,000 *per annum* for a term of 20 years, that is to commence at the end of 30 years, is equal to the exceſs of the value of an immediate annuity of the ſame magnitude, or £395,000 *per annum*, for a term of 50 years above the value of an immediate annuity of the ſame magnitude for a term of 30 years. Now it appears from Mr. Smart's fourth table of compound intereſt, page 78, that the value of an immediate annuity of £395,000 *per annum* for a term of 50 years, when the intereſt

A computation of the value of the ſecond part of the ſaid compound life-annuity of 10*l*. *per annum*.

of money is 4 per cent. is £395,000 × 21 482,184,62, or £8,485,462.9249, and that the value of an immediate annuity of £395,000 *per annum* for a term of 30 years is £395,000 × 17.292,033,30, or £6,830,353.1535. Therefore the value of a remote annuity of £395,000 *per annum* for a term of 20 years, that is to commence at the end of 30 years, is equal to £8,485,462.9249 — £6,830,353.1535, or £1,655,109.7714. Therefore the ſum which all the 69,400 life-annuitants, taken together, ought to pay to the Government for the ſaid ſecond annuity of £395,000 *per annum*, which they are to receive amongſt them during the ſaid ſecond term of 20 years. which is to commence at the end of the firſt term of 30 years, is £1,655,109.7714; and conſequently the ſum which each of the ſaid 69,400 life-annuitants ought to pay for his contingent ſhare of the ſaid annuity of £395,000 *per annum* during the ſaid ſecond term of 20 years, accompanied with the benefit of ſurvivorſhip during the ſaid term, in the manner above-deſcribed, is $\frac{£1,655,109.7714}{69,400}$, or £23.848,843. Q E I.

A computation of the value of the third part of the ſaid compound life-annuity of 10*l. per annum.*

CCCXXXI. Laſtly, the value of a remote life-annuity of £82.291,666 *per annum*, for the life of a perſon of the age of 35 years, that is to commence at the end of 50 years, or whereof the firſt payment is to be received at the end of 51 years, may be determined by the help of Art. CCXLI in the manner following.

The number of perſons repreſented in Monſieur de Parcieux's table of the probabilities of the duration of human life as living at the age of 35 years, is 694; and the number of perſons repreſented in the ſame table as living at the age of 85 years, or at the end of 50 years, is 48. Therefore, if $\overset{£}{V}$ be put for the value of an immediate annuity of one pound a year for the life of a perſon of the age of 85 years, it follows from Art. CCXLI that the value of a remote annuity of one pound a year for the life of a perſon of the age of 35 years, that is to commence at the end of 50 years, or when the ſaid perſon ſhall be 85 years old, will be equal to $\frac{48}{694} \times \frac{1}{r^{50}} \times \overset{£}{V}$. Now $\overset{£}{V}$, or the value of an immediate annuity of one pound a year for the life of a perſon of the age of 85 years, when the intereſt of money is 4 per cent. appears above in Table XVI, page 225, to be = £2.424,216. And it appears by Mr. Smart's ſecond table of compound intereſt, pages 60 and 62, that, when the intereſt of money is 4 per cent. or r is = 1.04, the fraction $\frac{1}{r^{50}}$,

or

or $\frac{1}{1.04^{50}}$, is = .140,712,62. Therefore $\frac{48}{694} \times \frac{1}{r^{50}} \times \frac{£}{V}$ is = $\frac{48}{694} \times .140,712,62 \times £2.424,216 = \frac{48}{694} \times .341,117,784,805,92,£ = £\frac{16.373,653,670,684,16}{694} = £0.023,593,160$; that is, the value of a remote life-annuity of one pound a year for the life of a perſon of the age of 35 years, that is to commence at the end of 50 years, or when the ſaid perſon ſhall have attained the age of 85 years, or ſo that he ſhall receive the firſt payment thereof at the age of 86 years, is = £0.023,593,160. Therefore the value of a remote life-annuity of £82.291,666 *per annum* for the life of a perſon of the age of 35 years, that is to commence at the end of 50 years, or when the ſaid perſon ſhall have attained the age of 85 years, is = 82.291,666 × £0.023,593,160 = £1.941,520,442. Q E I.

CCCXXXII. Therefore the ſum of the values of theſe three annuities, or the value of the whole life-annuity of £10 a year ſuppoſed to be granted to each of the aforeſaid 69,400 life-annuitants of the age of 35 years, is = £172.920,333 + £23.848,843 + 1.941,520 = £198.710,696. Q E I.

The value of the whole of the ſaid compound life-annuity of 10*l.* *per annum.*

CCCXXXIII. The annual intereſt of £198.710,696, at 4 per cent. is $\left(\frac{4}{100} \times £198.710,696, \text{ or } \frac{£794.842,784}{100}, \text{ or}\right)$ £7.948,427,84. Therefore a perpetual annuity of £7.948,427,84 *per annum* is equal in value to a life-annuity of £10 *per annum*, of the compound kind above-deſcribed, for the life of a perſon of the age of 35 years. Therefore, if each of the above-mentioned 69,400 life-annuitants of the age of 35 years, to whom we have ſuppoſed life-annuities of £10 each, of the compound kind above-mentioned, to have been granted, had been poſſeſſed of a perpetual annuity in the publick funds, of £7.948,427,84 *per annum*, and were to reſign, or relinquiſh, to the publick his right to ſuch perpetual annuity, in exchange for, and as the price of, the ſaid life-annuity of £10 a year, of the compound kind above-deſcribed, ſuch a bargain would be fair and equal on both ſides. Therefore the ſum of all the ſaid 69,400 life-annuities of £10 each, added together, would be equal in value to 69,400 perpetual annuities of £7.948,427,84 *per annum*, or to a portion of the perpetual annuities, now due to the publick creditors, of 69,400 × £7.948,427,84, or) £551,620.892,096 *per annum*. In order therefore to convert a portion of the perpetual annuities, now due to the publick creditors,

The ſaid compound life-annuity of 10*l.* *per annum* for the life of a perſon of the age of 35 years, is equal in value to a perpetual annuity of £7.948,427,84 *per annum.*

creditors, amounting to £551,620.892,096 *per annum*, into life-annuities of £10 each, of the compound kind above-deſcribed, it would be neceſſary for the Government to apply to this purpoſe an additional ſum of money every year, during the ſpace of 30 years, that is equal to the difference between £551,620.892,096, and £694,000, or that is equal to £142,379.107,904. And conſequently, ſince an additional ſum of £142,379.107,904 *per annum* would be ſufficient to enable the Government to convert into life-annuities, of the compound kind above-mentioned, for the lives of perſons of the age of 35 years, a portion of the perpetual annuities, now due to the publick creditors, amounting to £551,620.892,096 *per annum*; we may conclude that an additional ſum of a million of pounds every year would be ſufficient to enable the Government to convert into life-annuities of the ſame kind, for the lives of perſons of the ſame age of 35 years, a portion of the ſaid perpetual annuities that is greater than £551,620.892,096 *per annum* in the ſame proportion in which £1,000,000 is greater than £142,379.107,904, that is, a portion of the ſaid perpetual annuities amounting to $\left(\frac{1,000,000}{142,379.107,904} \times £551,620.892,096\right.$, or to $\left.\frac{551,620,892,096.000,000}{142,379.107,904}\right)$, or £3,874,310.635,995, or £3,874,310.6360 *per annum*. This therefore is the quantity which was propoſed in Art. CCCXXIII as neceſſary to be determined in order to aſcertain the effect of the application of a million of pounds, ſterling, *per annum* to the diminution of the national debt, during a period of 60 years, according to this ſeventh method.

The quantity of the perpetual annuities mentioned in Art. 323 as neceſſary to be determined, is the ſum of £3,874,310.6360 *per annum*.

CCCXXXIV. Let us therefore now ſuppoſe that the whole of the ſaid ſum of a million of pounds *per annum* is employed in the diminution of the national debt according to this ſeventh method, and conſequently that the ſum of £3,874,310 *per annum* of the perpetual annuities, now due to the publick creditors, is converted into life-annuities, of the compound kind above-deſcribed, for the lives of perſons of the age of 35 years. And let us ſuppoſe the number of life-annuitants between whom the ſaid ſum of £3,874,310 of the perpetual annuities, (ſo converted into life-annuities,) together with the ſum of £1,000,000 *per annum*, (which is to be added to it, on account of the ſaid converſion,) is to be divided, to be (as on the former ſuppoſition made in Art. CCCXXIII,) 69,400. And let the life-annuities granted to theſe 69,400 life-annuitants be ſuppoſed to be all equal to each other.

Of the original quantity of each man's life-annuity, upon a ſuppoſition that the whole ſum of £3,874,310 *per annum* of the perpetual annuities is converted into theſe compound life-annuities, and that the number of the life-annuitants continues, as before, to be 69,400.

CCCXXXV. Then,

CCCXXXV. Then, ſince the ſum that is to be paid every year by the Government to the ſaid 69,400 life-annuitants, or their ſeveral executors, or other perſonal repreſentatives, during a term of 30 years, is £3,874,310 + £1,000,000, or £4,874,310; and this ſum is to be equally divided between all the ſaid 69,400 life-annuitants, or their repreſentatives; it follows that the annuity which each of the ſaid life-annuitants, or his repreſentative, will be intitled to receive at the end of every year during the ſaid term of 30 years, will be $\frac{£4,874,310}{69,400}$, or £70.235,014 *per annum*.

The original quantity of ſuch compound life-annuity is £70.235,014 *per annum*.

CCCXXXVI. At the end of the ſaid term of 30 years the ſaid 69,400 life-annuitants will be reduced to 39,500; and in the courſe of 20 years more the ſaid 39,500 life-annuitants will be further reduced to 4,800. But it is ſuppoſed that during this latter term of 20 years the benefit of ſurvivorſhip is to take place between the ſaid life-annuitants. Therefore the ſame ſum of money which was divided, at the end of the laſt year of the firſt term of 30 years, amongſt the 39,500 life annuitants who were then living, will, at the end of the laſt year of the ſaid ſecond term of 20 years, be divided amongſt only 4,800 perſons. That ſum is 39,500 times £70.235,014, or £2,774,283.053,000. Therefore $\frac{£2,774,283.053,000}{4,800}$, or £577.975,636, will be the ſum which each of the ſaid 4,800 ſurviving life-annuitants will be intitled to receive at the end of the ſaid ſecond term of 20 years, or when he ſhall have attained the age of 85 years. And this ſum of £577.975,636 each of the ſaid 4,800 ſurviving life-annuitants will be intitled to receive likewiſe at the end of every following year during his life; but without any further augmentation.

The ſaid original life-annuity of £70.235,014 *per annum*, will increaſe by the benefit of ſurvivorſhip, in the courſe of the aforeſaid ſecond term of years, conſiſting of 20 years, to £. 577.975,636 *per annum*.

CCCXXXVII. We muſt now inquire what are the ſavings which will accrue to the publick by the deaths of any of the ſaid life-annuitants in any part of the ſaid period of 60 years.

Of the ſavings that will accrue to the publick in the courſe of the whole period of 60 years.

Now it is evident that during the whole of the firſt term of 30 years the deaths of the ſeveral life-annuitants who will die in the courſe of the ſaid term, will produce no ſaving to the publick; becauſe the executors, or other perſonal repreſentatives, of the deceaſed life-annuitants will be intitled to their annuities. But at the end of the ſaid term of 30 years a very great ſaving will accrue to the publick all at once. For then the annuities of the deceaſed life-annuitants are no longer to be paid to their repreſentatives,

The firſt and greateſt ſaving will be at the end of the firſt term above-

mentioned, conſiſting of 30 years.

repreſentatives, but are to become the property of the publick and to be employed in the reduction of the national debt in either the firſt or the ſecond method above-deſcribed. The number of life-annuitants that will have died in the courſe of the ſaid term of 30 years is the difference between 69,400, the original number of the ſaid life-annuitants, and 39,500, the number of them who will be living at the end of the ſaid term of 30 years; which difference is 29,900. Therefore the ſaving that will accrue to the publick after the expiration of the ſaid term of 30 years will be 29,900 annuities of £70.235,014, or the ſum of £2,100,026.918,600. This ſaving will accrue to the publick at the end of the 31ſt year; and the like ſaving will accrue to the publick at the ends of the 32d year, the 33d year, the 34th year, the 35th year, and of every following year during the whole period of 60 years, to the laſt year of it incluſively; ſo that there will be thirty ſuch ſavings of £2,100,026.918,600, in the ſaid time. Now all theſe ſavings are to be employed, as faſt as they ariſe, for the purpoſe of diminiſhing the national debt, in either the firſt, or the ſecond, method above-mentioned: and conſequently the quantity of the capital of the national debt that will have been extinguiſhed by them in the courſe of the ſaid latter thirty years of the ſaid period of 60 years, will be equal in value to the ſum that would be produced by means of an annual revenue of £2,100,026.918,600 that ſhould be received for 30 years together, and ſhould, at the end of every year, or as ſoon as it was received, be immediately lent out at intereſt at the rate of 4 per cent. together with the intereſt that had been already produced by it; which ſum, according to Mr. Smart's third table of compound intereſt, page 70, is 56.084,937,76 × £2,100,026.918,600, or £117,779,879.024,005,586,336, or (neglecting all the decimal fraction .024,005,586,336, except the four firſt places of figures,) £117,779,879.0240. Therefore the quantity of the capital of the national debt which will be extinguiſhed at the end of the ſaid period of 60 years by means of theſe 30 annual ſavings of £2,100,026.918,600 each, which will accrue to the publick at the ends of the latter 30 years of the ſaid period, will be equal in value to the ſum of £117,779,879.0240; and conſequently the quantity of the intereſt of the ſaid debt, or of the perpetual annuities that are now employed in paying it, which will be redeemed to the publick at the end of the ſaid period of 60 years by means of the ſaid ſavings, will be $\left(\frac{4}{100} \times £117,779,879.0240,\right.$ or $\frac{£471,119,516.0960}{100}$, or$\left.\right)$ £4,711,195 1609.

This ſaving will be £2,100,026.918,600 per annum.

The quantity of the capital of the national debt which will be extinguiſhed by means of this ſaving at the end of the whole period of 60 years, will be equal in value to £117,779,879.0240. And the quantity of the intereſt of the ſaid debt that will then be redeemed to the publick by means of the ſaid ſaving, will be £4,711,195 1609.

CCCXXXVIII. Beſides this very large ſaving of £2,100,026.918,600 *per annum*, (which will accrue to the publick at the end of the firſt 30 years in conſequence of the deaths of the 29,900 life-annuitants who will have died in the courſe of the ſaid 30 years,) there will be other ſavings that will accrue to the publick, after the expiration of the ſaid ſecond term of 20 years, by the deaths of the 4,800 then ſurviving life-annuitants in the courſe of the remaining 10 years of the ſaid period of 60 years. We muſt therefore now inquire what theſe ſavings will amount to in each of the ſaid 10 years.

Of the ſavings that will accrue to the publick after the expiration of the aforeſaid ſecond term, conſiſting of 20 years, by the deaths of life-annuitants.

CCCXXXIX. Now, in order to diſcover what theſe ſavings will amount to in each of the ſaid 10 years, it will be neceſſary to determine, by the help of Monſieur de Parcieux's table of the probabilities of the duration of human life, what numbers of the ſaid 4,800 life-annuitants (who are ſuppoſed to be living at the end of the ſaid ſecond term of 20 years,) will die in each of the ſaid 10 years. Now it appears by Monſieur de Parcieux's table that, out of 4,800 perſons of the age of 85 years, all living at the ſame time, the numbers living at the ſubſequent ages of 86 years, 87 years, 88 years, 89 years, &c. will be as follows; to wit,

The numbers of life-annuitants who will be living at the ends of the laſt ten years of the aforeſaid period of 60 years.

3,800,		700,
2,900,		400,
2,200,		200,
1,600,		100,
1,100,	and	000.

Therefore the numbers of the ſaid 4,800 life-annuitants (who will have lived to the end of the ſaid ſecond term of 20 years, or to the age of 85 years,) that will die in the 10 ſucceeding years, will be as follows; to wit,

The numbers of the ſaid life-annuitants who will die in each of the ſaid ten years

1,000,		400,
900,		300,
700,		200,
600,		100,
500,	and	100.

The ſavings accruing to the publick at the ends of each of the ſaid ten years, by the deaths of the ſaid life-annuitants.

CCCXL. It has been ſhewn above in Art. cccxxxvi, that each of the ſaid 4,800 life-annuitants (who will be living at the end of the ſaid ſecond term of 20 years, or at the age of 85 years,) will be intitled to receive at the end of every following year of his life the ſum of £577.975,636. Therefore the ſaving that will accrue to the publick at the end of the firſt of the ſaid 10 remaining years, by the deaths of the 1000 life-annuitants who will have died in the courſe of the ſaid firſt year, will be 1000 payments of £577.975,636 each, or the ſum of £577,975.6360; and the ſaving that will accrue to the publick at the end of the ſecond of the ſaid remaining 10 years by the deaths of the ſaid 1000 perſons who will have died in the ſaid firſt year and likewiſe of the 900 perſons who will have died in the ſaid ſecond year, will be 1000 + 900, or 1900, payments of £577.975,636 each, or the ſum of (1900 × £577.975,636, or) 1,098,153.708,400. And in like manner it may be ſhewn that the ſavings that will accrue to the publick at the ends of the 3d, 4th, 5th, 6th, and other following years of the ſaid remaining 10 years, will be as follows; to wit,

At the end of the third year,

$$\overline{1900 + 700} \times £577.975,636,$$

or $2,600 \times £577.975,636, = £1,502,736.6536.$

At the end of the fourth year,

$$\overline{2,600 + 600} \times £577.975,636,$$

or $3,200 \times £577.975,636, = £1,849,522.0352.$

At the end of the fifth year,

$$\overline{3,200 + 500} \times £577.975,636,$$

or $3,700 \times £577.975,636, = £2,138,509.8532.$

At the end of the ſixth year,

$$\overline{3,700 + 400} \times £577.975,636,$$

or $4,100 \times £577.975,636, = £2,369,700.1076.$

At

At the end of the ſeventh year,

$\overline{4{,}100 + 300} \times £577.975{,}636,$

or $4{,}400 \times £577.975{,}636, = £2{,}543{,}092.7984.$

At the end of the eighth year,

$\overline{4{,}400 + 200} \times £577.975{,}636,$

or $4{,}600 \times £577.975{,}636, = £2{,}658{,}687.9256.$

At the end of the ninth year,

$\overline{4{,}600 + 100} \times £577.975{,}636,$

or $4{,}700 \times £577.975{,}636, = £2{,}716{,}485.4892.$

And at the end of the tenth year,

$\overline{4{,}700 + 100} \times £577.975{,}636,$

or $4{,}800 \times £577.975{,}636, = £2{,}774{,}283.0528.$

Of the quantity of the national debt that will have been extinguiſhed at the end of the whole aforeſaid period of 60 years, by means of the ſaid ſavings.

CCCXLI. Theſe ſavings are to be employed in diminiſhing the national debt in either the firſt, or the ſecond, method above-mentioned: by which means the portion of the national debt that will have been extinguiſhed at the end of the ſaid period of 60 years, in conſequence of each of theſe ſavings, except the laſt, to wit, £2,774,283.0528, (which will accrue at the end of the laſt year of the ſaid period,) will be equal in value to a ſum that is greater than ſuch ſaving, namely, to the ſum which is the amount to which ſuch ſaving will have increaſed at the end of the ſaid period of 60 years, if we ſuppoſe it to have been improved in the mean time at compound intereſt at the rate of 4 per cent. We muſt therefore, in the next place, compute the amounts of theſe ſavings at the end of the ſaid period of 60 years, if ſo improved at compound intereſt in the mean time, or during the remaining years of the ſaid term after the times at which they will have reſpectively accrued, that is, during 9 years, 8 years, 7 years, 6 years, and ſo on, to the laſt year of the ſaid period. Now theſe amounts will be as follows.

The amounts of all the ſaid ſavings, except the laſt, at the end of the ſaid period of 60 years, if improved in the mean time by compound intereſt at the rate of 4 per cent.

CCCXLII. The amount of the firſt of the foregoing ſavings, (which will accrue to the publick at the end of the 51ſt year of the ſaid period of 60 years,) to wit, £577,975.6360, improved at compound intereſt at the rate of 4 per cent. during 9 years, is = £577,975.6360 × 1.423,311,81 = £822,639.5486.

The amount of the ſecond ſaving, £1,098,153.7084, improved in the ſame manner during 8 years, is = £1,098,153.7084 × 1.368,569,05 = £1,502,899.1774.

The amount of the third ſaving, £1,502,736.6536, improved in the ſame manner during 7 years, is = £1,502,736.6536 × 1.315,931,78 = £1,977,498.9194.

And the amounts of the ſix following ſavings, improved in the ſame manner during the remaining years of the ſaid period of 60 years after the times at which they will have reſpectively accrued to the publick, will be as follows.

£1,849,522.0352	×	1.265,319,02	=	£2,340,235.4090.
£2,138,509.8532	×	1.216,652,90	=	£2,601,824.2145.
£2,369,700.1076	×	1.169,858,56	=	£2,772,213.9555.
£2,543,092.7984	×	1.124,864,00	=	£2,860,633.5375.
£2,658,687.9256	×	1.081,600,00	=	£2,875,636.8603.
£2,716,485.4892	×	1.040,000,00	=	£2,825,144.9087.

The addition of the ſaid nine amounts, together with the ſaving that will accrue at the end of the laſt, or 60th, year of the ſaid period, into one ſum.

CCCXLIII. Having thus found the amounts to which the ſeveral yearly ſavings made at the ends of the fifty-firſt year and every following year of the ſaid period of 60 years, except the laſt, will have increaſed at the end of the ſaid term by being improved in the mean time at compound intereſt at the rate of 4 per cent. we muſt now add up all theſe amounts, together with the ſaving of £2,774,283.0528, (which will accrue at the end of the laſt year,) into one ſum: which may be done as follows.

£822,639 5486,

£
822,639.5486,
1,502,899.1774,
1,977,498.9194,
2,340,235.4090,
2,601,824.2145,
2,772,213.9555,
2,860,633.5375,
2,875,636.8603,
2,825,144.9087,
2,774,283.0528.
23,353,009.5837.

Therefore the ſum of the amounts of all theſe ten ſavings at the end of the ſaid period of 60 years, is £23,353,009.5837.

CCCXLIV. This ſum of the amounts of theſe laſt ten ſavings made in the laſt ten years of the ſaid period of 60 years, muſt now be added to the amount of the former great ſavings of £2,100,026.918;600 *per annum*, which began to accrue to the publick at the end of the firſt term of 30 years; which amount was ſhewn above (in Art. CCCXXXVII,) to be £117,779,879.0240. Therefore the ſum total of the amounts of all the ſavings at the end of the whole period of 60 years will be £117,779,879.0240 + £23,353,009.5837, or £141,132,888.6077; the intereſt of which at 4 per cent. is $\left(\frac{4}{100} \times £141,132,888.6077,\right.$ or $\left.\frac{£564,531,554.4308}{100},\right.$ or) £5,645,315.5443 *per annum*. Therefore the quantity of the capital of the national debt which will have been extinguiſhed, at the end of the ſaid period of 60 years, by means of all the ſaid ſavings, will be equal in value to the ſum of £141,132,888.6077; and the quantity of the intereſt of the ſaid debt, or of the perpetual annuities now employed in paying the ſaid intereſt, which would be thereby redeemed to the publick at the end of the ſaid period of 60 years, (ſo as to be free, at the end of the next, or 61ſt, year, to be diſpoſed of in any way the Parliament ſhould think fit,) would be the ſum of £5,645,315.5443 *per annum*, or (neglecting the fraction .5443) the ſum of £5,645,315 *per annum*. Q E I.

The addition of the laſt ſum total to the amount of the former great ſavings which began to accrue to the publick at the end of the firſt term above-mentioned, conſiſting of 30 years.

The quantity of the capital of the national debt that will have been extinguiſhed at the end of the whole period of 60 years by means of all the ſavings above-mentioned.

The quantity of the intereſt of the ſaid debt that will have been then redeemed to the publick by means of all the ſaid ſavings.

CCCXLV. To

The whole quantity of the intereſt of the national debt that will have been redeemed to the publick at the end of the ſaid whole period of 60 years by the faithful application of a million of pounds, ſterling, *per annum* according to this ſeventh method.

CCCXLV. To this ſum of £5,645,315 *per annum*, which will thus have been redeemed to the publick at the end of the ſaid period of 60 years by means of all the foregoing ſavings, we muſt add the ſum of £3,874,310, *per annum*, which was the quantity of the perpetual annuities, due to the publick creditors, which were ſuppoſed to be converted into life-annuities of the compound kind above-deſcribed; and the ſum thence ariſing, to wit, the ſum of £9,519,625 *per annum*, will be the whole quantity of the intereſt of the national debt, or of the perpetual annuities employed in the payment of the ſaid intereſt, which will be redeemed to the publick at the end of the ſaid term of 60 years, (ſo as to be free, at the end of the next, or 61ſt, year, to be diſpoſed of by the Parliament in any other way,) by the faithful application of a million of pounds, ſterling, a year to that uſeful purpoſe during that period according to this ſeventh method.

This ſum is the ſame with the quantity of the intereſt of the national debt that would be redeemed to the publick in the ſame time by the application of the ſame annual ſum of a million according to either of the ſix foregoing methods.

CCCXLVI. This ſum of £9,519,625 *per annum* of the perpetual annuities, (now due to the publick creditors,) which will be thus redeemed to the publick in the courſe of 60 years by means of an annual ſum of a million of pounds, ſterling, employed for this purpoſe according to this ſeventh method, is equal to the quantity of the ſaid perpetual annuities which would have been redeemed to the publick in the ſame ſpace of 60 years by employing the ſame annual ſum of a million of pounds, ſterling, for the ſame purpoſe in either of the ſix preceeding methods. For we have ſeen above that the ſaid quantity is £9,520,000 *per annum*, or, more accurately, £9,519,627 *per annum* in the firſt, ſecond, and third methods, and £9,519,599 *per annum* in the fourth method, and £9,519,852 *per annum* in the fifth method, and £9,519,590 *per annum* in the ſixth method; all which quantities differ ſo very little from each other, that they may be conſidered as equal.

A compariſon of this ſeventh method of applying a given annual ſum of money to the diminution of the national debt, with the former methods of doing the ſame thing, with reſpect to the advantage mentioned in Art. 254.

CCCXLVII. This ſeventh method of employing a given ſum of money every year in the diminution of the national debt, "by converting ſome of the perpetual annuities that now pay the intereſt of it, into life-annuities of the foregoing compound kind, or that ſhall continue at all events for a certain number of years (ſo as to be payable during the ſaid term either to the life-annuitants themſelves, or their executors, or other perſonal repreſentatives,) and afterwards ſhall be accompanied, during a ſecond term of years, with the benefit of ſurvivorſhip, and then, after the expiration of the ſaid ſecond term, ſhall continue at their laſt, or greateſt, magnitude during the remaining years of the lives of the ſaid life-annuitants," partakes, in a conſiderable degree, of the advantage mentioned in Art. CCLIV as belonging to the third method, to wit, the advantage of

ſecuring

ſecuring to the publick the faithful application of ſuch annual ſum to the purpoſes of its deſtination, without ſuſpenſion or interruption. For it would then be impoſſible to with-hold any part of the life-annuities ſo created from the perſons to whom they would be due, without an abſolute breach of the publick faith; which (as we before obſerved,) may be conſidered as a moral impoſſibility. And this advantage will relate not only to ſo much of the publick revenue as will at any time be due to the life-annuitants themſelves who will be ſtill alive, (as was the caſe in the fourth method above-mentioned, by which ſome of the perpetual annuities were converted into ſimple life-annuities, without a concomitant term of years, or any benefit of ſurvivorſhip;) but it will relate alſo, during the aforeſaid firſt term of years, to that part of the publick revenue which was allotted to the payment of the annuities of the deceaſed life-annuitants; becauſe the annuities of ſuch deceaſed life-annuitants will be due, during the ſaid firſt term of years, to their reſpective executors, or other perſonal repreſentatives: and it will relate alſo, during the ſecond term of years above-mentioned, to the annuities of all the life-annuitants who were living at the beginning of the ſaid ſecond term, or at the end of the aforeſaid firſt term; becauſe, during the whole of the ſaid ſecond term, the annuities of the life-annuitants who ſhall die in the courſe of the ſaid ſecond term, are to accrue to their ſurviving companions. But it will not relate to the great annual ſaving which will accrue to the publick at the end of the ſaid firſt term, nor to the latter annual ſavings which will accrue to the publick at the end of the ſaid ſecond term by the deaths of the life-annuitants who ſhall die after the end of the ſaid ſecond term. For this part of the publick revenue, which will be thus ſaved every year to the publick, will be as liable to be diverted, by the miniſters of ſtate and the Parliament, from its original deſtination of diminiſhing the national debt, as the money that ſhould be allotted to the diminution of the national debt in either the firſt or the ſecond method above-deſcribed: whereas in the third method of applying a given ſum of money every year to the diminution of the national debt, to wit, "by converting ſome of the perpetual annuities, that now pay the intereſt of it, into greater temporary annuities, that ſhould continue for a limited number of years, and allotting the ſaid annual ſum to the payment of the additions thereupon made to the perpetual annuities ſo converted," the whole of the ſaid annual ſum would be effectually ſecured from being diverted from its original and proper deſtination, and applied to any other purpoſe whatſoever, during the whole of the ſaid term.

CCCXLVIII. Theſe

A remark on the ſeven foregoing methods of applying a given annual ſum of money to the diminution of the national debt.

CCCXLVIII. Theſe are the methods that, I imagine, the Government would be moſt likely to adopt, if they were earneſtly to ſet about this good work of reducing the national debt to a more moderate quantity by ſetting apart and employing a given ſum of money every year, out of the publick revenue, for that purpoſe. And I have therefore thought it might be uſeful to ſet them forth in a very full and plain manner, and illuſtrate the effects of each of them by an example. And, in order the better to compare theſe effects with one another, I have thought it beſt, in the ſaid examples, to ſuppoſe the ſame annual ſum of money, to wit, a million of pounds, ſterling, *per annum*, to be employed for the ſame number of years, to wit, a period of 60 years, in each of the ſaid methods. And the reſult is, that all theſe methods are equally efficacious towards the great purpoſe of diminiſhing the national debt, if they are purſued with equal ſteadineſs and fidelity.

Some of the foregoing methods could not, probably, be carried into execution to any great extent.

CCCXLIX. But, though all theſe methods would be equally efficacious towards the reduction of the national debt, they are not all equally fit to be adopted, or capable of being carried into execution to any great extent. For it is probable that ſome of them would be thought convenient by only a ſmall number of the preſent publick creditors, and conſequently could be adopted only to a ſmall extent; the conſent of the ſaid creditors being neceſſary to all the foregoing methods, except the firſt. Thus, for example, the fourth method, "by which ſome of the perpetual annuities, now due to the publick creditors, would, with the conſent of the ſaid creditors, be converted into mere life-annuities, without a term of years certain annexed to them, or any benefit of ſurvivorſhip," would probably ſuit only a ſmall number of the preſent owners of the perpetual annuities; as, for inſtance, the owners of £180,393.5 *per annum* of the ſaid perpetual annuities, inſtead of the owners of £1,803,935, *per annum* of the ſaid annuities, as was ſuppoſed above in Art. CCLIX; and conſequently an annual ſum of only £100,000 *per annum*, and not of £1000,000 *per annum*, could be employed in this method. And, indeed it does not ſeem probable that any one of the five latter methods of employing a given ſum of money in the diminution of the national debt could be carried into execution to the extent ſuppoſed in the foregoing examples, or ſo as to employ ſo large a fund as a million of pounds, ſterling, *per annum*; becauſe it is not likely that any one of the ſaid methods ſhould ſuit the convenience of a ſufficient number of the publick creditors, or proprietors of the perpetual annuities now due from the Government, to make ſuch a meaſure practicable. But, perhaps, if the Government were to adopt *all the five* methods, and to propoſe to the people, in the firſt place, that a part of the ſaid annual ſum of a million of pounds, ſterling, as, for inſtance, half of it, or

But it ſeems likely that each of the five latter methods might be found practicable and convenient to a certain moderate extent.

or £500,000 *per annum*, ſhould be employed in converting ſome of the perpetual annuities, now due to the publick creditors, into greater temporary annuities, that ſhould continue only during a certain limited number of years, as, for example, 60 years, according to the third method above-deſcribed; —— and, in the ſecond place, that another part of the ſaid annual million, as, for inſtance, £100,000 *per annum*, ſhould be employed in converting a ſecond portion of the ſaid perpetual annuities into ſimple life-annuities, according to the fourth method above-deſcribed; —— and, thirdly, that another part of the ſaid annual million, as, for inſtance, £200,000 *per annum*, ſhould be employed in converting a third portion of the ſaid perpetual annuities into mixt annuities, or life-annuities, with a term of years certain annexed to them, or that ſhould continue during the lives of certain perſons of known ages, and likewiſe, in caſe thoſe life-annuitants ſhould die within a certain moderate number of years, (as, for inſtance, 30 years,) ſhould continue during the remainder of the ſaid term of years, and be payable to the executors, or other perſonal repreſentatives, of the ſaid deceaſed life-annuitants, according to the fifth method above-mentioned; —— and, fourthly, that another part of the ſaid annual million, as, for inſtance, £100,000 *per annum*, ſhould be employed in converting a fourth portion of the ſaid perpetual annuities into life-annuities, with the benefit of ſurvivorſhip during a certain moderate number of years, as, for example, 30 years, according to the ſixth method above-mentioned; —— and, fifthly and laſtly, that the remaining part of the ſaid annual million ſhould be employed in converting a fifth portion of the ſaid perpetual annuities into life-annuities accompanied with both the foregoing advantages, that is, with a permanent intereſt in the ſaid annuities to be enjoyed at all events by either the ſaid life-annuitants or their repreſentatives, during a certain moderate term of years, (as, for inſtance, 30 years,) and likewiſe, after the expiration of the ſaid term, with the benefit of ſucceeding by ſurvivorſhip to each other's annuities during a ſecond term of years, as, for example, 20 years; ——; I ſay, if the Government were to make theſe propoſals to the people, it ſeems not unlikely that they might meet with the concurrence of a ſufficient number of the publick creditors to enable the Government to carry them into execution to a ſufficient extent to exhauſt the whole of the ſaid ſuppoſed fund of a million of pounds *per annum*. But, if they did not meet with that concurrence to ſo great an extent, the remaining part of the ſaid annual million, which could not be employed in either of thoſe five methods, might be employed, for the ſame good purpoſe of diminiſhing the national debt, in either the firſt or the ſecond method above-deſcribed; and with equal advantage to the publick, if employed in either of thoſe methods with equal ſteadineſs and fidelity.

Of the diſadvantage that may ariſe to the public by granting life-annuities to perſons of different ages for the ſame price.

CCCL. When life-annuities have, on former occaſions, been eſtabliſhed in England by the Government, it has, I believe, been cuſtomary to take the ſame price for annuities of the ſame yearly amount from perſons of all ages, or, at leaſt, of ſeveral different ages within certain, pretty diſtant, limits. Now this ſeems to be a very injudicious practice, and would be likely to prove diſadvantageous to the nation, even though the price taken for the ſeveral annuities ſo granted ſhould be a juſt medium between the true values of the ſame annuities for the oldeſt and the youngeſt lives for which they were made grantable: becauſe it is probable that the greater part of the perſons who would purchaſe ſuch annuities would be nearer to the youngeſt age for which they would be grantable than to the oldeſt. Thus, for example, if a thouſand annuities of 10 pounds a year each, were to be offered to ſale by the Government for the lives of any perſons between the ages of 20 years and 40 years at the price which is the true value of a like annuity of £10 a year for the life of a perſon of the age of 30 years, it is probable that the greater part of theſe annuities would be purchaſed for the lives of perſons under 30 years of age, and conſequently for leſs than their true value: and therefore the Government, or Nation, would loſe more by the ſale of theſe annuities to perſons under the age of 30 years, than they would gain by the few annuities, of the ſame price, or claſs, which would be ſold to perſons who were above the age of 30 years, but under that of 40; and conſequently they would, upon the whole, be loſers by ſuch a bargain. It would therefore, I apprehend, be more prudent in the Government, when annuities are to be granted to perſons of different ages, to ſell them at their true and proper prices according to the ſeveral ages of the perſons for whoſe lives they are granted, having recourſe, for the determination of the ſaid prices, to the tables above-computed and ſet forth in pages 221, 222, &c.—232, and conſidering every ſuch life-annuitant as being of the age to which he had attained on his laſt preceeding birth-day.

This diſadvantage would be ſtill greater in the caſe of life-annuities with the benefit of ſurvivorſhip.

CCCLI. The diſadvantage to the publick, mentioned in the foregoing article as reſulting from the practice of granting life-annuities to perſons of different ages at the ſame price, would, it is evident, be conſiderably greater in the caſe of life-annuities accompanied with the benefit of ſurvivorſhip, than in the caſe of ſimple life-annuities: becauſe in the caſe of ſurvivorſhip thoſe life-annuitants whoſe ages would have but little exceeded the youngeſt age of the claſs, and who would conſequently have paid too little for their annuities, would, by means of this benefit of ſurvivorſhip, if they happened to live a good number of years, enjoy much larger annuities than in the other caſe of ſimple life-annuities; and conſequently the loſs of the publick would be proportionably greater in the former caſe than in the latter. It would therefore be very highly

expedient,

expedient, if the Government were to reſolve to eſtabliſh any life-annuities with the benefit of ſurvivorſhip, to take care that there ſhould be as many diſtinct claſſes of life-annuitants, intitled to the ſaid benefit of ſurvivorſhip with reſpect to each other, as there were different ages, differing from each other by a year, amongſt the ſaid life-annuitants, ſo that thoſe perſons only who were of the ſame age, or within a year of the ſame age, ſhould belong to the ſame claſs, and pay the ſame price for their annuities. This might be done, as I imagine, without occaſioning much difficulty or confuſion, by dividing the whole annual ſum which it was propoſed to apply to the converſion of ſome of the perpetual annuities into life-annuities with the benefit of ſurvivorſhip, into as many parts as there were different years in the ages of the life-annuitants for whoſe lives the ſaid annuities were intended to be granted. Thus, for example, if it were the intention of Government that perſons of any age from the age of 10 years to the age of 59 years, incluſively, ſhould be admitted to become purchaſers of theſe life-annuities with benefit of ſurvivorſhip, and the whole annual ſum intended to be employed in converting ſome of the perpetual annuities into theſe life-annuities with benefit of ſurvivorſhip, was £100,000 *per annum*; it would be expedient to divide the ſaid ſum of £100,000 *per annum* into fifty equal parts of £2000 *per annum* each, and to apply one of theſe parts, or the annual ſum of £2000, to the converſion of ſome of the perpetual annuities into theſe life-annuities with benefit of ſurvivorſhip, for the lives of perſons of the age of 10 years; and to apply another ſum of £2000 *per annum* to the converſion of ſome of the perpetual annuities into theſe life-annuities, with benefit of ſurvivorſhip, for the lives of perſons of the age of 11 years; and to employ the like annual ſums of £2000 each, in converting ſome of the perpetual annuities into theſe life-annuities, with the benefit of ſurvivorſhip, for the lives of perſons of the ſeveral ages of 12 years, 13 years, 14 years, 15 years, and every following number of years to 59 years incluſively. And the only difficulty that would attend this diviſion of the ſaid life-annuitants into the ſaid fifty claſſes according to their ages, would be to determine what would be the proper prices of the life-annuities that ſhould be granted to each claſs of life-annuitants, or what would be the quantity of the perpetual annuities, now due to the publick creditors, which the additional ſum of £2000 *per annum* would enable the Government to convert into life-annuities of the kind intended, accompanied with the benefit of ſurvivorſhip, for the lives of perſons of each of the ſaid fifty ages: which might be done by making a computation for each of the ſaid ages ſimilar to the computations entered into above in the examples of the fifth and ſeventh methods above-mentioned. And theſe computations the Government might eaſily procure to be made for the purpoſe by perſons that were but moderately converſant with this doctrine of Life-annuities, if they ſhould be of opinion that theſe methods of diminiſhing the national debt by eſtabliſhing life-annuities with benefit of ſurvivorſhip, were worth adopting.

Of the manner in which ſuch life annuities might be eſtabliſhed, ſo as to avoid that diſadvantage.

Of the frequent alienations of the Sinking Fund, that have been made for many years paſt.

CCCLII. It has been obſerved above, in Art. CCLIV, that our miniſters of ſtate and our parliaments have, ever ſince the year 1733, (and we might have ſaid, ever ſince the year 1727;) gone into a moſt pernicious practice, of diverting the Sinking Fund (which had been appropriated in the year 1717 to the gradual diſcharge of the national debt,) from the original purpoſes of its deſtination, and applying it to the current ſervices of the year. The conſequence of this practice has been the increaſe of the national debt to a much greater quantity than it would otherwiſe have arrived at, notwithſtanding the three wars of 1739, 1755, and 177-, in which we have been unfortunately, and, perhaps, unneceſſarily, engaged.

A miſtaken opinion that has ſometimes been advanced in juſtification of the ſaid alienations.

Yet it has been often alledged in juſtification of this practice, "That it is "a matter of indifference to the nation, and to the increaſe or diminution "of its debts, whether, when any given ſum of money is wanted for any "particular ſervice, the ſaid ſum be borrowed at intereſt, and a new tax "be laid for the payment of the ſaid intereſt, or whether the ſaid ſum "be taken at once out of the ſinking fund and no new tax be laid to "ſupply its place." "Thus, for example, ſay theſe gentlemen, ſuppoſe "it ſhould be neceſſary at any time to apply the ſum of a million of "pounds, ſterling, to ſome particular purpoſe not included in the con-"ſtant and ordinary expences of the ſtate, as, for inſtance, to the building "ſome new ſhips of war for the Royal Navy, or purchaſing naval ſtores; "and ſuppoſe that the ſinking fund produces an overplus of ſomething "more than a million of pounds, one year with another, above the "neceſſary expences of the ordinary government in time of peace; why "ſhould we not take the ſaid million, (which is wanted for the ſaid "purpoſe,) out of the ſinking fund, and thereby only ſuſpend, or rather "leſſen, for one year the operation of the ſaid fund in diminiſhing the "national debt, inſtead of borrowing the ſaid ſum of a million at intereſt "of new creditors, and leaving the whole of the ſinking fund to be "applied to the purpoſes of its deſtination? By the former method of "proceeding the national debt will not, it is true, have been diminiſhed "in the courſe of the year by the ſum of £1,000,000, as it would other-"wiſe have been; but then, on the other hand, it will not have been "increaſed by a new loan of £1,000,000, as it will be in the latter "caſe: and therefore, upon the whole, the quantity of the ſaid debt "at the end of the year will be the ſame in both caſes." This, I believe, is the manner in which the ſaid remark is uſually ſtated. But it is, as Dr. Price has obſerved, a very great and dangerous ſophiſm.

Remarks on the ſaid miſtaken opinion.

For it is tantamount to ſaying, that it comes to the ſame thing, with reſpect to the diminution of the national debt, whether the Government does, or does not, raiſe new taxes towards defraying the extraordinary expences of the ſtate; which, it is evident, cannot be true. If, indeed, the aforeſaid million were to be borrowed at intereſt, and no new tax were to be laid to pay the ſaid intereſt, but the ſaid intereſt were to be taken every

every year out of the ſinking fund, the above-mentioned reaſoning would be juſt, and ſuch a proceeding would have exactly the ſame effect in retarding the diſcharge of the national debt as if the Government had taken the whole million itſelf out of the ſinking fund at once. For a million of pounds, ſterling, paid at once, is equal in value to the intereſt of a million of pounds, ſterling, to be paid every year for ever. But, when a new tax is provided to pay the intereſt of the new loan, the ſaid two methods of procuring the ſaid million, or other ſum that is wanted, can be no longer equivalent to each other, but the former method, to wit, "that of borrowing the money, and at the ſame time laying a new tax to pay the intereſt of it," muſt tend leſs than the other method, to wit, "that of taking the whole ſum at once out of the ſinking fund," to retard the diſcharge of the national debt, and therefore will be preferable to it with reſpect to that important operation. This is ſo evident that, I confeſs, I am ſurprized it can ever have been doubted of. But, ſince it has been doubted of, and ſince it is a truth of very conſiderable importance, I will endeavour to illuſtrate it by an example: which may be done as follows.

An example, to illuſtrate the falſhood of the opinion ſtated and confuted in the preceeding article.

CCCLIII. Let it be ſuppoſed (as in the ſeveral examples above-mentioned,) that the intereſt of money is 4 per cent. and that the 4 per cent. annuities, now due to the publick creditors, ſell at the publick market at their *par*, or nominal value. And let it further be ſuppoſed that the Parliament had ſet apart the ſum of a million of pounds, ſterling, a year out of the ſinking fund, for the diminution of the national debt, and that the ſaid £1,000,000 *per annum* were to be faithfully applied to that purpoſe during a period of 20 years, in the firſt of the ſeven methods above-deſcribed, that is, in paying off ſome of the 4 per cent. annuities, now due to the publick creditors, at their *par*, or nominal value; by which means it is evident that a very conſiderable portion of the national debt will be extinguiſhed at the end of the ſaid 20 years. And let it be ſuppoſed that during every one of the ſaid 20 years the Parliament ſhall find it neceſſary to employ the ſame ſum of £1,000,000 in ſome other important ſervices, and that, being unwilling to ſuſpend, or impede, the operation of the ſinking fund, they ſhall forbear taking this million out of it, and ſhall, inſtead thereof, borrow a million every year of new creditors at 4 per cent. and at the ſame time impoſe every year a new perpetual tax of £40,000 *per annum* to pay the intereſt of the million ſo borrowed. And then let us inquire what will be the effect of this method of proceeding at the end of the ſaid 20 years, with reſpect to the increaſe or diminution of the national debt. This may be done in the manner following.

CCCLIV. The

CCCLIV. The quantity of the capital of the 4 per cent. annuities which will be extinguiſhed at the end of the ſaid 20 years by employing the ſaid £1,000,000 every year for that purpoſe in the firſt of the ſeven methods above-deſcribed, will be equal to the ſum to which an annual revenue of £1,000,000 will amount at the end of 20 years, if every payment of the ſaid revenue, together with the intereſt ariſing from the ſaid payment, ſhould, immediately after it is received, be lent out at intereſt at the rate of 4 per cent. Now it appears from Mr. Smart's third table of compound intereſt, page 68, that this ſum is = £1,000,000 × 29.778,078,58, or £29,778,078.58. Therefore the ſum of £29,778,078.58 of the capital of the 4 per cent. annuities, now due to the publick creditors, would be extinguiſhed by means of the ſaid £1,000,000 *per annum* in the courſe of the ſaid 20 years; or the national debt would be diminiſhed, by means of the ſaid £1,000,000 *per annum*, at the end of the ſaid 20 years, by the ſum of £29,778,078.58. But on the other hand it will be increaſed in the courſe of the ſaid 20 years by means of the aforeſaid loans of a million each, (which we have ſuppoſed to be made every year with new funds to pay the intereſt of them,) by only the ſum of £20,000,000. Therefore it will have been more diminiſhed than increaſed, and conſequently will, upon the whole, have been diminiſhed, in the courſe of the ſaid 20 years, by the exceſs of £29,778,078.58 above £20,000,000, that is, by the ſum of £9,778,078.58; which is not very much leſs than ten millions. Such will be the good effect, with reſpect to the diminution of the national debt, of the ſaid firſt method of proceeding, "by borrowing the money that is wanted every year, and laying new taxes to pay the intereſt of it."

CCCLV. But, if the other way of proceeding be adopted, or the million of pounds, that is wanted every year for the ſuppoſed ſervices, be taken out of the ſinking fund, it is evident, the operation of the ſaid million, in diminiſhing the national debt, will be thereby totally ſuſpended, and the ſaid debt will be exactly the ſame at the end of the ſaid 20 years as at the beginning of them.

CCCLVI. Therefore the difference between the quantity of the national debt at the end of 20 years, if the former method be taken, and the quantity of it at the ſame time, if the latter method be adopted, is no leſs a ſum than £9,778,078.58, or not much leſs than ten millions of pounds. Q E I.

CCCLVII. If

CCCLVII. If we had taken a longer period than 20 years, the difference of the two quantities of the national debt at the end of it, arising from theſe different methods of raiſing the ſaid annual ſum of a million of pounds, ſterling, would have much greater than £9,778,078.58. In a period of 30 years it would have been (£56,084,937.76 — £30,000,000, or) £26,084,937.76; in a period of 40 years it would have been (£95,025,515.72 — £40,000,000, or) £55,025,515.72; in a period of 50 years it would have been (£152,667,083.68 — £50,000,000, or) £102,667,083.68; and in a period of 60 years it would have been (£237,990,685.24 — £60,000,000, or) £177,990,685.24; as will appear from Mr. Smart's third table of compound intereſt, pages 68, 70, and 72.

Other examples for the ſame purpoſe.

CCCLVIII. The difference between theſe two methods of raiſing the money that may from time to time become neceſſary to defray the extra-ordinary expences of the ſtate, is a matter of ſuch importance that it can hardly be too much inſiſted on. And it ſeems to have been well underſtood, and properly attended to, by our miniſters of ſtate in the reign of king George the 1ſt, when the ſinking fund was firſt eſtabliſhed. For at that time it was cuſtomary for the Parliament to borrow money to ſupply the occaſional exigencies of the ſtate, and lay new taxes to pay the intereſt of the money ſo borrowed, inſtead of taking the ſaid money out of the ſinking fund and thereby interrupting the operation of that uſeful eſtabliſhment. Of this ſalutary practice Dr. Price in his excellent *Appeal to the Publick on the Subject of the National Debt*, 2d edition, publiſhed in the year 1772, page 29, note *b*, has given us ſeveral examples. He there informs us, that

The falſhood of the ſaid miſtaken opinion ſeems to have been well underſtood in the reign of king George the 1ſt.

The Sinking Fund was faithfully applied to the diſcharge of the national debt to the end of that reign.

	£
In 1718 was borrowed towards the ſupplies — — —	505,995;
In 1719 — — — — — —	312,737;
In 1720 — — — — —	500,000;
In 1721 — — — — — —	1,000,000;
In 1725 — — — — — —	500,000;
In 1726 — — — — — —	370,000;
And In 1727 — — — — —	1,750,000.

In all theſe inſtances (if I underſtand the doctor rightly,) the Parliament not only borrowed theſe ſeveral ſums, rather than take them out of the ſinking fund, (as they might have done,) but laid new taxes to pay the intereſt of them. And in the year 1726, (the nation being then under the apprehenſion of a war,) the land-tax was raiſed from two ſhillings in the pound to four ſhillings in the pound, rather than take any money from the ſinking fund. And in the following year, 1727, (in which the nation ſtill

continued

continued under the apprehenſion of a war,) king George the 1ſt in his ſpeech at the opening of the parliament, (on the 17th of January, $172\frac{6}{7}$,) after congratulating the gentlemen of the Houſe of Commons on the great addition that would be made that year to the *Sinking Fund*, warns them againſt being led by the NECESSITIES OF THE NATION to a diverſion of it. "Let all that wiſh well (are the words of the ſpeech,) "to the peace and quiet of my government, have the ſatisfaction to ſee "that our PRESENT NECESSITIES ſhall make no interruption in the "progreſs of that deſireable work of gradually diſcharging the national "debt. I hope therefore you will make a proviſion for the immediate "application of the produce of the Sinking Fund to the uſes for which "it was ſo wiſely contrived, and to which it ſtands now appropriated." To which the Houſe of Commons in their addreſs to the king in anſwer to his ſaid ſpeech, (which was carried, upon a debate and diviſion, by 251 voices againſt 81, and was preſented to the king on the 19th of January, $172\frac{6}{7}$,) reply in the words following, to wit, "And that all who "wiſh well to the peace and quiet of your Majeſty's government, may "have the ſatisfaction to ſee, that our PRESENT NECESSITIES ſhall "make no interruption in the progreſs of that deſireable work of gra-"dually diſcharging the national debt, we will conſider of the moſt proper "methods for immediately applying the produce of the Sinking Fund "to the uſes for which it was ſo wiſely contrived, and to which it ſtands "now appropriated." By this ſpeech of the King and addreſs of the Houſe of Commons it ſeems clear that the advantages attending a faithful application of the Sinking Fund to the purpoſes of its deſtination, were at that time well underſtood and generally allowed.

A change of conduct in the miniſters of ſtate and the Parliament, with reſpect to the Sinking Fund, took place in the year 1728.

CCCLIX. But ſoon after the death of king George the 1ſt a different opinion ſeems to have prevailed with many people, and, moſt unhappily for the kingdom, a different practice began to be adopted by the Government. Dr. Price informs us, in the paſſage above-mentioned, that in the years 1728 and 1729 the Sinking Fund was charged with the intereſt of the money borrowed in thoſe years; which was juſt as pernicious a meaſure, with reſpect to the interruption of the great object of diminiſhing the national debt, as if the Parliament had taken the whole ſums themſelves at once from the Sinking Fund, inſtead of borrowing them: as we have already obſerved above in Art. CCCLII, page 380, line 4 from the bottom, &c. It was, however, a leſs open and manifeſt violation of the Sinking Fund than the other meaſure of taking the groſs ſums themſelves out of it; and therefore probably it gave leſs offence to that part of the nation which had moſt approved of the inſtitution of the Sinking Fund, and it contributed leſs to expoſe Sir Robert Walpole (who was chancellor of

of the Exchequer at the inſtitution of the Sinking Fund in the month of March, $17\frac{16}{17}$,* and was the principal adviſer and promoter of that uſeful meaſure, and who was likewiſe in the ſame great office at the time of the aforeſaid miſapplications of the ſaid fund in the years 1728 and 1729,) to the charge of *inconſiſtency* and of becoming the deſtroyer of his own favourite and juſtly-applauded meaſure. And accordingly we find that, while this appearance of delicacy with reſpect to the violation of the Sinking Fund continued, the King and Parliament, both, ſpoke of it with reſpect, and pretended to be unwilling to break into it. For in the year 1729, when (the nation being ſtill threatened with a war,) extra-ordinary ſupplies were wanted, and the Sinking Fund would have afforded all that was neceſſary, king George the ſecond, in his ſpeech to the Parliament at the opening of the ſeſſion, on the 21ſt of January, $172\frac{8}{9}$, expreſſed himſelf to the members of the Houſe of Commons in theſe words; "And, as the produce of the Sinking Fund has exceeded our expecta- "tions, I muſt recommend it to your care to make a farther application "of it to its proper uſes." To which the Commons, in their addreſs to the King upon this ſpeech, make the following anſwer, to wit, "That "they will not fail to make the proper diſpoſition of the growing produce "of the Sinking Fund." And accordingly a million of the South-Sea annuities was ordered to be paid off that year out of the Sinking Fund; but at the ſame time the intereſt of the new money that was borrowed this year, was charged upon the ſaid fund, and became a perpetual incumbrance on it. And the ſame thing had been done in the preceeding year, 1728, with reſpect to the intereſt of the money borrowed in that year.

But yet an appearance of an unwillingneſs to break into it was kept up by the king and Parliament for about five years longer.

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* The Sinking Fund was eſtabliſhed by Stat. 3 George I, Cap. vii, Sect. 37, which is in theſe words. "And be it enacted and declared by the authority afore- "ſaid, That all the monies to ariſe from time to time, as well of, or for, the ſaid "exceſs, or ſurplus, by virtue of the ſaid act made for redeeming the Funds of the "Governour and Company of the Bank of England; and of, or for, the ſaid exceſs, "or ſurplus, by virtue of the ſaid act made for redeeming the Funds of the ſaid "Governour and Company of Merchants of Great-Britain trading to the South- "Seas, and other parts of America, and for encouraging the Fiſhery; as alſo of and "for the ſaid exceſs, or ſurplus, of the ſaid duties and revenues by this act appro- "priated, as aforeſaid; and the ſaid overplus monies of the ſaid general yearly Fund "by this act eſtabliſhed, or intended to be eſtabliſhed, as aforeſaid; *ſhall be appro- "priated, reſerved, and employed, to and for the diſcharging the principal and intereſt of "ſuch national debts and incumbrances as were incurred before the five and twentieth day "of December, one thouſand, ſeven hundred, and ſixteen*, and are declared to be national "debts, and are provided for by act of Parliament, in ſuch manner and form as ſhall "be directed or appointed by any future act or acts of Parliament to be diſcharged "therewith or out of the ſame, *and to, or for, none other uſe or purpoſe whatſoever*."

The words of the clauſe in the ſtatute of the 3d of Geo. I, by which the Sinking Fund was eſtabliſhed.

In the year 1733 the Parliament took a groſs ſum of half a million of pounds ſterling, out of the Sinking Fund, in order to keep the land-tax at only one ſhilling in the Pound.

CCCLX. But in a few years time this appearance of delicacy with reſpect to the miſapplication of the Sinking Fund was laid aſide, and the miniſters of ſtate and parliament openly took out of it whatever groſs ſums of money they thought neceſſary for the publick exigencies. The manner in which this unfortunate change of conduct happened, is related by Dr. Price in his tract above-mentioned, intitled *An Appeal to the Publick on the Subject of the National Debt*, in theſe words. "In the year 1732 the "land-tax had been reduced to one ſhilling in the pound; and, in order "to ſupply the deficiency ariſing from hence, half a million had been "procured for the current ſervice by the revival of the ſalt-duties, which, "but two years before, had been repealed, becauſe reckoned too burthen-"ſome to the poor.—In the year 1733, in order to keep the land-tax as "low as it had been the year before, it was neceſſary either to borrow "another half-million, or to take it from the *Sinking Fund*. The latter

Dr. Price's account of the debates upon this occaſion.

"method was choſen; and propoſed by Sir Robert Walpole to the Houſe "of Commons.—Long and warm debates enſued.—A propoſal to alienate, "in a time of profound peace, a fund which the law had made ſacred, "and the alienation of which no poſſible exigence of publick affairs could "juſtify, only for the ſake of keeping the land-tax for one year at one "ſhilling in the pound, juſtly kindled the indignation of the patriotick "party. They urged, in oppoſition to it, the prohibition of the law, "the faith of Parliament, and the ſecurity of the kingdom. The pro-"poſer of the alienation was reminded of his inconſiſtency and treachery "in endeavouring to beat down that very monument of glory which he "had boaſted of having erected for himſelf; and Sir John Barnard warned "him that he was drawing upon himſelf the curſes of poſterity—But all "arguments were vain.—The miniſtry pleaded that the landed intereſt "wanted eaſe; that there was no occaſion for being in a hurry to pay the "national debt; and that the circumſtances of the kingdom had altered "ſo much ſince the eſtabliſhment of the *Sinking Fund*, that the competition "then among the publick creditors was, not who ſhould be *firſt*, but who "ſhould be *laſt*, paid. Thus argued, among others, Sir Walpole. His "reaſons prevailed; and the Houſe of Commons, not uſed to refuſe him "any thing, conſented.

And the ſame pernicious practice has continued ever ſince.

"The practice of alienating the Sinking Fund having been thus begun, "it went on of courſe. In the next year, or 1734, £ ,200,000 was taken "from it. In 1735 it was even anticipated and mortgaged.

"Thus then expired, after an exiſtence of about eleven years, the "*Sinking Fund*,—that ſacred bleſſing—once the nation's only hope; pre-"maturely and cruelly deſtroyed by its own parent.——Could it have "eſcaped the hands of violence, it would have made us the envy and the "terrour of the world, by leaving us at this time [in the year 1772] "not.

" not only TAX-FREE, but in poſſeſſion of a treaſure greater, perhaps, " than was ever enjoyed by any kingdom.—But let me not dwell on a " recollection ſo grievous!"

CCCLXI. Dr. Price informs us further in the excellent tract above-mentioned, that about the year 1726 an opinion had been propagated, that, notwithſtanding the eſtabliſhment of the Sinking Fund, the publick debts had been for ſome years increaſing rather than decreaſing. " And " this, ſays he, occaſioned the publication of a very curious and important " pamphlet in defence of the Sinking Fund and the miniſtry, intitled, " *An Eſſay on the Publick Debts of this Kingdom.* I have now by me the " fourth edition of this pamphlet; and I wiſh I could put it into every " hand in the kingdom. It contains an excellent account of the import- " ance of diſcharging the publick debts, and of the proviſions made for " that purpoſe by the inſtitution of the Sinking Fund. It proves particu- " larly, in oppoſition to the opinion I have mentioned, that the publick " debts *had decreaſed*; and that, of the 52 millions then due, 50 millions " would in 28 years be extinguiſhed by the *Sinking Fund.* The ſame " explanation is given of the nature of this fund with that which I have " given: the ſame repreſentation is made of its powers; and the ſame " arguments are uſed to demonſtrate the evil of alienating it, in order to " avoid making new loans charged on new funds.—And he concludes " with this general inference from all his obſervations, "" That the pro- "" viſion which had been made of the *Sinking Fund*, was an expedient "" from which the full and effectual payment of the principal of the "" national debt, in a few years, might, with great aſſurance, be "" expected."" Dr Price's account of an excellent pamphlet on the publick debts of this kingdom, that was publiſhed in the year 1726.

CCCLXII. This pamphlet, Dr. Price further informs us, was anſwered in a pamphlet intitled, *A State of the National Debt*; and this produced another pamphlet by the author of the firſt, (or the *Eſſay on the Publick Debts,*) which was intitled, *A Defence of an Eſſay on the Publick Debts of this Kingdom, in anſwer to a pamphlet, intitled, A State of the National Debt.* And in this laſt pamphlet, intitled, *A Defence of an Eſſay, &c.* there are theſe words. " The ſucceſs of my *Eſſay on the Publick Debts*, " and the ſatisfaction of hearing from the throne, that my zeal for the pre- " ſervation of the Sinking Fund cannot have been diſagreeable to his Ma- " jeſty, gives me a pleaſure that, alone, is an ample reward for my attempts " to ſerve my country." From theſe words it ſeems probable that this pamphlet (of which I have now in my hands the ſecond edition, which was publiſhed in the year 1726;) had been very much read and approved of, and had tended very much to confirm the miniſters of ſtate in their reſolu- Of the good reception the ſaid pamphlet ſeems to have met with from the publick.

tion to continue, for that year at leaſt, to apply the Sinking Fund to the purpoſe of its deſtination, and to induce them to adviſe the king to make uſe of the words above mentioned in his ſpeech to the Parliament on the 17th of January, $172\frac{6}{7}$, to wit, " Let all that wiſh well to the peace and " quiet of my government have the ſatisfaction to ſee, that our preſent " neceſſities ſhall make no interruption in the progreſs of that deſireable " work of gradually diſcharging the national debt. I hope, therefore, you " will make a proviſion for the immediate application of the produce of " *the Sinking Fund* to the uſes for which it was ſo wiſely contrived, and to " which it ſtands now appropriated."

A conjecture concerning the author of the ſaid pamphlet.

CCCLXIII. The aforeſaid excellent pamphlet, intitled, *An Eſſay on the Publick Debts of the Kingdom*, was publiſhed without a name. But it ſeems probable that it was written by Sir Nathaniel Gould, an eminent merchant, who was one of the directors of the Bank in the year 1726, when the ſecond edition (and, perhaps, alſo the firſt edition of it,) was publiſhed. And the other pamphlet above-mentioned, which was written in anſwer to it and was intitled, *A State of the National Debt*, ſeems to have been written by the famous Mr. William Pulteney, who was afterwards created Earl of Bath. This may be conjectured with a conſiderable degree of probability from the following extract from Chandler's Collection of the Proceedings of the Houſe of Commons, vol. vii. page 23. " On " the 23d of February, $172\frac{7}{8}$, the Commons being in a committee of " ſupply, Mr. William Pulteney [who was then in oppoſition to the mi- " niſtry,] obſerved, "' That, notwithſtanding the great merit that ſome "' perſons had built upon the eſtabliſhment of the *Sinking Fund*, it appeared "' that the national debt had been increaſed ſince the ſetting up of that "' *Pompous Project*."' Upon which Sir Nathaniel Gould, an eminent " merchant, ſaid, "' That he apprehended that gentleman had his notions "' out of a treatiſe, intitled, *A State of the National Debt, &c.* ſuppoſed "' to be written by that very gentleman: but that, if he [Sir Nathaniel "' Gould] underſtood any thing, it was *Numbers*; and he durſt pawn his "' credit and reputation to prove that author's calculations and inferences "' to be falſe and erroneous."' To this Mr. Pulteney replied, "' That he "' took them to be right; and he would likewiſe pawn his credit and "' reputation to make good his aſſertion."' Upon this Sir Robert Walpole " took up the cudgels, and ſaid, "' He would maintain what Sir "' Nathaniel Gould had advanced."' Several warm expreſſions having " paſſed on both ſides, Mr. Hungerford interpoſed, in a jocular ſpeech, " that put the Houſe in good humour; and ſo the diſpute ended."

CCCLXIV. As

CCCLXIV. As this pamphlet, intitled, *An Eſſay on the Publick Debts of the Kingdom*, (which, for the reaſon juſt now given, I conjecture to have been written by Sir Nathaniel Gould,) is, in Dr. Price's opinion, of ſuch importance as to deſerve to be put into every hand in the kingdom; and, as it tends ſtrongly to illuſtrate and enforce the utility of the meaſure I have ventured to recommend above in Art. CCXLIX and CCLIV, (pages 295 and 298, 299,) "of appropriating, in the ſtricteſt manner poſſible, ſome part, at leaſt, of the Sinking Fund to the purpoſes of its original deſtination, or the gradual diſcharge of the national debt;" I am perſuaded my readers will not be ſorry to have an opportunity of peruſing it; and I therefore have determined to reprint it, word for word, as a part of the preſent Treatiſe. It is as follows.

AN

A N

E S S A Y

ON THE

PUBLICK DEBTS

OF

THIS KINGDOM.

WHEREIN

The Importance of discharging them is considered; the Provisions for that Purpose by the SINKING FUND, and the Progress therein hitherto made, are stated and explained; the Sufficiency of those Provisions is demonstrated; some general Mistakes about the Nature and Efficacy of this Expedient examined and removed; and the Progress of the SINKING FUND described and computed from *Midsummer*, 1727.

To which is subjoined,

An Enquiry into the General Convenience of reducing farther the Interest of our Publick Debts below 4 *per Cent. per Annum.*

In a Letter to a Member of the House of Commons.

Reprinted from the Second Edition, which was published by J. PEELE, in Pater-Noster-Row, in the Year 1726.

N. B. This Pamphlet is supposed to have been written by Sir NATHANIEL GOULD, an eminent Merchant and a Director of the Bank.

THE

PREFACE.

THERE may perhaps appear ſomething too aſſuming in the attempts of an author, to inform the publick, or to direct their ſentiments about matters of general importance, to admit of a better reception or entertainment from the town than what performances of this kind have of late years generally met with. I have thought therefore, that it may not be improper to mention my reaſon for the publication of the following ſheets, as an apology for it with the reader; to whom I can with great truth and ſincerity repreſent, that I ſhould never have thought any knowledge which I had, or any diſcoveries in my power, of our circumſtances with regard to our preſent debts, worth the publick notice, if I had not frequently met with ſome miſtakes on this ſubject, which appeared to me very generally to prevail, and firmly to be inſiſted on and believed much to the diſadvantage of our publick credit, and which at the ſame time I have flattered myſelf, might be confuted and removed, from ſuch informations only as I ſhould be able, on this occaſion, to collect. I have ſo often heard it affirmed, that our publick debts have increaſed upon us ſince the proviſions made for the diſcharge of them, that it has ſometimes ſeemed to me to be the more common opinion even of thoſe perſons who are moſt intereſted to be rightly informed in this particular; and have almoſt as often heard it from hence inferred, that thoſe proviſions are therefore inſufficient to anſwer the expectations we are ſuppoſed to have from them. And from the bad influence that the belief of this aſſertion, and the inference from it, muſt have on our publick credit, eſpecially when it falls in with any general apprehenſions for the publick peace or welfare on any other account, I have been induced to think, that as this fact is not true, nor the inference from it rightly made, it would be of general convenience that they were publickly contradicted, and proved to be otherwiſe; and that this were better done from that leſs exact and partial information which I have been able to come at upon this ſubject, than not done at all, or perpetually put off in expectation of its being ſome time done by ſuch perſons who have the exacteſt knowledge of our circumſtances in this reſpect, or the beſt capacity for improving it for this purpoſe.

Nor ſhould I have been diverted from communicating the few or partial diſcoveries that the following ſheets may be thought to contain, by being told, that ſuch miſrepreſentations of our circumſtances were made with deſign only, and by perſons who better than myſelf knew the ſtate of our affairs in this reſpect; becauſe in this caſe in particular, it appears to be the publick intereſt, that the truth ſhould be as generally known as may be, and that every perſon who is or may be an adventurer in our publick funds, ſhould, as diſtinctly as can be, underſtand the proviſions that have been made for ſupporting the credit of them.

What I have farther added beyond the general deſign, by which I was at firſt engaged to write upon this ſubject, the reader will judge of on the peruſal of it. I am not inſenſible that there are ſeveral parts of this performance open to exceptions; but I have more hopes of the reader's indulgence to theſe faults as they ſhall occur to him, than I have that he will forgive the recital of them here, or my detaining him by endeavouring in this place to explain away or obviate any exceptions of this kind. I ſhall therefore mention but two particulars, in which I may be thought more than once to have offended. One is, that I have not every where uſed the utmoſt exactneſs in ſuppoſing, ſtating, or deſcribing the publick debts, or the variations in them. To this fault, as often as I have been guilty of it from any other cauſe than my want of materials for that purpoſe, I have been chiefly induced by the views of being thereby more intelligible; having preſumed that it would be better to omit any ſuch degrees of exactneſs in this reſpect, as were more than ſufficient to anſwer the general deſign of this eſſay, which would at the ſame time render it more tedious and perplexing. I have alſo, for much the ſame reaſon, been induced to content myſelf with the uſe of ſome words in what has ſeemed to me to have been their more ordinary acceptation, when applied to this ſubject; which in a longer or more elaborate enquiry, I ſhould have thought myſelf obliged to define and explain diſtinctly before I ventured upon the uſe of them.

AN ESSAY ON THE PUBLICK DEBTS OF THIS KINGDOM.

In a LETTER to a Member of the Houſe of Commons.

SIR,

UPON recollecting the converſation that was the occaſion of your deſiring my thoughts in writing on the ſubject of our Publick Debts; I have concluded, that I ſhould beſt anſwer your expectations from me in this affair, by confining my thoughts,

1. To the conſideration of what advantage to the publick may be reaſonably expected from the diſcharge of thoſe debts, and the redemption of the duties provided for the payment of their intereſt.

2. To an enquiry into the reaſons we have at preſent to expect or hope that theſe debts, or any conſiderable part of them, will within any reaſonable compaſs of time be diſcharged and paid off. And,

3. To ſuch reflections as have occurred to me upon thoſe meaſures that may for the future be entered upon, for the more ſpeedy and effectual diſcharge of our preſent debts, from the income of the Sinking Fund already provided for that purpoſe; or for ſtill farther increaſing the annual income of that fund by ſuch reductions as may yet be made in the intereſt or annuities payable for the principal ſums of which the preſent debt conſiſts.

Of the advantages that will arise to the publick from the diſcharge of the national debts.

As to the firſt of theſe, or the advantage ariſing to the publick by the diſcharge of the preſent debts; there ſeems to be but little room to enlarge, after the conſideration of that great annual revenue at preſent levied and applied to the payment of our debts, which, after the total diſcharge of them, will, without any loſs or injury to private perſons, be redeemed to, and become the property of, the publick. The preſent yearly expence to the Government, on account of our publick debts, computing the annual income of the Sinking Fund and the yearly intereſt of thoſe debts together, will be found to amount to little leſs than, if not to exceed, the ſum of £3,000,000. A revenue exceeding the whole farther annual expence of our civil and military government in a time of peace; and which, together with the ordinary ſupplies which our Government requires in a time of peace, may perhaps be a fund ſufficient to anſwer our utmoſt probable expences during the moſt expenſive war.

I do not think myſelf at liberty to ſuppoſe, or promiſe it as one advantage ariſing to the publick from the diſcharge of the preſent debts, that the ſeveral duties appropriated to the payment of them will, as ſoon as they are redeemed, be immediately removed or determined; for reaſons, which in the following ſheets I ſhall have a further occaſion to mention; when I ſhall recommend it to be conſidered, whether the revenues ariſing from thoſe duties, or the greateſt part of them, are not raiſed with more eaſe, greater equality, and more to the common benefit of the ſubjects of Great-Britain, than ſome part of the ſupplies that are annually voted for the current ſervice of the year; and conſequently, how far it may be reaſonable to ſubſtitute a great part of the revenues ariſing from thoſe duties, after the redemption of them, in the place of our annual taxes. But it will, I preſume, appear no ſmall convenience to the publick, ariſing from the redemption of the aforeſaid duties, that, when they ſhall be no longer appropriated to the payment of our debts, the principal difficulty will be removed, which has at any time obſtructed the removal or leſſening any of theſe duties, though the convenience of the publick may, upon other accounts, have perſuaded to it; either as ſuch duties may have appeared to give too great perplexity to perſons employed in trade, or to prevent or obſtruct any profitable branch of our commerce with foreign countries; as they may have been thought to require too ſtrict an enquiry,

or

or too great ſeverity or expence in the collecting them; as by being laid on any commodities univerſally neceſſary, they may have ſeemed too great a burthen on the pooreſt of our inhabitants; or as by bearing too great a proportion to the bulk of the commodities on which they have been laid, they may have made the gain ariſing from defrauding the publick, or the temptation to attempt it, bear too great a proportion to the hazard of being diſcovered; or, as in any other reſpect they may be found to be attended with general inconvenience, or unreaſonable hardſhip on particular perſons, employments or conditions of life amongſt us.

And, however it ſhall be determined, after the diſcharge of our preſent debts, as to the continuance or removal of the whole or any part of the duties appropriated for the payment of them; the revenues ariſing from them, being redeemed, will become the property of the publick, and, if not from thenceforth removed, will be employed in the room of, and take away the occaſion for, ſuch other taxes as ſhall then appear a greater burthen to, or to be more unequally levied upon, the ſubjects of this kingdom.

Having mentioned the quantity of annual expence to the Government, occaſioned by our publick debts, it ſeems unneceſſary to proceed further in proving the importance of diſcharging them, or to deſcend to or enumerate any further inconveniencies, that upon this account we labour under. An uncomfortable employment! and which, I hope, I ſhall be excuſed from, for this further reaſon; that the inconvenience of our preſent debts, and the importance of diſcharging them, are ſo univerſally believed and felt, and ſo unanimouſly agreed to, that I know none of my fellow-ſubjects who want to be convinced of them. I ſhall proceed therefore to what I propoſed in the

Second place, to make out the probability, and repreſent the reaſons we have to hope, that the preſent publick debts will, within the compaſs of a few years, be effectually and honourably diſcharged.

Of the probability that the ſaid debts will be diſcharged in a few years by means of the Sinking Fund.

What I have chiefly propoſed under this head, is to deſcribe and explain, as far as my materials for that purpoſe will carry me, thoſe meaſures which have been already taken for the diſcharge of our publick debts by the proviſion of the Sinking Fund. To which attempt, though this proviſion has already been made as publick as our acts of parliament, and though the operation and progreſs of it, in the diſcharge of our debts, is without any difficulty to be computed, I find myſelf induced, from that general ſuſpicion of the inefficacy of this proviſion to anſwer the ends propoſed by it; and which ſeems to have prevailed amongſt ſome people, who have either not had leiſure for that purpoſe, or who have declined the trouble of collecting the materials for, or making thoſe computations from

from them which are requiſite, in order to their ſatisfaction about the uſe and efficacy of the Sinking Fund.

Of the eſtabliſhment of the Sinking Fund in the month of March, 1716.

The firſt material proviſion that was made for diſcharging the principal of our preſent debts, was enacted in the third year of his preſent Majeſty's reign, by three ſeveral acts of parliament at that time made; the firſt of which (in the order that they ſhould have been printed amongſt the ſtatutes publiſhed for that ſeſſions) is intitled, *An act for redeeming ſeveral funds of the Governour and Company of the Bank of England, purſuant to former proviſoe of redemption; and for ſecuring to them ſeveral new funds and allowances redeemable by Parliament; and for obliging them to advance further ſums, not exceeding £2,500,000 at five per cent. as ſhall be found neceſſary to be employed in leſſening the national debts and incumbrances; and for continuing certain proviſions formerly made for the expences of his Majeſty's civil government, and for the payment of annuities formerly purchaſed at the rate of five per cent. and for other purpoſes in this act mentioned*, page 341. The ſecond, intitled, *An act for redeeming the yearly fund of the South-Sea Company (being after the rate of ſix pound per cent. per annum, and ſettling on the ſaid Company a yearly fund after the rate of 5 per cent per annum, and to raiſe for an annuity or annuities, at 5 per cent per annum, any ſum not exceeding £2,000,000, to be employed in leſſening the national debts and incumbrances, and for making the ſaid new yearly fund and annuities to be hereafter redeemable in the time and manner thereby preſcribed*, page 375. And the third, intitled, *An act for redeeming the duties and revenues which were ſettled to pay off principal and intereſt on the orders made forth on four lottery acts, paſſed in the 9th and 10th years of her late Majeſty's reign; and for redeeming certain annuities payable on orders, according to a former act in that behalf; and for eſtabliſhing a general yearly fund, &c.* page 291.

Of the ſeveral ſums of which the Sinking Fund is compoſed.

The *Sinking Fund* of late years, (called ſo from its being underſtood to be appropriated to the ſinking and diſcharging, as far as it will go, the principal ſums of the preſent publick debt,) is made up of money ariſing yearly into the Exchequer, as the ſurplus of the produce of three ſeveral funds eſtabliſhed by the three aforeſaid acts of parliament, by the names of the Aggregate Fund, the South-Sea Fund, and the General Fund; the ſurpluſſes of which three funds, or what they annually produce more than the yearly ſums to the payment of which they are firſt appropriated, are by the laſt of the aforeſaid acts of parliament reſerved for, and made applicable only to, the diſcharge of the principal and intereſt of ſuch debts as had been before the year 1716 contracted and provided for by Parliament. The yearly ſums to the payment of which thoſe funds are firſt appropriated (except the ſum of £700,000 *per annum* to his Majeſty for the expence of his civil government) are generally the intereſt, or annuities, payable for ſeveral principal ſums, of which our publick debts conſiſt.

As

As often therefore as any of thoſe principal ſums are paid off, or the proprietors of any part of the publick debt are induced to accept of a leſs intereſt or annuity for the ſame principal ſums, the Sinking Fund is underſtood to increaſe by the yearly addition of the intereſt of ſuch ſums as are paid off, or the abatement of the annuity for ſuch part of the publick debt as is agreed to be continued at a lower rate of intereſt. But the duties, the ſurplus of which the Sinking Fund conſiſts of, not bringing in every year an equal ſum of money, the ſurplus likewiſe is not every year alike; and therefore, in order to compute in what time the preſent publick debt may be diſcharged by the Sinking Fund, it is neceſſary to enquire from what different annual ſums have of late years been produced by it, what yearly ſum it is reaſonable to ſuppoſe may for the future be produced by it at a medium, or one year with another. To which yearly ſum we are from time to time to add the yearly intereſt or annuity of ſuch principal ſums, part of the preſent publick debt, as may be paid off by it; and all ſuch abatements of intereſt of all or any part of the publick debts, as are already agreed hereafter to take place, or may for the future be agreed to by any of the creditors to the publick.

There are a great many particulars which you will ſee, Sir, I muſt want the knowledge of, in order to make this ſuppoſition with any great exactneſs. I could wiſh here to be able to ſtate the produce of the ſeveral particular duties, the exceſſes of which conſtitute and ſupply the Sinking Fund; the different ſums produced by them in different years, from the times they were ſeverally granted; to aſſign the moſt probable cauſes of their variation, and from thence infer the probability of their producing more or leſs for the future: but however unprovided I am with materials for an inquiry of this kind, it may be yet worth while to proceed in deſcribing the proportions in which any determined yearly ſum (though by miſtake) ſuppoſed to be the preſent yearly produce at a medium of the Sinking Fund, will increaſe, when applied to the payment of the publick debts; as thoſe proportions will be the ſame with thoſe in which any other ſum, with more truth or probability ſuppoſed to be produced one year with another by the Sinking Fund, will increaſe when applied to the ſame purpoſe.

Of the annual amount of the Sinking Fund.

The beſt account I have been able to get of the produce of the Sinking Fund for ſome years laſt paſt lies now before me, and ſtates the produce of the ſurpluſſes of the ſeveral funds, commonly called the Aggregate Fund, the General Fund, and the South-Sea Fund, (the ſum of which ſurpluſſes our acts of parliament call the *Sinking Fund*) to be from the 31ſt of December, 1722, to the ſame time in the year 1723, £619,000 and upward; and the produce of the ſame ſurpluſſes from thence to the 31ſt of December, 1724, to amount to upwards of £653,000. This amount

amount of the produce of the Sinking Fund for the two years above-mentioned, exceeds the produce of the ſame fund for ſome years before, by a greater ſum than can be accounted for by the diſcharge or reduction of the intereſt of any part of the publick debt before that time; and which therefore I am inclined to attribute to ſeveral proviſions about that time made by the Legiſlature, for preventing frauds in the payment, and for the more fully and effectually collecting of ſeveral duties which in part ſupply the revenues appropriated to the payment of our publick debts; and of which proviſions I would hope we may long enjoy the benefit in the increaſe of the Sinking Fund. And from hence, I ſhould think, we might venture to expect an annual produce from the Sinking Fund for the future, equal to the produce of the ſame fund at a medium for the two years above-mentioned, ending in December 1724; and increaſing by the yearly addition of the intereſt of ſuch principal ſums as may be henceforth paid off, and of the abatements of the intereſt, or annuities, of any of the publick debts when the ſame ſhall take place, that already are or may hereafter be agreed for.

Of the ſavings of intereſt that will accrue to the Sinking Fund in the year 1727.

The abatements of intereſt in the year 1727 are ſo conſiderable, and the time when they are to take place ſo near, that I believe it will be thought reaſonable to ſtep forwards to the time when the Sinking Fund will be increaſed by the addition of thoſe abatements; and from that time to conſider the progreſs that may be made in diſcharging the preſent publick debt by the Sinking Fund.

From Midſummer in the year 1727, it is already provided, that the Sinking Fund be increaſed by the reduction of the intereſt from 5 to 4 *per cent.* or an abatement of 1 *per cent. per annum* on the principal ſums following.

	l.	*s.*	*d.*
On £13,061,878, being the amount of the publick debt to the South-Sea Company, excluding £3,839,363, part of it, for which an annuity at 4 *per cent.* only is at preſent payable,	130,618	15	7
On 16,901,241*l.* 17*s.* of the South-Sea annuities, - -	169,012	8	4
On 3,775,027*l.* 17*s.* 10½, part of the debt to the Bank of England, - - - -	37,750	5	6½
On £4,000,000, farther part of the debt to the Bank of England, purchaſed by them of the South-Sea Company, - - - - - - -	40,000	0	0

To

	l.	s.	d.
To this, if the annual income of the Sinking Fund on the 31st of December, 1724, be added, supposed to be	600,000	0	0
And the increase of it by the discharge of £600,000 *per annum* of such Exchequer bills as remained uncancelled on the 31st of December, 1724, and are made payable out of the Sinking Fund from the said 31st of December, to the 24th of June, 1727, viz. the interest and charge of circulating £1,500,000 Exchequer bills at 3 *per cent.* - - - - - - - -	45,000	0	0
	1,022,381	9	5

The amount of the said several annual sums will be upward of £1,022,000, the produce of the Sinking Fund from the 24th of June, 1727.

The produce of the *Sinking Fund* from the 24th of June, 1727, will be upwards of £1,022,000 *per annum*; and the whole of the national debts at that time will be less than 50 millions.

The publick debts on the 31st of December, 1724, are stated to amount to £52,363,471, or thereabouts: from which, if it be allowed me to deduct £1,500,000 Exchequer bills above supposed to be paid off by the Sinking Fund on the 24th of June, 1727, and such further principal sums as provision is made for the discharge of otherwise than by the Sinking Fund, the remainder to be paid off on the 24th of June, 1727, will be considerably less than 50 millions. Which sum however, (that I may not be thought to strain matters in favour of this scheme of discharging the publick debts by a Sinking Fund) I will suppose to be the principal debt to be paid off on the 24th of June, 1727, and the annual produce of the Sinking Fund to be from the same time one million only. I will likewise suppose, (as is most generally true) that the above-mentioned principal sum of 50 millions, will from the same time carry interest after the rate of 4 per cent. And, because there are some persons so sanguine as to imagine, that by force of our Sinking Fund, or some schemes formed upon it, the same debt may be still further reduced to a lower rate of interest, and the Sinking Fund increased further by such reduction; I will likewise suppose such a scheme to have taken effect, and the above-mentioned principal sum to carry 3 per cent. interest only, and the annual produce of the Sinking Fund to be increased, by an abatement of 1 per cent. interest on 50 millions, to £1,500,000. Upon both which suppositions, I shall subjoin a computation, describing in what number of years, from Midsummer 1727, the above-mentioned principal sum of 50 millions, or any particular part of it, may be discharged and paid off; in which, when I had not time to correct them, I discovered a small mistake or two; which I hope the reader will excuse, when I have assured him, that they no where misrepresent the time in which the aforesaid debt, or any part of it, may be paid off, by so much as two days.

Computations of the numbers of years in which the aforesaid national debt of 50 millions may be paid off by means of the Sinking Fund, upon suppositions of its bearing the two different rates of interest of 4 per cent. and 3 per cent.

Computation at 4l. per Cent.

	Payments made at Midsummer every year.			*Total of all the payments from the beginning in every year.*		
	L.	*S.*	*D.*	*L.*	*S.*	*D.*
1728	1,000,000	0	0	1,000,000	0	0
	40,000	0	0			
29	1,040,000	0	0	2,040,000	0	0
	41,600	0	0			
30	1,081,600	0	0	3,121,600	0	0
	43,264	0	0			
31	1,124,864	0	0	4,246,464	0	0
	44,994	11	$2\frac{3}{8}$			
32	1,169,858	11	$2\frac{3}{8}$	5,416,322	11	$2\frac{3}{8}$
	46,794	6	$10\frac{1}{8}$			
33	1,216,652	18	$0\frac{1}{2}$	6,632,975	9	$0\frac{7}{8}$
	48,666	2	$3\frac{3}{4}$			
34	1,265,319	0	$4\frac{1}{4}$	7,898,294	9	$5\frac{1}{8}$
	50,612	15	$2\frac{1}{2}$			
35	1,315,931	15	$6\frac{3}{4}$	9,214,226	4	$11\frac{7}{8}$
	52,637	5	5			
36	1,368,569	0	$11\frac{3}{4}$	10,582,795	5	$10\frac{5}{8}$
	54,742	15	$2\frac{7}{8}$			
37	1,423,311	16	$2\frac{1}{8}$	12,006,107	2	$0\frac{3}{4}$
	56,932	9	5			
38	1,480,244	5	$7\frac{1}{2}$	13,486,351	7	$8\frac{1}{4}$
	59,209	15	$2\frac{1}{2}$			
39	1,539,454	0	10	15,025,805	8	$6\frac{1}{4}$
	61,578	3	$2\frac{3}{4}$			
40	1,601,032	4	$0\frac{3}{4}$	16,626,837	12	7
	64,041	5	$9\frac{1}{4}$			
41	1,665,073	9	10	18,291,911	2	5
	66,602	18	$9\frac{1}{2}$			
42	1,731,676	8	$7\frac{1}{2}$	20,023,587	11	$0\frac{1}{2}$
	69,267	1	$1\frac{3}{4}$			

Computation at 3l. per Cent.

	Payments made at Mid-summer every Year.			*Total of all the payments from the beginning in every year.*		
	£.	*S.*	*D.*	*£.*	*S.*	*D.*
1728	1,500,000	0	0	1,500,000	0	0
	45,000	0	0			
29	1,545,000	0	0	3,045,000	0	0
	46,350	0	0			
30	1,591,350	0	0	4,636,350	0	0
	47,740	10	0			
31	1,639,090	10	0	6,275,440	10	0
	49,272	14	0			
32	1,688,363	4	0	7,963,803	14	0
	50,650	17	$4\frac{1}{2}$			
33	1,739,014	1	$4\frac{1}{2}$	9,702,817	15	$4\frac{1}{2}$
	52,170	8	$5\frac{1}{4}$			
34	1,791,184	9	$9\frac{3}{4}$	11,494,002	5	$1\frac{1}{4}$
	53,735	10	$8\frac{1}{4}$			
35	1,844,920	0	6	13,338,922	5	$7\frac{1}{4}$
	55,347	12	0			
36	1,900,267	12	6	15,239,189	18	$1\frac{1}{4}$
	57,008	0	$6\frac{3}{4}$			
37	1,957,275	13	$0\frac{3}{4}$	17,196,465	11	2
	58,718	0	$6\frac{1}{2}$			
38	2,015,993	13	$6\frac{1}{4}$	19,212,459	4	$8\frac{1}{4}$
	60,479	16	$2\frac{1}{2}$			
39	2,076,473	9	$8\frac{3}{4}$	21,289,032	14	5
	62,544	4	1			
40	2,139,017	13	$9\frac{3}{4}$	23,428,050	8	$2\frac{3}{4}$
	64,170	10	$7\frac{3}{4}$			
41	2,203,188	4	$5\frac{1}{2}$	25,631,238	12	$7\frac{1}{4}$
	66,095	12	$11\frac{1}{4}$			
42	2,269,283	17	$4\frac{3}{4}$	27,900,522	10	0
	68,078	10	$3\frac{3}{4}$			

Computation at 4l. per Cent.

	Payments made at Midsummer every year.			*Total of all the payments from the beginning in every year.*		
	£.	S.	D.	£.	S.	D.
1743	1,800,943	9	$9\frac{1}{4}$	21,824,531	0	$9\frac{3}{4}$
	72,037	14	$9\frac{1}{2}$			
44	1,872,981	4	$6\frac{3}{4}$	23,697,512	5	$3\frac{1}{2}$
	74,919	4	$11\frac{3}{4}$			
45	1,947,900	9	$6\frac{1}{2}$	25,645,412	14	10
	77,916	0	$4\frac{1}{2}$			
46	2,025,816	9	11	27,671,229	4	9
	81,032	13	$2\frac{3}{4}$			
47	2,106,849	3	$1\frac{1}{4}$	29,778,078	7	$10\frac{1}{4}$
	84,273	19	$3\frac{3}{4}$			
48	2,191,123	2	5	31,969,201	10	$3\frac{1}{4}$
	87,644	18	6			
49	2,278,768	0	11	34,247,969	11	$2\frac{1}{4}$
	91,150	14	1			
50	2,369,918	15	0	36,617,988	6	$2\frac{1}{4}$
	94,796	15	0			
51	2,464,715	10	0	39,082,703	16	$2\frac{1}{4}$
	98,588	12	$4\frac{3}{4}$			
52	2,563,304	2	$4\frac{3}{4}$	41,646,007	18	7
	102,532	3	$3\frac{1}{2}$			
53	2,665,836	5	$8\frac{1}{4}$	44,311,844	4	$3\frac{1}{4}$
	106,633	9	$0\frac{1}{2}$			
54	2,772,469	14	$8\frac{3}{4}$	47,094,313	19	0
	110,898	15	$9\frac{1}{2}$			
55	2,883,368	10	$6\frac{1}{4}$	49,977,682	9	$6\frac{1}{4}$
	115,334	14	$9\frac{3}{4}$	22,317	10	$5\frac{3}{4}$
56	2,998,703	5	4	50,000,000	0	0
	1296	14	8			
	3,000,000	0	0			

Computation at 3l. per Cent.

	Payments made at Mid-summer every year.			*Total of all the payments from the beginning in every year.*		
	£.	*S.*	*D.*	*£.*	*S.*	*D.*
1743	2,337,362	7	$8\frac{1}{2}$	30,237,884	17	$8\frac{1}{2}$
	70,120	17	5			
44	2,407,483	5	$1\frac{1}{2}$	32,645,368	2	10
	72,224	9	$11\frac{1}{2}$			
45	2,479,707	15	1	35,125,075	17	11
	74,391	4	$7\frac{3}{4}$			
46	2,554,098	19	$8\frac{3}{4}$	37,679,174	17	$7\frac{3}{4}$
	76,622	19	$4\frac{1}{4}$			
47	2,630,721	19	1	40,309,896	16	$8\frac{3}{4}$
	78,921	13	2			
48	2,709,643	12	3	43,019,540	8	$11\frac{1}{4}$
	81,289	6	2			
49	2,790,932	18	5	45,810,473	7	$4\frac{3}{4}$
	83,727	19	9			
50	2,874,660	18	2	48,685,134	5	$6\frac{3}{4}$
	86,239	16	$8\frac{1}{2}$	1,314,865	14	$5\frac{1}{4}$
51	2,960,900	14	$10\frac{1}{2}$	50,000,000	0	0
$\frac{1}{2}$	39,099	5	$1\frac{1}{2}$			
	3,000,000	0	0			

You

Obſervations on the preceeding computations.

You will be pleaſed to obſerve, Sir, that the annual income of the Sinking Fund, in this manner applied to the diſcharge of the principal of the publick debts, increaſes yearly in the ſame manner and proportion as a principal ſum put out and continued at compound intereſt, or intereſt upon intereſt, at ſuch a rate of intereſt as the principal ſum to be paid off is ſuppoſed to carry: that the increaſe of it in every year, is by the intereſt of that principal ſum which was paid off in the year next before it; and that the whole of the increaſe of it in any one year, from the beginning to apply it in diſcharge of the principal debt, is the ſum of the intereſt of all the principal ſums that have been in the year before paid off by it: and that the whole of the debt propoſed to be paid off by a Sinking Fund in this manner applied, will be compleatly diſcharged the year before the Sinking Fund itſelf is increaſed, by the addition of the whole intereſt of the debt to be paid off.

From which obſervations, it will be eaſy to compute the progreſs of any other annual ſum, greater or leſs, than what I have ſuppoſed to be the produce of the Sinking Fund in the year 1727, in the payment of a principal ſum of 50 millions, at 4 per cent. or any other rate of intereſt, or any other principal ſum which you may think it more reaſonable (as our affairs now ſtand) to provide for the payment of, by the common rules for calculating the increaſe of principal ſums continued at compound intereſt.

In the uſe of which rules, you will find, Sir, if you ſhould think it more reaſonable to ſet the income of the Sinking Fund, from the year 1727, at £800,000, or (as ſome perſons have repreſented it) at £1200,000 *per annum*, that a debt of 50 millions, carrying 4 per cent. intereſt, would in the firſt of theſe caſes be paid off in about 32 years, and in the other in 25 years and one month; or if in either caſe the Sinking Fund ſhould be ſuppoſed to be increaſed by £500,000 *per annum* added to it, from the intereſt of the ſame debt, reduced to 3 per cent. from the ſame time, it would appear that it might be fully diſcharged and paid off, by a Sinking Fund of £1,300,000 *per annum* in about 25 years, and by a Sinking Fund of £1,700,000 in 21 years and 8 months, or thereabouts.

A miſtaken opinion has of late been propagated, that the publick debts have increaſed ſince the eſtabliſhment of the Sinking Fund.

But to whatever may in this manner be obſerved or proved, relating to the efficacy or progreſs of the Sinking Fund increaſing annually by addition of the intereſt of ſuch debts as are diſcharged by it, I have heard it objected and ſtrongly inſiſted on to be true, that our publick debts have been far from decreaſing or made leſs ſince the contrivance and application of this expedient for that purpoſe; but, on the contrary, have been growing upon us, and are now conſiderably greater than they were about the time when the ſurpluſſes of ſeveral funds were firſt appropriated to the diſcharge

diſcharge of thoſe debts. And this melancholy circumſtance the ſame perſons aggravate, with obſerving, that the increaſe of our debts has been in a time of almoſt uninterrupted peace; and infer, that our debts muſt increaſe ſtill faſter upon us, in caſe of any publick troubles.

I have often wondered how ſo uncomfortable a miſtake could ſo generally prevail, againſt the teſtimony that the memory of every perſon at all acquainted with publick tranſactions of this kind muſt bear, that our publick loans of late years (except ſuch as have been made on funds provided to diſcharge the monies advanced upon them within the year,) have not been equal to the ſums that have within the ſame time been paid off; till upon further enquiry upon this ſubject, I have had put into my hands copies of accounts, ſuppoſed to be made up at the Exchequer, ſtating the totals of the publick debts for different years to be greater conſiderably from the year 1720, than in that year, and in that year to be more than in any year before it. From which accounts I cannot but think this miſtake muſt ariſe and prevail with perſons who ſatisfied themſelves with obſerving the totals only, and have not attended to the particular articles of which they were made up; but in examining the particular articles of which thoſe totals are made up, they will find that the great increaſe of figures in the deſcription of our preſent incumbrances, is not owing to any real increaſe of their true quantity.

An account of the circumſtances that have given riſe to the ſaid miſtaken opinion.

In an account now before me, of the amount of the publick debts on the 31ſt of December in ſeveral years, beginning in 1717, and ending in the year 1724, the amount of the publick debts in the firſt of thoſe years is deſcribed to be £47,894,950, and in the laſt to be £52,363,471. Of which great increaſe in the deſcription of our debts, the chief reaſons are; firſt, the ſubſcription of ſeveral irredeemable annuities for different terms of years into the South Sea Company's ſtock, in the years 1719 and 1720; by which thoſe annuities were converted into a redeemable debt from the Government, and purchaſed back from the proprietors at higher rates, or a greater number of years purchaſe, than were paid by the proprietors for the ſame annuities when they were firſt purchaſed from the Government. Before theſe ſubſcriptions made, this part of our publick incumbrances is deſcribed in the aforeſaid account, by the principal ſums originally advanced by the proprietors on the purchaſe of them; and afterwards by the quantity of redeemable debt, for which by virtue of the aforeſaid ſubſcriptions they were exchanged; which generally exceeds by four years and one half's purchaſe the ſum originally contributed by the proprietors of thoſe annuities, and which upon the whole of the ſaid annuities at thoſe different times ſubſcribed, amounts to about £3,155,858. This in the preſent view muſt, I think, be admitted to be no real increaſe of the publick incumbrances, or at leaſt not properly brought into the account

1ſt, The purchaſe of certain irredeemable annuities for different terms of years.

account of thoſe years in which the aforeſaid ſubſcriptions were made; thoſe ſubſcriptions being well enough known and underſtood to have been of great advantage to the publick, and very much to have facilitated the diſcharge of the whole of our preſent debts; and it being very obvious, that whatever real incumbrance has been growing upon us on account of thoſe annuities, it is to be attributed only to the increaſing value of thoſe annuities, and to be computed from the times of their being valued at higher prices, and not from the times of the ſubſcriptions above-mentioned, by which the further increaſe of their value was moſt fortunately prevented; and about which, all that we have to wiſh is, that it had been done ſooner.

2dly, Army-debentures.

Another article increaſing in the aforeſaid accounts of the publick debts from the year 1717 to the year 1724, is of army-debentures, or annuities charged and made payable out of the fund commonly called the General Fund, after the rate of 4 per cent. for ſuch principal ſums, as in purſuance of ſeveral acts of parliament for appointing commiſſioners to ſtate the debt due to the army, have been certified to have been due for ſervices in the late war, and before the year 1717. This, Sir, from 40,157*l.* 8*s.* 5*d.* which on the 31ſt of December, 1717, is only ſtated to be due from the publick under this article, is on the 31ſt of December, 1724, by the aforeſaid accounts deſcribed to amount to upwards of £2,140,157. But as this debt was due before the year 1717, in the preſent inquiry, whether the publick debts are ſince that time increaſed or no, this ſum is (now that the quantity of it is determined,) to be reckoned in the amount of the publick debt, as well in the year 1717 as in the year 1724; or, in other words, to be conſidered as due from the publick from the time it was contracted, and not from the time only when it was certified to be due.

3dly, Exchequer notes.

Another article increaſing the total amount of the publick debts in the year 1724 beyond that of the year 1717, in the aforeſaid accounts, is, that of £1,000,000 of Exchequer notes made out and lent to the South-Sea Company in the year 1720, and in that year added to the amount of the publick debts. This ſum, on the re-payment of it by the South-Sea Company, would have been deducted from the amount of the publick debts in that year in which it was repaid, if it had not been provided by a ſubſequent act of parliament, that the aforeſaid Exchequer notes ſhould be cancelled and paid out of the Sinking Fund; and that the ſum of £1,000,000, due from the South-Sea Company, ſhould be applied, when paid, to the diſcharge of a farther million of Exchequer notes made forth in the year 1722, and upon which money was raiſed for the diſcharge of a like ſum in arrear to the navy; which ſaid ſum of £1,000,000 being in this manner ultimately ſupplied out of the Sinking Fund, it is neceſſary to

ſuppoſe

ſuppoſe it to have been due from the publick before the year 1716, the Sinking Fund being, as I have above obſerved, about that time appropriated to the diſcharge of ſuch debts only as were due before that year; and conſequently this ſum of £1,000,000 being in the year 1717 owing, and in arrear from the Government, ſhould alſo in our preſent inquiry about the increaſe of the publick debts, be in that year added to the amount of them.

As ſhould alſo, for much the ſame kind of reaſons, the following leſs material articles, viz. Other leſs material articles,

	l.	*s.*	*d.*
Navy-annuities, a debt, though before due, not brought into the publick accounts till the year 1718,	110,312	0	0
A further proviſion for the ſufferers at Nevis and St. Chriſtophers, about — — — —	41,000	0	0
The increaſe of a deficiency on the Eaſt-India Company's fund ſtated in the publick accounts, to be from the year 1717 to the year 1720, about — —	67,500	0	0
A ſum in the year 1723, raiſed for immediate ſervice on the credit of Exchequer notes, the payment of which was at the ſame time provided for by a tax on the eſtates of Roman Catholicks, — —	100,000	0	0

To theſe articles are to be added the three firſt above-mentioned, viz.

The increaſe computed on the ſubſcription of irredeemables, — — — — —	3,155,858	0	0
Of army debentures, — — —	2,100,000	0	0
And the ſum raiſed for diſcharging arrears to the navy,	1,000,000	0	0
The amount of which ſums together is, — —	6,574,670	0	0

And this ſum, Sir, muſt be added to the above-mentioned total of our publick debts in the year 1717, before the comparing it with the total of the ſame debts in the year 1724 will truly determine how far our debts are increaſed or grown leſs from one time to another. Let this then be done. — — —

	47,894,950	0	0
	6,574,670	0	0
	54,469,620	0	0
And the aforeſaid amount of our debts in 1724,	52,363,471	0	0
deducted from it, — — — —	2,106,149	0	0

The national debts have really been diminiſhed by the ſum of £2,106,149 from the year 1717 to the year 1724.

Another method of eſtimating the diminution of the national debts in the ſame period.

And it will appear, that our debts are not in reality increaſed from the year 1717, to the year 1724; but, on the contrary, are diminiſhed by the ſum of £2,106,149, or thereabouts.

The ſame thing will appear from enumerating the particulars of the real increaſe or decreaſe of our debts from one time to the other; of which, Sir, the following is very nearly a true account, viz.

	l.	*s.*	*d.*
Money at different times borrowed on the duty on coals for building churches, more than in the mean time has been paid off by the particular proviſion made for that purpoſe, — — —	92,778	2	0
Money borrowed for the ſervice of the year 1719, more than paid off by the proviſion made for that purpoſe on the 31ſt of December, 1724, — —	439,300	0	0
Money borrowed on the plate-act for the ſervice of the year 1720, — — — — —	312,000	0	0
Total,	844,078	2	0

And this ſum of 844,078 *l.* 2*s.* is the whole ſum that our debts can, with any propriety, be ſaid to be increaſed by from the year 1717. Such other ſums as have been ſince that time borrowed having been employed in aid of the Sinking Fund, and applied in the diſcharge of ſome other debts at a higher intereſt; of which the following (except what of this kind has been already mentioned) is likewiſe a true account, viz.

	l.	*s.*	*d.*
Borrowed in the year 1719 by lottery, — —	500,000	0	0
Advanced in the ſame year by the South-Sea Company on the increaſe of their ſtock and funds, about	544,142	0	0
Advanced in the year 1723, towards the diſcharge of the lottery annuities unſubſcribed to the South-Sea Company, about — — — —	1,000,000	0	0
Total,	2,044,142	0	0

By which ſum, together with the Sinking Fund, have been paid off from 1717 to 1724, viz.

	l.	*s.*	*d.*
Of Exchequer notes, — — —	2,924,612	0	0
Of lottery annuities unſubſcribed, — —	1,204,786	0	0

Bank

	l.	*s.*	*d.*
Bank annuities unſubſcribed, — — —	235,297	0	0
Deficiency of the Eaſt-India Company's fund, —	191,028	0	0
Beſides, there has been in the ſame time paid in part of a principal debt contracted by two lotteries in the years 1713 and 1714, by proviſion for that purpoſe at the ſame time made, about — — —	429,490	0	0
Total,	4,985,213	0	0
From whence the total of the laſt above-mentioned loans being deducted, viz. — — —	2,044,142	0	0
The remainder — — — — will be the ſum of what has been paid off from the year 1717 to the year 1724 by the Sinking Fund, or otherwiſe without the aſſiſtance of thoſe loans.	2,941,071	0	0
And from thence — — — —	2,941,071	0	0
Let us farther deduct the total of the aforeſaid articles by which our debts have really, in the mean time, been increaſed, viz. — — —	844,078	0	0
And that remainder — — —	2,096,993	0	0

will be the ſum by which our debts, within the aforeſaid ſeven years, appear by this computation really to have been diminiſhed; differing indeed from that ſum which I have from the firſt computation ſtated to be the decreaſe of the publick debts in the ſame time by near £10,000. But which difference, if I pretended to the utmoſt exactneſs, might be removed, by either adding to the laſt remainder, or reckoning amongſt the particulars by which our debts have decreaſed within the time aforeſaid, the value of ſuch annuities for lives as within that time have reverted to the Crown.

The ſaid debts have, according to this ſecond eſtimation, been diminiſhed in the ſame period by the ſum of £2,096,993.

This ſum, perhaps, eſpecially if it be farther reduced by the deduction of one million at two different times borrowed, to ſupply the deficiencies of the proviſion for the expence of his Majeſty's civil government, conſidered as part of, and an addition to, our publick debts, may be thought too inconſiderable a diminution of our debts to be boaſted of as the effects of this expedient for ſo great a length of time. But, as it is no real objection to the truth of thoſe computations which I have made, of the progreſs of the Sinking Fund from the year 1727, I preſume it will likewiſe be no diſcouragement to our dependance on this proviſion for the payment of our debts; eſpecially after we have conſidered the great addition that will be made to the Sinking Fund in the year 1727, and

have farther obſerved the much greater diſpatch which a yearly ſum applied to the payment of any determined debt at intereſt, and increaſing annually in the manner above-deſcribed, will make in the diſcharge of ſuch a debt in a few years after the firſt application of it to that purpoſe, than it will do when it firſt begins in that manner to be applied.

Another miſtaken opinion which has lately been advanced concerning the Sinking Fund.

The little progreſs, however, hitherto made in the diminution of our debts, leads me to the examination of another opinion, which I think I have obſerved to prevail with the ſame perſons, who affirm our publick debts to have increaſed upon us; which is, that upon the ſuppoſition that ſuch debts are really increaſing upon us by new loans equal to or exceeding the diſcharges made in the ſame time by the Sinking Fund, the Sinking Fund is in ſuch caſe making no effectual progreſs at all in the diminution of our debts. It is perhaps the more material to conſider here, how far this opinion is true, for this reaſon, that though this ſuppoſition on which it is founded has not been true hitherto, it muſt be, however, admitted to be not improbable, that ſome future exigencies of the Government may make ſuch new loans neceſſary, as may exceed any ſums in the ſame compaſs of time produced by or applied to the diſcharge of our debts from the Sinking Fund. And in this caſe, upon the ſuppoſition that ſuch new loans are made upon further funds found out for payment of the intereſt of the money ſo to be advanced upon them, this opinion, that the Sinking Fund, applied as aforeſaid, would be making no effectual advance to the compleat diſcharge of the whole of our publick debts, would not be true. This will be beſt explained, if during the time that the above ſuppoſed Sinking Funds are employed in the diſcharge of the aforeſaid debt of 50 millions, the whole of our debts ſhould be ſuppoſed, by new loans upon further funds borrowed at 4 or 3 per cent. intereſt, to be increaſed by a further ſum of 50 millions, and that ſum to be diſcharged in the ſame manner, and by the ſame Sinking Funds, after the diſcharge of the firſt 50 millions; or if the account of the progreſs of the above ſuppoſed Sinking Funds be carried on, till inſtead of 50 millions they ſhall have diſcharged a principal debt of 100 millions.

Computations of the times in which another debt of 50 millions of pounds, (contracted upon new funds during the time of diſcharging the firſt debt of 50 millions by means of the Sinking Fund,) might be paid off by means of the ſaid fund, if the intereſt of money is 4 and 3 per cent.

Computation

Computation at 4l. per Cent.

	Payments made at Mid-summer every year.			*Total of all the payments from the beginning in every year.*		
	L.	*S.*	*D.*	*L.*	*S.*	*D.*
1755				49,977,682	9	$6\frac{1}{4}$
56	2,998,703	5	4	52,976,385	14	$10\frac{1}{4}$
	119,948	2	$7\frac{1}{4}$			
57	3,118,651	7	$11\frac{1}{4}$	56,095,037	2	$9\frac{1}{2}$
	124,746	1	$1\frac{1}{4}$			
58	3,243,397	9	$0\frac{1}{2}$	59,338,434	11	10
	129,735	17	$11\frac{1}{2}$			
59	3,373,113	7	0	62,711,547	18	10
	134,924	10	8			
60	3,508,037	17	$8\frac{1}{2}$	66,219,585	16	6
	140,321	10	$3\frac{1}{2}$			
61	3,648,359	7	$11\frac{1}{2}$	69,867,945	4	$5\frac{1}{2}$
	145,934	7	6			
62	3,794,293	15	$5\frac{1}{2}$	73,662,238	19	11
	151,771	15	0			
63	3,946,065	10	$5\frac{1}{2}$	77,608,304	9	$4\frac{1}{2}$
	157,842	12	$5\frac{1}{4}$			
64	4,103,908	2	$10\frac{3}{4}$	81,712,212	12	$3\frac{1}{4}$
	164,156	6	6			
65	4,268,064	9	$4\frac{3}{4}$	85,980,277	1	8
	170,722	11	$6\frac{1}{2}$			
66	4,438,787	0	$11\frac{1}{4}$	90,419,064	2	$7\frac{1}{4}$
	177,551	9	$6\frac{3}{4}$			
67	4,616,338	10	6	95,035,402	13	$1\frac{1}{4}$
	184,653	10	$9\frac{3}{4}$			
68	4,800,992	1	$3\frac{3}{4}$	99,836,394	14	5
	192,039	13	$7\frac{3}{4}$	163,605	5	7
69	4,993,031	14	$11\frac{1}{2}$	100,000,000	0	0
	6,968	5	$0\frac{1}{2}$			
	5,000,000	0	0			

Computation at 3l. per Cent.

	Payments made at Mid-summer every Year.			*Total of all the payments from the beginning in every year.*		
	L.	*S.*	*D.*	*L.*	*S.*	*D.*
1750				48,685,134	5	$6\frac{3}{4}$
51	2,960,900	14	$10\frac{1}{2}$	51,646,034	0	$5\frac{1}{4}$
	88,827	0	$5\frac{1}{2}$			
52	3,049,727	15	4	54,695,761	15	$9\frac{1}{4}$
	91,491	16	$1\frac{3}{4}$			
53	3,141,219	11	$5\frac{3}{4}$	57,836,981	7	3
	94,236	11	$7\frac{3}{4}$			
54	3,235,456	3	$1\frac{1}{2}$	61,072,437	10	$4\frac{1}{2}$
	97,063	13	$8\frac{1}{4}$			
55	3,332,519	16	$9\frac{3}{4}$	64,404,957	7	$2\frac{1}{4}$
	99,975	11	$10\frac{3}{4}$			
56	3,432,495	8	$8\frac{1}{2}$	67,837,452	15	$10\frac{3}{4}$
	102,974	17	3			
57	3,535,470	5	$11\frac{1}{2}$	71,372,923	1	$10\frac{1}{4}$
	106,064	2	2			
58	3,641,534	8	$1\frac{1}{2}$	75,014,457	9	$11\frac{3}{4}$
	109,246	0	$7\frac{3}{4}$			
59	3,750,780	8	$9\frac{1}{4}$	78,765,237	18	9
	112,523	8	$3\frac{1}{4}$			
60	3,863,303	17	$0\frac{1}{2}$	82,628,541	15	$9\frac{1}{2}$
	115,899	2	$3\frac{3}{4}$			
61	3,979,202	19	$4\frac{1}{4}$	86,607,744	15	$1\frac{3}{4}$
	119,376	1	10			
62	4,098,579	1	$2\frac{1}{4}$	90,706,323	16	4
	122,957	7	$4\frac{1}{2}$			
63	4,221,536	8	$6\frac{3}{4}$	94,927,860	4	$10\frac{3}{4}$
	126,646	1	$9\frac{1}{4}$			
64	4,348,182	10	4	99,276,042	15	$2\frac{3}{4}$
	130,745	9	6	723,957	4	$9\frac{1}{4}$
65	4,478,927	19	10	100,000,000	0	0
	21,072	0	2			
	4,500,000	0	0			

From

From hence, Sir, it preſently appears that the above ſuppoſed Sinking Funds, in this manner increaſing by the addition of the intereſt of the principal ſums in every year paid off, and conſequently by additions in every year greater than thoſe made to it in the year before, will be ſufficient not only to diſcharge our preſent debts, but any probable addition in the mean time to be made to them by further loans on new-invented funds, in a few years after the preſent debts ſhall be diſcharged: and that the time required for the diſcharge of our debts, increaſed by any addition in this manner made, will by no means be lengthened out, or the payment of the whole of our debts by the Sinking Fund retarded or delayed in proportion to the addition to or increaſe of the debt itſelf: the total payment of our publick debts becoming by no means deſperate from any Sinking Fund, however leſs than thoſe above ſuppoſed, upon account of any determined increaſe of or additions made to them; unleſs thoſe additions are ſuppoſed to be continued increaſing in every year in the ſame or a greater proportion to one another than that in which the additions yearly made to the Sinking Fund increaſe. This is ſo true, that ſuppoſitions about the increaſe of the publick debt might be carried to the utmoſt extravagance, and ſtill appear to be provided for by the abovementioned Sinking Fund of £1,000,000, increaſing at the rate of 4 per cent. compound intereſt; which, if it were worth while, might be ſhewed to be ſufficient, in about 105 years, to pay off a debt of 1575 millions, allowing for the increaſe of the preſent debt of 50 millions, by an addition of 15 millions in every year in which that Sinking Fund ſhould be ſo applied. Nor will this at all ſurprize perſons who have been accuſtomed to attend to the increaſe of money put out at compound intereſt, or quantities continued in geometrical progreſſion; an enquiry into which will remove all doubts about the truth of what I have here advanced. It would however be true, that if at any time, on the diſcharge of any part of the principal of the preſent debt, the intereſt were not added to, and applied in the further diſcharge of, the remaining debt, but another equal or greater principal ſum ſhould be borrowed on the ſame annuity; the progreſs of the Sinking Fund would by ſuch meaſures, if the ſame ſum were borrowed, be ſtopped; and, if a greater, be put backwards: but as long as theſe meaſures are not taken, or the Sinking Fund diverted or applied to any other purpoſe than the diſcharge of our debts; the full and effectual payment of all our debts by this expedient, is by no means to be deſpaired of from the increaſe of them by new loans on further duties.

Obſervations on the great efficacy of the Sinking Fund in diminiſhing the national debts, notwithſtanding new ſums of money ſhould in the mean time be borrowed by the publick upon new funds to pay the intereſt of them.

But this is upon a ſuppoſition that the Sinking Fund is never charged with the intereſt of the new loans.

And that the Sinking Fund will, from time to time, be applied to the diſcharge of the publick debts, and not be diverted or applied to any other purpoſe whatſoever, is what, I think, we may ſecurely promiſe ourſelves; from conſidering that the aforeſaid fund has been appropriated to that purpoſe by the Legiſlature, and our publick faith in the ſame manner engaged

The publick faith is engaged to the proprietors of the national debt, that the Sinking Fund ſhall never be applied to any other purpoſe but that of diſcharging the ſaid national debt.

engaged to the creditors of the Government, that the ſurplus of the aforeſaid duties ſhould be applied to the diſcharge of the principal of their debts, as the funds themſelves to the payment of the intereſt or annuities contracted for; which faith of the publick in this manner engaged, I think we have all the reaſon in the world to believe will be as inviolably obſerved in this as in any other part of their contract with the proprietors of the publick debts.

The clauſe of the act of parliament, 13 Geo. 1, cap. 7, ſect 37, by which the Sinking Fund is ſo appropriated.

This appropriation of the Sinking Fund to the purpoſe aforeſaid, you will find, Sir, to have been made by the aforeſaid acts of parliament. In the laſt of which, taking them in that order in which I have referred to them, page 320, after reciting that by the two other acts of parliaments, the ſurpluſſes of the Aggregate and South-Sea Funds are provided to be reſerved to the diſpoſition of Parliament only; it is enacted, That the ſurpluſſes of the General Fund thereby created, ſhould in like manner be accounted for and reſerved for the diſpoſition of Parliament. And then it is further enacted in the words following, "That all the monies to ariſe "from time to time, as well of or for the ſaid exceſs or ſurplus, by virtue "of the ſaid act made for redeeming the funds of the Governour and "Company of the Bank of England [viz. the Aggregate Fund] and of "or for the ſaid exceſs or ſurplus, by virtue of the ſaid act for redeeming "the funds of the ſaid Governour and Company of Merchants trading to "the South-Seas, &c. and of or for the ſaid exceſs or ſurplus of the ſaid "duties and revenues by this act appropriated as aforeſaid, [viz. the "General Fund] and the ſaid overplus monies of the ſaid general yearly "fund by this act eſtabliſhed or intended to be eſtabliſhed as aforeſaid, "ſhall be appropriated, reſerved, and employed to and for the diſcharging "the principal and intereſt of ſuch national debts and incumbrances as "were incurred before the 25th of December, 1716, and are declared to "be national debts, and are provided for by act of parliament in ſuch "manner and form as ſhall be directed and appointed by any future act "or acts of parliament to be diſcharged therewith or out of the ſame, and "to and for none other uſe, intent or purpoſe whatſoever."

The ſaid clauſe ought to be conſidered as a ſolemn contract entered into by the Government with the publick creditors.

By theſe words, I think, the ſurpluſſes therein mentioned, of which the annual income of the Sinking Fund is made up, ſufficiently appear to have been appropriated by the legiſlative power to the payment of our publick debts, till they ſhall be intirely diſcharged and paid off. Nor can this proviſion well be underſtood as made by the Government for what then appeared for publick convenience only, and conſequently to be altered by ſubſequent acts whenever it ſhall appear, or be pretended to be otherwiſe; but muſt, I think, be conſidered as a contract by the Government with the publick creditors, if the occaſion of theſe acts of parliament be attended to. In which caſe it will appear, that the ſeveral proviſions by

by theſe acts made, were enacted and propoſed to the creditors aforeſaid, as inducements to them to accept of an intereſt, or annuity, for their debts by one ſixth part leſs than that which till that time they had received; of which the moſt obvious inducement was, that what was thus deducted from the yearly intereſt of their debts, ſhould be applied for the better ſecuring and gradual diſcharge of the principal of the ſaid debts. To which ſecurity, amongſt the other benefits by the ſame acts of parliament propoſed to them, they muſt, I think, be conſidered to have intitled themſelves by their ſubſcriptions afterwards made, ſubſequent to, and in conſideration of, ſuch propoſals made to them by the Legiſlature. And whoever will be at the trouble of turning over the ſeveral ſubſequent acts of parliament relating to the publick debts, will find this proviſion for the application of the Sinking Fund frequently repeated and confirmed: and in caſes where by act of parliament application of monies in the Sinking Fund to the diſcharge of debts that were leſs obviouſly, or leſs generally, known to have been within the deſcription of the debts intended by the proviſion above-recited, ſuch debts have been, by the recitals, declared and explained to have been debts incurred before the 25th of December, 1716, and provided for by Parliament in a manner that has plainly intimated it to be underſtood by the Legiſlature, that the above-recited proviſion was an engagement, or contract, of the Government with the publick creditors, about the punctual obſervation of which from time to time they were intitled to have all poſſible ſatisfaction; or at leaſt, that the punctual application of the above-mentioned ſurpluſſes to the diſcharge of our preſent debts, was regarded by them as a matter of the higheſt conſequence to the publick welfare. And as long as the publick welfare ſhall be in the leaſt regarded, and this continues to be the only expedient for removing ſuch heavy incumbrances on our affairs, and redeeming ſo conſiderable a revenue to the uſe of the publick, I think we may confidently expect, that no perſons whatſoever, whoſe hands the adminiſtration of our affairs may at any time for the future be committed to, can ever be induced to approve of, or recommend, the application of the produce of the Sinking Fund, in any poſſible exigence of our affairs, to any other uſes than thoſe to which it ſtands now appropriated, though there were no other conſiderations to enforce it.

Therefore it cannot be ſuppoſed that any miniſters of ſtate whatſoever will ever preſume to divert the produce of the Sinking Fund from the purpoſes of its original deſtination.

For let us inquire a little, what publick exigencies can be ſuppoſed to happen, that can make it at any time adviſeable to divert or apply the produce of the Sinking Fund to any other purpoſe till after the entire payment of our publick debts. Let the expence that the circumſtances of our affairs may at any time make neceſſary, be, or be ſuppoſed to be, ever ſo much more than what can be conveniently raiſed within the year; it muſt, I think, always appear more eligible in regard to the publick intereſt, as well as more eaſy to thoſe perſons in the adminiſtration, to whom the care

In any ſuppoſed exigencies of ſtate, that are likely to ariſe, it will be more expedient to raiſe money by new loans, with new funds, or taxes, to pay the intereſt of them, than to break in upon the Sinking Fund.

care of providing the neceſſary ſupplies ſhall at any time be allotted, to raiſe what ſhall be further wanted by increaſing the publick debt with further loans upon intereſt provided for by new duties, than to ſupply the ſame ſums in any way from the produce of the Sinking Fund.

An illuſtration of this propoſition by an example.

The computation that I have laſt made was to ſhew, that the time in which the above ſuppoſed Sinking Fund of £1,000,000 will be ſufficient to compleat the diſcharge of the publick debts, will by no means increaſe equally to the increaſe of the principal ſum of thoſe debts by further loans on new funds: but it may be of further uſe to ſhew, how much leſs the increaſe of the publick debts, by borrowing further ſums at intereſt provided for by new funds, will retard the diſcharge of the whole of the publick debts, than the ſupplying the ſame ſums in any way from the produce of the Sinking Fund would do. Let us ſuppoſe, for inſtance, that the Government were obliged for 25 years together to increaſe the preſent debt, by a million borrowed in every year at an intereſt of 4 per cent. provided for by further funds, the above-made computation will ſhew that that additional debt of 25 millions would be paid off by a Sinking Fund of one million, applied as is therein ſuppoſed, in little more than 7 years after the diſcharge of the preſent 50 millions. But if the ſame ſum were to be ſupplied out of the produce of the Sinking Fund, it is obvious that the payment of the publick debts muſt ſtand ſtill for 25 years, and be by more than two thirds of that time retarded beyond the time in which they would otherwiſe be diſcharged, though increaſed as aforeſaid; and the greater the ſum is ſuppoſed to be, that in theſe different ways is to be ſupplied, the greater will be the proportion in which the payment of our debts will be delayed, by ſupplying ſuch expences from the Sinking Fund, more than by the other way: or if the ſums in theſe different ways ſupplied ſhould be ſuppoſed leſs, the difference of the delay in theſe two caſes will be indeed leſs: but on ſuppoſition of the ſmalleſt ſum to be theſe two different ways ſupplied, the delay ariſing to the diſcharge of the publick debt by this miſapplication of the Sinking Fund, will be at leaſt three times as great as that which will be occaſioned by increaſing the publick debt in the other method.

The borrowing money, and charging the intereſt of it on the Sinking Fund, inſtead of laying new taxes for that purpoſe, would be as pernicious an alienation of the Sinking Fund as the taking groſs ſums out of it

The borrowing money on the income of the Sinking Fund in any form, if no more were in any one year borrowed than what had been by the Sinking Fund the year before paid off; and if that money be ſuppoſed to be borrowed at the ſame rate of intereſt that was payable for the debt before paid off; will have the ſame effect in delaying the payment of the publick debts, as the miſapplication of the revenue of the Sinking Fund the year before would have had: but if greater ſums be at any time borrowed on that fund, the payment of the publick debts will not only be ſtopped, but put backwards; and that in a manner that obviouſly leads not that were equal to the money ſo borrowed.

not only to delay the payment of the publick debts, but the taking away intirely the only ſecurity yet provided, that they ſhall ever be paid off. For which reaſon I ſhall not trouble you, Sir, with any computation of the different degrees in which different ſteps in purſuing theſe meaſures will affect us; but at once ſuppoſe it impoſſible that any perſons can propoſe to borrow money, (or much leſs to ſucceed in it) on the credit of ſchemes that themſelves deſtroy all probability of the re-payment of it; which, ſuch meaſures as theſe, muſt evidently appear to do, to thoſe that conſider, that we have already had the greateſt advantage from the reduction of intereſt that can with reaſon be hoped for in the proviſion of the preſent Sinking Fund; which if we once part with in exchange for an increaſed principal debt at a lower rate of intereſt only, it will be madneſs to expect that either ſuch a lower rate of intereſt, or any alteration in our circumſtances for the better, will admit of the ſame kind of proviſion to be made again for the payment of our debts increaſed by ſuch meaſures as theſe are.

I cannot therefore, Sir, amongſt the ordinary viciſſitudes of the affairs of any nation, not even amongſt any long and expenſive wars, that it may be neceſſary for the defence and ſafety of theſe kingdoms to carry on with our neighbours, find out that exigence of our affairs that can make the miſapplication of the Sinking Fund appear neceſſary, or probable to be put in practice; while it is ſo certain, that the lands, eſtates, expence, or commerce of Great-Britain, will yet eaſily admit of farther duties ſufficient to furniſh new funds to anſwer the intereſt of ſuch ſums as any publick occaſions that I can repreſent to myſelf can call for. Nor can I fear, that ſuch duties will not be chearfully voted and ſubmitted to, when they ſhall appear neceſſary to prevent the miſapplication of an annual ſum employed in ſo uſeful and neceſſary a ſervice to the publick, as the reduction of our debts; while that appears to be retarded ſo much more by diſcontinuing the payment of thoſe debts, than by the increaſe of them.

Examination of another reaſon which has lately been advanced for ſuſpecting that the miniſters of ſtate will not long continue to apply the whole of the Sinking Fund to the diſcharge of the national debt.

There is another objection to the probability of the payment of our publick debts, which, if I did not frequently meet with it, I ſhould chuſe not to mention, from my apprehenſions, that in ſtating of it as I have met with it, I ſhould be obliged to mention my ſuperiours with leſs decency, than that grateful ſenſe of the happineſs we enjoy under the preſent reign would on all other occaſions lead me to, or than you, Sir, from the ſame motives would expect from me. But as you are pleaſed to admit you have often met with it from others, you will give me leave to mention it, in my way to anſwer it. The objection I mean is, That the continuance of our publick debts is, and always muſt be, the intereſt of perſons in the adminiſtration; that the greateſt profit of their employments ariſes from hence; and that the neceſſary power and influence to ſupport themſelves in thoſe

employments, depend greatly on their having reſerved to themſelves the diſpoſition of the various offices and employments in collecting and applying the revenues appropriated to the payment of the publick debts; which, when thoſe debts ſhall be diſcharged, can ſubſiſt no longer.

It muſt be obſerved, in anſwer to this reaſon, that both the king and the miniſters of ſtate have hitherto been anxiouſly careful to apply the Sinking Fund to its proper uſe, the diſcharge of the national debt.

Whatever truth we ſhould admit to be in this objection, we have the pleaſure of obſerving, that it appears to be equally true, from the frequent and earneſt recommendations from his Majeſty of the neceſſary meaſures for diſcharging the publick debts to the care and endeavours of the legiſlature; the ſeveral ſteps that have been taken by them; and the great and effectual proviſion that is already made for this purpoſe; that nothing can have been, or will be, more ſincerely intended and endeavoured by his Majeſty, or the perſons who have had, or ſhall have, the honour to be employed by him.

It may further be obſerved, in anſwer to the ſaid reaſon, that it is not neceſſary, or probable, that, when the national debt ſhall be all paid off, the taxes which now pay the intereſt of it will be aboliſhed.

But from the ſenſe I have juſt now profeſſed to have of the bleſſings we enjoy under the preſent government, I muſt confeſs, I ſhould with no pleaſure look forwards on that period of time, when his preſent Majeſty or his ſucceſſors ſhould be deprived of the means of ſupporting it, or even of rewarding and encouraging the fidelity and ſervices of their beſt ſubjects. The chief uſe therefore that I have propoſed to make of this objection, is to take an occaſion from it, of conſidering how far it is probable that ſuch a reform as is above ſuppoſed, of the various employments in collecting and receiving the preſent revenues, will take place on the diſcharge of the publick debts; or how far it is reaſonable that it ſhould do ſo. And this ſuppoſition being founded on a preſumption, that the particular duties now appropriated to the payment of the publick debts will, after the payment of them, be immediately removed, the reaſonableneſs of that preſumption will be the matter in queſtion.

The Government is now ſupported by other taxes, that are granted anew every year, to wit, the land-tax and the malt-tax.

For the purpoſe of this inquiry, Sir, I ſhould propoſe it to be conſidered, that the ſupport of our government neceſſarily requires a conſiderable annual expence, that is at preſent ordinarily ſupplied by other taxes than thoſe which have been provided to anſwer the payment of the publick debts; that the preſent ordinary proviſion for that annual expence has been hitherto determined, rather by the neceſſities of the publick, than by choice; and that it yet remains to be debated, how far the duties at preſent appropriated to the payment of our debts, or part of them, may, after the diſcharge of thoſe debts, be continued and made to anſwer the ordinary annual expence of our government, more to the advantage of the publick, with leſs burden and expence to the particular eſtates of his Majeſty's ſubjects in this kingdom, and conſiſtently with a more equal and reaſonable proportion of the burthen or expence by every ſubject ſubmitted to,

Theſe taxes might, when the national debt was all paid off, be dropt, and their

place ſupplied by ſome of the duties which now pay the intereſt of the ſaid debt.

to, to the benefit he receives from the ſupport of our government, than is now done by the preſent proviſion made for the aforeſaid ordinary annual expence.

It is in vain to ſuppoſe, that the neceſſary expences of a government are to be ſupplied by any taxes that are no ways burthenſome to the whole or ſome part of the community, and conſequently to which ſome objections may not be dreſſed up by perſons intereſted in avoiding them; which objections, however, when ſuch taxes appear neceſſary, it is unreaſonable to propoſe or aggravate. I ſhall not therefore point out any inequality or hardſhip that I may apprehend to be in the ordinary annual proviſion made amongſt us by a land-tax; but content myſelf with making ſome obſervations, tending to recommend the greateſt part of the duties now appropriated to the payment of our debts, as the moſt convenient and reaſonable taxes to ſupply the ordinary expence of our government, when redeemed by the payment of thoſe debts.

This would be found to be very equitable and convenient.

Upon enumerating the ſeveral duties which at different times have been provided to anſwer the demands of the publick creditors, it will appear that the greateſt part of them (whether collected by cuſtom or exciſe) have been laid upon commodities in general uſe and conſumption amongſt that part of the inhabitants of this country, whoſe circumſtances will admit of the expence.

The latter duties (which now pay the intereſt of the national debt,) are, for the moſt part, duties on the conſumption of commodities.

About theſe duties it will appear upon reflection to be generally true that they have been added to the price which thoſe commodities had before the impoſition of ſuch duties, and from thenceforth to be ultimately paid in the laſt price of ſuch commodities by the conſumer.

Upon which ſuppoſition, if the aforeſaid duties are either, by way of cuſtom or exciſe, generally collected throughout the country where ſuch commodities are conſumed; it is plain that the ſaid duties will generally be paid by every perſon reſiding in ſuch a country, nearly in proportion to his ordinary annual expence.

And this, Sir, is the ſhare or proportion which, of all others, I think moſt eligible to be taken from every perſon reſiding in a country where great part of the inhabitants ſubſiſt by commerce, towards the publick expences of the government of that country, when it can in this manner be done, without enquiring exactly into the expence of every particular inhabitant.

Theſe are the moſt convenient taxes that can be laid in a commercial country.

For,

An advantage belonging to theſe taxes.

For, firſt, in this way the publick expence is leaſt ſenſibly felt by thoſe who really contribute towards it; every perſon being voluntary in his expence, and gratifying himſelf while he is contributing from his eſtate to the expence of the government.

A ſecond advantage belonging to them.

2. Contributions in this manner generally made by the inhabitants of a country in proportion to their expence, will be likewiſe made in a near proportion to the real value of the property of the ſame inhabitants; perhaps, a nearer than it would be done by a law made, directing the publick expences to be levied in that proportion, from the great difficulty of finding out, and plain inconvenience of exactly inquiring into the real value of every man's property for a purpoſe of this kind, in a country ſo much engaged in traffick as our's is. Nor will taxes upon our expences vary much from taxes proportioned to the value of our property, (if long continued) from what may at firſt ſight appear a reaſon for that concluſion; I mean, the different choice of the thrifty and extravagant in the proportion of their expences; the firſt of which, by contributing little himſelf to the publick expence, is providing for larger contributions by his ſucceſſors; and the other, by contributing too largely in haſte, is incapacitating himſelf for contributing at all.

Advantages of theſe taxes above direct taxes on property.

I think alſo, that in thoſe particulars in which a tax proportioned to our expences, either does, or may be contrived to, vary from one intended to levy the ſame ſum in proportion to the value of property in Great-Britain, ſuch a tax on our expences appears the more eligible.

1. A tax proportioned to the expences of perſons reſiding in Great-Britain, will collect a proportion of the income of the various profitable profeſſions and employments amongſt us, and of the annual gains of foreign and inland commerce; all which being received and enjoyed by virtue of the laws, and under the protection of this government, ſhould, together with the annual income of our property, contribute towards it.

2. It will likewiſe collect and take in a proportion of the annual income of ſuch eſtates or employments as ſupply the expence of foreigners on different accounts reſiding in Great-Britain, as well as of ſuch of his Majeſty's ſubjects who chuſe to reſide here and ſupport their expences by the income of eſtates in Ireland, or any of our colonies or plantations in America or elſewhere; from whom, in return for the protection their eſtates receive from the arms or influence of Great-Britain, ſupported at our expence, no contributions in common with the inhabitants of this kingdom can be thought unreaſonable.

Contributions

Contributions thus made by perſons reſiding in Great-Britain, in proportion to their expences, will likewiſe include a proportion of the annual income of ſuch eſtates as may be brought hither by foreigners chuſing to ſettle amongſt us, or by any of our own countrymen returning with their gains from other countries.

In ſhort, it will include a proportion of all eſtates whatſoever, whether within or without the kingdom of Great-Britain, and whether diſcovered or not diſcovered, that any way ſupply the expences of our inhabitants, in a manner (as is above obſerved,) not grievous to, or liable to be complained of by, the contributors themſelves, and with the further good œconomy of ſparing on ordinary occaſions, and increaſing, that publick ſtock, that unmoveable part of our property within this kingdom, to which in times of extraordinary danger and expence we muſt neceſſarily have recourſe.

Duties of this kind may alſo be made inſtrumental to the judicious regulation of our trade.

It may likewiſe be conſidered, in recommendation of this manner of ſupplying the ordinary expences of our government by duties in the manner above ſuppoſed, levied in proportion to our expences, what farther conveniencies to the publick may be procured by ſuch duties, over and above ſuch a ſupply to its ordinary expences; ſuch as diſcouraging the conſumption of ſuch foreign commodities as may, in a manner plainly inconvenient to the publick, interfere with, or hinder the conſumption of, the produce or manufactures of our own country; abating the extraordinary price of foreign commodities, or the exorbitant gains of foreigners by the importation of them; the diminiſhing a trade carried on with any of our neighbours, the balance of which is too evidently in their favour; the encouraging any other more profitable branch of the Britiſh commerce; or the preventing the increaſe of any particular article of expence, that may too plainly tend to debauch the manners, or abate the induſtry, of his Majeſty's ſubjects. Of this kind many are the conveniencies that may be procured to a country, by the ſame meaſures that ſupply the ordinary expences of its government. And when it ſhall be conſidered to how many publick uſes of this ſort ſeveral of the duties appropriated to the payment of our debts are ſubſervient, beſides the annual income produced by them; I believe it will appear by no means eligible, and much leſs neceſſary, that the whole of thoſe duties ſhould, immediately after the payment of the publick debts, be removed and determined; when the ſame conveniencies may be ſtill preſerved to us by the continuance of them, and the income of thoſe duties be made to ſupply ſuch of our expences as are now provided for by leſs equal, or leſs beneficial, taxes.

We may therefore conclude that it will not be expedient, when the national debt ſhall be totally diſcharged, to aboliſh all the duties that now are employed in paying the intereſt of it.

Such conſiderations as theſe, I think, are ſufficient to remove the above-mentioned ſuppoſition, that the payment of our publick debts is inconſiſtent with the intereſt of a Britiſh miniſtry; in which, however, I could

It may further be obſerved, in anſwer to the reaſon advanced above in page 419, that the views mentioned in that reaſon are too remote to influence the miniſters of ſtate for many years to come.

could ſtill advance farther, by remarking how remote the views of any intereſt of this kind are placed, by the length of time that will be neceſſarily required for the diſcharge of our preſent debts from a Sinking Fund; and by obſerving, that the removal of any part of the preſent duties, which are any ways inconvenient to the publick, and are continued now only becauſe appropriated to the payment of ſome part of our debts, will by no means imply or even admit of a reduction of officers employed in the collection of thoſe kind of duties, either by way of cuſtom or exciſe, in the ſeveral ports or diſtricts in Great-Britain, in proportion to the income of ſuch aboliſhed duties; and from ſeveral other reflections that have occurred to me on this ſubject, if I did not think it unneceſſary any farther to follow ſo groundleſs and indecent a jealouſy of the integrity and publick ſpirit of ſuch of my countrymen, who ſhall for the future deſerve and attain to the favour and confidence of his Majeſty or his ſucceſſors.

Thus far I have been endeavouring to make out, that the proviſion already made of the preſent Sinking Fund is an expedient, from which we may with great confidence expect the full and effectual payment of the principal of our preſent debts within a few years. Upon which, Sir, if I have dwelt longer than you may have thought neceſſary, I hope you will be pleaſed to conſider in excuſe of it, how far I muſt have been led to do ſo, by attending to the happy influence that a general confidence in the efficacy of this expedient would have on the credit of our publick funds, eſpecially in caſe that the meaſures lately taken by ſome neighbouring princes ſhould make a rupture with them neceſſary to us; and how far ſuch a general opinion of the efficacy of this ſcheme has a tendency to forward and increaſe the ſucceſs of it.

An inquiry whether the Government ought, in prudence, to endeavour to procure a further reduction of the intereſt of the national debt.

I am now brought, Sir, to the laſt taſk that, in obedience to your commands, I have aſſigned myſelf; and am to inquire what meaſures it may be moſt for the intereſt of the publick to take in the application and uſe of the Sinking Fund from the year 1727. About which the only queſtion that can, as I think, be put is, Whether it ſhall be from thenceforth adviſeable for us to endeavour after a greater increaſe of the Sinking Fund, by a farther reduction of the intereſt of the publick debts? Or if it may not be then on the whole more for the publick intereſt, to endeavour only after ſuch an increaſe of the aforeſaid fund, as will be produced by the application of it from time to time to the diſcharge of the publick debts, and the addition of the yearly intereſt of ſuch of the ſaid debts as ſhall be from time to time paid off.

Before I proceed to any other conſideration which it may be thought material to attend to in determining this queſtion, I ſhall take leave to ſtate the greater effect the firſt of theſe different meaſures would have in accelerating

rating the payment of the publick debts than the other of them. And this I chuſe firſt to do, becauſe in an affair of this publick concern, and where we are not to be ſuppoſed to give ourſelves the trouble of the ſame exactneſs in computation that we ſhould uſe in our own private affairs, I am a little apprehenſive that people, when they turn their thoughts to this ſubject, are apt, upon any increaſe of the Sinking Fund, to promiſe themſelves a farther degree of diſpatch in the payment of the publick debts in proportion to ſuch increaſe. For an inſtance, to explain my meaning: I fear, that upon ſtating from the above-mentioned ſuppoſition, that the Sinking Fund of £1,000,000 was increaſed to £1500,000 *per annum*, by an abatement of 1 per cent. intereſt on 50 millions, the debt ſuppoſed to be paid off by it; on ſtating ſuch a caſe, I ſay, I fear it would be in haſte inferred, from the Sinking Fund's being increaſed to half as much again as it was before, that the publick debts would be likewiſe paid off by the Sinking Fund ſo increaſed half as ſoon again, or that the publick debts would be paid off by a Sinking Fund of one million and a half *per annum* in two third parts of the time that would be taken up in diſcharging it by a Sinking Fund of one million *per annum* only. But this inference would not be true, by whatever means the Sinking Fund were ſuppoſed to be ſo increaſed; and leaſt true, when the increaſe of the Sinking Fund is made by a reduction of the intereſt of the debt to be paid off by it.

Of the effect which an increaſe of the Sinking Fund from £1,000,000 *per annum* to £1500,000 *per annum*, by a reduction of the intereſt upon a debt of 50 millions from 4 per cent. to 3 per cent. would have in accelerating the time in which the ſaid debt would be compleatly diſcharged.

If the aforeſaid fund of £1,000,000 *per annum* were increaſed to £1500,000 by an addition made to it of £500,000 *per annum* provided by a new tax, or any otherwiſe than by an abatement of the intereſt of the 50 millions to be paid off, which ſhould continue to carry 4 per cent. intereſt, it would be true, that while the ſaid increaſed Sinking Fund is ſuppoſed to be applied to the diſcharge of that debt, it would pay off in every year half as much again as the Sinking Fund of one million only, beginning at the ſame time to be applied to the ſame purpoſe, would do in the ſame year; and at the end of any number of years, in which both funds are ſuppoſed to continue ſo applied, will have paid off a principal ſum exceeding the principal ſum paid off by the Sinking Fund of one million only, by one half part of the latter; or in other words, the principal ſum paid off by the aforeſaid greater fund will be to that paid off by the leſſer, either in an equal number of years from the time they begin to be applied, or in any one year equally diſtant from that time, in the proportion of three to two. And in this ſenſe the aforeſaid greater fund may be ſaid to pay off the publick debt half as faſt again, as in the ſame time it will pay off half as much again. But from hence it is not to be inferred, that the leſs fund will be half as long again as the greater in diſcharging the ſame principal ſum; or that the ſame principal ſum would be paid off by the greater fund in two thirds of the time that would be taken up in diſcharging it by the ſmaller fund: and of this the plain reaſon will

ſoon appear on inſpecting the above-made computations; from which it may be obſerved, That the Sinking Fund applied, as we have all along ſuppoſed it, is increaſing by an addition in every year made to it of the intereſt of that principal ſum which was paid off by it in the year before; from whence both the income of the fund itſelf, and the principal ſums annually paid off by it, are in every year greater than in the year before, and increaſing in every year by an addition greater than the addition made to it in the year before: from whence it neceſſarily follows, that in a ſeries of payments made by the Sinking Fund for any number of years carried on, the payments towards the latter end of ſuch ſeries muſt be conſiderably greater than thoſe before; and that the amount of the payments for any number of years ſeparated at the latter end from the reſt of the ſeries, muſt greatly exceed the amount of the payments for any equal number of years in any other part of the ſame ſeries. And from hence it muſt appear, that the exceſs of the payments made by the greater Sinking Fund above thoſe made by the leſs in the ſame number of years, will not be a rule for determining the time in which they muſt ſeverally be employed in diſcharging the ſame principal ſums.

The time of ſuch compleat diſcharge would not be thereby diminiſhed to two thirds of what it was before.

And it will be further from the truth, in the caſe of the Sinking Fund increaſed from an abatement of the intereſt of the debt to be paid off, by an addition of an annual income equal to one half part of its income before ſuch increaſe, to ſuppoſe, that from thenceforth the debt will be diſcharged in two third parts of the time which would have been otherwiſe required; becauſe the additions from time to time made to a Sinking Fund employed in the payment of a debt carrying 3 per cent. intereſt only, do not increaſe in the ſame or ſo great a proportion, as thoſe made annually to a Sinking Fund in the diſcharge of a debt at 4 per cent. From which circumſtance the leſs Sinking Fund increaſing by this greater ratio or proportion, would in a longer ſeries than I hope we have any thing to do with in the preſent caſe, have ſo conſiderable an advantage, as to overtake the greater Sinking Fund in its payments, and from thence to be every year diſcharging a greater debt.

But it would only be leſs than before by about one ſixth part of its former quantity.

But in the caſe we have ſuppoſed, of a debt of fifty millions, the time in which we have before computed that that debt, carrying 3 per cent. intereſt, may be paid off by a Sinking Fund of £1500,000, is 23 years and one half nearly; and by the Sinking Fund of £1,000,000, the debt continuing at 4 per cent. intereſt, it may be paid off in about 28 years; ſo that the time ſaved in the diſcharge of our debts by the reduction of them to 3 per cent. intereſt, appears, on the aforeſaid ſuppoſitions, to be 4 years and a half, or thereabouts; which is ſomething leſs than one ſixth part of the time in which the ſame debt might be diſcharged without any further reduction of the intereſt.

Another

Another way of ſtating the advantage to the publick in this contraction of the time which our debts may take up in the diſcharge of them, from 28 to 23 years and a half, would be to find out and aſſign that annual ſum, which, added to the above-ſuppoſed Sinking Fund of £1,000,000 at the publick expence, and without any further reduction of the intereſt of the debt to be paid off, would anſwer the ſame purpoſe as the addition of £500,000 to that fund taken from the income of the publick creditors, and contract the time in which the payment of 50 millions would be compleated, from 28 years to 23 and a half. And this, Sir, will be found to be almoſt £322,000; which yearly expence to the Government for 23 years and a half, would anſwer the ſame purpoſe as the above-ſuppoſed deduction of £500,000 *per annum* from the income of the publick debts. And this advantage I chuſe to ſtate diſtinctly as it is, before I proceed farther, becauſe I think in all the diſcourſe I have met with on this publick affair, I have ſeldom heard any diſtinction made about the convenience of the ſeveral reductions of intereſt from 6 to 5 per cent. and from thence to the rate of intereſt, at 4 per cent. which is ſhortly to take place; or relating to the further reduction to 3 per cent. which we ſeem to intend and be providing for; but on the contrary, they ſeem all to be conſidered and expected alike, as of equal advantage in diſpatching the diſcharge of the publick debts; though it be at the ſame time true, that by the firſt of theſe reductions we came only to have any Sinking Fund at all; and to the ſecond of theſe reductions, together with the proviſions at the ſame time made about the unredeemable annuities, we owe it, that the total payment of our debts by this expedient begins to appear practicable. But in thoſe circumſtances in which we now are, and with thoſe views which we at preſent have of the payment of our debts within no great length of time, from the proviſions already made for that purpoſe, by the reduction of intereſt hitherto effected or contracted for; I think we are at liberty, before any further ſteps of this kind, to conſider of ſome probable conſequences that may follow upon them; which to have produced as objections to any former reductions of publick intereſt, while they appeared ſo neceſſary, might have been thought impertinent or untimely.

Another way of eſtimating the advantage to the publick which would ariſe from ſuch a diminution of the time in which the national debt would be diſcharged.

It is by no means a juſt concluſion, that, becauſe the former reductions of the intereſt of the publick debts from 6 per cent. to 5 per cent. and from 5 per cent. to 4 per cent. have been beneficial to the nation, therefore a further reduction of the ſaid intereſt from 4 per cent. to 3 per cent. will alſo be beneficial to it.

It ſeems to me to have been an opinion of late years pretty generally agreed to, (perhaps as long ſince as the celebrated Mr. Locke's performance on that ſubject) That all attempts to reduce intereſt by compulſive methods, or by force of any laws made for that purpoſe, are not only unlikely to ſucceed, but on other accounts inconvenient to the publick: but I know not if the intereſt of the publick in the reduction of it by any other means effected, has been much conſidered; or if ſuch a reduction of intereſt is not uſually expected by us with general ſatisfaction, ariſing from our regarding it as the effect of the common and natural cauſes of a lower intereſt in every country, and ſuch alterations in our circumſtances as are truly enumerated amongſt the inſtances of publick proſperity.

A reduction of the intereſt of money, if it is brought about without compulſion, is commonly eſteemed a ſign of general proſperity.

Of the cauſes of the variations of the rate of intereſt for money.

Mr. Locke, in his aforeſaid treatiſe on this ſubject, mentions, as the natural cauſes of the variations of the rate of intereſt in any country, the variations of the proportion that the quantity of money bears in ſuch a country to the demand for it, ariſing either from the quantity of debts contracted amongſt the inhabitants, or of the trade carried on by them. To which occaſions of a demand for money, I think ſhould be generally added all other circumſtances in the affairs of ſuch a country, as may be there ſuppoſed ordinarily to contribute to, or be the occaſion of, a greater or leſs demand of that kind. And as a further natural and ordinary cauſe of a variation any where in the rate of intereſt, I ſhould chuſe to add ſuch alterations in the circumſtances or ſituation of the affairs of the country where ſuch a variation happens, as may make it more or leſs dangerous or ſecure to advance money upon loans in all or any of the different ſcenes of buſineſs, where negotiations of this kind are uſually carried on; by which laſt cauſe I am apt to think, that the more ſudden and ſenſible variations in the rate of intereſt have been chiefly and moſt frequently every where occaſioned. And when a lower rate of intereſt is ſuppoſed to be produced amongſt us, by ſuch cauſes as theſe are, it is perhaps moſt reaſonable that it ſhould be regarded with general ſatisfaction; as it is a proof of ſuch a ſituation of our affairs as is of itſelf, and independently on this conſequence from it, an inſtance and part of the deſcription of our general welfare and proſperity; and as the monied man himſelf has in this caſe an equivalent for what he may be ſuppoſed to loſe by the abatement of his income, in the greater ſafety, with which, on ſuch an occaſion, he lends his money, or the leſs hazard which he runs of the repayment of it, as well as in the greater frequency of opportunities which ſuch a ſituation in our affairs produces, of putting out and improving money with greater ſafety. But as far as a lower rate of intereſt may be produced amongſt us, either without ſuch compulſive methods, or the concurrence of ſuch natural and ordinary cauſes for it as are above mentioned, I apprehend that it yet remains to be inquired into, if it be the intereſt of the publick that it ſhould be ſo? and, as far as the ſucceſs of any meaſures entered upon for this purpoſe may be uncertain, if it be, with regard to the publick, adviſeable that ſuch a reduction of our intereſt ſhould be attempted or endeavoured after? And in this caſe it will be allowed me, that a lower rate of intereſt thus produced, or ſuppoſed to be ſo, is no longer to be conſidered as a proof of, or attended with the above-mentioned inſtances of, our publick happineſs; ſuch a rate of intereſt having no tendency in itſelf to increaſe our money, or the lenders ſecurity for the repayment of it, nor, conſequently, being of any effect to produce a real increaſe of our negotiations in advancing money; which, while no proviſion is made for increaſing either, our capacity or diſpoſition to lend money can by no means become greater, or more frequent, from the greater application to borrow money only.

When the intereſt of money is lowered in a country by the cauſes here mentioned, it may be conſidered as a ſign of general proſperity.

But when the intereſt of money is lowered by the operation of other cauſes than thoſe above-mention'd, it is not certain that it ought to be ſo conſidered.

I ſhall

I shall therefore endeavour, Sir, to describe such transactions amongst us with respect to our publick debts, as I apprehend may have been supposed to have had a great share and influence in producing amongst us lately very great and general variations in the rate of interest, and from which a still further reduction of the rate of interest may be yet expected; I mean those great adventures in the publick funds, of late years so apparently undertaken with a view to such gains, as might be quickly made by the different prices of them, and which have so much contributed to the late great and sudden variations in the market prices of these securities. In the infancy of these adventures, the chief or only motives to them probably were those pieces of intelligence about the situation of our publick affairs, from the publication of which the adventurer might reasonably infer the general satisfaction or diffidence of the proprietors of the publick debts in their several securities. And as far as intelligence of this kind was true, and the general sense of the proprietors upon the publication of it rightly conjectured or inferred, the rise or fall of stocks produced by these adventures might be regarded as an event, which in a longer time or in a less proportion would have happened, if these adventures had not been made; and in this view may not improperly have been called the growth or declension of our publick credit. But as this practice grew upon us, it is not to be wondered at, if from the general industry of great numbers to be first acquainted with every material occurrence to the publick, and to be earliest in the improvement of their information in adventures of this nature, several variations in the prices of our funds have been produced by transactions in them, undertaken upon false or uncertain intelligence, and groundless inferences and conjectures from it; which variations have not been afterwards to be accounted for from any real alteration in the posture of our affairs, or the general sentiments of the proprietors of the publick debts; and from which therefore the real state of publick credit at such a time would be uncertainly, if not falsely, inferred or determined. The later variations in the prices of our stocks would still more improperly described to be the growth or declension of our publick credit; which credit, since the restoration of our tranquillity, and during the absence of our apprehensions for the publick safety, can only with propriety be said not to have been disputed or called in question, and which cannot, I think, be supposed to have been of late at all attended to by the purchasers of our publick securities, at premiums and advanced prices far beyond those sums for the re-payment of which the credit of the Government is any ways depended on. In short, by whatever names we have been accustomed, or may chuse, to describe the rise or fall of our stocks, I submit it to such persons who have made any observations on the late transactions in Exchange-Alley, if they have not (and especially the rise of them) been generally occasioned by such adventures made in them, as persons have been induced to from the hopes of gain, from a further

Of the effects of stock-jobbing on the prices of the publick stocks.

further ſpeedy variation in the price of them, without any regard to the continuance of it; and if theſe variations are not of late come to be expected from any the moſt inconſiderable occaſions, or perhaps for no reaſon at all, but what is to be inquired for in the market, and amongſt the accounts and contracts depending there.

While this diſpoſition continues amongſt numbers to be conſtantly adventuring in the publick funds, and conſequently upon expectations that muſt be generally ſupported by the moſt inconſiderable reaſons, it is hardly to be doubted but that at any time in the abſence of our apprehenſions of any general danger, the intelligence being ſpread amongſt them that any ſcheme or propoſals were to be ſet on foot, by which the riſe of ſtocks was either intended or ſuppoſed, would generally determine theſe adventurers to expect and provide for ſuch a riſe of ſtocks, and by their contracts founded on theſe expectations in a great meaſure to produce it: to effect which purpoſe, I hardly think it material, that any further reaſonable proviſion ſhould be made in the propoſals or ſchemes themſelves, or that any thing would be further neceſſary for this purpoſe, than declaring the riſe of ſtocks to be intended by them. Such a riſe of ſtocks I am almoſt inclined to believe might be the firſt effect of any intelligence communicated in Exchange-Alley at ſuch a time as I have above ſuppoſed, that ſome propoſals were ſhortly to be made to all or great part of the publick creditors, to agree to the further reduction of their intereſt or annuities, as diſagreeable as this muſt at firſt appear to the greateſt part of the creditors themſelves. But how far ſuch a riſe of ſtocks may be in this caſe expected, and how far it may proceed in forwarding any propoſals of this nature, I ſubmit to be conjectured from the following conſiderations.

If the government were to propoſe to reduce the intereſt of the publick debts (with the conſent of the owners of them,) from 4 per cent. to 3, or 3½, per cent. ſuch a propoſal would, probably, occaſion a riſe in the price of ſtocks.

Firſt, Such a propoſal muſt ſuppoſe and lead our expectations to a riſe of ſtocks in general; without which, or at leaſt if the contrary ſhould happen, ſuch a propoſal could by no means be executed or complied with, it being neceſſary to the ſucceſs of this propoſal, that the market price ſhould, at the time of making it, offer the proprietor as much, or more, as, if he declined to comply with it, would be payable to him by the Government. And as the greateſt part of the proprietors of the publick debts have been at different times incorporated for the purpoſe of carrying on certain trades, from the profits of which (as I would willingly hope) 1 per cent. or more, has been annually divided over and above the income of their intereſt in the publick debts; if their annuity from the Government, when reduced and diminiſhed, continues to be valued as before, the price of that part which is not liable to any diminution from theſe propoſals, may well enough be expected to riſe in ſome proportion to ſuch a reduction of their annuity. Thus, if to the proprietors of South-Sea ſtock, for inſtance, it were propoſed that their annuities in the year 1727, ſhould be

Reaſons in ſupport of this opinion.

be from thence reduced to 3 per cent. upon the ſuppoſition that their ſhares in the publick debt ſhould, after ſuch a reduction, continue to be valued, as before, at par, it might be as reaſonable to expect that the 1 per cent. continuing to be divided on every hundred pound ſtock, ſhould be from thenceforth valued in the price of it at *33l. 6s. 8d.* as it was before to expect it ſhould be ever valued at *25l.* And from the riſe of that part of our publick ſecurities which fall under this conſideration, ſome advance in our other ſecurities may likewiſe be expected, as the money received on the ſale of thoſe ſtocks which ſhall firſt, and in the greateſt proportion, riſe on this occaſion, is generally obſerved to be applied to the purchaſe of that part of our publick debts which is conceived to be leſs liable to variation in the prices of them.

Secondly, Such perſons as are obſerved to be conſtantly adventuring in the ſtocks from expectations of gain, either from the riſe or fall of them, muſt be generally ſuppoſed to be determined to theſe adventures by the loweſt degree of probability, that they ſhall ſucceed in them; and it is hardly therefore to be doubted, but that the ordinary adventures in our ſtocks would be made upon expectations of the riſe of them, upon the publication of any propoſals from authority that ſuppoſed the riſe of ſtocks, or implied that it was expected by our ſuperiours.

Thirdly, The riſe of ſtocks upon this occaſion would be further favoured, by the diſpoſition of thoſe proprietors who are not ordinarily engaged in adventures of this kind, to wait for the utmoſt advantage to be made of the riſe of ſtocks, whatever might be their ſentiments about continuing proprietors of the publick debts when reduced to a lower intereſt.

Fourthly, A riſe of ſtocks on this, as well as former occaſions, may be ſtill further advanced, by the ſpreading of falſe computations of the value of our ſtocks, and idle opinions about credit and circulation, and by the force of a general example, aſſiſted by the confidence of the proprietors of our publick debts in the authority by which theſe propoſals may be recommended.

And when the ſtocks ſhall be ſufficiently advanced to colour any propoſals of this nature, it is perhaps not impoſſible that the concurrence of the proprietors to ſuch propoſals ſhould be obtained; though at the ſame time they may be generally diſſatisfied with the lower rate of intereſt propoſed to them, and ſeverally determined on that account to quit their intereſt in the publick debts on the next convenient opportunity. For it is to be conſidered,

That

That the reaſon for their objecting publickly or declaring their ſentiments againſt any propoſals of this kind, is removed by the price of their ſecurities at market, where they are offered for the preſent as much, or more, for them, as, if they thought proper to decline theſe propoſals, would be payable to them by the Government; and that the general diſſatisfaction of the proprietors ſhould not determine them to take advantage of the then market price for the ſale of their ſecurities, and by that means occaſion the fall of ſtocks, and prevent the ſucceſs of theſe propoſals, may in a great meaſure be accounted for from a general inclination to have the utmoſt poſſible advantage from the riſe of ſtocks, from the difficulty of finding on ſuch an occaſion any immediate employment for their money, and their impatience of its lying by them unemployed, joined with that dependance which men generally have on their own foreſight and ſkill in the choice of the fitteſt opportunity for this purpoſe; from which motives, while the bulk of our publick ſecurities may be ſuppoſed to be kept from market, that part of them which ſhall be brought there, by the more wary or determined of the proprietors, will be found for a time provided for, by thoſe conſiderable ſums which the eſtates and credit that perſons engaged in ſuch adventures in the ſtocks as I have above deſcribed, will for a time ſupply the market with.

From the foregoing reaſons it ſeems probable that the conſent of the publick creditors might be obtained to ſuch a reduction of the intereſt of their debts.

I think, Sir, from hence it appears poſſible, that a propoſal for reducing the intereſt of our publick debts, though without any reaſonable foundation, may, as our affairs now ſtand, ſucceed even ſo far as to obtain the concurrence of the proprietors. And the inference which I would from thence make is, that it belongs to them, by whoſe influence or advice ſuch a propoſal ſhall at any time be made, firſt to conſider if there be a ſolid foundation for it, or if it be likely ſtill farther to ſucceed; and not to depend on the conſent of the proprietors, in this manner obtained, as a ſufficient proof that ſuch a propoſal was reaſonable, or as a ſecurity for the ſtill further ſucceſs of it.

Nevertheleſs ſuch a meaſure might not, upon the whole, be adviſeable.

Whenever therefore the further reduction of the intereſt of our publick debts ſhall be attempted, it ſhould be firſt enquired if the real proportion of our ordinary neceſſities for money to our capacity and diſpoſition to ſupply them, have been ſo far altered as to admit of it; and if ſuch an alteration has proceeded from thoſe reaſonable and general cauſes of it, which are likely long to continue and ſupport it. Nor will ſuch an alteration be ſafely inferred from the market-prices of our ſtocks, any further than thoſe prices are determined and produced by ſuch purchaſes only as are made with a view to the improvement of the money laid out in theſe ſecurities from the intereſt or income of them; by which purchaſes of late years the prices of our ſtocks have ſo ſeldom been determined, that perhaps it may be more reaſonable in this caſe to conclude from an enquiry into the

rate

rate of intereſt ordinarily reſerved on private loans, or into thoſe other tranſactions in which we are uſually directed by computations upon the cuſtomary rate of intereſt amongſt us. Theſe tranſactions, however, muſt, as well as the prices of ſtocks, in this caſe be conſidered and attended to on this account, that perſons in the diſpoſition of their money will, as often as they think themſelves equally ſecure, be determined by the greater intereſt they are offered for it; from whence it muſt be expected, that when the income of our publick ſecurities, compared with the prices they are ſold for, offer a leſs improvement for our money than may with equal ſecurity be made of it by private loans or otherwiſe, the general induſtry of mankind to make the beſt improvement of their eſtates, will quickly reduce either the price of our publick ſecurities, or the rate of intereſt in ſuch private tranſactions as aforeſaid.

The reduction of the intereſt of the national debts, from 4 per cent. to $3\frac{1}{2}$ or 3 per cent ought not to be attempted till a proportional reduction of the intereſt of money lent upon private mortgages ſhall have previouſly taken place.

I queſtion therefore, if any attempts to reduce the intereſt of our publick debts below 4 per cent. at preſent will be of any laſting convenience to the publick, or ever can be ſo till ſuch a lower rate of intereſt ſhall be preceeded by its being cuſtomarily accepted of upon private loans on unqueſtionable ſecurities. For let it be conſidered how ſuch a lower rate of intereſt can otherwiſe appear to be founded on any real variation in the proportion of our neceſſities for money to our capacity or diſpoſition to ſupply them; or if the contrary does not appear, from a higher rate of intereſt ordinarily paid at the ſame time upon private loans. And while this continues to be the caſe, how reaſonable is it to apprehend, that when the money and credit of thoſe adventurers, who firſt advanced the price of ſtock, ſhall be withdrawn, the ſame ordinary neceſſities for money, without any increaſe of the proviſion for ſupplying them, will bring the proprietors of the reduced ſecurities to expect and look out for the former annual income for their money, and thereby occaſion a declenſion in the price of theſe ſecurities proportioned to the diminution of their intereſt?

How far the continuance of thoſe adventures, by which the price of ſtocks is ſuppoſed firſt on ſuch an occaſion to be advanced, may be depended on for the ſupport of it, may be collected from the motives by which the adventurers were firſt engaged in them; and is from thence to be expected. but till the utmoſt probable riſe of ſtocks from ſuch propoſals has been effected: after which that the former ſupplies from their credit and eſtates ſhould be withdrawn from market, is not all that is in this caſe to be apprehended, it being further probable that they will be from thence employed in depreciating thoſe ſecurities which were at firſt advanced by them, with a view to the ſame kind of profits from the fall, as they before expected from the riſe of ſtocks. From which, together with the freſh neceſſities which the more inconſiderate of theſe adventures will naturally produce, it would not be at all ſtrange if the price of ſtocks ſhould be carried lower,

beyond the declenſion of them in proportion to the diminution of their income, and the rate of intereſt for a time become higher than it would have been, if ſuch an attempt for the reducing it had not been made. And this conſequence of a riſe of ſtock from adventures of this kind muſt ſome time or other be expected, as far as that rate of intereſt which our real neceſſities would produce is varied from or miſrepreſented by ſuch adventures. It may not probably immediately ſucceed a riſe of ſtocks by theſe means effected; theſe adventures in a time of general tranquillity, may for a conſiderable time be protracted by further views, or the market ſupplied by a ſucceſſion of them; and this has often been the caſe, till upon the arrival of ſome intelligence about the ſituation of our affairs, which we call bad news, theſe views have been given over, and the declenſion of our ſtocks on that occaſion attributed to, and accounted for only from, that intelligence: but if the real occaſion of the great variation in our ſtocks at ſuch a time were further enquired for, it would be found to be the precipitate ſale of great quantities of ſtock, which, with ſuch views as aforeſaid, had been before bought up; and that this declenſion would as certainly, if not ſo ſuddenly, have happened from the ſame occaſion, without the intervention of ſuch intelligence, when theſe views ſhould have been on any other account given over; or when (as, I think, I have heard ſome perſons acquainted with theſe tranſactions expreſs themſelves) *the game had been played out.*

It is very much to be wiſhed that the market-price of the publick debts may be always either equal to, or greater than, their *par*, or original, or nominal, value.

It is true, indeed, that the Government by thoſe terms on which they borrow, I mean, by engaging only for the payment of the intereſt or annuities contracted for till the repayment of the principal, avoid all inconvenience from the intereſt of money advancing after their contract for the reduction of it, and leave the entire diſappointment upon the proprietors. But I ſubmit it, how far the publick can be conſidered to be unconcerned in a diſappointment of the publick creditors, obviouſly owing to their concurrence with propoſals recommended to them by authority; or in that general mutiny and diſcontent, which will be the neceſſary conſequence of ſuch a diſappointment: which from a remarkable inſtance of this nature, after the execution of the late South-Sea ſcheme in the year 1720, we muſt have obſerved to have been once regarded by the Legiſlature, as of ſufficient moment to induce them to releaſe the moſt conſiderable advantage that the publick had agreed for from that ſcheme, though ſet on foot upon the propoſals of the creditors themſelves. And if ever the publick creditors ſhould be generally diſappointed by a conſiderable diſcount upon their ſecurities, obviouſly owing to their concurrence to an abatement of intereſt recommended by publick authority, and propoſed for the convenience of the publick; I doubt if their expectations of relief from the Government could be thought leſs reaſonable. I cannot, for my own part, but think, that the general ſubmiſſion of the publick creditors in their contracts with the

the Government, to wait for the re-payment of their principal till the publick convenience will admit of it, and waving any ſuch agreement about a determined time for the re-payment of it, as in private contracts is ordinarily provided, would be far from removing their expectations of redreſs under a diſappointment of this nature; and rather apprehend, that this ſubmiſſion would be urged on ſuch an occaſion, as a meritorious inſtance of their confidence in the care and protection of our Government, and as a reaſon for their expecting in return for it, that what may be then called Publick Credit ſhould be kept up, and their ſecurities by all poſſible means preſerved at *par*, till the time when they could be diſcharged.

The ſucceſs attending the reduction of ſo great a part of our debt from 6 to 5 per cent. and from thence afterwards to 4 per cent. cannot certainly be looked on as a foundation for expecting the ſame event of our endeavours to reduce intereſt ſtill further. As to the firſt of theſe reductions, we ſhall find it, on looking back, to have been attempted quickly after ſuch an alteration in the circumſtances of our affairs, as furniſhed the beſt foundation for our hopes of ſucceeding in it: at the end of a long and expenſive war, that threatened us with the loſs of every thing valuable, but more particularly of that part of our property which had been advanced for the ſervices of the publick; at a time when thoſe neceſſities of the Government were removed, which had obliged us for ſeveral years before to be continually increaſing the publick debts, and at the ſame time admitted of no proviſions for the diſcharging of them; at a time when the laſting proſperity of Great-Britain was lately ſecured to us by his preſent Majeſty's acceſſion to the throne, and, ſoon after, by the entire defeat of the laſt attempt that was likely to be made to diſturb or prevent the preſent happy eſtabliſhment. From ſuch a foundation as was then laid for the growing wealth of theſe kingdoms, from the increaſe of our people, our commerce and manufacture, and for the particular ſecurity and greater confidence of the publick creditors, it was moſt reaſonable to expect, that the abatement of publick intereſt then propoſed ſhould take place; eſpecially, when theſe propoſals were attended with the proviſions that were then firſt made, for ſecuring and rendering practicable the diſcharging the principal of the publick debts. Nor do I think it unreaſonable to have expected, that by degrees, and from the fruits and ſenſible effects of this happy alteration in our circumſtances, the further reduction of publick intereſt to 4 per cent. which has ſince been agreed for, and which in the year 1727 is generally to take place, might likewiſe be effected. And, though it may be doubted, whether the effect of this laſt reduction of publick intereſt has been yet fully tried, upon recollecting how little the intereſt of the proprietors of the publick debts in this reduction was attended to by themſelves, at the time when it was agreed to; and how poſſible it is, that a far greater number of the proprietors of the preſent funds may have propoſed to quit their intereſt in them, when the

Of the cauſes of the ſucceſs of the firſt of the two late attempts to reduce the intereſt of the publick debt; by which it was reduced from 6 per cent. to 5 per cent.

Of the further reduction of the ſaid intereſt from 5 per cent. to 4 per cent. which is to take place in the year 1727.

reduction is actually to take place, than will be able to find cuſtomers for it, unleſs at a conſiderable diſcount: yet when, on the other hand, it ſhall be conſidered, how far this reduction has been precedented by conſiderable loans amongſt us at the ſame rate of intereſt, as well before the exigencies of our government during the late war with France, as ſince his Majeſty's acceſſion to the throne; I hope we may chearfully conclude, that this reduction of the publick intereſt may well enough be ſupported by the regular application of the annual income of the Sinking Fund to the diſcharging of our preſent debts. But no difficulty of finding employment for money at 4 per cent. intereſt, nor any private loans at a lower rate of intereſt amongſt us, can yet, I think, lead us to expect that a further reduction of publick intereſt to 3 per cent. will be for any length of time ſubmitted to.

There is no reaſon to think that a further reduction of the ſaid intereſt to 3 per cent. will, for many years to come, be practicable.

And while this continues to be the caſe, and from 4 to 5 per cent. intereſt is every day offered upon unexceptionable ſecurities, I ſhould think it a more reaſonable uſe made of recollecting the late reductions of our publick intereſt, to place them to the account of our preſent happy circumſtances; and, before we proceed to expect from the late alteration in our affairs a further reduction of intereſt, to conſider how far we are indebted to it upon that account already.

I have indeed ſometimes heard it ſaid, that the laſt reduction of our annuities to 4 per cent. ſtill wants to be taken care of; and that the price of our publick ſecurities proportioned to that rate of intereſt, is only to be ſupported by ſuch adventures as will be encouraged by keeping in view the proſpect of a ſtill further reduction of thoſe annuities to be attempted. From theſe perſons I very much differ; and cannot but think that this laſt reduction (if no new troubles preſently fall out) would be effectually ſupported by the future regular application of the Sinking Fund; from which, in a number of years, I ſhould rather expect that a further reduction of intereſt may be naturally and reaſonably produced, if the effect of this proviſion be not before-hand too far preſumed upon and anticipated. But whoever really thinks that the further reduction of publick intereſt muſt be kept in view, in order to ſupport the reductions already made, evidently ſuppoſes us to be proceeding in meaſures with regard to our publick debts, in which we muſt ſomewhere ſtop, and whenever we do ſo, repent of every ſtep we have taken in advancing thither.

Examination of an argument in favour of a farther reduction of the ſaid intereſt, drawn from the low rate of intereſt in ſome of the towns of Holland.

For want of examples amongſt ourſelves, as I ſuppoſe, to ſupport our expectations of the further reduction of publick intereſt, I have ſometimes heard the preſent low rate of intereſt in ſome of the trading towns of our neighbours the Hollanders, quoted to prove the probability, that the ſame, or ſomething near the ſame, low rate of intereſt may be made to take place amongſt us too. But I ſee not why the low rate of intereſt in that country

ſhould

ſhould be more regarded as the ſtandard for the rate of intereſt amongſt us, than the higher rate of intereſt in other neighbouring countries, unleſs on account of our greater commerce and negotiation with the Hollanders. And after I have admitted that the rate of intereſt amongſt them is on this account moſt likely to have ſome influence upon our's; I muſt expect it ſhould be allowed me, that this lower rate of intereſt than our's having for ſeveral years prevailed amongſt the Hollanders, has already had its effect with us in the reduction of our intereſt to that rate which we now conſider it to be brought to; and that the further effect or influence of their example in the reduction of our intereſt, is only to be expected from the further reduction of the rate of intereſt below what it is at preſent ſuppoſed to be amongſt them.

That the circumſtances of our affairs are the ſame in all thoſe particulars that lead to a low intereſt in any country with thoſe of our before-named neighbours, is by no means, in the preſent inquiry, to be preſumed; if they were ſo, our rate of intereſt muſt now be pretty nearly the ſame with theirs. But if our rate of intereſt conſiderably exceeds theirs, and has (which, as I have been informed, is true) for a long ſucceſſion of years conſtantly done ſo; it muſt be inferred, that our circumſtances, in ſome particulars that influence the rate of intereſt, differ much from theirs. And the conſtancy with which our rate of intereſt has been obſerved for a long time to have exceeded theirs, is enough to ſatisfy us that the occaſion of it is to be enquired for in ſome difference in our circumſtances which has continued with equal conſtancy, and for the ſame length of time; and not amongſt any projects or contrivances at different times ſet on foot by either of us, to anſwer any purpoſe of this nature.

The true reaſon of the difference between the rates of intereſt given for money in England and in Holland.

The true and general reaſon of this difference between our rate of intereſt and theirs, has, perhaps, been long ſince aſſigned by the above-mentioned Mr. Locke, and ſeems moſt probably to be the very different proportion which the lands or property of any other kind producing a certain annual income amongſt the Hollanders, taxed as that kind of property has been with them, bear to the great ſtocks and other perſonal eſtates of the inhabitants of that country, from that which the value of lands and other property of the ſame kind here bear to the perſonal eſtates in this kingdom. To this difference it ſeems owing, that while the Hollander can find little other employment for the money he can ſpare from his own adventures within his own country, than in ſupplying the neceſſities which their commerce from time to time produces, the monied inhabitant of this country, beſides the opportunities offered him from the ordinary neceſſities of perſons engaged in trade, is hardly ever without propoſals for the employment of his money in ſupplying the wants of the proprietors of our lands, by either purchaſing or advancing money upon their eſtates; and

and from hence is in a condition to demand and obtain a greater reward for the uſe of money than the Hollander can do, where the demand for it in his own country is ſo much leſs. This difference between us, as far as it will be allowed to have been one cauſe why our intereſt has hitherto exceeded theirs, will be allowed alſo as a reaſon why it ſhould continue to do ſo, till the inhabitants, wealth and commerce of Great-Britain, ſhall have increaſed in the ſame proportion to the extent and value of our lands, as it may be obſerved they have done in the Seven Provinces.

The expectation of a further reduction of the intereſt of the publick debt has contributed to increaſe the pernicious practice of *ſtock-jobbing*.

I cannot forbear thinking, that upon this occaſion it deſerves moſt ſeriouſly to be conſidered, how far our late expectations of continued attempts to reduce the rate of intereſt, has contributed to promote and increaſe the aforeſaid traffick in Exchange-Alley; a practice that, in the midſt of thoſe reproaches which it lies under by the name of *ſtock-jobbing*, and the moſt ſerious complaints of its ill conſequences to the kingdom upon every declenſion of our ſtocks, ſeems to me to be ſtill growing upon us. Whenever it ſhall be ſeriouſly intended to prevent or reſtrain this practice, I believe it will appear, that whatever contrivances may be provided for prohibiting the contracts in Exchange-Alley in the manner they are now made, or altering the manner of conveying our intereſt in ſtocks from one perſon to another, while they increaſe the difficulty of the moſt innocent and neceſſary tranſactions in the publick funds, will have little further effect on this practice, than to force it into ſome other channel, and perhaps increaſe the profit and employment of the banker only, by making his credit or aſſiſtance further neceſſary; and that the moſt reaſonable method of preventing it, will be removing the encouragement and temptation to it. And though our complaints of theſe adventures are then only generally made, when they ſeem to contribute to the declenſion of our ſtocks, a little enquiry will convince us, that the foundation for ſuch a *fall* of ſtocks was really laid by thoſe adventures which ſeemed to contribute to, and attended, the *riſe* of them. If during a time of general tranquillity, from unlimited expectations of the perpetual advance of publick credit, countenanced amongſt us beyond any ſufficient foundation for it, perſons are induced to ſpread their eſtates upon the utmoſt price of our publick funds, in ſuch a manner that a variation of 2 or 3 per cent. in the price of them threaten them with the loſs of the greateſt part of their eſtates; what can be expected, but from the earlieſt appearance of publick troubles, an idle rumour, (though improbable to be true,) or the apprehenſion of any ill accident, (though moſt unlikely to fall out,) ſhould determine them, in this ſituation of their affairs, to conſult their ſafety with the utmoſt precipitation, and crowd the market with the ſtock of which before they continued proprietors upon ſuch deſperate terms. A fall of ſtocks by this means occaſioned, with perſons leſs exactly acquainted with the reaſon of it, ſerves as a confirmation of every falſe report at the ſame time publiſhed

to

to the prejudice of our affairs: from hence ſtill further quantities of ſtock are brought to market, and a further declenſion in the price of them occaſioned; from whence, to greater apprehenſions of publick danger, and from thence to the further fall of ſtocks, by turns producing and increaſing each other, we may have often been obſerved to proceed without any poſſibility of putting a ſtop to either of them. Upon ſuch an occaſion as this it has often, and perhaps conſtantly, happened, that ſeveral perſons propoſing to themſelves gain from the calamities of the publick, have, on a preſumption of the fall of ſtocks, contracted for the delivery of ſtock which they had not, and could propoſe to furniſh only by the purchaſe of what the growing apprehenſions of others ſhould afterwards bring to market; and of theſe adventures it has been uſual, on the fall of ſtocks, principally to complain. This is a practice, which has doubtleſs often contributed to the misfortunes of the publick on an occaſion of this nature; but which, I doubt not, would in a great meaſure be prevented for the future, if the exceſſive adventures in the purchaſe of ſtocks, in expectations of gain from the riſe of them, were firſt prevented; to which the contrary practice is chiefly owing. For it will, on enquiry, be found, that ſtock-jobbing begins from, and people have been uſually initiated into this practice by, general expectations of the riſe of ſtocks; in which when they are once habituated, and the expectations of gain from the variations in the price of ſtock are become the only end of their tranſactions, their deſpair of advantage by the riſe of ſtock is quickly changed for hopes of profit from the fall of it.

Beſides, the unreaſonable prices to which the extravagant adventures for the riſe of ſtock have carried them, muſt firſt have taught the more wary and leſs credulous adventurers to expect the fall of them, and have been the occaſion of that ſucceſs that has encouraged the continuance of theſe adventures for the fall of ſtocks: nor would the performance of their contracts, for the delivery of ſtock, which they had not, at certain times, be generally practicable, but from the contracts for the ſame time ordinarily made for purchaſing ſtock without providing the money to be paid for it.

And if it be to theſe ſanguine expectations of the riſe of ſtocks, and the adventures founded on them, that the frequent and exceſſive variations in the price of them are really and ultimately owing; how dearly do the publick pay, in every inſtance of perplexity in our affairs, for any convenience to be reaped from, or uſe to be made of, this prevailing humour in the abſence of our apprehenſions of publick danger?

The

The riſe of our ſtocks, produced by the aſſiſtance of ſuch inconſiderate adventures in the purchaſe of them as have been above deſcribed, is at leaſt, of itſelf, and without attending to any conſequence from it, a matter of abſolute indifference to the publick in the abſence of general danger: but the conſequence of it in the declenſion of our ſtocks, upon the approach of publick troubles, is by no means ſo; then it is that the general diffidence in our ſecurities and wreck of publick credit is of the utmoſt diſſervice to us, by rendering difficult, if not impracticable, the raiſing ſuch ſupplies as an occaſion of this kind may neceſſarily call for; and as the variation in our publick funds at ſuch a time may be regarded by our neighbours as the meaſure of our apprehenſions from their attempts upon us, and encourage them in their preſumption on the unſettled circumſtances of our affairs: all which difficulties in our affairs on ſuch an occaſion, attended with falſe and groundleſs reports and apprehenſions of our danger, general mutiny and diſcontent, ſeditious exceptions to the conduct of our ſuperiours, and great diſtreſs and interruption to our commerce, I cannot but think we in a great meaſure owe to ſuch inconſiderate purchaſes of our publick funds during the general tranquillity; and that they might in a great degree for the future be prevented, if, by removing all encouragement to the extraordinary riſe of ſtocks, the publick funds were ſuffered to fall generally into the hands of ſuch perſons, who, ſatisfied with their income, ſhall purchaſe them as a ſupply for their ordinary expences, with money which they are not ſoon likely to have any other occaſion for.

Ill conſequences that reſult from the variations in the price of the publick funds occaſioned by ſtock-jobbing contracts.

The proprietors of our debts have, as ſuch, not deſerved ſeverity from the publick; but, as ſubjects of this kingdom, are intitled to have their intereſt regarded by the Government, as far as the publick convenience will admit of it. And in this view there may be ſome room to conſider the unequal hardſhip to the publick creditors, by the loſs of a fourth part of the annual income of their eſtates, implied in the ſucceſs of an attempt to reduce their annuities to 3 per cent. And while the convenience to the publick, to be obtained by ſuch a reduction, is ſuppoſed to be the earlier diſcharge of the publick debts; the hardſhip appears greater from this circumſtance, that what ſhall be thus annually deducted and taken from their income, will not go ſo far in anſwering this purpoſe, as two thirds of the ſame yearly ſum any other way ſupplied, and for this general convenience, more equally levied upon the ſubjects of this kingdom: £322,000 *per annum*, or thereabouts, raiſed at the general expence, and added to the Sinking Fund of £1,000,000, being, as I have before obſerved, ſufficient, on the above-made ſuppoſitions, to effect the total diſcharge of the publick debts, as ſoon as the addition of £500,000 *per annum* deducted from the intereſt of thoſe debts when reduced to 3 per cent. For it ſhould be attended to, that though the gain or convenience to the publick is to be computed upon ſuch of our debts only as from time to time remain unſatisfied,

The proprietors of the publick debts ought to be treated with lenity and tenderneſs, as perſons who have deſerved well of the publick.

unſatisfied, and as long as they remain ſo; yet ſuppoſing the continuance and general ſucceſs of this reduction of our intereſt, the loſs to the publick creditors is from the time of ſuch a reduction to be computed on the whole of the annuities that ſhall be reduced. If this, as a hardſhip on the publick creditors, ſhould not be proper in this caſe to be conſidered; it may be ſo, however, for the purpoſe of collecting what their ſentiments on this affair muſt ſome time or other be. The loſs and inconvenience to them by this reduction will be ſo ſenſibly felt, that no miſrepreſentations can poſſibly long miſlead them: the continuance of their ſubmiſſion to former abatements of their intereſt has been already accounted for, by the then late alteration in our circumſtances for the better, making the purchaſer of our reduced annuities a large amends in his greater ſecurity, and being convenient to the monied men in general in the frequency of opportunities of improving their eſtates with ſafety. But will the preſent happy ſituation of our affairs admit of a further equal alteration in our circumſtances for the better, or that ſhall in the ſame proportion increaſe our ſecurity in advancing money upon the publick credit? The former reductions of our annuities may have been recommended to the publick creditors, as the only means that could render the diſcharge of our debts practicable to the Government; but, as far as they are intereſted in it, is not that end ſufficiently obtained? or is the proſpect that the payment of the publick debts may be thereby effected ſooner, by 4 years and a half in 28, than it would be otherwiſe, of conſequence enough to the proprietor of any part of them, to induce him for that purpoſe only to part with for the future one fourth part of the annual income of his eſtate? However the reduction of publick intereſt hitherto effected may have contributed to the ſecurity of the proprietors, from the next reduction it is perhaps not unreaſonable to apprehend a contrary effect; and next to the great difficulty in the diſcharging of our debts, the moſt reaſonable foundation of our apprehenſions may be, its becoming, in the opinion of ſome perſons, a matter of too much indifference to the publick whether they are ever diſcharged or no. When the publick debts, by the further reduction of their intereſt, ſhall ſit ſo eaſy upon us, as to require but one moiety of the annual proviſion at firſt made for the payment of it, and leave the other at the ſervice of the publick, the danger ſeems to me by no means inconſiderable, that it may ſoon after be determined to employ the annual income of the Sinking Fund in the room of, and to eaſe the publick of ſome other taxes by which our ordinary expences are ſupplied; and that it may be thought as reaſonable to reſt contented with the recovery of half the annual income of the publick funds without any expence to us, as to redeem the whole of them with the trouble and expence of really diſcharging ſo conſiderable a debt.

The further reduction of the intereſt of the publick debts would be a great hardſhip on the publick creditors, and would raiſe great diſcontents amongſt them.

And if this reduction of intereſt be ſucceſsfully carried on, the loſs and inconvenience aforeſaid cannot be confined to the proprietors of our debts only,

And it would alſo be highly inconvenient to many other perſons beſides the publick creditors.

only, or to their property in the publick funds; for if it were ſo, it is plain, the price of them, after ſuch a reduction, muſt be abated in proportion to it: it muſt therefore, if it ſucceeds, take place in the intereſt of all private loans, in the profits of all the different employments of our money, and by degrees muſt affect the profits of our commerce, and ſpread itſelf throughout the kingdom: an effect, which when not produced by, or attended with the increaſe of our wealth, the revival of commerce amongſt us, the ſucceſſion of general tranquillity to a dangerous or unſettled ſituation of our affairs, or other like inſtances of general proſperity, I know not how to regard otherwiſe, than as an uncomfortable and general inconvenience in a country where the perſonal eſtates are ſo conſiderable as here they are; which, if it ſhould be thought not material to attend to, as a hardſhip or inconvenience merely to particular perſons, ſhould at leaſt put us in mind of the oppoſition that muſt ſooner or later be expected amongſt us to meaſures from which a great reduction of intereſt is apprehended.

An examination of the advantages which ſome people imagine would arife from ſuch a reduction.

Againſt this great and general inconvenience to the proprietors of perſonal eſtates from a lower intereſt, I would willingly place any further publick or private advantage that may ariſe from it, beſides haſtening the payment of our publick debts. The chief, if not the only, advantages of this kind that I have met with by any perſons propoſed from a lower intereſt, have been the increaſe of our foreign commerce, and the advance of the value of our lands and irredeemable annuities of any kind.

The increaſe of our foreign commerce.

As to the firſt of theſe, it muſt be admitted, that caſes may be put about the particular circumſtances of any country in which a lower rate of intereſt would have a tendency to increaſe their commerce; as it might be an inducement to ſuch perſons who could no longer ſupport themſelves, or were not contented with the income of their eſtates at ſuch a lower rate of intereſt, to engage in trade; and as it might be the means of furniſhing others with money for the purpoſe of undertaking any particular branch of trade, at ſuch intereſt and upon ſuch terms as the profit of ſuch a trade would only anſwer. But all the advantage of this kind that in our circumſtances, and in the preſent caſe, we have to expect, is to be collected only by an inquiry into the preſent ſtate of our commerce; from whence, if it cannot be made to appear that there is at preſent any profitable branch of foreign commerce neglected by us, the profit of which will, over and above the hazard and other expences of adventuring, exactly bear 3 per cent. intereſt for the money employed in it, but will not anſwer four; I ſhould think we have more reaſon to apprehend ſome ill conſequences from a ſudden reduction of intereſt amongſt us with relation to our foreign commerce, which are by no means inconſiderable: ſuch as

It is rather to be apprehended that ſuch a reduction will be hurtful to our foreign commerce, and produce the following ill conſequences.

The

The raſhly engaging unexperienced perſons in unprofitable adventures, to their own and the nation's prejudice. 1ſt ill conſequence.

The increaſing our adventures in the ſeveral branches of our preſent commerce beyond the demand for our commodities, or the poſſibility of vending them with advantage at foreign markets; and thereby rendering the whole of our foreign commerce for the future leſs profitable; and by this means, 2d ill conſequence.

The furniſhing a temptation to the more ſkilful and experienced perſons, at preſent employed in our foreign commerce, to remove with their effects and ſettle in other countries, from whence the commerce they are beſt acquainted with may be carried on with more advantage. An inconvenience which we have the greateſt reaſon to guard againſt at this particular juncture, when our neighbours in the different parts of Europe are ſo generally attempting to rival us in our foreign commerce. 3d ill conſequence.

And if amongſt theſe conſequences of the ſudden reduction of our intereſt, the money of foreigners, which either our Government or private perſons amongſt us at preſent have the uſe of, (to whom moſt certainly a higher rate of intereſt than they can have at home for it, muſt have been the general inducement for their truſting it here) ſhould be called from us, and applied to other uſes; a higher rate of intereſt than before may not only be apprehended, but an abſolute impoſſibility of ſupplying the ordinary demands of our commerce for ſome time at any rate at all. 4th ill conſequence.

As to the proprietors of our lands and irredeemable annuities, I am content to admit that they may reaſonably expect a higher price to be offered for their eſtates, in ſome meaſure proportioned to, and regulated by, a lower rate of intereſt produced by, and in proportion to, any ſolid and reaſonable cauſes for it. But I think it has been with truth obſerved by Mr. Locke on this occaſion, that in this higher price of their eſtates, thoſe proprietors are only intereſted who have contracted, or want to contract, debts upon their eſtates; it being of no conſequence to the perſon who neither ſells nor mortgages his eſtate, or intends to do ſo, what price he may procure for it; and it being as plain, that the perſon who on this occaſion receives a higher price on the ſale of his eſtate, from thenceforth ſtands in the place of the monied man, poſſeſſed of a greater ſum of money indeed than he could have had before, but which will produce no greater annual income, nor, generally ſpeaking, go farther in any proviſion he has intended for himſelf or family, nor in any other uſe that he can apply it to (except the diſcharge of ſuch debts as he may have contracted) than a leſs ſum would have done when the rate of intereſt was higher. The principal, if not the only, general advantage of a lower intereſt to Of the increaſe of the value of lands, or the price they will ſell for, by ſuch a reduction.

This is no real publick advantage.

the proprietors of land, is therefore ſo far as they have contracted debts; which advantage to them, and to all other perſons who have contracted debts, is exactly balanced in the publick accounts by an equal loſs and inconvenience to their creditors.

Such a reduction would likewiſe, probably, be attended with a diminution of the general expences of the people, and, conſequentially, of the publick revenues ariſing from taxes on the conſumption of commodities.

I will deſire your attention, Sir, but to one conſequence more, which I think will naturally and neceſſarily follow a further reduction of our intereſt, if it can by any means be effected, or for any length of time prevail amongſt us, without the concurrence of what I have hitherto ſuppoſed to be the natural and only reaſonable cauſes of it, viz. a conſiderable diminution of our expences, which the publick, as our affairs now ſtand, and the proprietors of land in particular, ſeem to me not a little intereſted to prevent. A fourth part of the income and uſual profits of the perſonal eſtates in this kingdom, withdrawn and deducted from the whole of our ordinary annual expence, muſt occaſion a very conſiderable diminution in it, when not ſupplied by the increaſe of thoſe perſonal eſtates, or the growing wealth of our inhabitants, and muſt from thence occaſion a conſiderable diminution in the price and conſumption of our commodities. And this I apprehend will be the ſooner and more ſenſibly felt, as the intereſt of money and the profits of perſonal eſtates are more generally the funds for, and ſupply the expence of, the inhabitants of this metropolis of the kingdom, than of any other part of it; and as a variation in our faſhionable expences here is moſt likely to ſpread itſelf, by the force of our example, throughout the other parts of this kingdom, where perhaps there may not be the ſame occaſion for it; from hence it deſerves well to be conſidered, if the publick may not loſe as much, or more, in their revenues ariſing from different commodities conſumed amongſt us, as may be ſaved by the reduction of our intereſt; or if the proprietors of land may not at laſt find themſelves obliged to furniſh from their own revenues thoſe ſupplies for the ſervice of the Government, which have been hitherto furniſhed by our expences. And if it be poſſible that this diminution of our expences ſhould proceed further, in reducing the price of labour, and from thence of our neceſſary proviſions and the produce of our lands, the proprietors of thoſe eſtates muſt in their turn ſuffer from the reduction of their annual revenues.

And it might be neceſſary to increaſe the land-tax to make good the deficiencies of theſe taxes.

From ſuch reflections as theſe, Sir, it has ſeemed to me not unreaſonable, that we ſhould at leaſt for ſome time reſt contented with ſuch reductions of publick intereſt as have been hitherto made: from whence I have been further induced to think, that it would be of conſiderable convenience to the publick, if the application of the preſent Sinking Fund, which ſtands now appropriated to the diſcharge of the publick debts in general, were by act of parliament determined as to the courſe and order in which thoſe debts ſhould be for the future diſcharged by it. Theſe meaſures

It might be of great publick benefit to determine immediately, by an act of parliament, the order in which the publick debts ſhould be ſucceſſively diſcharged by the application of the Sinking Fund.

meaſures with regard to the publick debts may poſſibly have not been hitherto propoſed, on account of that advantage which the publick may have been ſuppoſed for the future to be in a condition to make in the further reduction of publick intereſt, while they reſerved to themſelves the preference of one creditor to another in the order of diſcharging them; but I ſubmit it, how far this advantage would be prudently exchanged for the following conveniencies to the publick, from determining the order in which the Sinking Fund ſhould be applied in the diſcharge of our preſent debts.

Advantages that would ariſe from ſuch a meaſure.

Firſt, The annual income of the Sinking Fund will, by this means, be more fully appropriated to the payment of the publick debts, and the application of it to that purpoſe more effectually ſecured, by entitling every particular creditor to expect the application of it in the order that ſhall be ſo determined.

Secondly, It will be of conſiderable uſe in fixing the credit of the publick funds, and the confidence of the proprietors on ſuch foundations, as will ſupport them in any time of publick difficulty, by removing all grounds for thoſe apprehenſions, which, on ſuch occaſions, are obſerved (greatly to the diſadvantage of the Government) to prevail amongſt us, that the income of the Sinking Fund will be applied to ſome other purpoſes than the diſcharge of our debts; and by giving every particular creditor an opportunity of computing and ſatisfying himſelf in the value of his intereſt in the publick funds, from the knowledge of that time when his principal will be punctually paid off.

Thirdly, It will in a great meaſure prevent ſtock-jobbing, by removing the temptation to it from the great variations in the market-prices of our debts, from ſuch extravagant premiums paid for them in a time of peace, as if the income of them was conceived to be an irredeemable annuity; and ſuch diſcounts on the other hand allowed upon them, in a time of the leaſt general apprehenſion, as if they were regarded as debts almoſt deſperate.

Fourthly, It will lay a further foundation for a greater equality in the prices of our publick debts, by giving an opportunity to the proprietors to ſuit their own convenience in the purchaſe of ſuch part of thoſe debts as are determined to be payable, as near the time as poſſible when they expect any occaſion for their money; and prevent in a great meaſure the neceſſities of the proprietors being brought to market, eſpecially in the manner in which, when any declenſion in the price of ſtocks is apprehended, it may be obſerved often to be done long before they have any real occaſion for their money.

Fifthly,

Fifthly, It will tend to the increaſe of our credit, and the facilitating both of publick and private loans at the preſent, or as far as is reaſonable to wiſh for it, at a lower rate of intereſt, by capacitating ſuch of the creditors, whoſe debts ſhall be in a leſs remote order of payment, to lend out ſuch ſums as they may have by them reſerved for diſtant uſes, in expectation of being ſupplied for ſuch diſtant occaſions by the payment of their ſhare in the publick debts in the order and at the time appointed for it.

And laſtly, ſuch a determination of the order in which the Sinking Fund ſhould be applied in the diſcharge of our publick debts, and the notice the creditors would thereby have when they ſhould be paid off, would give them an opportunity of looking out for, and providing, the moſt apt and convenient employment for their money againſt the time of receiving it; a convenience to the creditors themſelves, which, as the publick is always intereſted in the innocent improvement of our eſtates, may, I think, be eſteemed a general advantage.

A recapitulation of the ſeveral propoſitions that have been advanced and recommended in the courſe of this Eſſay.

I ſhall conclude, by putting together what I have been endeavouring to repreſent about the reduction of our publick intereſt, viz. That the general and uſual rate of intereſt in every country is determined by the proportion that the ordinary neceſſities or demands for money amongſt the inhabitants bear to their capacity and diſpoſition to ſupply them; That any other rate of intereſt produced without a variation in the proportion aforeſaid, or a foundation laid for it, is not likely to continue; That we ſeem here to have had the effect of the late happy alteration in our publick circumſtances, in ſuch reductions of our intereſt as have been made already; That the proſpects of a ſtill further reduction of publick intereſt are a continued encouragement to adventures, which, though they may be made to contribute to the producing ſuch a reduction for a time, are not to be depended on for the ſupport of it; That theſe adventures are themſelves at all times a general inconvenience, and particularly prejudicial to the publick on the approach of troubles. I have likewiſe endeavoured to repreſent, that the further reduction of publick intereſt is neither equally neceſſary, nor of equal advantage to the publick, as either of thoſe that have been already made; nor does it want to be explained, that the ſame addition to the Sinking Fund, to be now made by the next reduction of the publick intereſt below 4 per cent. will diminiſh the remaining income of the creditors in a greater proportion than thoſe before made, and be a greater inconvenience to them. I have recommended it to be conſidered, how far a reduction beginning with the publick intereſt muſt, if it ſucceeds, neceſſarily ſpread itſelf, and affect the reſt of our perſonal eſtates; and from thence the oppoſition that meaſures for reducing intereſt will ſome time meet with, where ſome real alteration in our circumſtances does not perſuade to it. I have propoſed it to be inquired, if there be any other general advantage to be obtained by a lower rate of intereſt amongſt us, than in regard

regard to our publick debts only. And from ſuch conſiderations, would ſubmit it, if it might not be convenient, that not only our meaſures for further reducing intereſt, but our expectations of it were at leaſt for ſome time ſuſpended; till after the regular application of the Sinking Fund now provided for a few years, and the intermiſſion of ſuch extraordinary adventures as aforeſaid in our publick ſtocks, we might with more certainty collect what lower of intereſt our real circumſtances will admit of.

In what little I have ſaid about the conſequences of a lower intereſt on our commerce and expences, I have referred myſelf, Sir, to ſentiments in which I have had the honour to agree with you, and muſt not pretend to have made out any thing to general ſatisfaction: if I had attempted to do ſo, I ſhould have been carried too far beyond the deſign of this eſſay; and ſhould have been obliged to examine ſome prevailing opinions on this ſubject, which ſeem to me ſo far from being reaſonable or true, that I have ſometimes thought that part of them which the private intereſt of particular perſons have not introduced amongſt us, to have been taken up merely on account of their reſemblance to paradoxes, and for that reaſon affording the greateſt amuſement in converſation.

I would not have it, from any thing I have ſaid, inferred, that I am in general againſt any expedient for the much ſpeedier diſcharge of our preſent debts; I ſhould be glad if any reaſonable method for this purpoſe could be thought of; nor would any new burthen, or variation in the preſent burthen, on the ſubjects of this kingdom, implied in any propoſals for this purpoſe, be with me an objection to them, if the means were but found out of proportioning ſuch a new burthen, either to the property or expences of our inhabitants, in ſuch a manner as would be generally ſubmitted and agreed to: and that ſuch an expedient were found out, I wiſh for this general reaſon, that whatever in publick affairs is thought of great and general importance to be done at all, ſhould be done, if poſſible, as ſoon as it appears to be ſo; that the moſt eligible methods for effecting it are ſuch as may be carried on, and finiſhed under the direction of the ſame perſons who were firſt engaged in them; and that the ſucceſs of ſuch meaſures ſhould be as little as poſſible hazarded by the different ſentiments of their ſucceſſors. But this conſideration will not go far in recommending the further increaſe of the Sinking Fund, by reducing the intereſt of the publick debts; which, upon the ſuppoſitions on which my calculations have been made, would not, if the Sinking Fund were increaſed by reducing their intereſt to 2 per cent. be paid off in leſs than 20 years and a half, or thereabouts; if to 1 per cent. in leſs than 18 years and 4 months; or if the creditors would be ſatisfied without any intereſt at all till the payment of their principal, in leſs than ſixteen years and eight months.

I am, SIR, &c.

[*End of Sir Nathaniel Gould's Eſſay on the Publick Debts of this Kingdom.*]

Article CCCLXV. In the foregoing excellent pamphlet of Sir Nathaniel Gould, pages 420 and 421, the judicious author, having occaſion to make mention of the land-tax, takes notice of the hardſhip ariſing from its inequality; but adds that, in his opinion, there was no great occaſion to dwell upon that circumſtance, becauſe there was *then* a proſpect that, by the operation of the Sinking Fund, the national debt would be wholly diſcharged in the courſe of a moderate number of years, after which he adviſes, and likewiſe ſuppoſes, that the taxes which had been before appropriated to the payment of the ſaid debt, would be employed in defraying the ordinary expences of Government, ſo as to make the annual grants of the land-tax and malt-tax become no longer neceſſary. But now (in April, 1782,) that pleaſing proſpect is at an end, and, inſtead of it, there is reaſon to apprehend, that it will be neceſſary, not only to continue the land-tax upon its preſent footing of four ſhillings in the pound, or to raiſe by it the ſum of two millions of pounds, ſterling, *per annum*, for a great number of years to come, (perhaps, for a hundred years,) but even to increaſe the ſaid tax to double its preſent quantity, or to raiſe by it the ſum of four millions of pounds, ſterling, *per annum*, in order to make good the payment of the intereſt of the prodigious ſum of money to which the national debt will, probably, have increaſed before the bleſſing of a general peace ſhall be compleatly reſtored to us;——more eſpecially, if our miniſters of ſtate ſhould reſolve to apply a million, or twelve hundred thouſand, pounds a year out of the Sinking Fund to the gradual diminution of the capital of the ſaid debt in the manner that has been above recommended. The land-tax muſt, therefore, in our preſent unfortunate circumſtances, be conſidered as a *permanent* part of the publick revenue; and conſequently the very great inequality with which it is aſſeſſed on the inhabitants of different counties in the kingdom, and even on thoſe of different diſtricts of the ſame county, is an object well worthy the attention of the Legiſlature; and, if it be really a grievance and a meaſure of great injuſtice (as, I muſt confeſs, it has always appeared to me to be, and as Sir Nathaniel Gould in the paſſages above referred to, in pages 420, 421, ſeems to have conſidered it,) it is highly fit to be ſpeedily corrected and redreſſed. I therefore hope I ſhall be excuſed, if, before I return from this political digreſſion concerning the Sinking Fund and the national debt, to the Doctrine of Life-annuities, (which is the proper ſubject of this book,) I lay before the reader ſome reflections on this ſubject of the inequality of the land-tax, and on the expediency of aſſeſſing it for the future in a more equal manner, which were firſt drawn up and publiſhed in the Publick Advertiſer in the month of January, 1780, together with a propoſal to make the intereſt of all future ſums of money that ſhould be borrowed by the publick, liable to pay land-tax in the ſame proportion with the rents of lands, to the end that all the ſubjects of the Crown, thoſe who are poſſeſſed of perſonal

perſonal property as well as thoſe who have landed eſtates, may both contribute, and be ſeen and known to contribute, in the ſame degree, in proportion to their reſpective annual incomes, to the publick expences which the exigencies of the ſtate make neceſſary. Theſe reflections are as follow:

On the expediency of an EQUAL ASSESSMENT of the LAND-TAX, and of making the annual intereſt of all new publick loans of money ſubject to the ſame tax.

CCCLXVI. It ſeems to be an indiſputable maxim, founded both on equity and good policy, that, when taxes are neceſſary to the exigencies of the ſtate, perſons poſſeſſed of equal incomes ſhould contribute equally towards them. And, whenever this maxim is departed from, even though the ground of ſuch departure may be juſt by virtue of ſome original compact in favour of the perſons who pay leſs than their ſhare of the publick taxes, it is ſure to create ſome jealouſy and uneaſineſs in the other members of the ſtate, and thereby to render the property ſo exempted ſomewhat leſs ſecure than it otherwiſe would be.

Of the very unequal aſſeſſment of the land tax.

We have, indeed, in our government, two ſtriking inſtances of a departure from this prudent and equitable maxim, which are frequently the ſubjects of very great complaints: I mean, the ſhamefully unequal aſſeſſment of the land-tax on the lands and houſes of the kingdom, and the total exemption of all the intereſt of the publick funds from that and every other tax. By the unequal aſſeſſment of the land-tax, ſome people pay more than four ſhillings in the pound upon the rents of their lands, while others pay only eight-pence or nine-pence, or in ſome places, (as the two northern counties and the new buildings at Marybone) not more than four-pence upon theirs. And this is done by acts of parliament renewed every year, and not by any permanent and original act of parliament that could be conſidered as a plighting of the national faith to the purchaſers of land, that their lands ſhould always be taxed according to the rule of aſſeſſment then obſerved. On the contrary, the perſons who have been lightly taxed have always feared, and thoſe who have been heavily taxed have always hoped, that the Parliament would, one day or other, have a ſufficient regard to juſtice to correct this groſs inequality, and to impoſe the land-tax according to a new and equal aſſeſſment; or rather, indeed, according to a certain proportion of the rents received by every man; or, if the land is kept in the owner's hands, of the rents which were received for it when it was laſt lett, or which it might eaſily be lett for, in the judgment of the commiſſioners of the land-tax; as is done, if I miſtake not, in the caſe of

the late houſe-tax. And they all have hoped, at times, (though now, I believe, that hope is at an end) that the land-tax would be reduced to two ſhillings in the pound, and ſometimes even that it would be intirely taken off, or (to ſpeak more correctly) permitted to expire without being re-impoſed; as I am fully perſuaded it might have been, if prudent and œconomical meaſures had been purſued by our ſeveral miniſters of ſtate for theſe laſt forty years, and both this and the two laſt wars had been avoided. There is, therefore, as I apprehend, no weight in the reaſoning of thoſe who ſay that ſuch a correction of the inequality of the land-tax would be unjuſt with reſpect to thoſe purchaſers of land who have bought their land at a greater price than they otherwiſe would have done, upon an expectation that the land-tax would continue to be raiſed according to the then preſent mode of aſſeſſment. The nation is not bound to continue in the practice of impoſing this tax unequally, becauſe theſe gentlemen have flattered themſelves that they would do ſo.

I am the more confirmed in this opinion of the injuſtice of continuing the land-tax on its preſent unequal footing, becauſe it was that of Dr. Benjamin Franklin, whom I conſider as one of the moſt judicious and wiſeſt men now living upon earth, and of whoſe talents we now feel the force, ſince, by our attack upon one of the charters of the Americans, and our other alarming acts of authority againſt them, we have driven him to employ them againſt us. This truly great man uſed always to quote the continuance of the land-tax, upon its preſent very unequal footing, as a proof of the little regard that was had to juſtice and common ſenſe in our national deliberations. And I remember once in particular, that when it was ſaid that ſome people thought the correction of this inequality would be unjuſt with reſpect to thoſe who had purchaſed land upon a ſuppoſition of its continuance, he replied with ſome quickneſs, "Unjuſt! yes, it would be unjuſt; for it would be doing but half of what ſtrict juſtice would require; which would be to create a counter-inequality in the aſſeſſment of the land-tax, whereby the lands which had been heavily taxed ſhould hereafter be taxed lightly, and thoſe which had been lightly taxed ſhould hereafter be taxed heavily, for the ſpace of about fourſcore years, or for a time that ſhould be equal to the time during which the preſent unequal aſſeſſment had been permitted to continue." This counter-inequality he, perhaps, would not ſeriouſly have wiſhed to ſee eſtabliſhed; but, I think, *that* manner of expreſſing himſelf ſhewed ſtrongly his opinion of the propriety of an exact equality for the future, and his contempt of the arguments derived from its ſuppoſed injuſtice with reſpect to purchaſers.

This firſt deviation, therefore, from the rule of equal taxation in our government ought, as I conceive, to be corrected in the next land-tax act which ſhall be paſſed, there being no valid nor juſt objection to be made to ſuch correction.

CCCLXVII. But

CCCLXVII. But the caſe is different with reſpect to the other inſtance of a deviation from the ſame rule, to wit, the exemption of the intereſt of the publick debts from the land-tax and all other taxes. For this intereſt cannot be made ſubject to the land-tax without a direct breach of the national faith to the proprietors of it, there being expreſs clauſes in the ſeveral acts of parliament by which the loans that conſtitute theſe debts have been eſtabliſhed, which provide that the ſeveral annuities granted to the perſons who have advanced their money to the Government, ſhall be *free from all taxes, charges, and impoſitions whatſoever.** And the money ſo advanced to Government has been advanced at a lower rate of intereſt in conſequence of theſe clauſes. And much of it, I believe, has been lent to Government in the war of 1741, at the moderate intereſt of about four per cent. which, if it had been left ſubject to the land-tax (as all other perſonal eſtate is, according to the ſtrict letter of the land-tax acts, though, from the difficulty of coming at it, the tax is ſeldom actually paid upon it;) could not have been obtained for leſs than five per cent. The owners of this debt may, therefore, be ſaid to have paid the land-tax upon it in the very act of lending it on the terms propoſed to them; ſince, in conſideration of their exemption from that and other taxes, they conſented to take four, inſtead of five, per cent. for their money. Nevertheleſs, in proceſs of time, theſe original compacts grow to be in a manner forgot by the generality of mankind, who are apt to conſider this exemption of the ſtock-holders from paying the land-tax, as an unjuſt diſtinction in their favour: and the land-holders in general are apt to hold this language, partly, perhaps, from ignorance of the aforeſaid original clauſe of exemption, and partly from the bias of ſelf-intereſt, which makes them wiſh to ſee the ſtock-holders bear a ſhare of the burthen which they labour under, whether they have, or have not, been ſo exempted. I have known men of very good underſtandings and education talk in this manner, and, when they have been told of the ſaid clauſe of exemption in the ſeveral acts of parliament for borrowing the ſaid money, either refuſe to believe that there was ſuch a clauſe, or, if convinced of the exiſtence of it, deny its efficacy and validity, in point of juſtice and good policy, to entitle the ſtock-holders to be ſo exempted. This opinion and inclination in the land-owners of the kingdom, and perhaps in other claſſes of men that are not themſelves ſtock-holders, certainly contributes to make the publick funds leſs ſecure than they would be, if they had not been ſo exempted in their firſt eſtabliſh-

Of the exemption of the publick funds from the payment of taxes.

The ground of it.

The ſaid exemption is odious to the land-holders, and thereby leſſens the ſecurity of the ſaid funds.

* Thus, for example, in the ſtatute of the 21ſt of George the Second, chap. ii, ſect. 15, it is enacted, "That all the ſeveral and reſpective annuities, payable in purſuance of this act, after the rate of four pounds *per centum per annum*, on all and every the principal ſums for which the ſame are payable, *ſhall be free from all taxes, charges, and impoſitions whatſoever.*"

ment, but had been left liable (like all other property, both real and perſonal,) to pay their proportion of the land tax: for then, as both the claſſes would conſtantly and viſibly have contributed at the ſame time to the relief of the exigencies of the ſtate, the land-holders would have had no pretence to grudge the ſtock-holders the enjoyment of the intereſt of their money, which would, in every view, have been as much their rightful property as the rents of freehold land are the property of their reſpective owners.

It would tend to the ſecurity of the publick funds, if the proprietors of them would conſent to make the intereſt of them liable to the land-tax.

This diminution of the ſecurity of the publick funds, ariſing from their being thus exempted from taxes, appears to me to be a matter of ſo much importance, (as their ſecurity evidently depends, in a conſiderable degree, on the opinion the nation at large entertains of the juſtice of continuing them) that I ſhould be ready, with reſpect to my little property in them, to give up my right to the ſaid exemption, and make it liable to pay the land-tax at that which may be ſuppoſed to be the medium rate at which it is levied upon the lands of the kingdom, which I have heard people eſtimate at about eighteen-pence in the pound, or, at moſt, two ſhillings, when the land-tax is called four ſhillings in the pound, or (to ſpeak more correctly) when the ſum raiſed by the land-tax in England and Wales, (excluſive of Scotland) is about two millions: for it is ſuppoſed that an equal aſſeſſment of the land-tax in England and Wales at eighteen-pence or two ſhillings in the pound, would raiſe the ſaid ſum of two millions. This, therefore, would be the rate at which property in the publick funds ought to be charged to the land-tax, if it could, conſiſtently with juſtice and the preſervation of the national faith, be charged to it at all. And at this rate, I ſay, I would freely conſent to have my property there charged, notwithſtanding my aforeſaid right to an exemption, if all the other proprietors of ſtock, or a majority of them, were willing to do the ſame: and this, not ſo much from a motive of publick ſpirit (though I hope that motive is not without its influence) as from a deſire of rendering the remaining part of the intereſt due to me, the eighteen ſhillings and ſixpence in the pound, more ſecure: for then nothing but the moſt ſhameleſs and bare-faced injuſtice could ever prompt any land-holder to wiſh for, or encourage, any attempt to get rid of the publick debt by any other method than that of fairly paying it off.

But this is hardly to be expected.

But ſuch a general conſent of the ſtock-holders to let their ſtock be made liable to pay the land-tax, is what we can hardly expect to ſee; more eſpecially as many of them are foreigners, reſident in Holland and Switzerland, who will probably conceive themſelves to be no way obliged to pay taxes for the ſupport of our government.

Nor can ſuch a meaſure be taken, conſiſtently with juſtice, without their conſent.

And to ſubject their ſtock to the land-tax by act of parliament, without their conſent, or even with the conſent of ſome, but not all of them, would undoubtedly be a breach of the national faith, and an act of bankruptcy, *pro tanto*. With reſpect, therefore, to the immenſe publick debt already ſubſiſting, I do not expect, or wiſh, to ſee any endeavours uſed by publick

publick authority to carry this meaſure, of making it liable to the land-tax, into execution. But with reſpect to the loans of the preſent year, 1780, and of the following years, (if more ſuch are to be expected,) I muſt own I ſhould like to ſee it tried. It would certainly have the good effect of increaſing the ſecurity of the money ſo lent, for the reaſon already ſuggeſted: and it would be no immediate diminution of the intereſt received and enjoyed for the ſaid money; becauſe the intereſt which would be given by Government for the money, when liable to this tax, would be proportionably greater than if lent under a clauſe of exemption. And yet Government would be no loſer by it, ſince it would receive back, in the ſhape of a tax on the ſaid intereſt, the additional intereſt it would be obliged to give on this account. This, it may be ſaid, is mere trifling and doing nothing, ſince you give with one hand what you take back with the other. But the advantage reſulting from it is this: the land-holder, and the ſtock-holder, whoſe ſtock is originally made liable to this tax, will ever after run the ſame fortune, and experience the ſame increaſe or diminution of their reſpective incomes, as the affairs of the ſtate are proſperous or unfortunate. If, contrary to all preſent appearance, the land-tax ſhould ever again be leſs than four ſhillings in the pound, the ſtock-holder upon this new eſtabliſhment would enjoy a proportional diminution of the tax upon his property: and, on the other hand, if it ſhould become neceſſary, in the courſe of this moſt ruinous and unhappy war, to make the land-tax double or treble of what it now is, (which ſeems to be a much more likely event than the former) the ſtock-holder will pay a double or treble tax as well as the owner of land, and viſibly bear his proportion of the common burthen, and thereby eſcape the dangerous envy of being conſidered as a kind of foreigner, or neutral perſon unconcerned in the welfare or calamities of his country.

But it would be a uſeful meaſure in the government, to make the intereſt of all *future* publick loans of money liable to the land-tax.

An ingenious and publick-ſpirited writer of ſome letters that have appeared in the Whitehall Evening Poſt, dated from Windſor, and which have been lately collected and publiſhed in a pamphlet printed for Dodſley, has recommended a meaſure of this kind, with reſpect to the publick debt already exiſting, and has given very powerful reaſons in ſupport of it, which, together with the many other important particulars contained in thoſe letters, are well worth the moſt ſerious attention of the publick. But, ſuch a ſtep would be attended with conſiderable difficulties, I cannot but doubt whether it be, upon the whole, adviſeable.

But the meaſure which I have here ventured to recommend, and which relates only to the ſtock hereafter to be created, would be attended with the ſame advantages, as far as it went, and would not be liable to any of the ſame objections.

And,

It would, probably, be expedient to admit the owners of ſuch ſtock in the publick funds as ſhould be made liable to pay the land-tax, to a right of voting for members of parliament.

And, perhaps, if this meaſure were adopted, either with reſpect to the intereſt of future loans of money, or to that of the money already due to the publick creditors, it might be adviſeable at the ſame time to admit the ſtock-holders who would thus become contributors to the land-tax, to a right of voting for members of the Houſe of Commons, by whom the ſaid tax is granted. Every proprietor of ſuch ſtock, who was of the male ſex, and a native of Great Britain, or Ireland, or any of the Britiſh dominions, and had been in poſſeſſion of an annuity of 10 pounds a year, ſtanding in his own name, in any of the publick funds for more than a whole year, and had reſided for more than a year together in any particular county in England, Wales, or Scotland, might, as I imagine, without any inconvenience, be permitted to vote at the election of the knights of the ſhire, or commiſſioners of the ſhire, in which he had ſo reſided. Perhaps the offer of ſuch a privilege might induce ſome of the preſent proprietors of the publick funds to conſent that the intereſt of their ſhares of the national debt ſhould for the future be made liable to the land-tax: more eſpecially, if they ſhall apprehend themſelves to be under a kind of neceſſity (from the enormity of the preſent load upon the publick revenue,) of making ſome ſacrifice of this nature, or of giving up a part of their annual income, ariſing from the funds, in order to preſerve the remainder; which the *Earl of Stair*, in his very able and moſt intereſting pamphlet on the State of the Publick Revenue, publiſhed about January, 1782, and intitled, *Facts, and Conſequences*, &c. poſitively declares it will be abſolutely neceſſary that they ſhould do.

When the ſtock-holders ſhould have thus become liable to the land-tax, and, in conſideration of their thereby contributing, like the owners of land, to the common burthens of the nation, ſhould have been admitted to a ſhare in the election of the national repreſentative, the ſecurity of their property in the publick funds would be rendered as compleat as any laws, or publick regulations, can make it: though they would ſtill have reaſon to wiſh, at leaſt as heartily as any other claſs of men in the kingdom, that the bleſſing of peace may ſpeedily be reſtored to us, and that then the Government may adopt ſuch meaſures of vigour for increaſing the publick revenue, and ſuch meaſures of œconomy in the management of it, as may enable them gradually to diſcharge ſome part of this enormous debt, and thereby render the payment of the remainder of it leſs precarious.

[*End of the reflections on the Land-tax and the Intereſt of the National Debt.*]

CCCLXVIII. I now return from theſe political digreſſions to the Doctrine of Life-Annuities, which is the proper ſubject of this treatiſe.

And, as I have gone through every thing that has appeared to be neceſſary to the illuſtration of the doctrine of annuities for ſingle lives, I ſhall now proceed to ſubjoin the like ſhort and convenient expreſſions for the values of annuities dependent on more than one life as were given above, in Art. 86 *et ſeq.* for the values of annuities depending on a ſingle life.

A ſhort expreſſion of the value of an annuity of one pound per annum *for a given number of years, depending on the joint continuance of two lives of given ages, according to a given table of the probabilities of the duration of human life, and a given rate of the intereſt of money.*

CCCLXIX. Let r be, as before, the ſum of one pound together with its intereſt for one year according to the given rate of intereſt. And let N be the number of years in the age of the younger of the two perſons on the joint continuance of whoſe lives the annuity is to depend; and $N+a$ the number of years in the age of the older of the ſaid perſons; and E the greateſt number of years through which it is ſuppoſed to be poſſible for human life to be extended, according to the table of the probabilities of the duration of human life adopted for the calculation; which number is in Monſieur de Parcieux's table 94 years. Let n be any number of years not greater than $E - \overline{N+a}$, or $E-N-a$, or than the greateſt number of years during which it is poſſible that the older of the two lives may be prolonged. And let the annuity of one pound *per annum* be granted for the term of n years, provided both the ſaid perſons of the ages of N years and $N+a$ years ſhall ſo long live, but otherwiſe to ceaſe upon the death of either of them. Let P be the number of perſons repreſented in Monſieur de Parcieux's table of the probabilities of the duration of human life, (or in ſuch other table of thoſe probabilities as is thought proper by the calculator to be adopted as the ground of his calculation,) as being all living together at the ſaid age of N years; and P^{I} the number of perſons repreſented in the ſaid table to be living at the age of $N+1$ years; and P^{II} the number living at the age of $N+2$ years; and P^{III} the number living at the age of $N+3$ years; and P^{IV}, P^{V}, P^{VI}, P^{VII}, P^{VIII}, P^{IX}, P^{X}, &c. the numbers living at the ſeveral following ages of $N+4$ years, $N+5$ years, $N+6$ years, $N+7$ years, $N+8$

$N+8$ years, $N+9$ years, $N+10$ years, &c. respectively. And let $\mathfrak{Q}$ be the number of persons represented in the said table as living at the age of $N+a$ years; and $\mathfrak{Q}^{\text{I}}$ the number of persons represented there as living at the age of $N+a+1$ years; and $\mathfrak{Q}^{\text{II}}$ the number living at the age of $N+a+2$ years; and $\mathfrak{Q}^{\text{III}}$ the number living at the age of $N+a+3$ years; and $\mathfrak{Q}^{\text{IV}}$, $\mathfrak{Q}^{\text{V}}$, $\mathfrak{Q}^{\text{VI}}$, $\mathfrak{Q}^{\text{VII}}$, $\mathfrak{Q}^{\text{VIII}}$, $\mathfrak{Q}^{\text{IX}}$, $\mathfrak{Q}^{\text{X}}$, &c. the numbers living at the several following ages of $N+a+4$ years, $N+a+5$ years, $N+a+6$ years, $N+a+7$ years, $N+a+8$ years, $N+a+9$ years, $N+a+10$ years, &c. respectively.

Then will the present value of an annuity of one pound a year, to be enjoyed during the space of n years, in case both the said lives, of the ages of N years and $N+a$ years, shall so long continue, be equal to the expression $\frac{\pounds 1}{}\times$ the series $\frac{P^{\text{I}}\times\mathfrak{Q}^{\text{I}}}{P\mathfrak{Q}\times r}+\frac{P^{\text{II}}\times\mathfrak{Q}^{\text{II}}}{P\mathfrak{Q}\times r^2}+\frac{P^{\text{III}}\times\mathfrak{Q}^{\text{III}}}{P\mathfrak{Q}\times r^3}+\frac{P^{\text{IV}}\times\mathfrak{Q}^{\text{IV}}}{P\mathfrak{Q}\times r^4}+\frac{P^{\text{V}}\times\mathfrak{Q}^{\text{V}}}{P\mathfrak{Q}\times r^5}+\frac{P^{\text{VI}}\times\mathfrak{Q}^{\text{VI}}}{P\mathfrak{Q}\times r^6}+\frac{P^{\text{VII}}\times\mathfrak{Q}^{\text{VII}}}{P\mathfrak{Q}\times r^7}+$ &c. continued to n terms, or to the term $\frac{P^n\times\mathfrak{Q}^n}{P\mathfrak{Q}\times r^n}$, or equal to the expression $\frac{\pounds 1}{P\mathfrak{Q}}\times$ the series $\frac{P^{\text{I}}\times\mathfrak{Q}^{\text{I}}}{r}+\frac{P^{\text{II}}\times\mathfrak{Q}^{\text{II}}}{r^2}+\frac{P^{\text{III}}\times\mathfrak{Q}^{\text{III}}}{r^3}+\frac{P^{\text{IV}}\times\mathfrak{Q}^{\text{IV}}}{r^4}+\frac{P^{\text{V}}\times\mathfrak{Q}^{\text{V}}}{r^5}+\frac{P^{\text{VI}}\times\mathfrak{Q}^{\text{VI}}}{r^6}+\frac{P^{\text{VII}}\times\mathfrak{Q}^{\text{VII}}}{r^7}+$ &c. continued to n terms, or to the term $\frac{P^n\times\mathfrak{Q}^n}{r^n}$. This is evident from Problem III, and its second Corollary, Art. XLIV, XLV, XLVI, and XLVIII, pages 42, 43, 44, 45, 46, 47, 48.

The expression of the value of an annuity of one pound a year for the *whole* joint continuance of two lives of given ages.

CCCLXX. If n years is the greatest number of years through which it is possible (according to the table of the probabilities of the duration of human life adopted in the calculation,) for the older of the two given lives, or the life of the age of $N+a$ years, to be extended, or, in other words, if n is equal to $E-N-a$, the said expression $\frac{\pounds 1}{P\mathfrak{Q}}\times$ the series $\frac{P^{\text{I}}\times\mathfrak{Q}^{\text{I}}}{r}+\frac{P^{\text{II}}\times\mathfrak{Q}^{\text{II}}}{r^2}+\frac{P^{\text{III}}\times\mathfrak{Q}^{\text{III}}}{r^3}+\frac{P^{\text{IV}}\times\mathfrak{Q}^{\text{IV}}}{r^4}+\frac{P^{\text{V}}\times\mathfrak{Q}^{\text{V}}}{r^5}+\frac{P^{\text{VI}}\times\mathfrak{Q}^{\text{VI}}}{r^6}+\frac{P^{\text{VII}}\times\mathfrak{Q}^{\text{VII}}}{r^7}+$ &c. continued to n terms, or to the

the term $\frac{P^n \times Q^n}{r^n}$, (which term will in this case be $\frac{P^{E-N-a} \times Q^{E-N-a}}{r^n}$,) will be the value of an annuity of one pound *per annum* for the whole joint continuance of the two given lives of N years and $N + a$ years: but, if n is less than the said complement of $N + a$ years to E, or to the utmost duration of human life, the said expression will be less than the value of an annuity of one pound *per annum* for the whole joint continuance of the two given lives of the ages of N years and $N + a$ years, and will be the value of an immediate, but imperfect, life-annuity of one pound *per annum* during n years of the joint continuance of the lives of two persons of the said ages. This is evident from Art. XLVIII, XLIX, LI, pages 48, 49, 50, 51, 52, 53.

An example of the computation of the value of an immediate and compleat life-annuity of one pound per annum *for the whole joint continuance of the lives of two persons of given ages, by means of the foregoing expression.*

CCCLXXI. Let it be required to find the value of an annuity of one pound *per annum* for the whole joint continuance of the lives of two persons of the ages of 75 years and 80 years, according to Monsieur de Parcieux's table of the probabilities of the duration of human life, and upon a supposition that the interest of money is 3 per cent.

Here n, or the number of years through which the annuity is to continue, in case both the lives (of which the older is of the age of fourscore years,) shall last so long, is the greatest possible number of years through which, according to Monsieur de Parcieux's table, a life of fourscore years of age can be extended, that is, (94 — 80 years, or) 14 years. Therefore the series $\frac{P^{\text{I}} \times Q^{\text{I}}}{r} + \frac{P^{\text{II}} \times Q^{\text{II}}}{r^2} + \frac{P^{\text{III}} \times Q^{\text{III}}}{r^3} + \frac{P^{\text{IV}} \times Q^{\text{IV}}}{r^4} + \frac{P^{\text{V}} \times Q^{\text{V}}}{r^5} + \frac{P^{\text{VI}} \times Q^{\text{VI}}}{r^6} + \frac{P^{\text{VII}} \times Q^{\text{VII}}}{r^7}$ + &c. in the foregoing expression must be continued to 14 terms; which terms may be computed as follows.

Here P is = 211, P^{I} = 192, P^{II} = 173, P^{III} = 154, P^{IV} = 136, P^{V} = 118, P^{VI} = 101, P^{VII} = 85, P^{VIII} = 71, P^{IX} = 59, P^{X} = 48, P^{XI} = 38, P^{XII} = 29, P^{XIII} = 22, and P^{XIV} = 16; and Q is = 118, Q^{I} = 101, Q^{II} = 85, Q^{III} = 71, Q^{IV} = 59, Q^{V} = 48,

$Q^{v} = 48$, $Q^{vi} = 38$, $Q^{vii} = 29$, $Q^{viii} = 22$, $Q^{ix} = 16$, $Q^{x} = 11$, $Q^{xi} = 7$, $Q^{xii} = 4$, $Q^{xiii} = 2$, and $Q^{xiv} = 1$. And r is $= 1.03$, and $\frac{1}{r} = \frac{1}{1.03} = .9708$, and $\frac{1}{r^2} = .9425$, $\frac{1}{r^3} = .9151$, $\frac{1}{r^4} = .8884$, $\frac{1}{r^5} = .8626$, $\frac{1}{r^6} = .8374$, $\frac{1}{r^7} = .8130$, $\frac{1}{r^8} = .7894$, $\frac{1}{r^9} = .7664$, $\frac{1}{r^{10}} = .7440$, $\frac{1}{r^{11}} = .7224$, $\frac{1}{r^{12}} = .7013$, $\frac{1}{r^{13}} = .6809$, and $\frac{1}{r^{14}} = .6611$. Therefore the expreſſion $\frac{\pounds 1}{P \times Q}$ x the ſeries

$$\frac{P^{i} \times Q^{i}}{r} + \frac{P^{ii} \times Q^{ii}}{r^2} + \frac{P^{iii} \times Q^{iii}}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4} + \frac{P^{v} \times Q^{v}}{r^5} + \frac{P^{vi} \times Q^{vi}}{r^6} + \frac{P^{vii} \times Q^{vii}}{r^7} + \frac{P^{viii} \times Q^{viii}}{r^8} + \frac{P^{ix} \times Q^{ix}}{r^9} + \frac{P^{x} \times Q^{x}}{r^{10}} + \frac{P^{xi} \times Q^{xi}}{r^{11}} + \frac{P^{xii} \times Q^{xii}}{r^{12}} + \frac{P^{xiii} \times Q^{xiii}}{r^{13}} + \frac{P^{xiv} \times Q^{xiv}}{r^{14}}$$

will be equal to $\frac{\pounds 1}{211 \times 118}$ x the ſeries

$$\begin{array}{llll}
192 \times 101 \times .9708 & + 173 \times 85 \times .9425 & + 154 \times 71 \times .9151 & \\
+ 136 \times 59 \times .8884 & + 118 \times 48 \times .8626 & + 101 \times 38 \times .8374 & \\
+ 85 \times 29 \times .8130 & + 71 \times 22 \times .7894 & + 59 \times 16 \times .7664 & \\
+ 48 \times 11 \times .7440 & + 38 \times 7 \times .7224 & + 29 \times 4 \times .7013 & \\
+ 22 \times 2 \times .6809 & + 16 \times 1 \times .6611 & = \frac{\pounds 1}{24,898} \times \text{the ſeries} &
\end{array}$$

$$\begin{array}{llll}
19,392 \times .9708 & + 14,705 \times .9425 & + 10,934 \times .9151 & \\
+ 8024 \times .8884 & + 5664 \times .8626 & + 3838 \times .8374 & \\
+ 2465 \times .8130 & + 1562 \times .7894 & + 944 \times .7664 & \\
+ 528 \times .7440 & + 266 \times .7224 & + 116 \times .7013 & \\
+ 44 \times .6809 & + 16 \times .6611 & = \frac{\pounds 1}{24,898} \times \text{the ſeries} &
\end{array}$$

$$\begin{array}{llll}
18,825.7536 & + 13,859.4625 & + 10,005.7034 & + 7,288.4336 \\
+ 4,885.7664 & + 3,213.9412 & + 2004.0450 & + 1233.0428 \\
+ 723.4816 & + 392.8320 & + 192.1584 & + 81.3508 \\
 & & & + 29.9596
\end{array}$$

$+ 29.9596 + 10.5776 = \frac{\pounds 1}{24,898} \times 62,746.5085 = \pounds 1 \times \frac{62746.5085}{24,898} = \pounds 1 \times 2.5201 = \pounds 2.5201$. Therefore the value of an annuity of one pound a year for the whole joint continuance of two lives of the ages of 75 years and 80 years, when the intereſt of money is 3 per cent. is $\pounds 2.5201$, or 2*l.* 10*s.* 5*d.* QE I.

Note. This value 2.5201 exceeds the value found for the ſame annuity in Art. XLIX, to wit, $\pounds 2.5197$, by $\pounds .0004$. But this very ſmall difference (which begins only in the fourth and laſt place of decimal fractions,) is rather to be conſidered as a proof of the agreement of the two calculations with each other than of an error in either of them, and probably has ariſen from the different order in which the arithmetical operations have been performed in them, the terms of the ſeries having been ſeparately divided by 24898 in Art. XLIX, and the ſeveral quotients thence ariſing added together into one ſum, whereas in this latter calculation the ſame terms have been all added together into one ſum without ſuch diviſion, and then the ſaid ſum has been divided by 24898.

An example of the computation of the value of an immediate, but imperfect, life-annuity, depending on the joint continuance of the lives of two perſons of given ages, by means of the ſame expreſſion.

CCCLXXII. Let it be required to find the value of an annuity of one pound *per annum* for the firſt five years of the joint continuance of the lives of two perſons of the ages of 75 years and 80 years, according to the ſame table of the probabilities of the duration of human life, and the ſame rate of intereſt, as in the laſt example.

For this purpoſe we need only take the firſt five terms of the foregoing ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ $+$ &c. and multiply their ſum into the fraction $\frac{\pounds 1}{P \times Q}$; and the product will be the value of the propoſed annuity. Theſe terms are $192 \times 101 \times .9708 + 173 \times 85 \times .9425 + 154 \times 71 \times .9151 + 136 \times 59 \times .8884 + 118 \times 48 \times .8626$; which are equal to

$19392 \times .9708 + 14{,}705 \times .9425 + 10{,}934 \times .9151 + 8024 \times .8884 + 5664 \times .8626 = 18{,}825.7536 + 13{,}859.4625 + 10{,}005.7034 + 7{,}288.4336 + 4{,}885.7664 = 54{,}865.1195$;

which being multiplied into the fraction $\frac{\text{£}1}{P \times Q}$, or $\frac{\text{£}1}{211 \times 118}$, or $\frac{\text{£}1}{24{,}898}$, is $= \text{£}1 \times \frac{54{,}865.1195}{24{,}898} = \text{£}1 \times 2.2035 = \text{£}2.2035$.

Therefore the value of an annuity of one pound *per annum* for the firſt five years of the joint continuance of the lives of two perſons of the ages of 75 years and 80 years, according to the foregoing ſuppoſitions concerning the duration of human life and the intereſt of money, is £2.2035, or 2*l.* 4*s.* c*d.*$\frac{3}{4}$. QEI.

CCCLXXIII. And, if the annuity to be purchaſed is a remote one, or is to be paid at the diſtance of more than one year, a ſhort expreſſion of its value, ſimilar to the foregoing expreſſions, may be found as follows.

A ſhort and general expreſſion of the value of a remote annuity of one pound per annum *for a given number of years, depending on the joint continuance of two lives of given ages, when the rate of the intereſt of money is alſo given.*

Let r be, as before, the value of one pound together with its intereſt for a year at the given rate of the intereſt of money. And let m be the number of years at the end of which the annuity is to commence, ſo that the firſt payment of it ſhall be made at the end of $m+1$ years. And let N be the number of years in the age of the younger of the two perſons upon the joint continuance of whoſe lives the annuity is to depend; and let $N+a$ be the number of years in the age of the older of the ſaid perſons; at the time of purchaſing the ſaid annuity. And let E be, as before, the whole number of years through which it is poſſible, according to the table of probabilities adopted in the calculation, for human life to be extended; which in Monſieur de Parcieux's table is 94 years. Then will $N+a+m$ be the number of years in the age of the older of the ſaid two perſons at the time of the commencement of the ſaid annuity, and $N+a+m+1$ the number of years in his age at the time when the firſt

firſt payment of the ſaid annuity will become due; and $E-|\overline{N}_{1}\overline{+a}+m$, or $E-N-a-m$, will be the greateſt poſſible number of years through which the life of the ſaid older perſon can be extended after he ſhall have attained the age of $N+a+m$ years, and the annuity ſhall have commenced. Let n be any number of years not greater than $E-N-a-m$; and let P be the number of perſons repreſented in Monſieur de Parcieux's table of the probabilities of the duration of human life, (or in ſuch other table of thoſe probabilities as ſhall be adopted as the ground of the calculation) as being alive at the age of N years, or of the younger of the ſaid two perſons, and Q be the number repreſented there as living at the age of $N+a$ years, or of the older of the ſaid perſons; and out of the P perſons repreſented in the ſaid table as living at the age of N years, let P_{m+1}, P_{m+11}, P_{m+111}, P_{m+1v}, P_{m+v}, &c. be the numbers of perſons repreſented therein as living at the ſubſequent ages of $N+m+1$ years, $N+m+2$ years, $N+m+3$ years, $N+m+4$ years, $N+m+5$ years, &c. or at the ends of $m+1$ years, $m+2$ years, $m+3$ years, $m+4$ years, $m+5$ years, &c. from the time of purchaſing the annuity; and out of the Q perſons repreſented therein as living at the age of $N+a$ years, let Q_{m+1}, Q_{m+11}, Q_{m+111}, Q_{m+1v}, Q_{m+v}, &c. be the numbers of perſons repreſented therein as living at the ſubſequent ages of $N+a+m+1$ years, $N+a+m+2$ years, $N+a+m+3$ years, $N+a+m+4$ years, $N+a+m+5$ years, &c. or at the ends of $m+1$ years, $m+2$ years, $m+3$ years, $m+4$ years, $m+5$ years, &c. from the time of purchaſing the annuity; and ſo on for all the following ages in the table.

Then will the value of an annuity of one pound *per annum*, to commence at the diſtance of m years, (ſo that the firſt payment of it ſhall be made at the end of $m+1$ years,) and to continue during n years, provided two perſons, who are of the ages of N years and $N+a$ years at the time of purchaſing the annuity, ſhall ſo long live, but which ſhall ceaſe as ſoon as either of the ſaid perſons ſhall die; — I ſay, the value of ſuch a remote annuity will be equal to the following expreſſion, to wit,

$$\frac{\pounds 1}{P\times Q}\times \text{the ſeries } \frac{P_{m+1}\times Q_{m+1}}{r^{m+1}}+\frac{P_{m+11}\times Q_{m+11}}{r^{m+2}}+\frac{P_{m+111}\times Q_{m+111}}{r^{m+3}}$$
$$+\frac{P_{m+1v}\times Q_{m+1v}}{r^{m+4}}+\frac{P_{m+v}\times Q_{m+v}}{r^{m+v}}+\text{ &c. continued to } n \text{ terms,}$$

or to the term $\frac{P_{m+n}\times Q_{m+n}}{r^{m+n}}$. This is evident from Art. 111, page 53.

An

An example of the computation of the value of a remote life-annuity, depending upon the joint continuance of two lives of given ages, by means of the foregoing expreſſion.

CCCLXXIV. Let it be required to find the value of an annuity of one pound *per annum* during the joint lives of two perſons of the ages of 75 and 80 years, but which ſhall not commence till five years after the purchaſe of it, ſo that the firſt payment of it ſhall not become due until the end of ſix years, or till the younger of the ſaid two perſons (if he ſhall be then living,) ſhall have attained the age of fourſcore and one years, and the older of the ſaid perſons (if he ſhall be then living,) ſhall have attained the age of fourſcore and ſix years; and which ſhall then continue during the whole remainder of the joint continuance of the lives of the ſaid two perſons; the intereſt of money being 3 per cent. (as in the laſt example,) and the probabilities of the duration of human life being ſuch as they are repreſented to be in Monſieur de Parcieux's table of them.

Here N, or the number of years in the age of the younger of the ſaid two perſons, is 75; and $N+a$, or the number of years in the age of the older of the ſaid two perſons, is 80; and m, or the number of years before the propoſed annuity is to commence, is 5; and conſequently $N+a+m$, or the number of years in the age of the older of the ſaid two perſons, at the time when the annuity is to commence, is 85. E, or the greateſt number of years through which human life can be extended, is, according to Monſieur de Parcieux's table of probabilities, 94 years; and conſequently $E-|\overline{N+a+m}$, or the greateſt number of years through which it is poſſible that the life of the older of the ſaid two perſons can be extended after the annuity ſhall have commenced, is $94-85$, or 9, years. Therefore 9 years is likewiſe the greateſt number of years through which it is poſſible that the lives of both the ſaid perſons ſhould continue together in being. Therefore n, or the greateſt number of years through which it is poſſible the annuity may continue, will be equal to 9 years; becauſe, the annuity, when once it has taken place, is ſuppoſed to continue during the whole remainder of the joint continuance of the lives of the ſaid two perſons. And conſequently the ſeries $\frac{P^{m+\mathrm{I}} \times Q^{m+\mathrm{I}}}{r^{m+\mathrm{I}}} + \frac{P^{m+\mathrm{II}} \times Q^{m+\mathrm{II}}}{r^{m+2}} + \frac{P^{m+\mathrm{III}} \times Q^{m+\mathrm{III}}}{r^{m+3}} + \frac{P^{m+\mathrm{IV}} \times Q^{m+\mathrm{IV}}}{r^{m+4}} + \frac{P^{m+\mathrm{V}} \times Q^{m+\mathrm{V}}}{r^{m+5}} + \&c.$ will conſiſt of nine terms. Theſe terms may be computed as follows.

Since

Since m is = v, we ſhall have $m+1$ = vi, and $m+11$ = vii, $m+111$ = viii, $m+iv$ = ix, $m+v$ = x, $m+vi$ = xi, $m+vii$ = xii, $m+viii$ = xiii, and $m+ix$ = xiv; and in like manner $m+1=6$, $m+2=7$, $m+3=8$, $m+4=9$, $m+5=10$, $m+6=11$, $m+7=12$, $m+8=13$, and $m+9=14$. Therefore the ſeries $\frac{Pm+1 \times Qm+1}{rm+1} + \frac{Pm+11 \times Qm+11}{rm+2} + \frac{Pm+111 \times Qm+111}{rm+3} + \frac{Pm+iv \times Qm+iv}{rm+4} + \frac{Pm+v \times Qm+v}{rm+5} + \frac{Pm+vi \times Qm+vi}{rm+6} + \frac{Pm+vii \times Qm+vii}{rm+7} + \frac{Pm+viii \times Qm+viii}{rm+8} + \frac{Pm+ix \times Qm+ix}{rm+9}$ is equal to $\frac{Pvi \times Qvi}{r^6} + \frac{Pvii \times Qvii}{r^7} + \frac{Pviii \times Qviii}{r^8} + \frac{Pix \times Qix}{r^9} + \frac{Px \times Qx}{r^{10}} + \frac{Pxi \times Qxi}{r^{11}} + \frac{Pxii \times Qxii}{r^{12}} + \frac{Pxiii \times Qxiii}{r^{13}} + \frac{Pxiv \times Qxiv}{r^{14}}$.

But r is, as before, = 1.03; and conſequently $\frac{1}{r}$ is = $\frac{1}{1.03}$ = .9708, and $\frac{1}{r^6}$ is = .8374, and $\frac{1}{r^7}$ = .8130, and $\frac{1}{r^8}$ = .7894, and $\frac{1}{r^9}$ = .7664, and $\frac{1}{r^{10}}$ = .7440, and $\frac{1}{r^{11}}$ = .7224, and $\frac{1}{r^{12}}$ = .7013, and $\frac{1}{r^{13}}$ = .6809, and $\frac{1}{r^{14}}$ =.6611. And P is = 211, and Q is = 118, and Pvi is = 101, $Pvii$ = 85, $Pviii$ = 71, Pix = 59, Px = 48, Pxi = 38, $Pxii$ = 29, $Pxiii$ = 22, and $Pxiv$ = 16; and Qvi is = 38, and $Qvii$ = 29, and $Qviii$ = 22, and Qix = 16, and Qx = 11, and Qxi = 7, and $Qxii$ = 4, and $Qxiii$ = 2, and $Qxiv$ = 1. Therefore the expreſſion $\frac{\mathcal{L}1}{P \times Q}$ × the ſeries $\frac{Pvi \times Qvi}{r^6}$

$$\frac{P^{VI} \times Q^{VI}}{r^6} + \frac{P^{VII} \times Q^{VII}}{r^7} + \frac{P^{VIII} \times Q^{VIII}}{r^8} + \frac{P^{IX} \times Q^{IX}}{r^9}$$

$$+ \frac{P^{X} \times Q^{X}}{r^{10}} + \frac{P^{XI} \times Q^{XI}}{r^{11}} + \frac{P^{XII} \times Q^{XII}}{r^{12}} + \frac{P^{XIII} \times Q^{XIII}}{r^{13}}$$

$$+ \frac{P^{XIV} \times Q^{XIV}}{r^{14}} \text{ is equal to } \frac{\pounds 1}{211 \times 118} \times \text{the series } 101 \times 38 \times .8374$$

$$+ 85 \times 29 \times .8130 + 71 \times 22 \times .7894 + 59 \times 16 \times .7664$$

$$+ 48 \times 11 \times .7440 + 38 \times 7 \times .7224 + 29 \times 4 \times .7013$$

$$+ 22 \times 2 \times .6809 + 16 \times 1 \times .6611 = \frac{\pounds 1}{211 \times 118} \times \text{the}$$

$$\text{series } 3838 \times .8374 + 2465 \times .8130 + 1562 \times .7894$$

$$+ 944 \times .7664 + 528 \times .7440 + 266 \times .7224 + 116 \times .7013$$

$$+ 44 \times .6809 + 16 \times .6611 = \frac{\pounds 1}{211 \times 118} \times \text{the series}$$

$$3213.9412 + 2004.0450 + 1233.0428 + 723.4816 + 392.8320$$

$$+ 192.1584 + 81.3508 + 29.9596 + 10.5776 = \frac{\pounds 1}{211 \times 118}$$

$$\times 7881.3890 = \frac{\pounds 1}{24,898} \times 7881.3890 = \pounds 1 \times \frac{7881.3890}{24,898} = \pounds 1 \times$$

$.3165 = \pounds 0.3165 = 6s.\ 4d.$ Therefore $6s.\ 4d.$ is the value of a remote annuity of one pound *per annum*, that is to commence at the diſtance of five years, (or whereof the firſt payment is to be made at the end of ſix years,) and that is to continue during the joint lives of two perſons who, at the time of purchaſing it, are of the ages of 75 and 80 years; according to Monſieur de Parcieux's table of the probabilities of life, and when the intereſt of money is 3 per cent. Q E I.

CCCLXXV. The foregoing examples are, I preſume, ſufficient to illuſtrate the manner of computing the values of annuities, whether immediate or remote, that are to depend on the joint continuance of two lives of given ages. But, when a whole table of the values of ſuch annuities is to be computed, it is not neceſſary to make a new calculation, ſimilar to that given above in Art. CCCLXXI, for every different year of human

human life; but the values of theſe annuities for different pairs of lives whoſe ages differ from each other by the ſame number of years, may be deduced one from another by an eaſy arithmetical proceſs ſimilar to that of Mr. Morgan, above explained in Art. c, ci, cii, pages 109, 110, 111, 112, by which the value of an annuity for a ſingle life of any given age is deduced from that of the like annuity for a life that is one year older. This method, (as well as that explained above in Art. c, ci, cii,) has been given by Mr. Morgan in his learned treatiſe on the Doctrine of Aſſurances and Annuities for Lives, page 73, and may be explained as follows.

A ſhort and eaſy method of deducing from the value of an annuity of one pound a year during the joint continuance of any two given lives, the value of a like annuity for the joint continuance of two other lives that are one year younger than the former lives.

CCCLXXVI. Let the number of years in the age of the younger of the two given lives be called N, and the number of years in the age of the older of the ſaid lives be called $N+a$. Then will the numbers of years in the ages of two lives that are younger than the given lives by a year, be $N-1$ years and $N+a-1$ years.

Let the value of an annuity of one pound a year for the joint continuance of the two given lives of the ages of N years and $N+a$ years be $\overset{\text{£.}}{V}$; and the value of the like annuity of one pound a year for the joint continuance of two other lives that are one year younger than the two given lives, and which conſequently are of the ages of $N-1$ years and $N+a-1$ years, be $\overset{\text{£.}}{Y}$.

Let P be the number of perſons who are ſuppoſed, in the table of the probabilities of the duration of human life which is adopted as the ground of the calculation, to be living at the age of the younger of the two given lives, that is, at the age of N years, and $P+d$ the number of perſons who are therein repreſented as living at the age which is younger than the former age by one year, that is, at the age of $N-1$ years; and let Q be the number of perſons who are therein repreſented as living at the age of the older of the two given lives, or at the age of $N+a$ years, and $Q+e$ the number of perſons repreſented as living at the age which is younger than the ſaid older age by one year, or at the age of $N+a-1$ years.

And let r be the value of one pound together with its intereſt for one year, according to the rate of intereſt ſuppoſed in the calculation.

Then will $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives that are of the ages of $N-1$ years and $N+a-1$ years, or are reſpectively younger than the two given lives by one year, be equal to $\frac{1}{r} \times \frac{P \times Q}{P+d \times Q+e} \times \frac{\pounds}{1+V}$, or to the quantity which ariſes by adding one pound to $\overset{\pounds}{V}$, the value of an annuity of one pound a year for the joint continuance of the two given lives of the ages of N years and $N+a$ years, and then multiplying the ſum $\overline{1+V}|\pounds$, firſt, into the fraction $\frac{1}{r}$, and, ſecondly, into the fraction $\frac{P \times Q}{P+d \times Q+e}$. This may be demonſtrated in the manner following.

DEMONSTRATION.

CCCLXXVII. It is evident from Art. CCCLXX, that $\overset{\pounds}{V}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the given ages of N years and $N+a$ years, is (if we make uſe of the notation uſed in Art. CCCLXIX, CCCLXX,) equal to the expreſſion $\frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} +$ &c. continued to $E-N-a$ terms, and that $\overset{\pounds}{Y}$, or the value of a like annuity of one pound a year for the joint continuance of two lives of the ages of $N-1$ years and $N+a-1$ years, is equal to the expreſſion $\frac{\overset{\pounds}{1}}{P+d \times Q+e} \times$ the ſeries $\frac{P \times Q}{r} \frac{P^{I} \times Q^{I}}{r^2} + \frac{P^{II} \times Q^{II}}{r^3} + \frac{P^{III} \times Q^{III}}{r^4} + \frac{P^{IV} \times Q^{IV}}{r^5} +$ &c. continued to $E-N-a+1$ terms.

Now

Now this laſt expreſſion (which is $= \overset{\pounds}{Y}$,) may be derived from the former (which is $= \overset{\pounds}{V}$,) by adding $\overset{\pounds}{1}$ to it, and then multiplying the ſum, firſt, into the fraction $\frac{1}{r}$, and, ſecondly, into the fraction $\frac{P \times Q}{\overline{P+d} \times \overline{Q+e}}$.

For, if we add $\overset{\pounds}{1}$ to the expreſſion $\frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} +$ &c. continued to $E - N - a$ terms, it will become equal to $\overset{\pounds}{1} + \frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. continued to $E - N - a$ terms, $= \frac{\overset{\pounds}{1}}{P \times Q} \times P \times Q + \frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. continued to $E - N - a$ terms, $= \frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $P \times Q + \frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. continued to $E - N - a + 1$ terms. And this quantity, being multiplied by $\frac{1}{r}$, will become equal to $\frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $\frac{P \times Q}{r} + \frac{P^{I} \times Q^{I}}{r^2} + \frac{P^{II} \times Q^{II}}{r^3} + \frac{P^{III} \times Q^{III}}{r^4} + \frac{P^{IV} \times Q^{IV}}{r^5}$ + &c. continued to $E - N - a + 1$ terms; and,

being further multiplied by the fraction $\frac{P \times Q}{P+d \times Q+e}$, it will become equal to $\frac{\overset{\pounds}{1}}{P+d \times Q+e} \times$ the series $\frac{P \times Q}{r} + \frac{P^{I} \times Q^{I}}{r^2} + \frac{P^{II} \times Q^{II}}{r^3} + \frac{P^{III} \times Q^{III}}{r^4} + \frac{P^{IV} \times Q^{IV}}{r^5} +$ &c. continued to $E - N - a + 1$ terms, that is, to $\overset{\pounds}{Y}$. Therefore $\frac{1}{r} \times \frac{P \times Q}{P+d \times Q+e} \times \frac{\overset{\pounds}{1}}{1+V}$ is $= \overset{\pounds}{Y}$. QED.

CCCLXXVIII. I will now proceed to give a few examples of this method of deducing the values of annuities for joint lives from thoſe of annuities for joint lives that are one year older, by which it will become familiar to the reader.

Examples of the foregoing method.

CCCLXXIX. Let us therefore ſuppoſe that the intereſt of money is $3\frac{1}{2}$ per cent. and that the two lives for whoſe joint continuance an annuity of one pound a year is to be granted are, ſucceſſively, of the ages of 83 years and 93 years, of 82 years and 92 years, of 81 years and 91 years, of 80 years and 90 years, of 79 years and 89 years, of 78 years and 88 years, of 77 years and 87 years, of 76 years and 86 years, of 75 years and 85 years, of 74 years and 84 years, of 73 years and 83 years, of 72 years and 82 years, of 71 years and 81 years, and of 70 years and 80 years; in all which pairs of lives the difference of the two ages is conſtantly 10 years. And let the probabilities of the duration of human life be ſuppoſed to be ſuch as they are repreſented to be in Monſieur de Parcieux's table of them.

CCCLXXX. Then, in the firſt place, we muſt compute the value of an annuity of one pound a year for the joint continuance of the two oldeſt lives, to wit, thoſe of the ages of 83 years and 93 years, by means of the expreſſion $\frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} +$ &c. continued to $E - N - a$ terms, which is given for that purpoſe in Art. CCCLXX.

Here

Here r is $= 1.035$; and E is 94 years; N, or the number of years in the age of the younger of the two lives, is 83; a, or the difference of the ages of the two lives, is 10 years; $N+a$, or the number of years in the age of the older of the two lives, is 93; $E-N-a$ is $94-93$ years, or 1 year; P, or the number of persons living at the age of the younger life, or the age of 83 years, is 71; P', or the number of persons living at the age of 84 years, is 59; Q, or the number of persons living at the age of the older of the two lives, or at the age of 93 years, is 2; and Q', or the number of persons living at the age of 94 years, is 1; and Q'', Q''', Q^{IV}, &c. or the numbers of persons living at the ages of 95 years, 96 years, 97 years, &c. are equal to 0. Therefore all the terms of the series $\frac{P' \times Q'}{r} + \frac{P'' \times Q''}{r^2} + \frac{P''' \times Q'''}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} +$ &c. after the first term $\frac{P' \times Q'}{r}$, are equal to 0; and consequently the expression $\frac{\pounds 1}{P \times Q} \times$ the series $\frac{P' \times Q'}{r} + \frac{P'' \times Q''}{r^2} + \frac{P''' \times Q'''}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} +$ &c. is $= \frac{\pounds 1}{P \times Q} \times \frac{P' \times Q'}{r}$

$= \frac{\pounds 1}{71 \times 2} \times \frac{59 \times 1}{1.035} = \frac{\pounds 1}{142} \times \frac{59}{1.035} = \pounds 1 \times \frac{59}{142 \times 1.035} = \pounds 1 \times \frac{59}{146.970} = \pounds 1 \times .401,442 = \pounds 0.401,442$. Therefore, according to Monsieur de Parcieux's table of the probabilities of the duration of human life, the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 83 years and 93 years, when the interest of money is $3\frac{1}{2}$ per cent. is $\pounds 0.401,442$, or 8*s*. $\frac{1}{4}d$. QEI.

CCCLXXXI. Having thus found the value of an annuity of one pound a year for the joint continuance of the eldest pair of lives by means of the expression $\frac{\pounds 1}{P \times Q} \times$ the series $\frac{P' \times Q'}{r} + \frac{P'' \times Q''}{r^2} + \frac{P''' \times Q'''}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} +$ &c. given for that purpose in

Art.

Art. CCCLXX, we may now proceed to find the values of a like annuity for the joint continuance of all the younger pairs of lives above-mentioned, by means of the expreſſion $\frac{1}{r} \times \frac{P\,Q}{\overline{P+d|} \times \overline{|Q+e}} \times \overline{1+V|}\,\pounds$. This may be done in the manner following.

CCCLXXXII. To find the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 82 years and 92 years, we muſt proceed as follows.

The value of an annuity of one pound a year for the joint continuance of two lives of the ages of 83 years and 93 years has been found to be $\overset{\pounds}{0.401,442}$. Therefore $\overset{\pounds}{V}$ is $= \overset{\pounds}{0.401,442}$. The numbers of perſons living at the ages of 83 years and 82 years are 71 and 85; and the numbers of perſons living at the ages of 93 years and 92 years are 2 and 4. Therefore P is $= 71$, and $P+d$ is $= 85$, and Q is $= 2$, and $Q+e$ is $= 4$. And r is $= 1.035$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d|} \times \overline{|Q+e}} \times \overline{1+V|}\,\pounds$ is $= \frac{1}{1.035} \times \frac{71 \times 2}{85 \times 4} \times \pounds 1.401,442 = \frac{1}{1.035} \times \frac{71}{85 \times 2} \times \pounds 1.401,442 = \frac{1}{1.035} \times \frac{71}{170} \times \pounds 1.401,442 = \frac{1}{1.035 \times 170} \times 71 \times \pounds 1.401,442 = \frac{1}{175.950} \times \pounds 99.502,382 = \frac{99.502,382}{175.950} = \pounds 0.565,515$. Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 82 years and 92 years, is $= \pounds 0.565,515$, or 11*s.* 3*d.* $\frac{3}{4}$. Q E I.

CCCLXXXIII. When the two lives are of the ages of 81 years and 91 years, we ſhall have $\overset{\pounds}{V} = \overset{\pounds}{0.565,515}$, and conſequently $\overset{\pounds}{1} + \overset{\pounds}{V} = \pounds 1.565,515$. And P will be $= 85$, and $P+d = 101$, and $Q = 4$, and $Q+e = 7$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d|} \times \overline{|Q+e}} \times \overline{1+V|}\,\pounds$ will be

be $= \frac{1}{1.035} \times \frac{85 \times 4}{101 \times 7} \times £1.565,515 = \frac{1}{1.035} \times \frac{340}{707} \times$

$£1.565,515 = \frac{340 \times 1.565,515}{1.035 \times 707} = \frac{£}{\frac{340 \times 1.565,515}{731.745}} = \frac{£}{\frac{532.275,100}{731.745}}$

$= £0.727,405$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 81 years and 91 years, is $= £0.727,405$, or 14*s*. 6*d*.$\frac{1}{2}$ QEI.

CCCLXXXIV. When the two lives are of the ages of 80 years and 90 years, we ſhall have $\overset{£}{V} = \overset{£}{0.727,405}$, and conſequently $\overset{£}{1} + \overset{£}{V}$ $= £1.727,405$. And P will be $= 101$, $P + d = 118$, $Q = 7$, and $Q + e = 11$. Therefore $\frac{1}{r} \times \frac{P \times Q}{P + d \times Q + e} \times \overline{1 + V}|£$ will be $= \frac{1}{1.035} \times \frac{101 \times 7}{118 \times 11} \times £1.727,405 = \frac{707}{1.035 \times 118 \times 11}$

$\times £1.727,405 = \frac{£}{\frac{1221.275,335}{1.035 \times 118 \times 11}} = \frac{£}{\frac{1221.275,335}{1.035 \times 1298}} = \frac{£}{\frac{1221.275,335}{1343.430}}$

$= \overset{£}{0.909,072}$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 80 years and 90 years, is $= £0.909,072$, or 18*s*. 2*d*.$\frac{1}{2}$. QEI.

CCCLXXXV. When the two lives are of the ages of 79 years and 89 years, we ſhall have $\overset{£}{V} = \overset{£}{0.909,072}$, and conſequently $\overset{£}{1} + \overset{£}{V}$ $= £1.909,072$. And P will be $= 118$, and $P + d = 136$, and $Q = 11$, and $Q + e = 16$. Therefore $\frac{1}{r} \times \frac{P \times Q}{P + d \times Q + e} \times \overline{1 + V}|£$

will

will be $= \frac{1}{1.035} \times \frac{118 \times 11}{136 \times 16} \times £1.909,072 = \frac{£ 1298 \times 1.909,072}{1.035 \times 136 \times 16}$

$= \frac{£ 1298 \times 1.909,072}{1.035 \times 2176} = \frac{2477.975,456}{2252.160} = £1.100,266.$ Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 79 years and 89 years, is £1.100,266, or 1*l.* 2*s.* 0*d.* Q E I.

CCCLXXXVI. When the two lives are of the ages of 78 years and 88 years, we shall have $\overset{£}{V} = \overset{£}{1.100,266}$, and consequently $\overset{£}{1} + \overset{£}{V}$ $= £2.100,266$. And P will be $= 136$, and $P + d = 154$, and $Q = 16$, and $Q + e = 22$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d|} \times \overline{|Q+e}} \times \overline{1+V|}\,£$

will be $= \frac{1}{1.035} \times \frac{136 \times 16}{154 \times 22} \times £2.100,266 = \frac{1}{1.035} \times \frac{2176}{3388}$

$\times £2.100,266 = \frac{£ 4570.178,816}{3506.580} = £1.303,315.$ Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 78 years and 88 years, is £1.303.315, or 1*l.* 6*s.* 0*d.* $\frac{3}{4}$. Q E I.

CCCLXXXVII. When the two lives are of the ages of 77 and 87 years, we shall have $\overset{£}{V} = \overset{£}{1.303,315}$, and consequently $\overset{£}{1} + \overset{£}{V} =$ £2.303,315. And P will be $= 154$, and $P + d = 173$, and $Q = 22$, and $Q + e = 29$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d|} \times \overline{|Q+e}} \times \overline{1+V|}\,£$

will be $= \frac{1}{1.035} \times \frac{154 \times 22}{173 \times 29} \times £2.303,315, = \frac{1}{1.035} \times \frac{3388}{173 \times 29}$

$\times £2.303,315, = \frac{1}{1.035} \times \frac{3388}{5017} \times £2.303,315, = \frac{3388}{5192.595} \times$

£2 303,315

$£2.303,315 = \frac{£7803.631,220}{5192.595} = £1.502,838$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 77 years and 87 years, is $£1.502,838$, or *1l.* 10*s.* $\frac{1}{2}d$. Q E I.

CCCLXXXVIII. When the two lives are of the ages of 76 and 86 years, we shall have $\overset{£}{V} = \overset{£}{1.502,838}$, and consequently $\overset{£}{1+V} = £2.502,838$. And P will be $= 173$, $P+d = 192$, and $Q = 29$, and $Q+e = 38$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d|} \times \overline{|Q+e}} \times \overline{1+V|}\,£$ will be $= \frac{1}{1.035} \times \frac{173 \times 29}{192 \times 38} \times £2.502,838 = \frac{1}{1.035} \times \frac{5017}{7296} \times £2.502,838 = \frac{£12556.738,246}{7551.360} = £1.662,844$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 76 years and 86 years, is $£1.662,844$, or *1l.* 13*s.* 3*d.* Q E I.

CCCLXXXIX. When the two lives are of the ages of 75 years and 85 years, we shall have $\overset{£}{V} = \overset{£}{1.662,844}$, and consequently $\overline{1+V|}\,£ = £2.662,844$. And P will be $= 192$, and $P+d = 211$, and $Q = 38$, and $Q+e = 48$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d|} \times \overline{|Q+e}} \times \overline{1+V|}\,£$ will be $= \frac{1}{1.035} \times \frac{192 \times 38}{211 \times 48} \times £2.662,844 = \frac{1}{1.035} \times \frac{7296}{211 \times 48} \times £2.662,844 = \frac{1}{1.035} \times \frac{7296}{10128} \times £2.662,844 = \frac{7296 \times £2.662,844}{10482.480} = \frac{£19,428.109,824}{10482.480} = £1.853,388$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 75 years and 85 years, is $£1.853,388$, or *1l.* 17*s.* 0*d.* $\frac{3}{4}$. Q E I.

CCCXC. When the two lives are of the ages of 74 years and 84 years, we ſhall have $\overset{£}{V} = \overset{£}{1.853,388}$, and conſequently $\overline{1+V}|\,£ = £2.853,388$. And P will be $= 211$, and $P+d = 231$, and $Q = 48$, and $Q+e = 59$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times |\overline{Q+e}} \times \overline{1+V}|\,£$ will be $= \frac{1}{1.035} \times \frac{211 \times 48}{231 \times 59} \times £2.853,388 = \frac{1}{1.035} \times \frac{10128}{13629} \times £2.853,388 = \frac{\overset{£}{28,899.113,664}}{14,106.015} = £2.048,708$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 74 years and 84 years, is £2.048,708, or 2*l.* 0*s.* 11*d.* $\frac{3}{4}$. Q E I.

CCCXCI. When the two lives are of the ages of 73 years and 83 years, we ſhall have $\overset{£}{V} = \overset{£}{2.048,708}$, and conſequently $\overline{1+V}|\,£ = £3.048,708$. And P will be $= 231$, and $P+d = 251$, and $Q = 59$, and $Q+e = 71$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times |\overline{Q+e}} \times \overline{1+V}|\,£$ will be $= \frac{1}{1.035} \times \frac{231 \times 59}{251 \times 71} \times £3.048,708 = \frac{1}{1.035} \times \frac{13629}{17821} \times £3.048,708 = \frac{\overset{£}{41,550.841,332}}{18,444.735} = £2.252,720$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 73 years and 83 years, is £2.252,720, or 2*l.* 5*s.* $\frac{1}{2}d$. Q E I.

CCCXCII. When the two lives are of the ages of 72 years and 82 years, we ſhall have $\overset{£}{V} = \overset{£}{2.252,720}$, and conſequently $\overline{1+V}|\,£ = £3.252,720$. And P will be $= 251$, and $P+d = 271$, and $Q = 71$, and $Q+e = 85$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times |\overline{Q+e}} \times \overline{1+V}|\,£$ will be

be $= \frac{1}{1.035} \times \frac{251 \times 71}{271 \times 85} \times \pounds 3.252,720 = \frac{1}{1.035} \times \frac{17821}{23035} \times$ $\pounds 3.252,720 = \frac{\pounds 57,966.723,120}{23,841.225} = \pounds 2.431,365$. Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 72 years and 82 years, is $= \pounds 2.431,365$, or 2*l*. 8*s*. 7*d*.$\frac{1}{2}$ QEI.

CCCXCIII. When the two lives are of the ages of 71 years and 81 years, we shall have $\overset{\pounds}{V} = \overset{\pounds}{2.431,365}$, and consequently $\overline{1+V}|\pounds = \pounds 3.431,365$. And P will be $= 271$, and $P+d = 291$, and $Q = 85$, and $Q+e = 101$. Therefore $\frac{1}{r} \times \frac{P \times Q}{P+d| \times |Q+e} \times \overline{1+V}|\pounds$ will be $= \frac{1}{1.035} \times \frac{271 \times 85}{291 \times 101} \times \pounds 3.431,365 = \frac{1}{1.035} \times \frac{23035}{29391}$ $\times \pounds 3.431,365 = \frac{\pounds 79,041.492,775}{30,419.685} = \pounds 2.598,366$. Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 71 years and 81 years, is $= \pounds 2.598,366$, or 2*l*. 11*s*. 11*d*.$\frac{1}{2}$. QEI.

CCCXCIV. When the two lives are of the ages of 70 years and 80 years, we shall have $\overset{\pounds}{V} = \overset{\pounds}{2.598,366}$, and consequently $\overline{1+V}|\pounds = \pounds 3.598,366$. And P will be $= 291$, and $P+d = 310$, and $Q = 101$, and $Q+e = 118$. Therefore $\frac{1}{r} \times \frac{P \times Q}{P+d| \times |Q+e} \times \overline{1+V}|\pounds$ will be $= \frac{1}{1.035} \times \frac{291 \times 101}{310 \times 118} \times \pounds 3.598,366 = \frac{1}{1.035} \times \frac{29,301}{36,580}$ $\times \pounds 3.598,366 = \frac{\pounds 105,759.605,106}{37,860.300} = \pounds 2.793,416$. Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 80 years, is $= \pounds 2.793,416$, or 2*l*. 15*s*. 10*d*.$\frac{1}{2}$. QEI.

End of the examples of the foregoing method.

CCCXCV. The foregoing examples are, I preſume, ſufficient to illuſtrate, and render familiar to the reader, the method given in Art. CCCLXXVI of deriving the values of annuities for two joint lives from thoſe of equal annuities for two joint lives one year older than the former by means of the expreſſion $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \overline{1+V}\,\mathcal{L}$.

A remark on the foregoing method of computing a table of the values of annuities for two joint lives.

And this is undoubtedly the beſt method that can be taken for the purpoſe of computing a table of theſe annuities. Yet it is liable to the inconvenience mentioned above in Art. CXCIV, page 208, as belonging to the other method above explained in Art. C, CI, CII, by which the values of annuities for ſingle lives are ſucceſſively deduced from the values of annuities for the next older lives by means of the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}\,\mathcal{L}$, to wit, that, as the values of theſe annuities are deduced by it one from another in regular ſucceſſion from the older ages to the younger, any error that ſhould happen to be made in computing the value of any one of them would affect the values of all the following life-annuities, which would belong to younger ages than thoſe in which the error aroſe. But this inconvenience Mr. Morgan has enabled us to remove by giving us a method of examining and proving the truth of our calculations, as faſt as we make them, which is ſimilar to the method given by him for the like purpoſe in the caſe of annuities for ſingle lives, which has been ſtated and explained above in Art. CXCV, CXCVI, CXCVII, and CXCVIII. This method is as follows.

Of Mr. Morgan's method of proving the truth of the computations of the values of annuities for two joint lives, that are made by means of the foregoing expreſſion, $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \overline{1+V}\,\mathcal{L}$, *as faſt as they are made.*

CCCXCVI. Let N be (as before, in Art. CCCLXXVI,) the number of years in the age of the younger of any two given lives, upon the joint continuance of which an annuity of one pound *per annum* is to depend; and $N+a$ be the number of years in the older of the ſaid lives. And let P, P^{I}, P^{II}, P^{III}, P^{IV}, P^{V}, P^{VI}, &c. be the numbers of perſons repreſented in the table of the probabilities of life that is adopted as the ground of the calculation, as living at the ages of N years, $N+1$ years,

years, $N+2$ years, $N+3$ years, $N+4$ years, $N+5$ years, $N+6$ years, &c. reſpectively; and Q, Q^{I}, Q^{II}, Q^{III}, Q^{IV}, Q^{V}, Q^{VI}, &c. be the numbers of perſons repreſented therein as living at the ages of $N+a$ years, $N+a+1$ years, $N+a+2$ years, $N+a+3$ years, $N+a+4$ years, $N+a+5$ years, $N+a+6$ years, &c. reſpectively. Alſo let A be the number of perſons repreſented in the ſaid table as living at the youngeſt age in the ſaid table; (which in Monſieur de Parcieux's table of theſe probabilities is the age of 3 years:) and let K be the number of perſons repreſented therein as living at the age which is greater than the ſaid youngeſt age by a years. And let the number of years by which the age of N years, or of the younger of the two given lives, exceeds the youngeſt age in the table, be m years. Then will the age of $N-m$ years be the youngeſt age in the table, at which the number of perſons repreſented in the table as living is A; and $N-m+a$ will be the number of years in the age which exceeds the ſaid youngeſt age by a years, and at which the number of perſons repreſented in the table as living is K.

Preliminary propoſitions neceſſary to the demonſtration of the principal propoſition on which Mr. Morgan's method is founded.

CCCXCVII. Theſe things being premiſed, it is evident, in the firſt place, That the preſent value of a ſingle future payment of one pound, to be received at the end of m years in caſe two perſons, who are now of the ages of $N-m$ years (or the youngeſt age in the table,) and $N-m+a$ years, ſhall both be then alive, is $\frac{\pounds 1}{r^m} \times \frac{P \times Q}{A \times K}$. This is evident from Problem III, Art. XLV, pages 44, 45, 46.

In the ſecond place, it follows from Art. CCCLXXIII, that the value of a remote annuity of one pound a year, that is to commence at the diſtance of m years, (or whereof the firſt payment is to be received at the end of $m+1$ years,) and that is to continue during the joint lives of two perſons of the ages of $N-m$ years (or the youngeſt age in the table,) and $N-m+a$ years, will be $\frac{\pounds 1}{A \times K} \times$ the ſeries $\frac{P^{I} \times Q^{I}}{r^{m+1}} + \frac{P^{II} \times Q^{II}}{r^{m+2}} + \frac{P^{III} \times Q^{III}}{r^{m+3}} + \frac{P^{IV} \times Q^{IV}}{r^{m+4}} + \frac{P^{V} \times Q^{V}}{r^{m+5}} +$ &c. continued to the end of the table, and conſequently will be $=$ £1 $\times$ the ſeries $\frac{P^{I} \times Q^{I}}{A \times K \times r^{m+1}} + \frac{P^{II} \times Q^{II}}{A \times K \times r^{m+2}} + \frac{P^{III} \times Q^{III}}{A \times K \times r^{m+3}} + \frac{P^{IV} \times Q^{IV}}{A \times K \times r^{m+4}}$

$+\frac{P^{IV}\times Q^{IV}}{A\times K\times r^{m+4}}+\frac{P^{V}\times Q^{V}}{A\times K\times r^{m+5}}+$ &c. continued to the end of the table. Let the value of this remote annuity be called $\overset{\pounds}{R}$.

And, in the third place, it is evident that the value of an immediate life-annuity of one pound a year (or one of which the firſt payment is to be received at the end of a year,) that is to continue during the joint lives of two perſons of the ages of N years and $N+a$ years, is $=$ $\frac{\overset{\pounds}{1}}{P\times Q}\times$ the ſeries $\frac{P^{I}\times Q^{I}}{r}+\frac{P^{II}\times Q^{II}}{r^{2}}+\frac{P^{III}\times Q^{III}}{r^{3}}+\frac{P^{IV}\times Q^{IV}}{r^{4}}+\frac{P^{V}\times Q^{V}}{r^{5}}+$ &c. continued to the end of the table. Let this value be (as before,) called $\overset{\pounds}{V}$.

The ſaid principal propoſition itſelf.

CCCXCVIII. Now, if the laſt of theſe three quantities (which is $=\overset{\pounds}{V}$) be multiplied into the firſt of them, to wit, $\frac{\overset{\pounds}{1}}{r^{m}}\times\frac{P\times Q}{A\times K}$, or the preſent value of a future ſingle payment of one pound, to be received at the end of m years in caſe of the joint continuance of two lives of the ages of $N-m$ years and $N-m+a$ years, the product thence ariſing will be equal to the ſecond of them, (to wit, £1 $\times$ the ſeries $\frac{P^{I}\times Q^{I}}{A\times K\times r^{m+1}}+\frac{P^{II}\times Q^{II}}{A\times K\times r^{m+2}}+\frac{P^{III}\times Q^{III}}{A\times K\times r^{m+3}}+\frac{P^{IV}\times Q^{IV}}{A\times K\times r^{m+4}}+\frac{P^{V}\times Q^{V}}{A\times K\times r^{m+5}}+$ &c. continued to the end of the table,) or to $\overset{\pounds}{R}$, or to the value of a remote annuity of one pound a year, that is to commence at the diſtance of m years, or ſo that the firſt payment of it ſhall be received at the end of $m+1$ years, and that ſhall continue during the joint lives of two perſons of the ages of $N-m$ years (or the youngeſt age in the table,) and $N-m+a$ years.

DEMONSTRATION.

For $\frac{\overset{\pounds}{1}}{r^{m}}\times\frac{P\times Q}{A\times K}\times\frac{\overset{\pounds}{1}}{P\times Q}\times$ the ſeries $\frac{P^{I}\times Q^{I}}{r}+\frac{P^{II}\times Q^{II}}{r^{2}}+\frac{P^{III}\times Q^{III}}{r^{3}}+\frac{P^{IV}\times Q^{IV}}{r^{4}}+\frac{P^{V}\times Q^{V}}{r^{5}}+$ &c. continued to the

the end of the table, is $= \frac{\text{£}1}{r^m} \times \frac{1}{A \times K} \times$ the ſame ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5}$ + &c. continued to the end of the table, $=$ £1 $\times$ the ſeries $\frac{P^{I} \times Q^{I}}{A \times K \times r^{m+1}} + \frac{P^{II} \times Q^{II}}{A \times K \times r^{m+2}} + \frac{P^{III} \times Q^{III}}{A \times K \times r^{m+3}} + \frac{P^{IV} \times Q^{IV}}{A \times K \times r^{m+4}} + \frac{P^{V} \times Q^{V}}{A \times K \times r^{m+5}}$ + &c. continued to the end of the table, $= \frac{\text{£}}{R}$. QED.

CCCXCIX. If, therefore, in computing a table of the ſeveral values of an annuity of one pound a year for the joint continuance of ſeveral ſucceſſive pairs of lives whoſe ages differ from each other by the ſame number of years, or a years, we at the ſame time compute the corre-

Mr. Morgan's method itſelf, derived from the foregoing propoſition.

ſponding values of the expreſſion $\frac{\text{£}1}{r^m} \times \frac{P \times Q}{A \times K}$, and of the expreſſion £1 $\times$ the ſeries $\frac{P^{I} \times Q^{I}}{A \times K \times r^{m+1}} + \frac{P^{II} \times Q^{II}}{A \times K \times r^{m+2}} + \frac{P^{III} \times Q^{III}}{A \times K \times r^{m+3}} + \frac{P^{IV} \times Q^{IV}}{A \times K \times r^{m+4}} + \frac{P^{V} \times Q^{V}}{A \times K \times r^{m+5}}$ + &c. continued to the end of the table, or of $\frac{\text{£}}{R}$, and then multiply the expreſſion $\frac{\text{£}1}{r^m} \times \frac{P \times Q}{A \times K}$ into $\frac{\text{£}}{V}$, or the value of the annuity for two lives which we have before computed, and it ſhall appear that the product thereby obtained is equal to $\frac{\text{£}}{R}$, or to £1 $\times$ the ſeries $\frac{P^{I} \times Q^{I}}{A \times K \times r^{m+1}} + \frac{P^{II} \times Q^{II}}{A \times K \times r^{m+2}} + \frac{P^{III} \times Q^{III}}{A \times K \times r^{m+3}} + \frac{P^{IV} \times Q^{IV}}{A \times K \times r^{m+4}} + \frac{P^{V} \times Q^{V}}{A \times K \times r^{m+5}}$ + &c. continued to the end of the table, we may ſafely conclude that the value of $\frac{\text{£}}{V}$ has been accurately computed.

CCCC. To

An explanation of the manner of applying the foregoing rule, or method, of Mr. Morgan to the proof of the truth of the computations of a table of the values of annuities for two joint lives.

CCCC. To make the manner of applying this rule of Mr. Morgan more apparent, it will be proper to ſet down again in regular order, in a new table, the values of the life-annuities for the joint continuance of two given lives, which we have computed above in Art. CCCLXXX, CCCLXXXI, CCCLXXXII, &c.—CCCXCIV, from Monſieur de Parcieux's table by means of the expreſſion $\frac{1}{1.035} \times \frac{P \times Q}{\overline{P+d}^{l} \times \overline{Q+e}} \times \overline{1+V}^{l}$ £. And, as, in obtaining the ſaid values one from another by means of that expreſſion, we proceeded upwards, or from the older lives to the younger, it will be convenient to ſet down the ſaid values in the ſame order in the ſaid new table in a column adjoining to two other columns that contain the numbers of years in the two ages correſponding to the ſaid values. After theſe three columns, (containing the years in the ages of the two given lives, and the values of the correſponding annuities,) I ſhall ſet down, in a fourth column, the ſeveral ſucceſſive values of the expreſſion $\frac{\text{£}}{1^{m}} \times \frac{P \times Q}{A \times K}$, or the preſent values of a ſingle payment of one pound, to be received at the ends of 81 years, 80 years, 79 years, 78 years, and every following leſſer number of years down to 70 years, in caſe two perſons of the ages of 3 years (which is the youngeſt age in the table,) and 13 years ſhall be then living. And then, in a fifth column, I ſhall ſet down the ſums of the terms in the foregoing ſeries of values contained in the fourth column, as they ariſe; ſo that every term in this fifth column that is even with any two given ages in the firſt and ſecond columns, ſhall be equal to the ſum of all the terms in the fourth column that correſpond to the ages that are older than the ſaid two given ages. Thus, for example, the term in the fifth column that is even with the ages of 75 years and 85 years in the firſt and ſecond columns, is equal to the ſum of all the terms in the fourth column that correſpond to the ages that are older than the ages of 75 and 85 years. Theſe ſums, contained in this fifth column, will be equal to the values of remote annuities of one pound a year for the joint continuance of the lives of two perſons of the ages of 3 years and 13 years, that are to commence at the diſtances of 80 years, 79 years, 78 years, 77 years, &c. down to 70 years, or ſo that the firſt payments of them ſhall become due at the ends of 81 years, 80 years,

80 years, 79 years, 78 years, &c. down to 71 years; and they will comprize all the different values of the quantity which in Art. cccxcvii is called $\overset{\mathcal{L}}{R}$, that relate to remote periods that are greater than 70 years. For they are the ſums of the ſucceſſive values of the expreſſion $\frac{\overset{\mathcal{L}}{1}}{r^m} \times \frac{P \times Q}{A \times K}$, which, it is evident, are the ſame with the terms of the ſeries $\frac{P^{\prime} \times Q^{\prime}}{A \times K \times r^{m+1}} + \frac{P^{\prime\prime} \times Q^{\prime\prime}}{A \times K \times r^{m+2}} + \frac{P^{\prime\prime\prime} \times Q^{\prime\prime\prime}}{A \times K \times r^{m+3}} + \frac{P^{\text{IV}} \times Q^{\text{IV}}}{A \times K \times r^{m+4}} + \frac{P^{\text{V}} \times Q^{\text{V}}}{A \times K \times r^{m+5}} +$ &c. continued to the end of the table, which ſeries is $= \overset{\mathcal{L}}{R}$.

And, laſtly, in a ſixth column, I ſhall ſet down the products that ariſe by multiplying the terms of the third column, or the values of the life-annuities for the joint continuance of two lives of the ſeveral ages ſet down in the firſt and ſecond columns, by the correſponding terms, or terms that are placed even with them, in the fourth column, or by the values of ſingle future payments of one pound, depending on the joint continuance of two lives of the ages of 3 years and 13 years.

And, when we have thus obtained the numbers that are to be placed in theſe ſeveral columns, we muſt compare thoſe in the ſixth column with thoſe that ſtand even with them in the fifth column: and ſo far as we find them to be equal to the ſaid numbers in the fifth column, we may conclude that the numbers in the third column, or the values of an annuity of one pound for two joint lives of the ages ſet down in the firſt and ſecond columns, have been rightly computed.

CCCCI. The only difficulty that can occur in forming a table of this kind is in the computation of the ſucceſſive values of the expreſſion $\frac{\overset{\mathcal{L}}{1}}{r^m} \times \frac{P \times Q}{A \times K}$. But this will be found to be a work of no great labour, and may be performed in the manner following. Of the computation of the ſucceſſive values of the expreſſion $\frac{\overset{\mathcal{L}}{1}}{r^m} \times \frac{P \times Q}{A \times K}$.

CCCCII. A is the number of perſons repreſented in Monſieur de Parcieux's table of probabilities as living at the age of 3 years, which is 1000; and K is the number of perſons therein repreſented as living at the age of 13 years, which is 860. Therefore $A \times K$ is $=$ 1000 × 860 $=$ 860,000. And r is $=$ 1.035; and therefore (by Mr. Smart's ſecond table of compound intereſt, page 60 *et ſeq.*) $\frac{1}{r}$ is $=$ .966,183,57;

and $\frac{1}{r^{81}}$ is $=$ .061,635,61; $\frac{1}{r^{80}}$ is $=$ .063,792,85;

$\frac{1}{r^{79}}$ is $=$ .066,025,60; $\frac{1}{r^{78}}$ is $=$ .068,336,50;

$\frac{1}{r^{77}}$ is $=$ .070,728,27; $\frac{1}{r^{76}}$ is $=$ .073,203,76;

$\frac{1}{r^{75}}$ is $=$ .075,765,90; $\frac{1}{r^{74}}$ is $=$ .078,417,70;

$\frac{1}{r^{73}}$ is $=$ .081,162,32; $\frac{1}{r^{72}}$ is $=$ .084,003,00;

$\frac{1}{r^{71}}$ is $=$ .086,943,11; $\frac{1}{r^{70}}$ is $=$ .089,986,12;

$\frac{1}{r^{69}}$ is $=$ .093,135,63; $\frac{1}{r^{68}}$ is $=$ .096,395,38;

and $\frac{1}{r^{67}}$ is $=$ .099,769,22. And the values of P and Q when m is $=$ 81, or the numbers of perſons repreſented in Monſieur de Parcieux's table of probabilities as living at the ages of (81 $+$ 3 years, or) 84 years and (81 $+$ 13 years, or) 94 years, are 59 and 1; and the following values of P and Q at the following leſſer ages of 83 years and 93 years, 82 years and 92 years, 81 years and 91 years, 80 years and 90 years, 79 years and 80 years, 78 years and 88 years, 77 years and 87 years, 76 years and 86 years, 75 years and 85 years, 74 years and 84 years, 73 years and 83 years, 72 years and 82 years, 71 years and 81 years, and 70 years and 80 years, are 71 and 2, 85 and 4, 101 and 7, 118 and 11, 136 and 16, 154 and 22, 173 and 29, 192 and 38, 211 and 48, 231 and 59, 251 and 71, 271 and 85, 291 and 101, and, laſtly, 310 and 118.

CCCCIII. Therefore

CCCCIII. Therefore the firſt value of the expreſſion $\frac{\text{£}\ 1}{rm} \times \frac{P \times Q}{A \times K}$, or that which it has when m is $= 81$, is $= \text{£}0.061,635,61 \times \frac{59 \times 1}{860,000}$ $= \frac{59 \times \text{£}\ 0.061,635,61}{860,000} = \frac{\text{£}\ 3.636,400}{860,000} = \text{£}0.000,004,22.$

The ſecond value of the expreſſion $\frac{\text{£}\ 1}{rm} \times \frac{P \times Q}{A \times K}$, or that which it has when m is $= 80$, is $= \text{£}0.063,792,85 \times \frac{71 \times 2}{860,000} =$ $\frac{142 \times \text{£}\ 0.063,792,85}{860,000} = \frac{\text{£}\ 9.058,584}{860,000} = \text{£}0.000,010,53.$

The third value of the expreſſion $\frac{\text{£}\ 1}{rm} \times \frac{P \times Q}{A \times K}$, or that which it has when m is $= 79$, is $= \text{£}0.066,025,60 \times \frac{85 \times 4}{860,000} =$ $\frac{340 \times \text{£}\ 0.066,025,60}{860,000} = \frac{34 \times \text{£}\ 0.066,025,60}{86,000} = \frac{\text{£}\ 2.244,870}{86,000} = \text{£}0.000,026,10.$

The fourth value of the expreſſion $\frac{\text{£}\ 1}{rm} \times \frac{P \times Q}{A \times K}$, or that which it has when m is $= 78$, is $= \text{£}0.068,336,50 \times \frac{101 \times 7}{860,000} =$ $\frac{707 \times \text{£}\ 0.068,336,50}{860,000} = \frac{\text{£}\ 48.313,905}{860,000} = \text{£}0.000,056,17.$

The fifth value of $\frac{\pounds 1}{rm} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.070,728,27 \times \frac{118 \times 11}{860,000} = \frac{1298 \times \pounds 0.070,728,27}{860,000} = \frac{\pounds 91.805,094}{860,000} = \pounds 0.000,106,75.$

The fixth value of $\frac{\pounds 1}{rm} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.073,203,76 \times \frac{136 \times 16}{860,000} = \frac{2176 \times \pounds 0.073,203,76}{860,000} = \frac{\pounds 159.291,381}{860,000} \pounds 0.000,185,22.$

The feventh value of $\frac{\pounds 1}{rm} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.075,765,90 \times \frac{154 \times 22}{860,000} = \frac{3388 \times \pounds 0.075,765,90}{860,000} = \frac{\pounds 256.694,869}{860,000} = \pounds 0.000,298,48.$

The eighth value of $\frac{\pounds 1}{rm} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.078,417,70 \times \frac{173 \times 29}{860,000} = \frac{5017 \times \pounds 0.078,417,70}{860,000} = \frac{\pounds 393.421,600}{860,000} = \pounds 0.000,457,46.$

The ninth value of $\frac{\pounds 1}{rm} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.081,162,32 \times \frac{192 \times 38}{860,000} = \frac{7296 \times \pounds 0.081,162,32}{860,000} = \frac{\pounds 592.160,286}{860,000} = \pounds 0.000,688,55.$

The

The tenth value of $\frac{\text{£}\ 1}{rm} \times \frac{P \times Q}{A \times K}$ is = £0.084,003,00 ×

$$\frac{211 \times 48}{860{,}000} = \frac{10{,}128 \times \text{£}\ 0.084{,}003}{860{,}000} = \frac{\text{£}\ 850\ 782{,}384}{860{,}000} = \text{£}0.000{,}989{,}28.$$

The eleventh value of $\frac{\text{£}\ 1}{rm} \times \frac{P \times Q}{A \times K}$ is = £0.086,943,11 ×

$$\frac{231 \times 59}{860{,}000} = \frac{13629 \times \text{£}\ 0.086{,}943{,}11}{860{,}000} = \frac{\text{£}\ 1184.947{,}646}{860{,}000} = \text{£}0.001{,}377{,}84.$$

The twelfth value of $\frac{\text{£}\ 1}{rm} \times \frac{P \times Q}{A \times K}$ is = £0.089,986,12 ×

$$\frac{251 \times 71}{860{,}000} = \frac{17821 \times \text{£}\ 0.089{,}986{,}12}{860{,}000} = \frac{\text{£}\ 1603.642{,}644}{860{,}000} = \text{£}0.001{,}864{,}70.$$

The thirteenth value of $\frac{\text{£}\ 1}{rm} \times \frac{P \times Q}{A \times K}$ is = £0.093,135,63 ×

$$\frac{271 \times 85}{860{,}000} = \frac{23035 \times \text{£}\ 0.093{,}135{,}63}{860{,}000} = \frac{\text{£}\ 2145.379{,}237}{860{,}000} = \text{£}0.002{,}494{,}62.$$

The fourteenth value of $\frac{\text{£}\ 1}{rm} \times \frac{P \times Q}{A \times K}$ is = £0.096,395,38 ×

$$\frac{291 \times 101}{860{,}000} = \frac{29391 \times \text{£}\ 0.096{,}395{,}38}{860{,}000} = \frac{\text{£}\ 2833.156{,}513}{860{,}000} = \text{£}0.003{,}294{,}36.$$

And

And the fifteenth value of $\frac{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.099,769,22 \times \frac{310 \times 118}{860,000} = \frac{36580 \times \overset{\pounds}{0.099,769,22}}{860,000} = \frac{\overset{\pounds}{3649.558,067}}{860,000} = \pounds 0.004,243,67.$

CCCCIV. Theſe ſeveral ſucceſſive values of the expreſſion $\frac{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$, if ranged in order, will be as follows.

$\pounds$
0.000,004,22,
0.000,010,53,
0.000,026,10,
0.000,056,17,
0.000,106,75,
0.000,185,22,
0.000,298,48,
0.000,457,46,
0.000,688,55,
0.000,989,28,
0.001,377,84,
0.001,864,70,
0.002,494,62,
0.003,294,36,
0.004,243,67.

Theſe numbers therefore will conſtitute the fourth column of the enſuing table.

CCCCV. The

CCCCV. The ſums that ariſe by the continual addition of theſe numbers will be as follows.

The ſums of the ſaid ſucceſſive values of the expreſſion $\frac{\pounds}{\frac{1}{ym} \times \frac{P \times Q}{A \times K}}$.

£
0.000,004,22,
0.000,014,75,
0.000,040,85,
0.000,097,02,
0.000,203,77,
0.000,388,99,
0.000,687,47,
0.001,144,93,
0.001,833,48,
0.002,822,76,
0.004,200,60,
0.006,065,30,
0.008,559,92,
0.011,854,28,
0.016,097,95.

Theſe numbers therefore will conſtitute the fifth column of the enſuing table.

CCCCVI. This

CCCCVI. This table is as follows.

TABLE XXVIII.

Conſiſting of ſix columns of numbers; in the firſt of which the numbers of years in the ſeveral ages of human life, that differ from each other by a year, from the age of 84 *years to the age of* 70 *years, incluſively, are ſet down in regular order; and in the ſecond column are ſet down the numbers of years in the ſeveral ages of human life, that differ from each other by a year, from the age of* 94 *years to the age of* 80 *years, incluſively; and in the third column are ſet down the ſeveral values of an annuity of one pound a year for the joint continuance of the lives of two perſons of the ages ſet down in the firſt and ſecond columns even with the ſaid values; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is* $3\frac{1}{2}$ *per cent. and in the fourth column are ſet down the preſent values of a ſingle payment of one pound, to be received at the ends of* 81 *years,* 80 *years,* 79 *years,* 78 *years, and every following leſſer number of years down to* 67 *years, incluſively, if two perſons of the ages of* 3 *years and* 13 *years ſhall both be living at the ends of the ſaid years; and in the fifth column are ſet down the numbers that ariſe by the continual addition of the numbers ſet down in the fourth column; ſo that each number in the ſaid fifth column is equal to the ſum of all the numbers in the ſaid fourth column that are placed above it, or that correſpond to the preceeding, or older, ages; and in the ſixth and laſt column are ſet down the products that ariſe by multiplying the terms of the third column, (or the values of a life-annuity of one pound a year for the joint continuance of two lives of the ages ſet down in the firſt and ſecond columns,) by the correſponding terms of the fourth column, reſpectively.*

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Present values of a single payment of one pound, to be received at the ends of 81 years, 80 years, 79 years, 78, 77, 76, &c. years, in case two persons of the ages of 3 years and 13 years shall both be living at the ends of those years respectively.*	*Sums of the values in the fourth column.*	*Products of the multiplication of the numbers in the third and fourth columns.*
		£	£	£	£
84	94	0.000,000	0.000,004,22	0.000,004,22	0,000,004,22
83	93	0.401,442	0.000,010,53	0.000,014,75	0.000,014,76
82	92	0.565,515	0.000,026,10	0.000,040,85	0.000,040,85
81	91	0.727,405	0.000.056,17	0.000,097,02	0.000,097,04
80	90	0.909,072	0.000,106,75	0.000,203,77	0.000,203,79
79	89	1.100,266	0.000,185,22	0.000,388,99	0.000,389,01
78	88	1.303,315	0.000,298,48	0.000,687,47	0.000,687,48
77	87	1.502,838	0.000,457,46	0.001,144,93	0.001,144,95
76	86	1.662,844	0.000,688,55	0.001,833,48	0.001,833,51
75	85	1.853,388	0.000,989,28	0.002,822,76	0.002,822,89
74	84	2.048,708	0.001,377,84	0.004,200,60	0.004,200,64
73	83	2.252,720	0.001,864,70	0.006,065,30	0.006,065,33
72	82	2.431,365	0.002,494,62	0.008,559,92	0.008,559,95
71	81	2.598,366	0.003,294,36	0.011,854,28	0.011,854,33
70	80	2.793,416	0.004,243,67		

SCHOLIUM.

CCCCVII. In the foregoing table I have computed only the firſt fourteen values (reckoning from the oldeſt ages to the younger,) of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 10 years. It would not have been difficult (as the reader muſt perceive,) to compleat the table by computing the values of an annuity of one pound a year for the joint continuance of all the younger lives whoſe ages differ from each other by the ſame difference of 10 years, down to the value of a like annuity for the joint continuance of two lives of the ages of 3 years and 13 years. But this was not neceſſary to the deſign with which the foregoing computations were undertaken; which was only to ſhew how ſuch a table might be formed by means of the expreſſion $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e} \times \overline{1+V}}\,\mathcal{L}$, and how the ſeveral numbers thereby obtained might be verified, as faſt as they were computed, by multiplying them into the correſponding values of the expreſſion $\frac{\frac{\mathcal{L}}{1}}{rm} \times \frac{P \times Q}{A \times K}$, or the correſponding numbers that are ſet down in the fourth column of the ſaid table, and comparing the products thence ariſing with the numbers in the fifth column. This, I apprehend, is made ſufficiently manifeſt by the computation of the fourteen values ſet down in the foregoing table: and therefore I have declined the trouble of continuing theſe computations any further.

CCCCVIII. Nevertheleſs, as it will be of great convenience to ſuch perſons as have occaſion to deal in the purchaſe of annuities for joint lives, to have a *compleat* table of the values of an annuity of one pound for the joint continuance of two lives whoſe ages differ from each other by 10 years, (of which the foregoing table contains only the firſt fourteen numbers,) and likewiſe to have other tables of the values of the like annuities for the joint continuance of two lives whoſe ages differ from each other by more, or leſs, than 10 years, I have cauſed the foregoing table to be compleated by another hand under the inſpection of the learned Mr. Morgan, above-mentioned, (the preſent actuary of the Society for Equitable Aſſurances on Lives and Survivorſhips,) and alſo nine other tables of the ſame kind to be computed from Monſieur de Parcieux's table

table of the probabilities of the duration of human life upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. (as it is ſuppoſed to be in the foregoing table,) to wit, a table of the values of an annuity of one pound a year for the lives of two perſons of equal ages, another of the values of the like annuity for the lives of two perſons whoſe ages differ from each other by five years, a third for two lives whoſe ages differ by 20 years, a fourth for two lives whoſe ages differ by 30 years, and a fifth, ſixth, ſeventh, eighth, and ninth, table, for two lives whoſe ages differ by 40 years, 50 years, 60 years, 70 years, and 80 years. Theſe tables have all been computed by means of the expreſſion $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \overline{1+V}\,\mathcal{L}$, in the ſame manner as the fourteen numbers above computed in Table XXVIII; and the computations have alſo been verified in the ſame manner, to wit, by computing the ſucceſſive values of the expreſſion $\frac{\mathcal{L}\,1}{rm} \times \frac{P \times Q}{A \times K}$, and by finding the ſums of thoſe ſucceſſive values, and, laſtly, by multiplying the values of the ſeveral annuities into the correſponding values of the expreſſion $\frac{\mathcal{L}\,1}{rm} \times \frac{P \times Q}{A \times K}$, and obſerving that the products thereby obtained were equal to the correſponding ſums of the ſucceſſive values of $\frac{\mathcal{L}\,1}{rm} \times \frac{P \times Q}{A \times K}$. But I have not thought it neceſſary to cauſe all theſe latter numbers (which ſerve only to prove the truth of the computations) to be printed; and therefore I ſhall preſent the reader with only the values of the annuities themſelves, as was done in the tables of the values of an annuity of one pound a year for a ſingle life given above in Art. ccl, Tables XII, XIII, XIV, XV,——XXIII, pages 221, 222, 223,—232. And, as it is moſt uſual, in exhibiting tables of the values of life-annuities, to begin with thoſe of the younger ages and proceed on to thoſe of older ages, I ſhall obſerve the ſame order in ſetting down the values contained in the following tables, notwithſtanding they were computed one from another by proceeding in a contrary order, or from the older ages to the younger. Theſe tables are as follows.

TABLE XXIX.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons of the ſame age, when the intereſt of money is $3\frac{1}{2}$ *per cent.------Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.*

Years in the age of the firſt life.	*Years in the age of the ſecond life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the firſt life.*	*Years in the age of the ſecond life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the firſt life.*	*Years in the age of the ſecond life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£			£
3	3	15.785,24	34	34	14.172,18	65	65	5.699,541
4	4	16.363,93	35	35	14.008,32	66	66	5.373,930
5	5	16.731,89	36	36	13.838,74	67	67	5.061,732
6	6	16.994,35	37	37	13.663,10	68	68	4.764,788
7	7	17.170,57	38	38	13.437,90	69	69	4.485,940
8	8	17.287,49	39	39	13.203,01	70	70	4.229,522
9	9	17.378,30	40	40	12.957,85	71	71	3.967,857
10	10	17.397,65	41	41	12.701,78	72	72	3.735,261
11	11	17.338,47	42	42	12.434,14	73	73	3.506,636
12	12	17.194,84	43	43	12.154,19	74	74	3.285,035
13	13	17.045,84	44	44	11.861,14	75	75	3.075,111
14	14	16.891,22	45	45	11.554,15	76	76	2.843,825
15	15	16.730,68	46	46	11.232,32	77	77	2.625,379
16	16	16.563,92	47	47	10.933,91	78	78	2.429,124
17	17	16.432,30	48	48	10.620,90	79	79	2.223,694
18	18	16.296,21	49	49	10.330,55	80	80	2.057,238
19	19	16.155,41	50	50	10.025,94	81	81	1.906,337
20	20	16.009,67	51	51	9.743,490	82	82	1.785,767
21	21	15.900,58	52	52	9.484,577	83	83	1.649,026
22	22	15.788,72	53	53	9.213,854	84	84	1.471,612
23	23	15.673,96	54	54	8.930,286	85	85	1.301,203
24	24	15.556,18	55	55	8.669,387	86	86	1.148,822
25	25	15.435,19	56	56	8.396,670	87	87	1.041,569
26	26	15.310,85	57	57	8.111,004	88	88	0.873,177
27	27	15.183,00	58	58	7.847,174	89	89	0.708,630
28	28	15.051,43	59	59	7.571,510	90	90	0.551,722
29	29	14.915,97	60	60	7.282,755	91	91	0.410,100
30	30	14.776,38	61	61	6.979,449	92	92	0.299,890
31	31	14.632,46	62	62	6.659,908	93	93	0.241,546
32	32	14.483,96	63	63	6.356,820	94	94	0.000,000
33	33	14.330,63	64	64	6.037,443			

TABLE XXX.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 5 years; when the interest of money is $3\frac{1}{2}$ per cent ----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	8	16.510,17	32	37	14.035,08	61	66	6.065,430
4	9	16.854,05	33	38	13.843,23	62	67	5.748,630
5	10	17.051,58	34	39	13.643,78	63	68	5.447,890
6	11	17.155,02	35	40	13.436,29	64	69	5.150,625
7	12	17.171,55	36	41	13.220,25	65	70	4.858,151
8	13	17.154,48	37	42	12.995,13	66	71	4.567,925
9	14	17.120,70	38	43	12.739,85	67	72	4.299,865
10	15	17.048,09	39	44	12.473,04	68	73	4.040,373
11	16	16.933,54	40	45	12.193,97	69	74	3.792,444
12	17	16.795,58	41	46	11.901,28	70	75	3.560,612
13	18	16.652,69	42	47	11.616,67	71	76	3.314,346
14	19	16.504,62	43	48	11.317,93	72	77	3.088,057
15	20	16.351,08	44	49	11.025,10	73	78	2.876,561
16	21	16.213,13	45	50	10.718,15	74	79	2.663,173
17	22	16.090,90	46	51	10.416,04	75	80	2.477,972
18	23	15.964,95	47	52	10.137,24	76	81	2.292,901
19	24	15.835,03	48	53	9.845,200	77	82	2.129,559
20	25	15.701,10	49	54	9.556,736	78	83	1.964,259
21	26	15.583,33	50	55	9.273,595	79	84	1.770,303
22	27	15.462,37	51	56	8.994,263	80	85	1.595,707
23	28	15.338,05	52	57	8.718,822	81	86	1.437,314
24	29	15.210,20	53	58	8.449,496	82	87	1.316,220
25	30	15.078,62	54	59	8.167,754	83	88	1.149,833
26	31	14.943,13	55	60	7.889,259	84	89	0.969,173
27	32	14.803,51	56	61	7.597,410	85	90	0.793,411
28	33	14.659,54	57	62	7.290,850	86	91	0.630,011
29	34	14.510,98	58	63	7.002,970	87	92	0.495,243
30	35	14.357,56	59	64	6.700,90	88	93	0.351,339
31	36	14.199,03	60	65	6.382,877	89	94	0.000.000

TABLE XXXI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 10 years; when the interest of money is 3½ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£			£
3	13	16.366,17	31	41	13.500,30	59	69	5.611,286
4	14	16.585.56	32	42	13.282,31	60	70	5.336,698
5	15	16.688,70	33	43	13.055,10	61	71	5.054,106
6	16	16.732,59	34	44	12.818,10	62	72	4.784,148
7	17	16.749,69	35	45	12.570,68	63	73	4.523,082
8	18	16.734,46	36	46	12.312,20	64	74	4.260,822
9	19	16.703,36	37	47	12.063,41	65	75	3.999,072
10	20	16.634,79	38	48	11.784,38	66	76	3.728,183
11	21	16.547,42	39	49	11.513,43	67	77	3.470,693
12	22	16.418,12	40	50	11.229,92	68	78	3.233,054
13	23	16.284,59	41	51	10.953,88	69	79	2.996,397
14	24	16.146,60	42	52	10.685,81	70	80	2.793,419
15	25	16.003,93	43	53	10.405,59	71	81	2.598,368
16	26	15.856,33	44	54	10.112,29	72	82	2.431,367
17	27	15.723,54	45	55	9.825,467	73	83	2.252,722
18	28	15.586,50	46	56	9.525,226	74	84	2.048,709
19	29	15.444,98	47	57	9.227,309	75	85	1.853,389
20	30	15.298,75	48	58	8.935,094	76	86	1.662,845
21	31	15.167,59	49	59	8.645,309	77	87	1.502,839
22	32	15.032,49	50	60	8.341,630	78	88	1.303,316
23	33	14.893,27	51	61	8.038,570	79	89	1.100,267
24	34	14.749,68	52	62	7.735,712	80	90	0.909,073
25	35	14.601,50	53	63	7.437,175	81	91	0.727,405
26	36	14.448,46	54	64	7.123,728	82	92	0.565,515
27	37	14.290,29	55	65	6.808,550	83	93	0.401,442
28	38	14.104,15	56	66	6.496,025	84	94	0.000,000
29	39	13.910,74	57	67	6.186,700			
30	40	13.709,61	58	68	5.894,503			

TABLE XXXII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 20 years; when the interest of money is 3½ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	23	15.560,29	27	47	12.402,10	51	71	5.515,096
4	24	15.772,84	28	48	12.146,36	52	72	5.249,786
5	25	15.876,39	29	49	11.900,87	53	73	4.984,023
6	26	15.925,03	30	50	11.644,53	54	74	4.719,681
7	27	15.929,42	31	51	11.398,29	55	75	4.469,896
8	28	15.902,97	32	52	11.162,98	56	76	4.202,853
9	29	15.861,35	33	53	10.917,97	57	77	3.943,096
10	30	15.784,00	34	54	10.662,54	58	78	3.706,498
11	31	15 667,99	35	55	10.417,61	59	79	3.462,599
12	32	15.510,67	36	56	10.162,63	60	80	3.246,444
13	33	15.347,68	37	57	9.896,823	61	81	3.039,033
14	34	15.178,75	38	58	9.625,224	62	82	2.848,658
15	35	15.003,54	39	59	9.342,070	63	83	2.646,551
16	36	14.821,70	40	60	9.046.436	64	84	2.409,133
17	37	14.651,58	41	61	8.737.294	65	85	2.173,497
18	38	14.452,13	42	62	8.413,488	66	86	1 953,728
19	39	14.244,52	43	63	8.095,180	67	87	1.766,127
20	40	14.028,29	44	64	7.761,737	68	88	1.527,607
21	41	13.821,31	45	65	7.411,735	69	89	1.292,917
22	42	13.605,70	46	66	7.064,716	70	90	1.065,724
23	43	13.381,12	47	67	6.733,989	71	91	0.849,497
24	44	13.146,84	48	68	6.408,777	72	92	0.646,370
25	45	12.902,34	49	69	6.102,706	73	93	0.444,598
26	46	12.646,97	50	70	5.807,267	74	94	0.000,000

TABLE XXXIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 30 years; when the intereſt of money is $3\frac{1}{2}$ per cent.----Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£			£
3	33	14.605,10	24	54	10.805,23	45	75	4.717,841
4	34	14.761,38	25	55	10.556,78	46	76	4.427,255
5	35	14.812,79	26	56	10.298,13	47	77	4.152,480
6	36	14.810,22	27	57	10.028,53	48	78	3.892,549
7	37	14.763,70	28	58	9.769,124	49	79	3.631,597
8	38	14.662,37	29	59	9.499,177	50	80	3.399,171
9	39	14.542,30	30	60	9.217,864	51	81	3.182,288
10	40	14.384,49	31	61	8.924,272	52	82	2.990,528
11	41	14.186,34	32	62	8.617,370	53	83	2.779,762
12	42	13.945,55	33	63	8.317,989	54	84	2.533,005
13	43	13.694,31	34	64	8.005,274	55	85	2.295,974
14	44	13.432,04	35	65	7.678,014	56	86	2.071,761
15	45	13.158,08	36	66	7.356,765	57	87	1.876,901
16	46	12.871,77	37	67	7.041,733	58	88	1.628,765
17	47	12.611,02	38	68	6.726,106	59	89	1.381,241
18	48	12.338,55	39	69	6.419,791	60	90	1.137,781
19	49	12.075,74	40	70	6.126,861	61	91	0.903,979
20	50	11.801,15	41	71	5.828,088	62	92	0.686,240
21	51	11.551,46	42	72	5.547,756	63	93	0.467,103
22	52	11.312,81	43	73	5.267,684	64	94	0.000,000
23	53	11.064,30	44	74	4.990,020			

TABLE XXXIV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 40 years; when the intereſt of money is 3½ per cent.--- Computed from Monſieur de Parcïeux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	43	12.995,63	21	61	8.987,406	39	79	3.742,057
4	44	13.020,79	22	62	8.674,708	40	80	3.511,390
5	45	12.944,45	23	63	8.369,404	41	81	3.291,726
6	46	12.812,26	24	64	8.050,492	42	82	3.092,312
7	47	12.655,71	25	65	7.716,753	43	83	2.873,803
8	48	12.464,90	26	66	7.388,815	44	84	2.619,179
9	49	12.274,57	27	67	7.067,833	45	85	2.369,586
10	50	12.047,58	28	68	6.755,442	46	86	2.133,182
11	51	11.804,02	29	69	6.453,925	47	87	1.931,165
12	52	11.543,45	30	70	6.166,483	48	88	1.669,911
13	53	11.271,87	31	71	5.873,947	49	89	1.412,744
14	54	10.988,55	32	72	5.600,948	50	90	1.159,767
15	55	10.714,92	33	73	5.329,415	51	91	0.919,313
16	56	10.429,73	34	74	5.061,816	52	92	0.697,812
17	57	10.145,47	35	75	4.801,680	53	93	0.473,412
18	58	9.870,862	36	76	4.525,229	54	94	0.000,000
19	59	9.584,837	37	77	4.259,330			
20	60	9.286,549	38	78	4.003,965			

TABLE XXXV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 50 years; when the intereſt of money is $3\frac{1}{2}$ per cent.--- Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	53	10.680,23	24	74	5.075,965
4	54	10.628,92	25	75	4.811,044
5	55	10.513,03	26	76	4.529,340
6	56	10.350,53	27	77	4.257,630
7	57	10.148,69	28	78	4.003,127
8	58	9.938,555	29	79	3.742,189
9	59	9.709,814	30	80	3.512,640
10	60	9.449,236	31	81	3.294,317
11	61	9.154,806	32	82	3.096,568
12	62	8.824,696	33	83	2.880,142
13	63	8.501,682	34	84	2.628,121
14	64	8.164,376	35	85	2.382,003
15	65	7.811,534	36	86	2.150,472
16	66	7.463,965	37	87	1.950,898
17	67	7.132,382	38	88	1.689,411
18	68	6.809,136	39	89	1.429,589
19	69	6.496,404	40	90	1.175,112
20	70	6.197,247	41	91	0.931,818
21	71	5.900,765	42	92	0 706,129
22	72	5.623,758	43	93	0.477,775
23	73	5.348,020	44	94	0.000.000

TABLE XXXVI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 60 years; when the intereſt of money is $3\frac{1}{2}$ per cent.--- Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	63	8.049,257	20	80	3.526,172
4	64	7.882,630	21	81	3.306,196
5	65	7.643,727	22	82	3.106,801
6	66	7.382,707	23	83	2.888,572
7	67	7.107,740	24	84	2.634,545
8	68	6.828,138	25	85	2.386,279
9	69	6.554,273	26	86	2.152,327
10	70	6.281,256	27	87	1.949,808
11	71	5.989,106	28	88	1.688,533
12	72	5.702,314	29	89	1.428,905
13	73	5.416,621	30	90	1.174,597
14	74	5.134,386	31	91	0.931,450
15	75	4.858,956	32	92	0.705,886
16	76	4.566,065	33	93	0.477,648
17	77	4.288,874	34	94	0.000,000
18	78	4.028,814			
19	79	3.761,967			

TABLE XXXVII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 70 years; when the intereſt of money is $3\frac{1}{2}$ per cent.--- Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	73	5.137,907	14	84	2.656,044
4	74	4.956,842	15	85	2.402,893
5	75	4.746,962	16	86	2.163,852
6	76	4.503,802	17	87	1.959,233
7	77	4.258,192	18	88	1.695,616
8	78	4.022,334	19	89	1.433,647
9	79	3.777,677	20	90	1.176,850
10	80	3.557,527	21	91	0.933,060
11	81	3.341,255	22	92	0.706,947
12	82	3.137,624	23	93	0.478,200
13	83	2.914,904	24	94	0.000,000

TABLE XXXVIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 80 years; when the intereſt of money is $3\frac{1}{2}$ per cent.--- Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	83	2.783,857	9	89	1.432,098
4	84	2.574,553	10	90	1.180,457
5	85	2.351,322	11	91	0.937,542
6	86	2.133,541	12	92	0.709,889
7	87	1.940,957	13	93	0.479,721
8	88	1.686,249	14	94	0.000,000

CCCCIX. According to Monſieur de Parcieux's table of the probabilities of the duration of human life, (which begins with the age of 3 years, and ends with the age of 94 years, and ſuppoſes the utmoſt poſſible extent of human life to be ſomewhat leſs than 95 years,) it is evident that only one pair of lives can be found whoſe ages will differ from each other by 90 years, to wit, two lives of the ages of 3 and 93 years. And the value of an annuity of one pound a year for the joint continuance of two lives of theſe ages, upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. is £0.468,599.

Remarks on the great number of tables (of the ſame kind as thoſe above-computed,) that would be neceſſary in order to exhibit the values of annuities for two joint lives in all their poſſible varieties.

CCCCX. It would evidently be a work of great uſe and convenience to thoſe perſons who have occaſion to ſell or purchaſe annuities for two joint lives, to compute as many more tables, of the ſame kind as the ten foregoing ones, as there may be taken different numbers of years for the difference of the ages of the two lives upon which the annuity is to depend; ſo that, for example, if the difference of the ages of any two lives was 23 years, or 27 years, inſtead of 20 years, or 30 years, the value of an annuity of one pound a year during the joint continuance of both the lives ſhould be accurately exhibited in ſome one or other of the tables, in the ſame manner as the values of the like annuities are exhibited in the foregoing tables when the difference of the ages is either 0, or 5 years, or 10 years, or 20, 30, 40, 50, 60, 70, or 80, years. But this would require an immenſe quantity of calculation. For, in order to have the exact values of an annuity of one pound a year for two joint lives in all the poſſible varieties in which the ages of the two lives may be combined together, when the intereſt of money is $3\frac{1}{2}$ per cent. (as it is ſuppoſed to be in the ten foregoing tables,) it would be neceſſary to compute no fewer than 93 different tables, or 83 more than the ten that are above computed. For, beſides the ſaid ten tables, which exhibit thoſe values when the ages of the two lives are equal, and when they differ from each other by 5 years, or 10 years, or 20 years, or 30 years, or 40 years, or 50 years, or 60 years, or 70 years, or 80 years, it would be neceſſary to compute an eleventh table of the ſame kind that ſhould exhibit the values of a like annuity for the joint continuance of two lives whoſe ages differed from each other by one year; and a twelfth table of the ſame kind when the difference of the ages is two years; and a thirteenth table of the ſame kind when the difference of the ages is three years; and, in general, a new table of the ſame

ſame kind for every new difference that can be taken between the ſaid two ages: which differences (if we reckon 0 for one of them, or include the caſe of two lives of equal ages,) will, all together, (or including the ten differences in the tables above-computed,) amount to the number of 93, if the youngeſt life ſet down in the table of probabilities is of the age of one year; or, if we make uſe of Monſieur de Parcieux's table of probabilities, (in which the youngeſt age is that of 3 years,) will amount to the number of 91. And, if we were deſirous of having the values of the like annuities for two joint lives in all their poſſible varieties when the intereſt of money is either 2 per cent. or $2\frac{1}{2}$ per cent. or 3 per cent. or 4 per cent. or $4\frac{1}{2}$ per cent. or 5 per cent. or 6 per cent. or 7 per cent. or 8 per cent. or 9 per cent. or 10 per cent. as well as when it is $3\frac{1}{2}$ per cent. (as we have the values of the like annuities for ſingle lives of all ages for all thoſe different rates of intereſt in the tables exhibited above in pages 221, 222, — — — 232,) it would be neceſſary to compute the ſame number of tables of this kind, to wit, 93, or 91, tables, for each of theſe rates of intereſt; which would make, in all, 12 times 93, or 12 times 91, ſuch tables, or more than eleven hundred ſuch tables. Now the computation of ſuch a prodigious number of tables would be a buſineſs of ſo much length and labour that it, probably, will never be undertaken; though, perhaps, it might be worth the while of the Government, or of ſome of the ſocieties for making inſurances upon lives, or of the dean and chapter of ſome rich cathedral church, whoſe lands are leaſed out upon lives, or of ſome other wealthy body of men, to whom ſuch tables might be peculiarly uſeful, to cauſe *two, or three*, ſets of theſe tables to be computed for *two, or three*, of the moſt common and uſeful rates of intereſt, as, for example, for $3\frac{1}{2}$ per cent. 4 per cent. and $4\frac{1}{2}$ per cent. If this were to be undertaken and carefully performed, under ſuch encouragement, it would, I doubt not, be allowed on all hands to be a work of great merit and of general advantage to the publick.

A method of finding by Interpolation *the values of ſuch annuities for two joint lives, as are not contained in any of the foregoing tables.*

CCCCXI. But in the mean while, and until ſuch tables ſhall be publiſhed, it will be deſireable to find out, if poſſible, ſome tolerably eaſy method of deriving the values of ſuch annuities for two joint lives as are *not* ſet down in the foregoing ten tables, from the values of thoſe which *are* therein exhibited. Now this may be done to a moderate degree of exactneſs, ſufficient for common purpoſes, by a kind of *Interpolation*, which may be explained in the following manner.

CCCCXII. The

The principles of the ſaid method of Interpolation.

CCCCXII. The principles upon which this method of interpolation is founded are as follows.

In the firſt place it is evident beyond a doubt, that the value of an annuity for the joint continuance of any two given lives is greater than the value of the like annuity for two other lives whereof the younger is of the ſame age with the younger of the two former lives, and the older is older than the older of the two former lives: or, in other words, the value of an annuity for two joint lives of N years and $N+n$ years is greater than the value of the ſame annuity for two joint lives of the ages of N years and $N+n+e$ years. Thus, for example, the value of an annuity of one pound a year for two joint lives of the ages of 23 years and 30 years is greater than the value of the like annuity for two joint lives of the ages of 23 years and 40 years.

In the ſecond place it ſeems highly probable, that, if we take a moderate number of lives that are ſucceſſively older the one than the other by one year, and combine them, one after another, with another life that is younger than any of them, the values of an annuity for theſe ſucceſſive pairs of joint lives, (in all of which the youngeſt life is of the ſame age,) will be nearly in arithmetical proportion, as well as the ages of the older lives in theſe ſucceſſive pairs of lives, which are ſuppoſed to increaſe by the equal difference of one year. Thus, if the younger life in each of theſe pairs of lives is of the age of N years, and the ſeveral older lives, with which this life is to be ſucceſſively combined, are of the ages of $N+n$ years, $N+n+1$ years, $N+n+2$ years, $N+n+3$ years, $N+n+4$ years, $N+n+5$ years, &c. it is probable that the values of an annuity

for two joint lives of the ages of N years and $N+n$ years,
and for two joint lives of the ages of N years and $N+n+1$ years,
and for two joint lives of the ages of N years and $N+n+2$ years,
and for two joint lives of the ages of N years and $N+n+3$ years,
and for two joint lives of the ages of N years and $N+n+4$ years,
and for two joint lives of the ages of N years and $N+n+5$ years, &c.

will form, pretty nearly, an arithmetical progreſſion, as well as the older ages, $N+n$ years, $N+n+1$ years, $N+n+2$ years, $N+n+3$ years, $N+n+4$ years, $N+n+5$ years, &c. themſelves: only the ſeries of thoſe values will be a decreaſing progreſſion, whereas the ſeries of the older ages is an increaſing one. This, I ſay, ſeems probable, (though it is not abſolutely evident,) and will be found to be ſufficiently near the truth, when the number of the terms in theſe progreſſions is not greater than 11, to be the foundation of a very uſeful method of approximating to the values of theſe joint annuities.

CCCCXIII. Theſe

CCCCXIII. Theſe things being premiſed, let it be propoſed to find the value of an annuity of one pound a year for the joint continuance of two lives of the ages of N years and $N+a$ years, in which a, the difference of the two ages, is not either 5 years, or 10 years, or 20 years, or 30 years, or 40 years, or 50 years, or 60 years, or 70 years, or 80 years, (which are the differences of the two ages in the foregoing tables,) but ſome intermediate number of years between ſome two of theſe differences, that are contiguous to each other. The ſaid method itſelf.

Let the greateſt difference (amongſt the differences ſet down in the foregoing tables, which are either 5 years, or 10 years, or ſome multiple of 10 years,) than which the given difference a is greater, be $10\times m$, or $10m$. Then, it is evident, the next greater difference, or the leaſt difference (amongſt the differences ſet down in the foregoing tables,) that is greater than the difference a, will be $10\times m+1$, or $10m+10$. Therefore (by the firſt principle above-mentioned,) the value of the propoſed annuity of one pound a year for the joint continuance of two lives of the ages of N years and $N+a$ years will be of an intermediate magnitude between the value of a like annuity for two joint lives of the ages of N years and $N+10m$ years, and the value of a like annuity for two joint lives of the ages of N years and $N+10m+10$ years; both which values may be found in ſome of the foregoing ten tables. And thus we may obtain, by means of the firſt principle above-mentioned, two limits of the value of the propoſed annuity, between which we may be ſure it is of an intermediate magnitude.

And, by the ſecond principle above-mentioned, we may make a nearer approximation to its true value by reaſoning as follows.

The eleven following values of an annuity of one pound a year for two joint lives, (of which eleven values we can find the firſt and the laſt in the ten foregoing tables,) will form, pretty nearly, a decreaſing arithmerical progreſſion; to wit, the values of the ſaid annuity for two joint lives of the ages of

N years and $N+10m$ years,
N years and $N+10m+1$ years,
N years and $N+10m+2$ years,
N years and $N+10m+3$ years,
N years and $N+10m+4$ years,
N years and $N+10m+5$ years,

N years and $N + 10m + 6$ years,
N years and $N + 10m + 7$ years,
N years and $N + 10m + 8$ years,
N years and $N + 10m + 9$ years,
and N years and $N + 10m + 10$ years.

Subtract therefore the laſt of theſe values from the firſt, and divide the difference by 10; and the quotient thence ariſing will be the quantity which muſt be continually added to the laſt value, or ſubtracted from the firſt value, in order to form an arithmetical progreſſion of terms between the firſt value and the laſt. Let theſe continual additions, or ſubtractions, be made. And, amongſt the terms thereby obtained, that which correſponds to the ages of N years and $N + a$ years (which latter age muſt be equal to one of the intermediate ages between $N + 10m$ years and $N + 10m + 10$ years,) will be a near value of the propoſed annuity of one pound a year for two joint lives of the ages of N years and $N + a$ years. Q E I.

Examples of the foregoing method of diſcovering the values of the above-mentioned intermediate, or omitted, annuities for two joint lives by Interpolation.

Firſt example. CCCCXIV. As an example of this method of interpolation, let the two lives for whoſe joint continuance an annuity of one pound a year is to be granted, be of the ages of 70 years and 77 years.

Then it is evident, in the firſt place, that the value of this annuity is not contained in any of the foregoing ten tables; becauſe 7 years is not the difference of the two ages in any of them.

But, in the ſecond place, we may obſerve that the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 77 years muſt be leſs than the value of a like annuity for the joint continuance of two lives of the ages of 70 years and 75 years, but greater than the value of the like annuity for the joint continuance of two lives of the ages of 70 years and 80 years.

In the third place we may obſerve that the values of the two latter annuities, (between which the value of the propoſed annuity lies,) to wit, the values of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 75 years and of a like annuity for the joint continuance of two lives of the ages of 70 years and 80 years, are both contained in the foregoing tables; the former of theſe values being contained in Table XXX, in which the difference of the ages is 5 years, and the latter of them being contained in Table XXXI, in which the

the difference of the ages is 10 years. The former of theſe values appears in Table XXX to be = £3.560,612; and the latter of them appears in Table XXXI to be = £2.793,419. Therefore the value of the propoſed annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 77 years is of an intermediate magnitude between £3.560,612 and £2.793,419.

In the fourth place we may reaſonably ſuppoſe that the values of an annuity of one pound a year for the joint continuance of the ſix following pairs of lives, to wit,

two lives of the ages of 70 years and 75 years,
two lives of the ages of 70 years and 76 years,
two lives of the ages of 70 years and 77 years,
two lives of the ages of 70 years and 78 years,
two lives of the ages of 70 years and 79 years,
and two lives of the ages of 70 years and 80 years,

will form, pretty nearly, an arithmetical progreſſion, or will decreaſe by nearly equal differences. Therefore, if we ſubtract the laſt of them, which is = £2.793,419, from the firſt, or £3.560,612, and divide the remainder, to wit, £0.767,193, by 5, the quotient, £0.153,438, will be the common difference by which theſe terms will decreaſe; and conſequently, if this quotient be either continually added to the laſt term, £2.793,419, or continually ſubtracted from the firſt term, £3.560,612, we ſhall thereby obtain the values of the intermediate terms of the progreſſion. If we proceed by addition, theſe intermediate terms will be as follows, to wit,

£		£		£
2.793,419	+	0.153,438,	or	2.946,857,
2.946,857	+	0.153,438,	or	3.100,295,
3.100,295	+	0.153,438,	or	3.253,732,
and 3.253,732	+	0.153,438,	or	3.407,171;

and, if we proceed by ſubtraction, they will be as follows, to wit,

£		£		£
3.560,612	—	0.153,438,	or	3.407,174,
3.407,174	—	0.153,438,	or	3.253,736,
3.253,736	—	0.153,438,	or	3.100,298,
and 3.100,298	—	0.154,438,	or	2.946,860.

Therefore the values of an annuity of one pound a year for the joint continuance of two lives of the ages of

70 years and 76 years,
70 years and 77 years,
70 years and 78 years,
and 70 years and 79 years,

Will be nearly equal to

£
3.407,174,
3.253,736,
3.100,298,
and 2.946,860;

of which values the ſecond, to wit, £3.253,736, is that we were in ſearch of. Q E I.

Note. This value, £3.253,736, (which is the third term of the arithmetical progreſſion, conſiſting of ſix terms, whoſe firſt and laſt terms are £3.560,612 and £2,793,419) might have been found ſeparately, or without finding the other intermediate terms of the ſaid progreſſion, by dividing the difference of the extreme terms, to wit, £0.767,193, by 5, ſo as to find the common difference of the terms, or £0.153,438, and then ſubtracting twice the ſaid difference, or £0.306,876, from the firſt, or greateſt, term, £3.560,612. For £3.560,612 — £0.306,876, is = £3.253,736.

Second example.

CCCCXV. As another example of this method of Interpolation, let us ſuppoſe the ages of the two lives, for the joint continuance of which an annuity of one pound a year is to be granted, to be 59 years and 70 years.

Here we muſt obſerve in the firſt place, that the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years muſt be ſomewhat leſs than the value of a like annuity for two joint lives of the ages of 59 and 69 years; which appears by Table XXXI to be = £5.611,286.

In

In the ſecond place we muſt obſerve that the value of the ſaid annuity of one pound a year for the joint continuance of two lives of the ages of 59 years and 70 years will be greater than the value of a like annuity for the joint continuance of two lives of the ages of 59 years and 79 years; which appears by Table XXXII to be = £3.462,599. Therefore the value of the ſaid annuity of one pound a year for two joint lives of the ages of 59 years and 70 years is greater than £3.462,599, but leſs than £5.611,286. And it will evidently be much nearer to the greater of theſe values, or £5.611,286, than to the leſſer value, £3.462,599.

In the third place, in order to make a nearer approach to the value of this annuity, we muſt ſuppoſe that the values of an annuity of one pound a year for the eleven following pairs of lives, to wit,

for two lives of the ages of 59 years and 69 years,
two lives of the ages of 59 years and 70 years,
two lives of the ages of 59 years and 71 years,
two lives of the ages of 59 years and 72 years,
two lives of the ages of 59 years and 73 years,
two lives of the ages of 59 years and 74 years,
two lives of the ages of 59 years and 75 years,
two lives of the ages of 59 years and 76 years,
two lives of the ages of 59 years and 77 years,
two lives of the ages of 59 years and 78 years,
and two lives of the ages of 59 years and 79 years,

will form, pretty nearly, an arithmetical progreſſion, or decreaſe by nearly equal differences. And, if they do ſo decreaſe, the difference between the firſt and ſecond of thoſe values will be nearly the tenth part of the difference between the firſt and the laſt values. Now the firſt of theſe values has been ſhewn to be = £5.611,286, and the laſt of them has been ſhewn to be = £3.462,599: and the difference of theſe values is £2.148,687; of which the tenth part is £0.214,868. Therefore the ſecond of the foregoing eleven values will be = £5.611,286 — £0.214,868, or £5.396,418. Therefore the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years, will be, nearly, equal to £5.396,418. Q E I.

CCCCXVI. There is another way of obtaining a near value of this annuity by means of the foregoing tables, beſides this of Interpolation; but which differs very little from it, and is founded on exactly the ſame principles. It is as follows.

It

It appears by Table XXX, (in which the difference of the ages of the two lives is 5 years,) that the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 59 years and 64 years is £6.700,900. And we have seen that the value of a like annuity for the joint continuance of two lives of the ages of 59 and 69 years is £5.611,286; which is less than £6.700,900 by the difference £1.089,614.

Now we may reasonably suppose that the values of an annuity of one pound a year for the joint continuance of the seven following pairs of lives, to wit,

two lives of the ages of 59 years and 64 years,
two lives of the ages of 59 years and 65 years,
two lives of the ages of 59 years and 66 years,
two lives of the ages of 59 years and 67 years,
two lives of the ages of 59 years and 68 years,
two lives of the ages of 59 years and 69 years,
and two lives of the ages of 59 years and 70 years,

will form, pretty nearly, an arithmetical progression, or will decrease by equal differences. And, if they do so decrease, the difference between the first of them (which is = £6.700,900,) and the last, (which is the value sought,) will be equal to six times their common difference; and consequently, if we can find the common difference of these values, the last of them may be derived from the first by subtracting from it six times the said common difference. And it may also be derived from the sixth value (which we have seen to be = £5.611,286,) by subtracting from it the said common difference itself. We must therefore inquire what is the said common difference.

Now, since the first of these values (which are supposed to form an arithmetical progression,) is = £6.700,900, and the sixth of them is = £5.611,286, it follows that the common difference of these values must be a fifth part of the difference of £6.700,900 and £5.611,286, that is, a fifth part of £1.089,614, and therefore will be = £0.217,922. Therefore six times the said common difference will be = (six times £0.217,922, or) £1.307,532; which, being subtracted from £6.700,900, or the first of the foregoing seven values, leaves £5.393,368 for the last of the said values, or the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years. Q E I.

Or, if we subtract the common difference, £0.217,922, itself, from £5.611,286, or the sixth of the foregoing values, the remaining quantity, £5.393,364,

£5.393,364, will be the laſt of the ſaid ſeven values, or the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years. QE I.

Note. Theſe values £5.393,368 and £5.393,364 are ſo nearly equal to the value found for this annuity in the laſt article, to wit, £5.396,418, that the difference is not worth attending to. But, if it were, we ought to conſider the value laſt obtained, to wit, £5.393,368, as being nearer to the exact value of the propoſed annuity than £5.396,418, becauſe it is obtained by means of an arithmetical progreſſion conſiſting of fewer terms than the other, in Art. ccccxv, by means of which the former value, £5.396,418, had been found.

CCCCXVII. As a third example of this method of Interpolation, let the ages of the two lives, upon the joint continuance of which an annuity of one pound a year is to depend, be 26 years and 53 years; the difference of which is 27 years. Third example.

Here we muſt obſerve, in the firſt place, that the value of this annuity is leſs than the value of a like annuity for two joint lives of the ages of 26 years and 46 years, and greater than the value of a like annuity for two joint lives of the ages of 26 years and 56 years; whence it follows, by Tables XXXII and XXXIII, that it is leſs than £12.646,97, and greater than £10.298,13.

In the ſecond place we may reaſonably ſuppoſe that the values of an annuity of one pound a year for the joint continuance of the following eleven pairs of lives, to wit,

two lives of the ages of 26 years and 46 years,
two lives of the ages of 26 years and 47 years,
two lives of the ages of 26 years and 48 years,
two lives of the ages of 26 years and 49 years,
two lives of the ages of 26 years and 50 years,
two lives of the ages of 26 years and 51 years,
two lives of the ages of 26 years and 52 years,
two lives of the ages of 26 years and 53 years,
two lives of the ages of 26 years and 54 years,
two lives of the ages of 26 years and 55 years,
and two lives of the ages of 26 years and 56 years,

will

will form, pretty nearly, an arithmetical progreſſion, or will decreaſe by nearly equal differences. And, if they do ſo decreaſe, their common difference will be nearly a tenth part of the difference of the extreme terms. Now the extreme terms are £12.646,97 and £10.298,13, the difference of which is £2.348,84; and the tenth part of this difference is £0.234,884. Therefore £0.234,884, is the common difference of the terms; and conſequently £10.298,13 + 3 × £0.234,884, (or £10.298,13 + £0.704,652,) or £11.002,78 will be the value of the laſt term but three, or of the eighth term, or will be the value of the propoſed annuity of one pound a year for two joint lives of the ages of 26 years and 53 years. Q E I.

CCCCXVIII. Theſe three examples will, I preſume, be ſufficient to illuſtrate this method of deriving the values of ſuch annuities for two joint lives as *are not* contained in the foregoing ten tables, from the values of the annuities that *are* contained in thoſe tables, by Interpolation. And therefore I ſhall not add any more examples of this method with *that* deſign.

Other examples of the foregoing method of Interpolation, *which ſerve to ſhew to what degree of exactneſs the near values of theſe annuities for two joint lives, that are obtained by means of it, may be ſuppoſed to co-incide with their true values.*

CCCCXIX. But it will be neceſſary to give a few more inſtances of this method of Interpolation with *another* view, namely, in order to ſhew that the values of annuities for two joint lives which are obtained by means of it, are pretty nearly equal to their true values, and may conſequently be uſed, on all common occaſions, inſtead of the ſaid true values, without any ſenſible inconvenience. Now this will beſt appear by computing, by means of this method of Interpolation, a few of the near values of an annuity of one pound a year for the joint continuance of two lives whoſe ages differ from each other by 5 years, and then comparing the ſaid near values of theſe annuities, thereby obtained, with the exact values of the ſame annuities exhibited above in Table XXX, page 493.

First examp'e. CCCCXX. Let it therefore be required, in the firſt place, to find, by this method of Interpolation, a near value of an annuity of one pound a year for the joint continuance of two lives of the ages of 10 years and 15 years; the exact value of which annuity appears in Table XXX to be £17.048,9.

Now

Now it appears by Table XXIX that the value of an annuity of one pound a year for two joint lives that are both of the same age of 10 years, is £17.397,65. And it appears from Table XXXI that the value of a like annuity of one pound a year for two joint lives of the ages of 10 years and 20 years is £16.634,79. Therefore, according to this method of Interpolation, the value of an annuity of one pound a year for two joint lives of the ages of 10 years and 15 years will be nearly equal to an arithmetical mean proportional between £17.397,65 and £16.634,79, and consequently will be nearly equal to $\left(\frac{\text{£}17.397,65 + \text{£}16.634,79}{2}\right.$, or $\left.\frac{\text{£}34.032,44}{2}\right)$, or £17.016,22. QEI.

The difference between this near value, £17.016,22, of the propoſed annuity, and £17.048,09, its exact value, is £0.031,87, which is leſs than one 534th part of the ſaid true value.

CCCCXXI. Let the two lives be of the ages of 20 years and 25 years. Second example.

Then will the exact value of an annuity of one pound a year for theſe two joint lives, according to Table XXX, be £15.701,10.

The near value of the ſame annuity will, according to the foregoing method of Interpolation, be an arithmetical mean proportional between £16.009,67, which appears by Table XXIX to be the value of a like annuity for two joint lives that are both of the age of 20 years, and £15.298,75, which appears by Table XXXI to be the value of a like annuity for two joint lives of the ages of 20 years and 30 years; and consequently the ſaid near value will be $\left(= \frac{\text{£}16.009,67 + \text{£}15.298,75}{2}\right.$, $\left. = \frac{\text{£}31.308,42}{2}\right) = $ £15.654,21. QEI.

The difference between this near value, £15.654,21, of the propoſed annuity, and its more exact value, £15.701,10, is £0.046,89, which is leſs than the 334th part of the ſaid exact value.

Third example. CCCCXXII. Let the two lives be of the ages of 30 years and 35 years.

Then will the exact value of an annuity of one pound a year for these two joint lives, according to Table XXX, be £14.357,56.

And the near value of the same annuity will, according to the foregoing method of Interpolation, be an arithmetical mean proportional between the value of a like annuity for two joint lives that are both of the same age of 30 years, which appears by Table XXIX to be £14.776,38, and the value of a like annuity for two joint lives of the ages of 30 years and 40 years, which appears by Table XXXI to be £13.709,61; and consequently it will be $= \left(\frac{£\ 14.776,38 + £\ 13.709,61}{2} = \frac{£\ 28\ 485,99}{2} =\right)$ £14.242,99. QEI.

The difference between this near value, £14.242,99, of the proposed annuity, and its more exact value, £14.357,56, is £0.114,57; which is less than the 125th part of the said exact value.

Fourth example. CCCCXXIII. Let the two lives be of the ages of 40 years and 45 years.

Then will the exact value of an annuity of one pound a year for these two joint lives, according to Table XXX, be £12.193,97.

The value of an annuity of one pound a year for two joint lives that are both of the age of 40 years, appears by Table XXIX to be £12.957,85; and the value of a like annuity for two joint lives of the ages of 40 years and 50 years appears by Table XXXI to be £11.229,92. Therefore the value of the like annuity for two joint lives of the ages of 40 years and 45 years is nearly equal to an arithmetical mean between £12.957,85 and £11.229,92, and consequently is nearly equal to $\left(\frac{£\ 12.957,85 + £\ 11.229,92}{2},\right.$ or $\frac{£\ 24.187,77}{2}$, or$\left.\right)$ £12.093,88. QEI.

The

The difference between this near value, £12.093,88, of the propoſed annuity, and its more exact value, £12.193,97, is £0.100,09, which is leſs than the 121ſt part of the ſaid exact value.

CCCCXXIV. In like manner, if the two lives are of the ages of 50 years and 55 years, the exact value of an annuity of one pound during their joint continuance, given in Table XXX, is £9.273,595; and the near value of the ſame annuity, obtained by the foregoing method of Interpolation, will be an arithmetical mean between £10.025,94 and £8.341,632, and conſequently will be equal to $\frac{\text{£}18.367,572}{2}$, or £9.183,786. QEI. Fifth example.

The difference between this near value, £9.183,786, of the propoſed annuity, and its more exact value, £9.273,595, is £0.089,809; which is leſs than the 103d part of the ſaid exact value.

CCCCXXV. If the two lives are of the ages of 60 years and 65 years, the exact value of the annuity will be £6.382,877; and the near value of it will be $= \left(\frac{\text{£}7.282,755 + \text{£}5.336,698}{2} = \frac{\text{£}12.619,453}{2} =\right)$ £6.309,726; which differs from the exact value, £6.382,877, by £0.073,151, which is leſs than the 87th part of the ſaid exact value. Sixth example.

CCCCXXVI. If the two lives are of the ages of 70 years and 75 years, the exact value of the annuity will be £3.560,612; and the near value of it will be $= \left(\frac{\text{£}4.229,522 + \text{£}2.793,419}{2} = \frac{\text{£}7.022,941}{2} =\right)$ £3.511,470; which differs from the exact value, £3.560,612, by £0.049,142; which is leſs than the 72d part of the ſaid exact value. Seventh example.

CCCCXXVII. And, if the two lives are of the ages of 80 years and 85 years, the exact value of the annuity will be £1.595,707; and the near value of it will be $= \left(\frac{\text{£}2.057,238 + \text{£}0.909,073}{2} = \frac{\text{£}2.966,311}{2} =\right)$ Eighth example.

£1.483,155; which differs from the exact value, £1.595,707, by £0.112,552, which is less than the 14th part of the said exact value.

Conclusions from the foregoing examples.

First conclusion. CCCCXXVIII. From these examples we may conclude that, when the difference of the ages of two lives is 5 years, and the age of the older of them is not greater than 75 years, the near value of an annuity of one pound a year for the joint continuance of both lives, that is derived, by the foregoing method of Interpolation, from the values, (given in Tables XXIX and XXXI,) of a like annuity for two joint lives both of the same age with the younger of the two proposed lives, and of a like annuity for two joint lives whose ages differ by 10 years, and of which the younger is of the same age with the younger of the two proposed lives; —— I say, we may conclude that the said near value of the said annuity, so obtained, will differ from the true value of the same annuity by less than the 72d part of the said true value; and that, while the age of the older life is not greater than 65 years, the difference of the said near and true values will be less than an 103d part of the said true value. For, I presume, it cannot be doubted that, what we have found to be true concerning the differences of these values in the foregoing examples of two lives of the ages, of 10 years and 15 years, of 20 years and 25 years, of 30 years and 35 years, of 40 years and 45 years, of 50 years and 55 years, of 60 years and 65 years, of 70 years and 75 years, and of 80 years and 85 years, will be true with respect to all other lives of ages that lie between the ages herein specified, and which differ from each other by the same difference of 5 years; as, for example, of two lives of the ages of 13 years and 18 years, 23 years and 28 years, 33 years and 38 years, 43 years and 48 years, &c. But, if any person should doubt of this conclusion, he may easily satisfy himself of the truth of it by applying this method of interpolation to the discovery of the near values of as many of these latter annuities as he shall think fit, by means of Tables XXIX and XXXI, in the manner above exemplified in the preceeding articles, and then comparing the near values, thereby obtained, with the true values of the same annuities exhibited in Table XXX.

Second conclusion. CCCCXXIX. And, secondly, we may conclude with a good degree of probability, that, since the near values of the annuities contained in Table XXX, for the joint continuance of two lives whose ages differ from each other by 5 years, which are obtained by this method of Interpolation, differ from the true values of the same annuities by such small quantities as a 72d, or a 103d, part, of the said true values, the near values of other annuities for two joint lives, where the difference of the ages is greater or less

leſs than 5 years, obtained by the ſame method of Interpolation, will differ from their true values by almoſt as ſmall quantities, or by quantities not very different from the 72d part, or the 103d part, of the ſaid true values. For in both caſes the near values of the annuities ſought are obtained by ſuppoſing that the principle laid down in Art. CCCCXII is nearly true, or that the values of eleven annuities of one pound a year for the joint continuance of eleven pairs of lives of the following ages, to wit,

N years and N years,
N years and $N + 1$ years,
N years and $N + 2$ years,
N years and $N + 3$ years,
N years and $N + 4$ years,
N years and $N + 5$ years,
N years and $N + 6$ years,
N years and $N + 7$ years,
N years and $N + 8$ years,
N years and $N + 9$ years,
and N years and $N + 10$ years,

form, pretty nearly, an arithmetical progreſſion, or decreaſe by nearly equal differences. If therefore we find upon trial (as we have done in the foregoing examples,) that the near value of an annuity for two joint lives of the ages of N and $N + 5$ years obtained in this manner, differs but by a ſmall quantity from its true value, we may conclude with a high degree of probability, that the near values of the other intermediate annuities, (as, for example, of annuities for two joint lives of the ages of N years and $N + 3$ years or of the ages of N years and $N + 7$ years,) that are obtained in the ſame manner, will differ from their true values either by as ſmall quantities, or by quantities that will be very little greater than the ſaid difference between the near value and the true value of the ſaid annuity for two joint lives of the ages of N years and $N + 5$ years, which is the middlemoſt annuity of the whole eleven.

CCCCXXX. And, if this concluſion be juſt, (which, I think, it is hardly poſſible to doubt of,) the near values of annuities for two joint lives, obtained by this method of Interpolation, may be conſidered as differing from their true values by only about the 72d part of the ſaid true values, when the age of the older life is not greater than 75 years, and by only about the 103d part of the ſaid true values when the age of the older life is not greater than 65 years, and by a much ſmaller part of the ſaid true values when the age of the older life is only 30 or 20 years. Third concluſion.

CCCCXXXI. Now

The values of annuities for two joint lives, obtained by the foregoing method of Interpolation, are ſufficiently exact for ordinary purpoſes.

CCCCXXXI. Now either the 103d part of the true value of an annuity, or the 72d part of it, is too ſmall a difference to be of much importance in the bargains that are made for the purchaſe of life-annuities. And conſequently this method of finding the values of annuities for two joint lives by Interpolation between the values of other contiguous annuities that have been already computed, may juſtly be conſidered as a very uſeful and a ſufficient ſupplement to the want of ſuch compleat tables of the values of theſe joint annuities, adapted to all the poſſible differences of ages in the two lives, as are mentioned in Art. ccccx.

End of the explanation and illuſtration of the foregoing method of finding the values of annuities for two joint lives by Interpolation.

CCCCXXXII. I ſhall now preſent the reader with another ſet of tables of the values of annuities for two joint lives, of the ſame kind as thoſe above exhibited in Art. ccccviii, and which I have procured to be computed, (like the former,) under the inſpection of the learned Mr. Morgan, from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is $4\frac{1}{2}$ per cent. Theſe tables are as follows.

TABLE

TABLE XXXIX.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons of the ſame age, when the intereſt of money is 4½ per cent.----Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the firſt life.	Years in the age of the ſecond life.	Values of an annuity of one pound a year for the joint continuance of both lives	Years in the age of the firſt life.	Years in the age of the ſecond life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the firſt life.	Years in the age of the ſecond life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	3	13.570,765	34	34	12.659,86	65	65	5.423,067
4	4	14.0[illegible]2,22	35	35	12.536,31	66	66	5.123,340
5	5	14 395,92	36	36	12.407,78	67	67	4.834,907
6	6	14.631,71	37	37	12.273,92	68	68	4.559,660
7	7	14.795,55	38	38	12.095,25	69	69	4.300,488
8	8	14.910,24	39	39	11.907,44	70	70	4.061,770
9	9	15.00[illegible],20	40	40	11.709,84	71	71	3.816,915
10	10	15.037,76	41	41	11.501,77	72	72	3.599,136
11	11	15.004,12	42	42	11.282,46	73	73	3.384,355
12	12	14.897,33	43	43	11.051,13	74	74	3.175,568
13	13	14.785,09	44	44	10.806,91	75	75	2.977,377
14	14	14.668,91	45	45	10.548,84	76	76	2.757,619
15	15	14.546,70	46	46	10.275,91	77	77	2.549,448
16	16	14.418,73	47	47	10.023,25	78	78	2.362,120
17	17	14.321,26	48	48	9.755,940	79	79	2.165,060
18	18	14.219,84	49	49	9.508,360	80	80	2.005,385
19	19	14.114,20	50	50	9.246,458	81	81	1.860,452
20	20	14.004,10	51	51	9.003,958	82	82	1.744,983
21	21	13.926,24	52	52	8.782,414	83	83	1.613,537
22	22	13.846,16	53	53	8.549,082	84	84	1.441,788
23	23	13.763,77	54	54	8.302,844	85	85	1.276,351
24	24	13.678,93	55	55	8.076,876	86	86	1.128,148
25	25	13.591,50	56	56	7.839,037	87	87	1.024,200
26	26	13.501,34	57	57	7.588,114	88	88	0.859,738
27	27	13.408,28	58	58	7.356,793	89	89	0.698,585
28	28	13.312,16	59	59	7.113,508	90	90	0.544,507
29	29	13.212,80	60	60	6.856,915	91	91	0.405,106
30	30	13.109,99	61	61	6.585,462	92	92	2.296,467
31	31	13.003,53	62	62	6.297,341	93	93	0.239,234
32	32	12.893,19	63	63	6.023,531	94	94	0.000,000
33	33	12.778,72	64	64	5.732,893			

TABLE XL.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 5 years; when the intereſt of money is 4½ per cent.----Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£			£
3	8	14.219,29	32	37	12.556,49	61	66	5.756,635
4	9	14.525,27	33	38	12.407,81	62	67	5.466,931
5	10	14.707,65	34	39	12.252,17	63	68	5.191,171
6	11	14.810,70	35	40	12.089,10	64	69	4.917,419
7	12	14.839,89	36	41	11.918,07	65	70	4.646,949
8	13	14.840,94	37	42	11.738,52	66	71	4.377,326
9	14	14.828,32	38	43	11.531,14	67	72	4.127,800
10	15	14.782,57	39	44	11.312,59	68	73	3.885,426
11	16	14.700,60	40	45	11.082,06	69	74	3.653,183
12	17	14.598,23	41	46	10.838,70	70	75	3.435,590
13	18	14.491,45	42	47	10.600,65	71	76	3.203,078
14	19	14.379,98	43	48	10.349,18	72	77	2.988,987
15	20	14.263,55	44	49	10.102,06	73	78	2.788,448
16	21	14.160,62	45	50	9.840,830	74	79	2.585,274
17	22	14.071,49	46	51	9.582,860	75	80	2.408,860
18	23	13.979,20	47	52	9.345,367	76	81	2.231,987
19	24	13.883,53	48	53	9.094,623	77	82	2.075,853
20	25	13.784,30	49	54	8.846,134	78	83	1.917,410
21	26	13.699,50	50	55	8.601,572	79	84	1.730,370
22	27	13.612,10	51	56	8.359,590	80	85	1.561,668
23	28	13.521,96	52	57	8.120,294	81	86	1.408,369
24	29	13.428,91	53	58	7.885,844	82	87	1.291,505
25	30	13.332,79	54	59	7.638,860	83	88	1.129,847
26	31	13.233,41	55	60	7.393,970	84	89	0.953,637
27	32	13.130,57	56	61	7.135,517	85	90	0.781,712
28	33	13.024,06	57	62	6.861,983	86	91	0.621,495
29	34	12.913,66	58	63	6.605,043	87	92	0.489,284
30	35	12.799,11	59	64	6.333,493	88	93	0.347,978
31	36	12.680,15	60	65	6.045,498	89	94	0.000,000

TABLE XLI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 10 years; when the intereſt of money is $4\frac{1}{2}$ per cent.----Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£			£
3	13	13.534,84	31	41	12.132,91	59	69	5.339,366
4	14	14.344,57	32	42	11.959,77	60	70	5.087,882
5	15	14.446,47	33	43	11.777,89	61	71	4.827,611
6	16	14.498,41	34	44	11.586,69	62	72	4.578,320
7	17	14.528,31	35	45	11.385,50	63	73	4.336,530
8	18	14.531,10	36	46	11.173,61	64	74	4.092,576
9	19	14.520,95	37	47	10.969,90	65	75	3.848,068
10	20	14.478,80	38	48	10.737,84	66	76	3.593,607
11	21	14.420,71	39	49	10.512,31	67	77	3.350,952
12	22	14.326,17	40	50	10.274,39	68	78	3.126,500
13	23	14.227,96	41	51	10.042,43	69	79	2.902,025
14	24	14.125,85	42	52	9.816,964	70	80	2.709,443
15	25	14.019,61	43	53	9.579,45	71	81	2.523,916
16	26	13.908,99	44	54	9.328,882	72	82	2.365,251
17	27	13.811,42	45	55	9.083,299	73	83	2.194,845
18	28	13.710,22	46	56	8.824,214	74	84	1.999,081
19	29	13.605,12	47	57	8.566,170	75	85	1.811,170
20	30	13.495 90	48	58	8.312,358	76	86	1.627,327
21	31	13.400,16	49	59	8.059,770	77	87	1.473,044
22	32	13.301,14	50	60	7.793,079	78	88	1.279,463
23	33	13.198,65	51	61	7.525,774	79	89	1.081,750
24	34	13.092,47	52	62	7.257,402	80	90	0.895,079
25	35	12.982,38	53	63	6.992,041	81	91	0.717,247
26	36	12.868,14	54	64	6.711,295	82	92	0.553,568
27	37	12.749,48	55	65	6.427,546	83	93	0.397,601
28	38	12.605,79	56	66	6.144,923	84	94	0.000,000
29	39	12.455.44	57	67	5.863,955			
30	40	12.297,99	58	68	5.597,977			

TABLE XLII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 20 years; when the intereſt of money is 4½ per cent.----Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£			£
3	23	13.566,18	27	47	11.235,80	51	71	5.249,784
4	24	13.764,63	28	48	11.025,14	52	72	5.006,611
5	25	13.869,97	29	49	10.823,13	53	73	4.761,978
6	26	13.928,96	30	50	10.610,56	54	74	4.517,666
7	27	13.950,52	31	51	10.406,55	55	75	4.286,359
8	28	13.946,14	32	52	10.212,00	56	76	4.037,427
9	29	13.929,47	33	53	10.008,01	57	77	3.794,414
10	30	13.882,15	34	54	9.793,822	58	78	3.572,788
11	31	13.801,26	35	55	9.588,700	59	79	3.343,173
12	32	13.684,05	36	56	9.373,704	60	80	3.139,598
13	33	13.561,85	37	57	9.148,020	61	81	2.943,841
14	34	13.434,35	38	58	8.916,202	62	82	2.764,131
15	35	13.301,20	39	59	8.672,806	63	83	2.572,536
16	36	13.162,05	40	60	8.416,828	64	84	2.345,809
17	37	13.033,30	41	61	8.147,138	65	85	2.119,938
18	38	12.878,23	42	62	7.862,472	66	86	1.908,778
19	39	12.715,57	43	63	7.581,639	67	87	1.728,597
20	40	12.544,83	44	64	7.285,200	68	88	1.497,798
21	41	12.382,05	45	65	6.971,573	69	89	1.269,895
22	42	12.211,23	46	66	6.659,066	70	90	1.048,546
23	43	12.031,83	47	67	6.360,345	71	91	0.834,287
24	44	11.843,25	48	68	6.065,302	72	92	0.638,300
25	45	11.644,84	49	69	5.786,990	73	93	0.440,344
26	46	11.435,90	50	70	5.517,470	74	94	0.000,000

TABLE XLIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 30 years; when the intereſt of money is 4½ per cent.----Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	33	12.892,77	24	54	9.910,650	45	75	4.518,077
4	34	13.047,91	25	55	9.702,397	46	76	4.247,671
5	35	13.112,32	26	56	9.484,074	47	77	3.991,243
6	36	13.130,47	27	57	9.254,869	48	78	3.748,011
7	37	13.110,84	28	58	9.034,348	49	79	3.502,708
8	38	13.043,28	29	59	8.803,302	50	80	3.284,035
9	39	12.959,63	30	60	8.560,832	51	81	3.079,666
10	40	12.842,64	31	61	8.305,940	52	82	2.899,153
11	41	12.689,53	32	62	8.037,502	53	83	2.699,676
12	42	12.497,79	33	63	7.774,945	54	84	2.464,370
13	43	12.296,06	34	64	7.498,687	55	85	2.237,648
14	44	12.083,66	35	65	7.207,393	56	86	2.022,653
15	45	11.859,89	36	66	6.920,335	57	87	1.835,846
16	46	11.623,98	37	67	6.638,708	58	88	1.596,108
17	47	11.410,32	38	68	6.353,241	59	89	1.356,043
18	48	11.185,18	39	69	6.076,191	60	90	1.119,059
19	49	10.967,99	40	70	5.810,588	61	91	0.890,743
20	50	10.739,19	41	71	5.538,183	62	92	0.677,405
21	51	10.532,33	42	72	5.282,17-	63	93	0.462,633
22	52	10.334,98	43	73	5.025,291	64	94	0 000,000
23	53	10.128,01	44	74	4.769,599			

TABLE XLIV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 40 *years; when the intereſt of money is* $4\frac{1}{2}$ *per cent.--- Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.*

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£			£
3	43	11.664,55	21	61	8.361,570	39	79	3.606,689
4	44	11.706,30	22	62	8.087,980	40	80	3.390,203
5	45	11.657,84	23	63	7.820,090	41	81	3.183,644
6	46	11.559,58	24	64	7.538,184	42	82	2.996,187
7	47	11.439,70	25	65	7.240,908	43	83	2.789,655
8	48	11.288,63	26	66	6.947,583	44	84	2.547,150
9	49	11.138,05	27	67	6.659,348	45	85	2.308,580
10	50	10.953,87	28	68	6.377,813	46	86	2.082,011
11	51	10.754,12	29	69	6.105,244	47	87	1.888,493
12	52	10.538,20	30	70	5.844,810	48	88	1.636,142
13	53	10.311,44	31	71	5.578,317	49	89	1.386,792
14	54	10.073,02	32	72	5.329,295	50	90	1.140,577
15	55	9.842,622	33	73	5.080,618	51	91	0.905,794
16	56	9.600,677	34	74	4.834,663	52	92	0.689,009
17	57	9.358,650	35	75	4.594,865	53	93	0.468,882
18	58	9.124,664	36	76	4.338,334	54	94	0.000,000
19	59	8.879,210	37	77	4.090,833			
20	60	8.621,334	38	78	3.852,445			

TABLE XLV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 50 years; when the intereſt of money is 4½ per cent.--- Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	53	9.769,927	24	74	4.848,025
4	54	9.740,534	25	75	4.603,719
5	55	9.652,684	26	76	4.342,180
6	56	9.522,340	27	77	4.089,075
7	57	9.355,740	28	78	3.851,485
8	58	9.181,316	29	79	3.606,634
9	59	8.989,406	30	80	3.391,199
10	60	8.767,436	31	81	3.185,907
11	61	8.513,132	32	82	3.000,036
12	62	8.224,342	33	83	2.795,513
13	63	7.940,830	34	84	2.555,537
14	64	7.642,510	35	85	2.320,372
15	65	7.327,995	36	86	2.098,610
16	66	7.016,758	37	87	1.907,556
17	67	6.718,992	38	88	1.655,071
18	68	6.427,600	39	89	1.403,200
19	69	6.144,730	40	90	1.155,588
20	70	5.873,408	41	91	0.918,076
21	71	5.603,355	42	92	0.697,208
22	72	5.350,681	43	93	0.473,203
23	73	5.098,132	44	94	0.000.000

TABLE XLVI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 60 years; when the intereſt of money is $4\frac{1}{2}$ per cent.--- Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	63	7.519,867	20	80	3.404,081
4	64	7.378,607	21	81	3.197,263
5	65	7.169,216	22	82	3.009,860
6	66	6.938,290	23	83	2.803,637
7	67	6.693,300	24	84	2.561,759
8	68	6.442,915	25	85	2.324,537
9	69	6.196,954	26	86	2.100,434
10	70	5.950,820	27	87	1.906,501
11	71	5.685,410	28	88	1.654,218
12	72	5.423,923	29	89	1.402,532
13	73	5.162,329	30	90	1.155,084
14	74	4.902,882	31	91	0.917,714
15	75	4.648,840	32	92	0.696,969
16	76	4.376,827	33	93	0.473,078
17	77	4.118,650	34	94	0.000,000
18	78	3 875,876			
19	79	3.625,462			

TABLE XLVII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 70 years; when the intereſt of money is $4\frac{1}{2}$ per cent.--- Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	73	4.898,531	14	84	2.582,455
4	74	4.734,189	15	85	2.340,580
5	75	4.541,851	16	86	2.111,582
6	76	4.310,868	17	87	1.915,651
7	77	4.088,644	18	88	1.661,118
8	78	3.868,953	19	89	1.407,168
9	79	3.639,892	20	90	1.157,288
10	80	3.433,726	21	91	0.919,296
11	81	3.230,664	22	92	0.698,015
12	82	3.039,329	23	93	0.473,624
13	83	2.828,899	24	94	0.000,000

TABLE XLVIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 80 years; when the intereſt of money is 4½ per cent.--- Computed from Monſieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	83	2.702,580	9	89	1.405,610
4	84	2.503,721	10	90	1.160,806
5	85	2.290,611	11	91	0.923,697
6	86	2.082,129	12	92	0.700,914
7	87	1.897,819	13	93	0.475,131
8	88	1.651,923	14	94	0.000,000

CCCCXXXIII. According to Monſieur de Parcieux's table of the probabilities of the duration of human life (which begins with the age of 3 years, and ends with the age of 94 years, and ſuppoſes the utmoſt poſſible extent of human life to be ſomewhat leſs than 95 years,) it is evident that only two pairs of lives can be found whoſe ages will differ from each other by 90 years, to wit, two lives of the ages of 3 years and 93 years, and two lives of the ages of 4 years and 94 years. And an annuity for the joint continuance of this latter pair of lives cannot, according to this table of probabilities, be of any value, becauſe it is ſuppoſed to be certain, according to this table, that the older life, to wit, the life of 94 years, will be extinct before the end of the year, or before the payment of the annuity will become due. Therefore the only two lives, whoſe ages differ from each other by 90 years, for the joint continuance of which an annuity can be of any value, are two lives of the ages of 3 years and 93 years. And the value of an annuity of one pound a year for the joint continuance of two lives of theſe ages, when the intereſt of money is $4\frac{1}{2}$ per cent. is £0.464,115.

The value of an annuity of one pound a year for two joint lives of the ages of 3 years and 93 years.

CCCCXXXIV. When the intereſt of money is $4\frac{1}{2}$ per cent. and the difference of the ages of the two lives is not either 0, or 5 years, or 10 years, or 20 years, or 30 years, or 40, 50, 60, 70, 80, or 90, years, but ſome intermediate number of years lying between ſome two of theſe differences that are contiguous to each other, we muſt have recourſe to the method of interpolation above explained in Art. CCCCXIII, in order to obtain a near value of an annuity of one pound a year for their joint continuance; as we did above when the intereſt of money was ſuppoſed to be $3\frac{1}{2}$ per cent. For this method of interpolation will be equally applicable to the diſcovery of theſe near values of annuities for two joint lives at one rate of the intereſt of money as at another.

When the value of an annuity for two joint lives, (upon a ſuppoſition that the intereſt of money is $4\frac{1}{2}$ per cent) is not contained in any of the foregoing tables, recourſe muſt be had to the method of *interpolation* above-deſcribed.

CCCCXXXV. We may therefore, by the help of the two foregoing ſets of tables, together with the method of interpolation above-explained, diſcover either the true values, or tolerably near values, of all annuities for the joint continuance of two lives of any ages whatſoever, when the intereſt of money is either $3\frac{1}{2}$ per cent. or $4\frac{1}{2}$ per cent.

End of the directions for finding the values of annuities for two joint lives when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.

Of the values of annuities for two joint lives, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent.

Theſe values may alſo be found to a tolerable degree of exactneſs by means of the two foregoing ſets of tables.

The conjectural ſuppoſition by means of which the ſaid values may be deduced from the values of the ſame annuities when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. which are given in the two foregoing ſets of tables.

CCCCXXXVI. And from the values of theſe annuities for two joint lives when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. we may find tolerably near values of the ſame annuities when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. by ſuppoſing that the values of an annuity for the joint continuance of any two given lives at the five following different rates of intereſt, to wit, 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent. (which differ from each other by only $\frac{1}{2}$ per cent.) form, pretty nearly, an arithmetical progreſſion, or decreaſe by nearly equal differences. For, if this ſuppoſition be true, the third value of the annuity, or that which it has when the intereſt of money is 4 per cent. will be nearly equal to an arithmetical mean proportional between the ſecond value of it, or that which it has when the intereſt of money is $3\frac{1}{2}$ per cent. and the fourth value of it, or that which it has when the intereſt of money is $4\frac{1}{2}$ per cent. which two extreme values may be found in the two foregoing ſets of tables. We, therefore, need only ſubtract the fourth value of the annuity, or that which it has when the intereſt of money is $4\frac{1}{2}$ per cent. from the ſecond value of it, or that which it has when the intereſt of money is $3\frac{1}{2}$ per cent. and divide the remainder by 2; and, if the quotient, thence ariſing, be ſubtracted from the ſecond value of the annuity, or that which it has when the intereſt of money is $3\frac{1}{2}$ per cent. the remainder will be nearly equal to the third value of it, or that which it has when the intereſt of money is 4 per cent. And, in like manner, if we ſubtract the ſaid quotient from the fourth value of the annuity, or that which it has when the intereſt of money is $4\frac{1}{2}$ per cent. the remainder will be the fifth value of the annuity, or that which it has when the intereſt of money is 5 per cent. and, if we add the ſaid quotient to the ſecond value of the annuity, or that which it has when the intereſt of money is $3\frac{1}{2}$ per cent. the ſum will be equal to the firſt value of it, or that which it has when the intereſt of money is 3 per cent.

An example of the method of deducing the former values from the latter,

Thus, for example, if it were required to aſſign the values of an annuity of one pound a year for two joint lives of the ages of 25 years and 35 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. by means of the two foregoing ſets of tables, which exhibit the values of this annuity only when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. we muſt proceed in the following manner.

The value of an annuity of one pound a year for the joint continuance of two lives of the ages of 25 years and 35 years, when the intereſt of money is $3\frac{1}{2}$ per cent. appears by Table XXXI, page 494, to be = £14.601,50;

£14.601,50; and the value of the ſame annuity, when the intereſt of money is $4\frac{1}{2}$ per cent. appears by Table XLI to be = £12.982,38. The difference between £14.601,50 and £12 982,3 is £1.619,12; and half this difference is £ .809,56. Therefore the value of this annuity when the intereſt of money is 4 per cent. will be nearly equal to £14.601,50 — £0.809,56, or £13.791,94; and the value of it when the intereſt of money is 5 per cent. will be nearly equal to £12.982,38 — £0.809,56, or £12.172,82; and the value of it when the intereſt of money is 3 per cent. will be nearly equal to £14.601,50 + £0.809,56, or £15.411,06. Therefore the values of an annuity of one pound a year for the joint continuance of two lives of the ages of 25 years and 35 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. are nearly equal to £15.411,06, £13.791,94, and £12.172,82. QEI.

CCCCXXXVII. In the foregoing article we have ſhewn how to derive a near value of any given annuity upon a ſuppoſition that the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. from the two values of the ſame annuity when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. which two values are either given exactly in ſome of the tables of the two foregoing ſets of tables, or may be derived, by the aforeſaid method of Interpolation, from the values which are there ſet down. And the method of doing this (and which we have deſcribed in the laſt article,) is very ſimilar to the aforeſaid method of Interpolation (deſcribed above in Art. CCCCXIII,) and may itſelf likewiſe be called with propriety *a method of Interpolation*, as it proceeds (like the former method,) on a ſuppoſition that the unknown value of the propoſed annuity, when the intereſt of money is 4 per cent. is nearly equal to an arithmetical mean between the two known values of the ſame annuity when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. or conſiſts in the *interpolation*, or *interpoſition*, of an arithmetical mean between thoſe two known values. Indeed, as to the manner of determining the other two unknown values of the propoſed annuity, or thoſe which it has when the intereſt of money is 3 per cent. and 5 per cent. the word *Interpolation* is not quite ſo proper for it; becauſe they are not found by *interpolating*, or *interpoſing*, any new terms between the two known values of the ſaid annuity (which it has when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) but by adding new terms, at both ends, to the arithmetical progreſſion, (conſiſting of three terms) whereof the two known values of the annuity (which it has when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) are the firſt and laſt terms, and the arithmetical mean between thoſe two known values is the middle term. With reſpect, therefore, to theſe values of the propoſed annuity, which it has when the intereſt of money is 3 per cent. and 5 per cent. and which are thus determined

Of the ſimilarity of this method of deducing the ſaid former values from the latter, to the method of *interpolation* deſcribed above in Art. 413.

mined to a moderate degree of exactneſs by *continuing* the aforeſaid arithmetical progreſſion (conſiſting of the aforeſaid three terms,) at both its ends, we ought rather to call this method of proceeding *the method of Continuation* than *the method of Interpolation.* And therefore, perhaps, it may be proper to call this whole method of finding all the ſaid three unknown values of the propoſed annuity, (which it has when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent.) taken together, *the method of Interpolation and Continuation.* But, by whatever name we call them, the two parts of this method are both founded on the ſame principle, or ſuppoſition, to wit, " that, when three rates of the intereſt of money are taken that are ſucceſſively greater the one than the other by only one half per cent. (as either the rates of 3 per cent. $3\frac{1}{2}$ per cent. and 4 per cent. or of $3\frac{1}{2}$ per cent. 4 per cent. and $4\frac{1}{2}$ per cent. or of 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent.) the values of an annuity for two joint lives of any given ages at theſe three rates of intereſt, will form, pretty nearly, an arithmetical progreſſion, or will decreaſe by nearly equal differences;" which ſuppoſition (though I do not know any method of demonſtrating it,) has ſo great an appearance of probability that, I imagine, it is hardly poſſible to doubt the truth of it.

This method may be called, with propriety, *the method of interpolation and continuation.*

Of the degree of exactneſs *to which the near values of annuities for two joint lives, when the intereſt of money is* 3 *per cent.* 4 *per cent. and* 5 *per cent. that are deduced, by the foregoing* method of Interpolation and Continuation, *from the values of the ſame annuities when the intereſt of money is* $3\frac{1}{2}$ *per cent. and* $4\frac{1}{2}$ *per cent. may, upon reaſonable grounds, be ſuppoſed to be true.*

CCCCXXXVIII. But, though it can hardly be doubted that the near values of an annuity for two joint lives of any given ages when the intereſt of money is 3 per cent. and 4 per cent. and 5 per cent. that are obtained by the foregoing method of *Interpolation and Continuation* from the values of the ſame annuity when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. are not very different from their true values, yet it would be deſireable (if it can be done,) to diſcover to *what degree of exactneſs* we may conſider theſe near values of the propoſed annuity as agreeing, or coinciding, with its correſpondent true values.

CCCCXXXIX. Now, to do this in the moſt ſatisfactory manner, it would be neceſſary to compute the exact values of an annuity of one pound a year for ſeven or eight pairs of joint lives of different ages when the

the intereſt of money is 3 per cent. and 4 per cent. and 5 per cent. and then to compare the ſaid exact values with the near values of the ſame annuity, at the ſame rates of the intereſt of money, that are derived, by means of the foregoing method of Interpolation and Continuation, from the values of the ſame annuity when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. which are given in the two foregoing ſets of tables: upon which compariſon I have no doubt that the differences between the ſaid near values and exact values would be found to be very inconſiderable. But this compariſon I am not at preſent able to make, not being poſſeſſed of the exact values of any annuities for two joint lives at the intereſt of 3 per cent. 4 per cent. or 5 per cent. computed from Monſieur de Parcieux's table of the probabilities of the duration of human life. And I do not think it an object of ſufficient importance to make it worth while, on this account only, to procure any of theſe exact values to be computed.

A probable method of diſcovering the ſaid degree of exactneſs.

CCCCXL. But we may form a very probable conjecture concerning the degree of exactneſs to which the aforeſaid near values of annuities for two joint lives at 3 per cent. 4 per cent. and 5 per cent. (which are obtained, by the foregoing method of *interpolation and continuation*, from the values of the ſame annuities at the intereſts of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) agree, or co-incide, with their exact values, by ſuppoſing what, I imagine, it is hardly poſſible to doubt of, to wit, that there is ſuch an analogy between the values of annuities for two joint lives and the values of annuities for ſingle lives of the ſame ages with the older of the two joint lives, that, if the near values of annuities for ſingle lives at the intereſts of 3 per cent. 4 per cent. and 5 per cent. (that are obtained from the exact values of the ſame annuities at the intereſts of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the foregoing method of *interpolation and continuation*,) are found to agree, or co-incide, with their true values to a certain degree of exactneſs, ſo as (for example,) to differ from the ſaid true values by only a 200th, or an 100th, part of the ſaid true values, it may be concluded that the near values of annuities for two joint lives (of which the older is of the ſame age with the ſingle life with which it is compared,) at the ſame intereſts of 3 per cent. 4 per cent. and 5 per cent. (that are obtained, by the ſaid method of interpolation and continuation, from the exact values of the ſame annuities at the intereſts of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) will likewiſe agree, or co-incide, with their true values to the ſame, or very nearly the ſame, degree of exactneſs as the near values and the true values of the correſponding annuities for ſingle lives were found to do, or ſo as to differ from the ſaid true values by only about a 200th, or a 100th, part of the ſaid true values. Thus, for example, if we ſhould derive the value of an annuity of one pound a year for a ſingle life of the age of 35 years, when the intereſt of money is 4 per cent. from the two values of the

the ſame annuity for the ſame life when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the foregoing method of interpolation, or by taking an arithmetical mean between thoſe two values; and, upon comparing this near value of the ſaid annuity with its exact value, (which is contained above in Table XVI, page 225,) we ſhould find the difference between them to be only a 200th, or a 100th, part of the ſaid exact value; we may conclude that the near value of an annuity of one pound a year for two joint lives of the ages of 25 years and 35 years, when the intereſt of money is 4 per cent. that is derived from the exact values of the ſame annuity for the ſame two joint lives, when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the foregoing method of interpolation, or by taking an arithmetical mean between the ſaid two exact values, will alſo differ from the true value of the ſame annuity by only about a 200th, or a 100th, part of the ſaid true value. If this concluſion be allowed to be juſt, (and it certainly appears ſo highly probable, that it ſeems difficult to doubt of the truth of it,) we may then try the degree of exactneſs to which this method of *interpolation and continuation* exhibits the values of annuities for two joint lives when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. by applying it to the determination of the values of the like annuities for ſingle lives (of the ſame ages as the older lives in the correſponding joint lives,) when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent and comparing the near values of the ſaid annuities for ſingle lives, thereby obtained, with the exact values of the ſame annuities for ſingle lives at the ſame rates of intereſt, which are exhibited above in Tables XIV, XVI, and XVIII. Of this I ſhall now proceed to give a few inſtances.

An example of the ſaid method.

CCCCXLI. In the firſt place therefore, let us ſuppoſe that the two lives for whoſe joint continuance an annuity of one pound a year is to be granted, are of the ages of 25 years and 35 years.

We have ſeen in Art. ccccxxxvi, that the two exact values of this annuity, when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. are £14.601,50 and £12.982,38, and that the near values of the ſame annuity, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. that are derived from the ſaid two exact values by the foregoing method of interpolation and continuation, are £15.411,06, £13.791,94, and £12.172,82. We are therefore to endeavour to determine, (by a reaſonable conjecture founded on the analogy juſt now ſuppoſed to take place between the values of annuities for joint lives and the values of annuities for ſingle lives,) *to what degree of exactneſs* theſe three near values, £15.411,06, £13.791,94, and £12.172,82, of the aforeſaid annuity when the

the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. agree, or co-incide, with the true values of the ſame annuity at the ſame rates of intereſt. Now, in order to lay a ground for a conjecture of this kind, we may proceed as follows.

The value of an annuity of one pound a year for a ſingle life of the age of 35 years, when the intereſt of money is 3 per cent. appears by Table XIV, page 223, to be £18.464,13; and the values of the ſame annuity, when the intereſt of money is $3\frac{1}{2}$ per cent. 4 per cent $4\frac{1}{2}$ per cent. and 5 per cent. appear by Tables XV, XVI, XVII, and XVIII, to be £17.206,612, £16.084,014, £15.078,39, and £14.174,54. Theſe are the true, or exact, values of this annuity at theſe five different rates of intereſt.

Now let the firſt, third, and fifth values of this annuity be derived from the ſecond and fourth values of it, to wit, £17.206,612 and £15.078,39, by the aforeſaid method of *interpolation and continuation*. This may be done in the manner following. The exceſs of the ſecond term, £17.206,612, of this progreſſion above the fourth term, £15.078,39, is = £2.128,222; and half this difference is £1.064,111. Therefore the firſt of the ſaid five terms (which are ſuppoſed, for the preſent purpoſe, to conſtitute a decreaſing arithmetical progreſſion,) will be = £17.206,612 + £1.064,111, or £18.270,723; and the third term will be = £17.206,612 — £1.064,111, or £16.142,501; and the fifth term will be £15.078,39 — £1.064,111, or £14.014,279; and conſequently the values of the propoſed annuity of one pound a year for the life of a perſon of the age of 35 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. will be nearly equal to £18.270,723, £16.142,501, and £14.014,279. Now, if we compare theſe three near values of this annuity, at theſe three different rates of intereſt, with the three exact values of the ſame annuity at the ſame rates of intereſt, (which we have ſeen to be £18.464,13, £16.084,014, and £14.174,54,) we ſhall find, that the firſt near value, to wit, £18.270,723, is leſs than the correſponding true value, £18.464,13, by the difference £0.193,407, which is leſs than the 95th part of the ſaid true value; and that the ſecond near value, to wit, £16.142,501, is greater than the correſponding true value, £16.084,014, by the difference £0.058,487, which is leſs than the 275th part of the ſaid true value; and that the third near value, £14.014,279, is leſs than the correſponding true value, £14.174,54, by the difference, £0.160,261, which is leſs than the 88th part of the ſaid true value. Thus it appears that the near values of an annuity of one pound a year for a ſingle life of the age of 35 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained by this method of *interpolation and continuation*, differ from the correſponding true values of

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it by leſs than the 95th, the 275th, and the 88th, parts of the ſaid true values reſpectively. And from hence we may conclude (by means of the analogy above-mentioned between the values of annuities for ſingle lives and the values of annuities for joint lives,) that the near values of an annuity of one pound a year for two joint lives of the ages of 25 years and 35 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. which were obtained in Art. ccccxxxv by the ſame method of *interpolation and continuation*, to wit, £15.411,06, £13.791,94, and £12.172,82, will likewiſe differ from the correſponding true values of the ſame annuity at thoſe three rates of intereſt, by only about the 95th part, the 275th part, and the 88th part, of the ſaid true values reſpectively.

Another example of the ſaid method.

CCCCXLII. In the ſecond place, Let us ſuppoſe the two lives to be of the ages of 25 years and 45 years.

The value of an annuity of one pound a year for two joint lives of theſe ages, when the intereſt of money is $3\frac{1}{2}$ per cent. appears by Table XXXII, page 495, to be £12.902,34; and the value of the ſame annuity for the ſame joint lives, when the intereſt of money is $4\frac{1}{2}$ per cent. appears by Table XLII to be £11.644,84. The difference of theſe values is £1.257,50; and half this difference is £0.628,75. Therefore, according to the foregoing method of *interpolation and continuation*, the value of the ſame annuity for the ſame joint lives, when the intereſt of money is 3 per cent. will be nearly equal to (£12.902,34 + £0.628,75, or) £13.531,09; and the value of it, when the intereſt of money is 4 per cent. will be nearly equal to (£12.902,34 — £0.628,75, or) £12.273,59; and the value of it, when the intereſt of money is 5 per cent. will be nearly equal to (£11.644,84 — £0.628,75, or) £11.016,09.

Now, in order to form a probable conjecture concerning the degree of exactneſs to which theſe near values of the aforeſaid annuity for two joint lives agree with its correſponding true values, let us derive the values of a like annuity of one pound a year for a ſingle life of the age of 45 years when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. from the values of it when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the ſame method of *interpolation and continuation*; and then compare the near values, thereby obtained, with the correſpondent true values of the ſame annuity at the ſame rates of intereſt, (to wit, 3 per cent. 4 per cent. and 5 per cent.) as exhibited above in Tables XIV, XVI, and XVIII. This may be done as follows.

The

The value of an annuity of one pound a year for a life of the age of 45 years, when the intereſt of money is 3½ per cent. appears by Table XV to be £14.716,120; and the value of the ſame annuity for the ſame life, when the intereſt of money is 4½ per cent. appears by Table XVII to be £13.163,92. The difference of theſe values is £1.552,200; and half the ſaid difference is £0.776,100. Therefore the value of an annuity of one pound a year for a life of the age of 45 years, when the intereſt of money is 3 per cent. will be nearly equal to (£14.716,120 + £0.776,100, or) £15.492,220; and the value of the ſame annuity for the ſame life, when the intereſt of money is 4 per cent. will be nearly equal to (£14.716,120 — £0.776,100, or) £13.940,020; and the value of the ſame annuity for the ſame life, when the intereſt of money is 5 per cent. will be nearly equal to (£13.163,92 — £0.776,100, or) £12.387,820.

Now the true values of this annuity, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. appear by Tables XIV, XVI, and XVIII, to be £15.608,74, £13.904,190, and £12.487,42. Therefore the firſt of the foregoing near values of the ſaid annuity, to wit, £15.492,220, falls ſhort of the correſponding true value of it, £15.608,74, by the difference, £0.116,520, which is leſs than the 133d part of the ſaid true value; and the ſecond near value of the ſaid annuity, to wit, £13 940,020, exceeds the correſponding true value of it, to wit, £13.904,190, by the difference, £0.035,830, which is leſs than the 388th part of the ſaid true value; and the third near value of the ſaid annuity, to wit, £12.387,820, is leſs than the correſponding true value of it, to wit, £12.487,42, by the difference, £0.099,600, which is leſs than the 125th part of the ſaid true value. We may therefore conclude (by means of the analogy above-mentioned between the values of annuities for ſingle lives and the values of annuities for joint lives,) that the three near values of an annuity of one pound a year for two joint lives of the ages of 25 years and 45 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. which were obtained above by the ſaid method of *interpolation and continuation*, to wit, £13.531,09, £12.273,59, and £11.016,09, will differ from the correſponding true values of the ſame annuity by only about the 133d part, the 388th part, and the 125th part, of the ſaid true values reſpectively.

CCCCXLIII. As a third example of this method of determining by analogy the degree of exactneſs of the values of annuities for two joint lives, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained by the foregoing method of *interpolation and continuation*, let us ſuppoſe the two lives to be of the ages of 20 years and 70 years. A third example of the ſame method.

The value of an annuity of one pound a year for two joint lives of theſe ages, when the intereſt of money is $3\frac{1}{2}$ per cent. appears by Table XXXV, page 495, to be £6.197,247; and the value of the ſame annuity for the ſame joint lives, when the intereſt of money is $4\frac{1}{2}$ per cent. appears by Table XLV to be £5.873,408. The difference of theſe values is £0.323,839; and half this difference is £0.161,919. Therefore, according to the foregoing method of *interpolation and continuation*, the values of the ſame annuity for the ſame joint lives, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. will be nearly equal to (£6.197,247 + £0.161,919, or) £6.359,166, (£6.197,247 — £0.161,919, or) £6.035,328, and (£5.873,408 — £0.161,919, or) £5.711,489, reſpectively.

Now, in order to form a conjecture concerning the degree of exactneſs of theſe near values of an annuity of one pound a year for theſe two joint lives of the ages of 20 years and 70 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. we muſt derive the values of a like annuity for a ſingle life of the age of 70 years, at the ſame rates of the intereſt of money, from the values of it, when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the ſame method of *interpolation and continuation*, and then compare the near values, thereby obtained, with the correſponding true values of the ſame annuity for a ſingle life of the ſame age of 70 years, at the ſame rates of intereſt, as they are exhibited above in Tables XIV, XVI, and XVIII. This may be done as follows.

The value of an annuity of one pound a year for a ſingle life of the age of 70 years, when the intereſt of money is $3\frac{1}{2}$ per cent. appears by Table XV to be £6.575,357; and the value of the ſame annuity for the ſame life, when the intereſt of money is $4\frac{1}{2}$ per cent. appears by Table XVII to be £6.220,54. The difference of theſe values is £0.354,817; and half the ſaid difference is £0.177,408. The ſum of £6.575,357 and £0.177,408 is £6.752,765; and the difference of £6.575,357 and £0.177,408 is £6.397,949; and the difference of £6.220,54 and £0.177,408 is £6.043,132. Therefore £6.752,765 will be a near value of an annuity of one pound a year for a ſingle life of the age of 70 years, when the intereſt of money is 3 per cent. and £6.397,949 will be a near value of the ſame annuity for the ſame life, when the intereſt of money is 4 per cent. and £6.043, 32 will be a near value of the ſame annuity for the ſame life, when the intereſt of money is 5 per cent.

Now the true values of an annuity of one pound a year for a ſingle life of the age of 70 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. appear by Tables XIV, XVI, and XVIII, to be £6.765,920, £6.393,749, and £6.055,280; which differ from the foregoing near values of the ſame annuity, obtained by the method of

interpolation

interpolation and continuation, to wit, £6.752,765, £6.397,949, and £6.043,132, by only £0.013,155, £0.004,200, and £0.012,148, which are less than the 514th part, the 1522d part, and the 498th part, of the said true values £6.765,920, £6.393,749, and £6.055,280, respectively. We may therefore conclude (by means of the analogy above-mentioned between the values of annuities for single lives and the values of annuities for joint lives,) that the three near values of an annuity of one pound a year for two joint lives of the ages of 20 years and 70 years, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. which were obtained above by the said method of *interpolation and continuation*, to wit, £6.359,166, £6.035,328, and £5.711,489, will differ from the corresponding true values of the same annuity by only about the 514th, the 1522d, and 498th, part of the said true values.

Conclusion drawn from the three foregoing examples, concerning the degree of exactness of the near values of annuities for two joint lives obtained by means of the foregoing method of *Interpolation and Continuation*.

CCCCXLIV. It appears from the three foregoing examples, that the near values of an annuity of one pound a year for a single life of either 35 years, or 45 years, or 70 years, of age, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained by the foregoing method of *interpolation and continuation*, differ but little from the corresponding true values of the same annuity; and further, that the differences of the near values of the said annuity, when the interest of money is 4 per cent. (which are obtained by *interpolation*, or by interposing arithmetical means between the values of the same annuity, when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) from its corresponding true values, are much smaller than the differences of the near values of the said annuity from its true values, when the interest of money is 3 per cent. and 5 per cent. in which cases the said near values are not obtained by *interpolation*, but by *continuation*. Therefore, if the foregoing analogy between the values of annuities for single lives and the values of annuities for joint lives be allowed to subsist, it will follow, that the near values of annuities for two joint lives, when the interest of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained, by the foregoing method of *interpolation and continuation*, from the values of the like annuities when the interest of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. will likewise differ but little from the true values of the same annuities; and that the differences of the near values of such annuities from their true values, will be still less, when the interest of money is 4 per cent. than when it is 3 per cent, or 5 per cent. But in all the three rates of interest the said near values of these annuities will be *near enough* to their true values for most of the common purposes of business.

End of the explanation and illustration of the foregoing method of Interpolation and Continuation.

Of the values of annuities for two joint lives, when the intereſt of money is either lower than 3 per cent. or higher than 5 per cent.

CCCCXLV. We have ſeen in the foregoing articles how, by the help of the two ſets of tables of the values of annuities for two joint lives, given above in Art. ccccviii and ccccxxxii, (which are founded on a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) we may find tolerably near values of the ſame annuities when the intereſt of money is 3 per cent. or 4 per cent. or 5 per cent. by means of the foregoing method of *Interpolation and Continuation*. But what ſhall we do, may the reader aſk, when the intereſt of money is not 3, 4, or 5, per cent. but 2 per cent. or 6 per cent. or 7 per cent. or 8, or 9, or 10, per cent? Will the ſame method of Interpolation and Continuation enable us to find the values of annuities for two joint lives at theſe ſeveral rates of intereſt to a tolerable degree of exactneſs? or, in other words, may it be ſuppoſed that the values of an annuity of one pound a year for two joint lives of given ages at the ſeveral following rates of intereſt of money, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. $5\frac{1}{2}$ per cent. 6 per cent. $6\frac{1}{2}$ per cent. 7 per cent. $7\frac{1}{2}$ per cent. 8 per cent. $8\frac{1}{2}$ per cent. 9 per cent. $9\frac{1}{2}$ per cent. and 10 per cent. (as well as at the intereſts of 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent.) will form, pretty nearly, an arithmetical progreſſion, or decreaſe by nearly equal differences? For, if this ſuppoſition is true, it is evident that, when any two of the terms of this progreſſion are known, all the other terms may be derived from them by additions, or ſubtractions, of the common difference, by which the terms decreaſe; and conſequently the values of the propoſed annuity at all the other rates of intereſt may be deduced in this manner from its values when the intereſt of money is $3\frac{1}{2}$ per cent. and 4 per cent. which are exhibited in the two foregoing ſets of tables. In anſwer to this queſtion I muſt obſerve, that it does not ſeem probable that this ſuppoſition will be nearly true in ſuch a variety of different rates of intereſt. For we have, in the laſt article, ſeen reaſon to conclude, that the near values of an annuity for two joint lives of given ages when the intereſt of money is 3 per cent. and 5 per cent which are obtained by means of this ſuppoſition, differ much more from its correſponding true values than the near value of it, (obtained by means of the ſame ſuppoſition,) when the intereſt of money is 4 per cent, differs from its true value. And therefore we have reaſon to conclude, that the near values of the ſame annuity when the intereſt of money is $2\frac{1}{2}$ per cent. or 2 per cent, or $5\frac{1}{2}$ per cent. or 6 per cent, or 7, 8, 9, or 10 per cent. that would be obtained by means of this ſuppoſition, would differ ſtill more from its correſponding true values, and, probably, would differ from them too much to make it adviſeable to neglect the differences and conſider the ſaid near values, in practice, as equal to the correſponding true values: at leaſt we may well ſuppoſe this to be the caſe at the very

very high intereſts of 8, 9, and 10 per cent. It ſeems deſirable, therefore, to diſcover, if poſſible, ſome other method of finding a tolerably near value of an annuity for two joint lives of given ages, when the intereſt of money is either lower than 3 per cent or higher than 5 per cent, that may exempt us from the neceſſity of computing its exact value by means of the expreſſion $\frac{\mathcal{L}}{\frac{1}{P \times Q}} \times$ the ſeries $\frac{P_I \times Q_I}{r} + \frac{P_{II} \times Q_{II}}{r^2} + \frac{P_{III} \times Q_{III}}{r^3} + \frac{P_{IV} \times Q_{IV}}{r^4} + \frac{P_V \times Q_V}{r^5} + \frac{P_{VI} \times Q_{VI}}{r^6} + \frac{P_{VII} \times Q_{VII}}{r^7} +$ &c. continued to the end of the table of the probabilities of the duration of human life; the computation of which, when the two lives are young, is very tedious. Now one method of doing this to a moderate degree of exactneſs (though not to ſo great a degree of exactneſs as might be deſired) is as follows.

A method of approximating to the value of an annuity of one pound a year for the joint continuance of two lives of given ages, without computing the expreſſion $\frac{1}{P \times Q} \times$ *the ſeries* $\frac{P_I \times Q_I}{r} + \frac{P_{II} \times Q_{II}}{r^2} + \frac{P_{III} \times Q_{III}}{r^3} + \frac{P_{IV} \times Q_{IV}}{r^4} + \frac{P_V \times Q_V}{r^5} +$ *&c. (continued to the end of the table of the probabilities,) which is accurately equal to it.*

CCCCXLVI. It is evident that the ſeries $\frac{P_I \times Q_I}{r} + \frac{P_{II} \times Q_{II}}{r^2} + \frac{P_{III} \times Q_{III}}{r^3} + \frac{P_{IV} \times Q_{IV}}{r^4} + \frac{P_V \times Q_V}{r^5} +$ &c. continued to the end of the table of probabilities, will conſiſt of the ſame number of terms as the ſeries $\frac{Q_I}{r} + \frac{Q_{II}}{r^2} + \frac{Q_{III}}{r^3} + \frac{Q_{IV}}{r^4} + \frac{Q_V}{r^5} +$ &c. continued likewiſe to the end of the table of probabilities: becauſe, when all the perſons who are repreſented in the table of probabilities as living at the age of the older of the two lives, and whoſe number is originally Q, ſhall be dead, the annuity (which is to depend on the joint continuance of both the lives,) muſt neceſſarily ceaſe, notwithſtanding ſeveral of the perſons who are there repreſented as living at the age of the younger of the two lives, and whoſe number is originally P, may be ſtill alive.

CCCCXLVII. Since

CCCCXLVII. Since therefore the two ſerieſes $\frac{Q_I}{r} + \frac{Q_{II}}{r^2} + \frac{Q_{III}}{r^3} + \frac{Q_{IV}}{r^4} + \frac{Q_V}{r^5} +$ &c. and $\frac{P_I \times Q_I}{r} + \frac{P_{II} \times Q_{II}}{r^2} + \frac{P_{III} \times Q_{III}}{r^3} + \frac{P_{IV} \times Q_{IV}}{r^4} + \frac{P_V \times Q_V}{r^5} +$ &c. conſiſt of the ſame number of terms;—and, ſince they alſo have the ſame quantities for the denominators of their ſeveral terms, to wit, the quantities r, r^2, r^3, r^4, r^5, &c.—and the numerators of the terms of the former ſeries, to wit, Q_I, Q_{II}, Q_{III}, Q_{IV}, Q_V, &c. are involved in, or are factors of, the numerators of the terms of the latter ſeries, to wit, $P_I \times Q_I$, $P_{II} \times Q_{II}$, $P_{III} \times Q_{III}$, $P_{IV} \times Q_{IV}$, $P_V \times Q_V$, &c.—it ſeems reaſonable to ſuppoſe that the value of the whole latter ſeries may, in ſome way or other, be deduced from the value of the whole former ſeries, to a tolerable degree of exactneſs, ſo as to make it unneceſſary to take the trouble of computing all the terms of the latter ſeries and then adding them up into one ſum. And, if this ſhall appear to be practicable, it will follow, that the value of an annuity of one pound a year for the joint continuance of two lives of any given ages, may be deduced from the value of an annuity of the like amount for the older of the ſame two lives ſingly; which value we ſhall always be able to find in one of the twelve tables above exhibited in Art. CCI, pages 221, 222, 223, &c. —— 232. And thus the ſaid tables of the values of annuities for ſingle lives may be made ſubſervient to the diſcovery of the values of the like annuities for two joint lives. For let $\overset{\text{£}}{A}$ be the value of an annuity of one pound a year for the older of the two lives ſingly; and let $\overset{\text{£}}{B}$ be the value of an annuity of one pound a year for the joint continuance of both lives. Then will $\overset{\text{£}}{A}$ be equal to $\frac{1}{Q} \times \overset{\text{£}}{1}$ × the ſeries $\frac{Q_I}{r} + \frac{Q_{II}}{r^2} + \frac{Q_{III}}{r^3} + \frac{Q_{IV}}{r^4} + \frac{Q_V}{r^5} +$ &c. and $\overset{\text{£}}{B}$ will be equal to $\frac{1}{P \times Q} \times \overset{\text{£}}{1}$ × the ſeries $\frac{P_I \times Q_I}{r} + \frac{P_{II} \times Q_{II}}{r^2} + \frac{P_{III} \times Q_{III}}{r^3} + \frac{P_{IV} \times Q_{IV}}{r^4} + \frac{P_V \times Q_V}{r^5} +$ &c. and conſequently the ſeries $\frac{Q_I}{r} + \frac{Q_{II}}{r^2} + \frac{Q_{III}}{r^3} + \frac{Q_{IV}}{r^4} + \frac{Q_V}{r^5} +$ &c. will be $= Q \times \overset{\text{£}}{A}$, and the

ſeries

ſeries $\frac{P_{\rm I} \times Q_{\rm I}}{r} + \frac{P_{\rm II} \times Q_{\rm II}}{r^2} + \frac{P_{\rm III} \times Q_{\rm III}}{r^3} + \frac{P_{\rm IV} \times Q_{\rm IV}}{r^4} + \frac{P_{\rm V} \times Q_{\rm V}}{r^5}$ + &c. will be $= P \times Q \times \overset{\pounds}{B}$. If therefore we can derive the latter ſeries from the former, we ſhall thereby obtain the value of $P \times Q \times \overset{\pounds}{B}$; which, being divided by $P \times Q$, will give us $\overset{\pounds}{B}$, or the value of an annuity of one pound a year for the two propoſed joint lives.

We muſt therefore endeavour to find a method of deriving the value of the ſeries $\frac{P_{\rm I} \times Q_{\rm I}}{r} + \frac{P_{\rm II} \times Q_{\rm II}}{r^2} + \frac{P_{\rm III} \times Q_{\rm III}}{r^3} + \frac{P_{\rm IV} \times Q_{\rm IV}}{r^4} + \frac{P_{\rm V} \times Q_{\rm V}}{r^5}$ + &c. from the value of the ſeries $\frac{Q_{\rm I}}{r} + \frac{Q_{\rm II}}{r^2} + \frac{Q_{\rm III}}{r^3} + \frac{Q_{\rm IV}}{r^4} + \frac{Q_{\rm V}}{r^5}$ + &c. which is equal to $Q \times \overset{\pounds}{A}$. Now this may be done to a certain moderate degree of exactneſs, by means of a conjectural ſuppoſition that has an appearance of great probability, in the manner following.

CCCCXLVIII. Put S for the value of the ſimple ſeries $\frac{Q_{\rm I}}{r} + \frac{Q_{\rm II}}{r^2} + \frac{Q_{\rm III}}{r^3} + \frac{Q_{\rm IV}}{r^4} + \frac{Q_{\rm V}}{r^5}$ + &c. and the Greek capital Σ for the value of the compound ſeries $\frac{P_{\rm I} \times Q_{\rm I}}{r} + \frac{P_{\rm II} \times Q_{\rm II}}{r^2} + \frac{P_{\rm III} \times Q_{\rm III}}{r^3} + \frac{P_{\rm IV} \times Q_{\rm IV}}{r^4} + \frac{P_{\rm V} \times Q_{\rm V}}{r^5}$ + &c. and put $g = \frac{Q_{\rm I}}{r}$, and $h = \frac{Q_{\rm II}}{r^2}$, and $i = \frac{P_{\rm I} \times Q_{\rm I}}{r}$, and $k = \frac{P_{\rm II} \times Q_{\rm II}}{r^2}$. And let G denote the value of an infinite ſeries of terms in geometrical proportion, of which $\frac{Q_{\rm I}}{r}$ and $\frac{Q_{\rm II}}{r}$, or g and h, are the two firſt terms, and the Greek capital Γ denote the value of an infinite ſeries of terms in geometrical proportion, of which $\frac{P_{\rm I} \times Q_{\rm I}}{r}$ and $\frac{P_{\rm II} \times Q_{\rm II}}{r^2}$, or i and k, are the two firſt terms.

Then

Then will the infinite geometrical ſeries G, or $g + b + \frac{bb}{g} + \frac{b3}{gg} + \frac{b4}{g3} + \frac{b5}{g4} + \frac{b6}{g5}$ + &c. be $= \frac{gg}{g-b}$, and the infinite geometrical ſeries Γ, or $i + \frac{kk}{i} + \frac{k3}{ii} + \frac{k4}{i3} + \frac{k5}{i4} + \frac{k6}{i5}$ + &c. be $= \frac{ii}{i-k}$; as will be evident from Art. LXXX, pages 91, 92.

CCCCXLIX. Now it ſeems reaſonable to ſuppoſe that the geometrical ſeries G, or $g + b + \frac{bb}{g} + \frac{b3}{gg} + \frac{b4}{g3} + \frac{b5}{g4} + \frac{b6}{g5}$ + &c. *ad infinitum*, will bear, pretty nearly, the ſame proportion to the ſimple ſeries S, or $\frac{Q\text{I}}{r} + \frac{Q\text{II}}{r2} + \frac{Q\text{III}}{r3} + \frac{Q\text{IV}}{r4} + \frac{Q\text{V}}{r5} + \frac{Q\text{VI}}{r6} + \frac{Q\text{VII}}{r7}$ + &c. continued to the end of the table of probabilities, (of which ſeries the two firſt terms, $\frac{Q\text{I}}{r}$ and $\frac{Q\text{II}}{r2}$, are the ſame with the two firſt terms, g and b, of the ſaid geometrical ſeries,) as the geometrical ſeries Γ, or $i + k + \frac{k2}{i} + \frac{k3}{ii} + \frac{k4}{i3} + \frac{k5}{i4} + \frac{k6}{i5}$ + &c. *ad infinitum*, bears to the compound ſeries Σ, or $\frac{P\text{I} \times Q\text{I}}{r} + \frac{P\text{II} \times Q\text{II}}{r2} + \frac{P\text{III} \times Q\text{III}}{r3} + \frac{P\text{IV} \times Q\text{IV}}{r4} + \frac{P\text{V} \times Q\text{V}}{r5} + \frac{P\text{VI} \times Q\text{VI}}{r6} + \frac{P\text{VII} \times Q\text{VII}}{r7}$ + &c. continued to the end of the table of probabilities, of which ſeries the two firſt terms $\frac{P\text{I} \times Q\text{I}}{r}$ and $\frac{P\text{II} \times Q\text{II}}{r2}$, are the ſame with the two firſt terms, i and k, of the ſaid geometrical ſeries. And, if this ſuppoſition is true, the ſeries Σ, or $\frac{P\text{I} \times Q\text{I}}{r} + \frac{P\text{II} \times Q\text{II}}{r2} + \frac{P\text{III} \times Q\text{III}}{r3} + \frac{P\text{IV} \times Q\text{IV}}{r4} + \frac{P\text{V} \times Q\text{V}}{r5} + \frac{P\text{VI} \times Q\text{VI}}{r6}$

$+ \frac{P^{VI} \times Q^{VI}}{r^6} + \frac{P^{VII} \times Q^{VII}}{r^7} +$ &c. will be nearly equal to $\frac{\Gamma \times S}{G}$.

But we have seen in Art. XLVII, that the series Σ, or $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5} + \frac{P^{VI} \times Q^{VI}}{r^6} + \frac{P^{VII} \times Q^{VII}}{r^7} +$ &c. is $= P \times Q \times \overset{\pounds}{B}$, and the series S, or $\frac{Q^{I}}{r} + \frac{Q^{II}}{r^2} + \frac{Q^{III}}{r^3} + \frac{Q^{IV}}{r^4} + \frac{Q^{V}}{r^5} + \frac{Q^{VI}}{r^6} + \frac{Q^{VII}}{r^7} +$ &c. is $= Q \times \overset{\pounds}{A}$. Therefore $P \times Q \times \overset{\pounds}{B}$ will be nearly equal to $\frac{\Gamma \times Q \times \overset{\pounds}{A}}{G}$.

But the series G is $= \frac{gg}{g-h}$, and the series Γ is $= \frac{ii}{i-k}$; as we have seen in Art. CCCCXLVIII. Therefore $P \times Q \times \overset{\pounds}{B}$ will be nearly equal to $\frac{Q \times \overset{\pounds}{A} \times \frac{ii}{i-k}}{\frac{gg}{g-h}}$, or $Q \times \overset{\pounds}{A} \times \frac{ii}{i-k} \times \frac{g-h}{gg}$, or $Q \times \overset{\pounds}{A} \times \frac{ii}{gg} \times \frac{g-h}{i-k}$; and $\overset{\pounds}{B}$, or the value of an annuity of one pound a year for the two joint lives whose ages correspond to the numbers P and Q, will be nearly equal to $\frac{Q \times \overset{\pounds}{A}}{P \times Q} \times \frac{ii}{gg} \times \frac{g-h}{i-k}$, or $\frac{\overset{\pounds}{A}}{P} \times \frac{ii}{gg} \times \frac{g-h}{i-k}$.

But g is $= \frac{Q^{I}}{r}$, and h is $= \frac{Q^{II}}{r^2}$, and i is $= \frac{P^{I} \times Q^{I}}{r}$, and k is $= \frac{P^{II} \times Q^{II}}{r^2}$. Therefore $i = g \times P^{I}$, and k is $= h \times P^{II}$. Consequently ii is $= gg \times P^{I} \times P^{I}$, and $i-k$ is $= g \times P^{I} - h \times P^{II}$, and $\frac{ii}{gg} \times \frac{g-h}{i-k}$ is $=$

is $= \frac{gg \times P^{I} \times P^{I}}{gg} \times \frac{g-h}{g \times P^{I} - h \times P^{II}} = P^{I} \times P^{I} \times \frac{g-h}{g \times P^{I} - h \times P^{II}}$.

Therefore $\frac{\overset{£}{A}}{P} \times \frac{ii}{gg} \times \frac{g-h}{i-k}$ is $= \frac{\overset{£}{A}}{P} \times P^{I} \times P^{I} \times \frac{g-h}{g \times P^{I} - h \times P^{II}} =$ $\overset{£}{A} \times \frac{P^{I}}{P} \times \frac{P^{I} \times \overline{g-h}}{g \times P^{I} - h \times P^{II}} = \overset{£}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{II} - h \times P^{II}}$. Therefore $\overset{£}{B}$, or the value of an annuity of one pound a year for the two joint lives whose ages correspond to the numbers P and $\mathcal{Q}$, will be nearly equal to $\overset{£}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, or may be derived from $\overset{£}{A}$, or the value of the like annuity for a single life of the age corresponding to $\mathcal{Q}$, by multiplying it into the fraction $\frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$. QE I.

Examples of the foregoing method of approximating to the values of annuities for two joint lives.

CCCCL. I shall now proceed to try the truth of this expression, $\overset{£}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, by applying it to the computation of the values of an annuity of one pound a year for the joint continuance of two lives of some of the ages set down in the tables above exhibited in Art. CCCCVIII and CCCCXXXII. And, if we shall find that the values of those annuities which we shall obtain by means of this expression, differ but little from the values of them contained in those tables, (which have been accurately computed,) we shall have reason to conclude that the same expression will give us the like approximations to the true values of annuities for two joint lives in other instances.

Examples of the said method, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent.

First example; in which the difference of the ages of the two lives is 10 years.

CCCCLI. Let us therefore suppose that the two lives for the joint continuance of which an annuity of one pound a year is to be purchased, are of the ages of 10 years and 20 years; and that the interest of money is $3\frac{1}{2}$ per cent. and that the table of probabilities of the duration of human life, by which the calculation is to be governed, is that of Monsieur de Parcieux.

Then

Then we ſhall have (by Table XV.) $\overset{£}{A}$, or the value of an annuity of one pound a year for a ſingle life of the age of 20 years, $= \overset{£}{19.440,616}$, and $P = 880$, $P^{I} = 872$, $P^{II} = 866$, $Q = 814$, $Q^{I} = 806$, $Q^{II} = 798$, $r = 1.035$, $\frac{1}{r}\left(=\frac{1}{1.035}\right) = 0.966,183$, $\frac{1}{r^2} = 0.933,510$; and conſequently $g\left(=\frac{Q^{I}}{r} = 806 \times .966,183\right) = 778.743,498$, and $h\left(=\frac{Q^{II}}{r^2}\right.$ $\left. = 798 \times .933,510\right) = 744.940,980$. Therefore $g \times P^{I}$ will be $(= 778.743,498 \times 872) = 679,064.330,256$, and $h \times P^{I}$ will be $= 744.940,980 \times 872) = 649,588.534,560$, and $h \times P^{II}$ will be $(= 744.940,980 \times 866) = 643,629.006,720$; and conſequently $g \times P^{I} - h \times P^{I}$ will be $(= 679,064.330,256 - 649,588.534,560) = 29,475.795,696$, and $g \times P^{I} - h \times P^{II}$ will be $(= 679,064.330,256 - 643,629.006,720) = 35,435.323,536$. Therefore $\frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$ will be $= \frac{29,475.795,696}{35,435.323,536} =$ (nearly) $\frac{29,475.795,696}{35,435} = .8318$; and $\frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$ will be $=$ (nearly) $\frac{P^{I}}{P} \times .8318 \left(= \frac{872}{880} \times .8318 = \frac{109}{110} \times .8318 = \frac{90.6662}{110}\right)$ $= .8242$. Therefore $\overset{£}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$ will be $\left(= \overset{£}{A} \times .8242\right.$ $\left. = \overset{£}{19.440,616} \times .8242\right) = \overset{£}{16.022,955}$. QE I.

The true value of this annuity for two joint lives appears by Table XXXI, page 494, to be $\overset{£}{16.634,79}$; which is greater than the near value of it juſt now found, to wit. $\overset{£}{16.022,955}$, by the difference $\overset{£}{0.611,835}$, which is ſomewhat leſs than a 27th part of the ſaid true value. Difference between the foregoing near value and the true value of this annuity.

CCCCLII. Now let the two lives be of the ages of 20 years and 30 years.

In this caſe $\overset{£}{A}$ will be the value of an annuity of one pound a year for a ſingle life of the age of 30 years, and therefore (by Table XV.) will be Second example; in which the difference of the two lives is likewiſe 10 years.

be = 18.068,798. And P will be = 814, P^{I} = 806, P^{II} = 798, Q = 734, Q^{I} = 726, Q^{II} = 718, r = 1.035, $\frac{1}{r}$ $\left(= \frac{1}{1.035}\right)$ = 0.966,183, and $\frac{1}{r^2}$ = 0.933,510. Therefore $\frac{Q^{I}}{r}$ will be (= 726 × .966,183) = 701.448,858, and $\frac{Q^{II}}{r^2}$ will be = (=718 × .933,510) = 670.260,180, that is, g will be = 701.448,858, and h will be = 670.260,180. Therefore $g \times P^{I}$ will be (= 701.448,858 × 806) = 565,367.779,548, and $h \times P^{I}$ will be (=670.260,180 × 806) = 540,229.705,080, and $h \times P^{II}$ will be (= 670.260,180 × 798) = 534,867.623,640; and consequently $g \times P^{I} - h \times P^{I}$ will be (= 565,367.779,548 − 540,229.705,080) = 25,138 074,468, and $g \times P^{I} - h \times P^{II}$ will be (= 565,367.779,548 − 534,867.623,640) = 30,500.155,808. Therefore $\frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$ will be = $\frac{25,138.074,468}{30,500.155,908}$ = (nearly) $\frac{25,138.074,468}{30,500}$ = .8241; and $\frac{P^{I}}{P}$ × $\frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$ will be $\left(= \frac{P^{I}}{P} \times .8241 = \frac{806}{814} \times .8241 = \frac{403}{407} \times .8241 = \frac{332\ 1123}{407}\right)$ = .8160. Therefore $\overset{£}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$ will be = $\overset{£}{A} \times .8160 = \overset{£}{18}.068,798 \times .8160 = \overset{£}{14}.7441$. QEI.

Difference between the foregoing near value and the true value of this annuity.

The true value of this annuity for the said joint lives of the ages of 20 years and 30 years appears by Table XXXI, page 494, to be £15.298,75; which is greater than the value just now found for it, to wit, £14.7441, by the difference £0.554,65, which is less than the 27th part of the said true value, £15.298,75.

Five more examples of the same method, in all which the difference of the ages of the two lives is the same as in the two former examples, to wit, 10 years.

CCCCLIII. If we derive in the same manner the values of an annuity of one pound a year for the joint continuance of the five following pairs of lives, to wit, two lives of the ages of 30 years and 40 years, two lives of the ages of 40 years and 50 years, two lives of the ages of 50 years and 60 years, two lives of the ages of 60 years and 70 years, and two lives of the ages of 70 years and 80 years, from the values of the same annuity for single lives of the same ages as the older lives in each of these pairs of lives respectively, that is, for single lives of the ages of 40 years, 50 years, 60 years,

years, 70 years, and 80 years, (which values appear by Table XV to be £16.104,542, £13.183,083, £10.104,074, £6.575,357, and £3.661,781,); —I say, if we derive near values of the said annuity for the said pairs of joint lives from these values of the same annuity for the said single lives by means of the aforesaid expression $\overset{£}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - b \times P_I}{g \times P_I - b \times P_{II}}$, those near values will be found to be as follows; to wit,

for two joint lives of the ages of 30 years and 40 years, £12.865,18;
for two joint lives of the ages of 40 years and 50 years, £10.917,67;
for two joint lives of the ages of 50 years and 60 years, £7.680,967;
for two joint lives of the ages of 60 years and 70 years, £5.074,649;
and for two joint lives of the ages of 70 years and 80 years, £2.645,883.

CCCCLIV. Now it appears from Table XXXI, that the true values of these last five annuities for joint lives are as follows, to wit,

The true values of the last five annuities.

for two joint lives of the ages of 30 years and 40 years, £13.709,61;
for two joint lives of the ages of 40 years and 50 years, £11.229,92;
for two joint lives of the ages of 50 years and 60 years, £8.341,630;
for two joint lives of the ages of 60 years and 70 years, £5.336,698;
and for two joint lives of the ages of 70 years and 80 years, £2.793,419.

CCCCLV. The difference between the near value, £12.865,18, of the first of these annuities, and its true value, £13.709,61, is £0.844,43; which is less than a 16th part of the said true value.

The differences between their foregoing near values and their true values.

The difference between the near value, £10.917,67, of the second of these annuities, and its true value, £11.229,92, is £0.312,25; which is less than a 35th part of the said true value.

The difference between the near value, £7.680,967, of the third of these annuities, and its true value, £8.341,630, is £0.660,663; which is less than a 12th part of the said true value.

The

The difference between the near value, £5.074,649, of the fourth of the foregoing annuities, and its true value, £5.336,698, is £0.262,049; which is less than the 20th part of the said true value.

And the difference between the near value, £2.645,883, of the fifth and last of the foregoing annuities, and its true value, £2.793,419, is £0.147,536; which is less than the 19th part of the said true value.

CCCCLVI. In all these examples it appears that the near values of annuities for two joint lives, obtained by means of the expression $\overset{£}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$, are less than their respective true values, and that the differences of the said near values and true values are in some cases about a 35th part of the said true values respectively, and in other cases a 27th, a 20th, a 19th, and a 16th part of the said true values, and in some cases almost a 12th part of them.

Five other examples of the foregoing method of finding near values of annuities for two joint lives; in which the difference of the ages of the two lives is 30 years.

CCCCLVII. Let us now examine the near values of some annuities for two joint lives, that are obtained by means of the said expression, $\overset{£}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$, when the difference of the ages of the two lives is considerably greater than 10 years. Let this difference, therefore, be 30 years; and let us derive, by means of the said expression, $\overset{£}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$, near values of an annuity of one pound a year for the five following pairs of joint lives, upon a supposition that the interest of money is (as it was supposed to be in the foregoing examples,) 3 ½ per cent. to wit,

two lives of the ages of 10 years and 40 years,
two lives of the ages of 20 years and 50 years,
two lives of the ages of 30 years and 60 years,
two lives of the ages of 40 years and 70 years,
and two lives of the ages of 50 years and 80 years,

from

from the values of a like annuity of one pound a year for ſingle lives of the ages of 40 years, 50 years, 60 years, 70 years, and 80 years, reſpectively, which are given above in Table XV, to wit, £16.104,542, £13.183,083, £10.104,074, £6.575,357, and £3,661,781. Theſe near values will be found to be as follows; to wit,

The near values of the ſaid annuities.

for two joint lives of the ages of 10 years and 40 years, £13.892,33;
for two joint lives of the ages of 20 years and 50 years, £11.068,05;
for two joint lives of the ages of 30 years and 60 years, £8 560,176;
for two joint lives of the ages of 40 years and 70 years, £5.931,793;
and for two joint lives of the ages of 50 years and 80 years, £3.320,420.

CCCCLVIII. Now it appears from Table XXXIII, that the true values of theſe laſt five annuities for two joint lives are as follows; to wit, Their true values.

for two joint lives of the ages of 10 years and 40 years, £14.384,49;
for two joint lives of the ages of 20 years and 50 years, £11.801,15;
for two joint lives of the ages of 30 years and 60 years, £9.217,864;
for two joint lives of the ages of 40 years and 70 years, £6.126,861;
and for two joint lives of the ages of 50 years and 80 years, £3.399,171.

CCCCLIX. The difference between the near value, £13.892,33, of the firſt of theſe annuities and its true value, £14.384,49, is £0.492,16; which is leſs than the 29th part of the ſaid true value. The differences between their ſaid near values and true values.

The difference between the near value, £11.068,05, of the ſecond annuity, and its true value, £11.801,15, is £0.733,10; which is leſs than the 16th part of the ſaid true value.

The difference between the near value, £8.560,176, of the third annuity, and its true value, £9.217,864, is £0.657,688; which is leſs than the 14th part of the ſaid true value.

The difference between the near value, £5.931,793, of the fourth annuity, and its true value, £6.126,861, is £0.195,068; which is leſs than the 31ſt part of the ſaid true value.

And

And the difference between the near value, £3.320,420, and the true value, £3.399,171, of the fifth and laſt annuity, is £0.078,751; which is leſs than the 43d part of the ſaid true value.

CCCCLX. In theſe five examples it appears that the near values of annuities for two joint lives, whoſe ages differ from each other by 30 years, obtained by means of the expreſſion $\overset{£}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$, are leſs than their reſpective true values, (as was the caſe with the near values of the annuities mentioned in Art. CCCCLI, &c. —— CCCCLVI) and that the differences of the ſaid near values and true values are in ſome caſes leſs than the 43d part of the ſaid true values reſpectively, and in other caſes a 31ſt, a 29th, and a 16th part of the ſaid true values, and in ſome caſes about a 14th part of them.

Theſe differences are rather ſmaller than the differences of the near values and true values of the annuities for two joint lives whoſe ages differ from each other by only 10 years, which are mentioned in Art. CCCCLVI.

Examples of the ſaid method, upon a ſuppoſition that the intereſt of money is 4½ per cent.

CCCCLXI. We will next examine the near values of annuities for two joint lives, which may be obtained by means of the expreſſion $\overset{£}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$, when the intereſt of money is 4½ per cent.

Examples in which the differences of the ages of the two lives is 10 years.

Let there be ſeven pairs of lives whoſe ages differ from each other by 10 years, to wit,

two lives of the ages of 10 years and 20 years,
two lives of the ages of 20 years and 30 years,
two lives of the ages of 30 years and 40 years,
two lives of the ages of 40 years and 50 years,
two lives of the ages of 50 years and 60 years,
two lives of the ages of 60 years and 70 years,
and two lives of the ages of 70 years and 80 years.

When

When the intereſt of money is $4\frac{1}{2}$ per cent. the near values of an annuity of one pound a year for the joint continuance of theſe ſeveral pairs of lives, that may be derived by means of the expreſſion $\overset{\pounds}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$, from the values of the like annuity of one pound a year for ſingle lives of the ages of 20 years, 30 years, 40 years, 50 years, 60 years, 70 years, and 80 years, (which values appear by Table XVII to be £16.623,93, £15.690,92, £14.254,42, £11.920,73, £9.346,46, £6.220,54, and £3.532,59,) will be found to be as follows; to wit,

The near values of an annuity of one pound a year for the ſaid ſeven pairs of joint lives.

The near value of an annuity of one pound a year for two joint lives of the ages of 10 years and 20 years, will be £14.655,20;

for two joint lives of the ages of 20 years and 30 years, £13.359,42;

for two joint lives of the ages of 30 years and 40 years, £11.594,96;

for two joint lives of the ages of 40 years and 50 years, £10.126,50;

for two joint lives of the ages of 50 years and 60 years, £7.329,817;

for two joint lives of the ages of 60 years and 70 years, £4.889,327;

and for two joint lives of the ages of 70 years and 80 years, £2.580,00.

The true values of the ſame annuity for the ſame pairs of joint lives.

CCCCLXII. Now it appears from Table XLI that the true values of theſe ſeven annuities of one pound a year for two joint lives are as follows; to wit,

for two joint lives of the ages of 10 years and 20 years, £14.478,80;

for two joint lives of the ages of 20 years and 30 years, £13 495,90;

for two joint lives of the ages of 30 years and 40 years, £12.297,99;

for two joint lives of the ages of 40 years and 50 years, £10.274,39;

for two joint lives of the ages of 50 years and 60 years, £7.793,079;

for two joint lives of the ages of 60 years and 70 years, £5.087,882;

and for two joint lives of the ages of 70 years and 80 years, £2.709,443.

The differences between the ſaid near values and true values.

CCCCLXIII. The difference between the near value, £14.655,20, of the firſt of theſe annuities, and its true value, £14.478,80, (which, it is worth obſerving, is, leſs than the near value, though in all the former examples the true values of the annuities have been greater than their near values,) is £0.176,40; which is leſs than the 82d part of the ſaid true value.

The difference between the near value, £13.359,42, of the ſecond annuity, and its true value, £13.495,90, is £0.136,48; which is leſs than the 98th part of the ſaid true value.

The difference between the near value, £11.594,96, of the third annuity, and its true value, £12.297,99, is £0.703,03; which is leſs than the 17th part of the ſaid true value.

The difference between the near value, £10.126,50, of the fourth annuity and its true value, £10.274,39, is £0.147,89; which is leſs than the 69th part of the ſaid true value.

The difference between the near value, £7.329,817, of the fifth annuity, and its true value, £7.793,079, is £0.463,262; which is leſs than the 17th part of the ſaid true value.

The difference between the near value, £4.889,327, of the ſixth annuity, and its true value, £5.087,882, is £0.198,555; which is leſs than the 25th part of the ſaid true value.

And the difference between the near value, £2.580,00, of the ſeventh and laſt annuity, and its true value, £2.709,443, is £0.129,443; which is leſs than the 20th part of the ſaid true value.

A remark on the ſaid differences.

Theſe differences are ſmaller than thoſe of the near values and the true values of the annuities mentioned in Art. CCCCLVII and CCCCLVIII, for two joint lives whoſe ages differ from each other by 30 years, when the intereſt of money is $3\frac{1}{2}$ per cent. and are ſmaller in a ſtill greater degree than the differences of the near values and the true values of the annuities for two joint lives, mentioned in Art. CCCCLVI, whoſe ages differ by only 10 years, when the intereſt of money is $3\frac{1}{2}$ per cent.

CCCCLXIV. We

CCCCLXIV. We will now examine the near values of ſome annuities for two joint lives, that are obtained by means of the expreſſion $\overset{\pounds}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - b \times P_I}{g \times P_I - b \times P_{II}}$, when the difference of the ages of the two lives is 30 years, and the intereſt of money (as in the two preceeding articles.) is $4\frac{1}{2}$ per cent.

Five other examples of the foregoing method of finding the values of annuities for two joint lives; in which the difference of the ages of the two lives is 30 years.

Now, when the intereſt of money is $4\frac{1}{2}$ per cent. the near values of an annuity of one pound a year for the five following pairs of joint lives, to wit,

two joint lives of the ages of 10 years and 40 years,
two joint lives of the ages of 20 years and 50 years,
two joint lives of the ages of 30 years and 60 years,
two joint lives of the ages of 40 years and 70 years,
and two joint lives of the ages of 50 years and 80 years,

which may be obtained by means of the expreſſion $\overset{\pounds}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - b \times P_I}{g \times P_I - b \times P_{II}}$, will be found to be as follows; to wit,

for two joint lives of the ages of 10 years and 40 years, £12.446,40;
for two joint lives of the ages of 20 years and 50 years, £10.248,45;
for two joint lives of the ages of 30 years and 60 years, £8.075,146;
for two joint lives of the ages of 40 years and 70 years, £5.655,547;
and for two joint lives of the ages of 50 years and 80 years, £3.214,679.

The near values of the ſaid five annuities.

CCCCLXV. Now it appears from Table XLIII, that the true values of theſe laſt five annuities for two joint lives are as follows; to wit,

Their true values.

for two joint lives of the ages of 10 years and 40 years, £12 842,64;
for two joint lives of the ages of 20 years and 50 years, £10.739,19;
for two joint lives of the ages of 30 years and 60 years, £8.560,832;
for two joint lives of the ages of 40 years and 70 years, £5.810,588;
and for two joint lives of the ages of 50 years and 80 years, £3.284,035.

CCCCLXVI. All theſe true values are greater than the near values of the ſame annuities obtained in Art. CCCCLXIV by means of the expreſſion $\mathcal{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, as was the caſe with reſpect to the true values of the annuities mentioned in all the preceeding examples, except the firſt example in Art. CCCCLXII, in which it appeared that the true value of an annuity of one pound a year for two joint lives of the ages of 10 years and 20 years, when the intereſt of money is $4\frac{1}{2}$ per cent, to wit, £14.478,80, was leſs than its near value, £14.655,20, obtained by means of the expreſſion $\mathcal{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$. This remarkable exception I do not know how to account for.

The differences between the ſaid near values and true values.

CCCCLXVII. The difference between the near value, £12.446,40, of the firſt of the five laſt-mentioned annuities for two joint lives, and its true value, £12.842,64, is £0.396,24; which is leſs than the 32d part of the ſaid true value.

The difference between the near value, £10.248,45, of the ſecond of the ſaid five annuities, and its true value, £10.739,19, is £0.490,74; which is leſs than the 21ſt part of the ſaid true value.

The difference between the near value, £8.075,146, of the third annuity, and its true value, £8.560,832, is £0.485,686; which is leſs than the 17th part of the ſaid true value.

The difference between the near value, £5.655,547, of the fourth annuity, and its true value, £5.810,588, is £0.155,041; which is leſs than the 37th part of the ſaid true value.

And the difference between the near value, £3.214,679, of the fifth and laſt annuity, and its true value, £3.284,035, is £0.069,356; which is leſs than the 47th part of the ſaid true value.

A table

A table of the foregoing near values of an annuity of one pound a year for different pairs of joint lives, and of the corresponding true values of the same annuity, and of the differences of the said near values and true values, and of the fractions that express the proportions of the said differences to the said true values.

CCCCLXVIII. If the near values and true values of all the annuities mentioned in the foregoing articles, from Art. CCCCLI to the last article, inclusive, be ranged in regular order in two contiguous columns, and their differences be set down in a third column adjoining to the second column, and the fractions that express the proportions of the said differences to the said true values, be set down in a fourth column adjoining to the said third column, the said several numbers will be as follows.

The near values of an annuity of one pound a year for two joint lives of several different ages, obtained by means of the expression $\text{£}\ A \times \frac{P_1}{P} \times \frac{g \times P_1 - h \times P_1}{g \times P_1 - h \times P_{11}}$, *when the interest of money is* $3\frac{1}{2}$ *per cent.*	*The true values of the same annuity for the same joint lives, when the interest of money is* $3\frac{1}{2}$ *per cent.*	*The differences of the said near and true values.*	*The proportions of the said differences to the said true values.*
£	£	£	
16.022,955	16.634,79	0.611,835	$\frac{1}{27}$
14.7441	15.298,75	0.554,65	$\frac{1}{27}$
12.865,18	13.709,61	0.844,43	$\frac{1}{16}$
10.917,67	11.229,92	0.312,25	$\frac{1}{35}$
7.680,967	8.341,630	0.660,663	$\frac{1}{12}$
5.074,649	5.336,698	0.262,049	$\frac{1}{20}$
2.645,883	2.793,419	0.147,536	$\frac{1}{19}$
£	£	£	
13.892,33	14.384,49	0.492,16	$\frac{1}{29}$
11.068,05	11.801,15	0.733,10	$\frac{1}{16}$
8.560,176	9.217,864	0.657,688	$\frac{1}{14}$
5.931,793	6.126,861	0.195,068	$\frac{1}{31}$
3.320,420	3.399,171	0.078,751	$\frac{1}{43}$

The near values of the same annuity for the same joint lives, obtained by means of the same expression $\mathcal{L}\ A \times \frac{P_1}{P} \times \frac{g \times P_1 - h \times P_1}{g \times P_1 - h \times P_{11}}$ *when the interest of money is* $4\frac{1}{2}$ *per cent.*	*The true values of the same annuity for the same joint lives, when the interest of money is* $4\frac{1}{2}$ *per cent.*	*The differences of the said near and true values.*	*The proportions of the said differences to the said true values.*
£	£	£	
14.655,20	14.478,80	0.176,40	$\frac{1}{82}$
13.359,42	13.495,90	0.136,48	$\frac{1}{98}$
11.594,96	12.297,99	0.703,03	$\frac{1}{17}$
10.126,50	10.274,39	0.147,89	$\frac{1}{69}$
7.329,817	7.793,079	0.463,262	$\frac{1}{16}$
4.889,327	5.087,882	0.198,555	$\frac{1}{25}$
2.580,00	2.709,443	0.129,443	$\frac{1}{20}$
£	£	£	
12.446,40	12.842,64	0.396,24	$\frac{1}{32}$
10.248,45	10.739,19	0.490,74	$\frac{1}{21}$
8.075,146	8.560,832	0.485,686	$\frac{1}{17}$
5.655,547	5.810,588	0.155,041	$\frac{1}{37}$
3.214,679	3.284,035	0.069,356	$\frac{1}{47}$

A second table of the same kind as the last, in which the near values of the annuities for two joint lives are derived from the former near values of them, exhibited in the last table, by multiplying the said former near values into the fraction $\frac{104}{100}$.

CCCCLXIX. If the foregoing near values of an annuity of one pound a year for two joint lives (which were obtained by means of the expression $\mathcal{L}\ A \times \frac{P_1}{P} \times \frac{g \times P_1 - h \times P_1}{g \times P_1 - h \times P_{11}}$) be increased in the proportion of 104 to 100, or be multiplied into the fraction $\frac{104}{100}$, we shall thereby obtain a second

cond ſet of near values of the ſaid annuity for the ſame joint lives, that will, for the moſt part, differ leſs than the foregoing near values of it from its correſponding true values. This will appear upon tryal, by computing theſe ſecond near values, (which will be equal to $A \times \frac{104}{100} \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$) and placing them one under another in a column (as the former near values were placed in the laſt article,) and then ſetting down the correſponding true values of the ſame annuity in a ſecond column adjoining to the former, and the differences between the ſaid near values and true values in a third column adjoining to the ſecond, and the fractions that expreſs the proportions of the ſaid differences to the ſaid true values in a fourth column adjoining to the third. This may be done in the manner following.

The near values of an annuity of one pound a year for two joint lives of ſeveral different ages, obtained by means of the expreſſion $A \times \frac{104}{100} \times \frac{P_1}{P} \times \frac{g \times P_1 - b \times P_1}{g \times P_1 - b \times P_{11}}$ *when the intereſt of money is* $3\frac{1}{2}$ *per cent.*	*The true values of the ſame annuity for the ſame joint lives.*	*The differences of the ſaid near and true values.*	*The proportions of the ſaid differences to the ſaid true values.*
£	£	£	
16.663,86	16.634,79	0.029,07	$\frac{1}{572}$
15.333,86	15.298,75	0.035,11	$\frac{1}{435}$
13.379,78	13.709,61	0.329,83	$\frac{1}{41}$
11.354,37	11.229,92	0.124,45	$\frac{1}{90}$
7.988,205	8.341,630	0.353,425	$\frac{1}{23}$
5.277,634	5.336,698	0.059,064	$\frac{1}{90}$
2.751,718	2.793,419	0.041,701	$\frac{1}{66}$
£	£	£	
14.448,02	14.384,49	0.063,53	$\frac{1}{226}$
11.510,77	11.801,15	0.290,38	$\frac{1}{40}$
8.902,583	9.217,864	0.315,281	$\frac{1}{29}$
6.169,064	6.126,861	0.042,203	$\frac{1}{145}$
3.453,236	3.399,171	0.054,065	$\frac{1}{62}$

The near values of the same annuity for the same joint lives, obtained by means of the same expression $\overset{£}{A} \times \frac{104}{100} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$, *when the interest of money is* $4\frac{1}{2}$ *per cent.*	*The true values of the same annuity for the same joint lives, when the interest of money is* $4\frac{1}{2}$ *per cent.*	*The differences of the said near values and true values.*	*The proportions of the said differences to the said true values.*
£	£	£	
15.241,40	14.478,80	0.762,60	$\frac{1}{18}$
13.893,89	13.495,90	0.397,99	$\frac{1}{33}$
12.057,85	12.297,99	0.240,14	$\frac{1}{51}$
10.531,56	10.274,39	0.257,17	$\frac{1}{39}$
7.623,009	7.793,079	0.170,070	$\frac{1}{45}$
5.084,900	5.087,882	0.002,982	$\frac{1}{1705}$
2.683,200	2.709,443	0.026,243	$\frac{1}{103}$
£	£	£	
12.944,256	12.842,64	0,101,61	$\frac{1}{126}$
10.658,38	10.739,19	0.080,81	$\frac{1}{132}$
8.398,151	8.560,832	0.162,681	$\frac{1}{52}$
5.881,768	5.810,588	0.071,180	$\frac{1}{81}$
3.343,266	3.284,035	0.059,231	$\frac{1}{55}$

A remark on the near values contained in the laſt table.

CCCCLXX. By theſe inſtances it appears that the expreſſion $\overset{£}{A} \times \frac{104}{100} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$ almoſt always gives us the value of an annuity of one pound a year for two joint lives to a conſiderably greater degree of exactneſs than the expreſſion $\overset{£}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$, and, in moſt caſes, to a ſufficient degree of exactneſs for practical purpoſes. And from its anſwering ſo well to the inſtances here given, it ſeems reaſonable to ſuppoſe that it will give the values of annuities for two joint lives to nearly the ſame

ſame degree of exactneſs in other inſtances, or where the ages of the two joint lives are different from thoſe above-ſuppoſed: and this it ſeems likely to do as well at one rate of the intereſt of money as at another; though for want of tables of the true values of annuities for two joint lives at any other rates of intereſt than $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. to try it by, the foregoing examples are confined to annuities at only thoſe two rates of intereſt. And therefore I think upon the whole, I may venture to recommend the ſaid expreſſion $\frac{\pounds}{A} \times \frac{104}{100} \times \frac{P_I}{P} \times \frac{g \times P_I - b \times P_I}{g \times P_I - b \times P_{II}}$, as a tolerably convenient approximation to the value of an annuity of one pound a year for two joint lives, when the intereſt of money is either leſs than 3 per cent. or greater than 5 per cent. and conſequently the value of ſuch annuity cannot be found in either of the two foregoing ſets of tables for two joint lives in Art. ccccviii and ccccxxxii, nor be derived from the values exhibited in thoſe tables by either the method of *Interpolation* explained in Art. ccccxii, ccccxiii, &c. —— ccccxxxi, or the method of *Interpolation and Continuation* explained in Art. ccccxxxvi, ccccxxxvii, &c.

I now proceed to conſider the values of annuities that depend on the joint continuance of *three* lives.

Of the values of annuities depending on the joint continuance of three lives.

A ſhort expreſſion of the value of an annuity of one pound a year depending on the joint continuance of three lives of given ages.

CCCCLXXI. Let r be, as before, the ſum of one pound, together with its intereſt for a year, according to any given rate of intereſt. And let N be the number of years in the age of the younger of the three perſons on the joint continuance of whoſe lives the annuity is to depend; and $N + a$ the number of years in the age of the next elder of the ſaid three perſons; and $N + a + b$ the number of years in the age of the oldeſt of the ſaid three perſons; and E the greateſt number of years through which it is ſuppoſed to be poſſible for human life to be extended, according to the table of probabilities of the duration of human life adopted for the calculation; which number is in Monſieur de Parcieux's table 94 years. Let n be any number of years not greater than $E - N + a + b$, or $E - N - a - b$, or than the greateſt number of years during which it is poſſible that the oldeſt of the three lives may be prolonged. And let an annuity of one pound *per annum* be granted for the term of n years, provided all the three perſons aforeſaid, of the ages of N years, $N + a$ years, and $N + a + b$ years, ſhall ſo long live, but otherwiſe to ceaſe upon the

death of either of them. Let P be the number of perſons repreſented in Monſieur de Parcieux's table of the probabilities of the duration of human life, (or in ſuch other table of thoſe probabilities as is thought proper by the calculator to be adopted as the ground of his calculation,) as being all living together at the ſaid firſt, or youngeſt, age of N years; and P^{I} the number of perſons repreſented in the ſaid table to be living at the age of $N + 1$ years; and P^{II} the number living at the age of $N + 2$ years; and P^{III} the number living at the age of $N + 3$ years; and P^{IV}, P^{V}, P^{VI}, P^{VII}, P^{VIII}, P^{IX}, P^{X}, &c. the numbers living at the ſeveral following ages of $N + 4$ years, $N + 5$ years, $N + 6$ years, $N + 7$ years, $N + 8$ years, $N + 9$ years, $N + 10$ years, &c. reſpectively. And let Q be the number of perſons repreſented in the ſaid table as living at the ſecond, or next older, age of $N + a$ years; and Q^{I} the number of perſons repreſented there as living at the age of $N + a + 1$ years; and Q^{II} the number living at the age of $N + a + 2$ years; and Q^{III} the number living at the age of $N + a + 3$ years; and Q^{IV}, Q^{V}, Q^{VI}, Q^{VII}, Q^{VIII}, Q^{IX}, Q^{X}, &c. the numbers living at the ſeveral following ages of $N + a + 4$ years, $N + a + 5$ years, $N + a + 6$ years, $N + a + 7$ years, $N + a + 8$ years, $N + a + 9$ years, $N + a + 10$ years, &c. reſpectively. And let R be the number of perſons repreſented in the ſaid table as living at the third, or oldeſt, age of $N + a + b$ years; and R^{I} the number of perſons repreſented there as living at the age of $N + a + b + 1$ years; and R^{II} the number living at the age of $N + a + b + 2$ years; and R^{III} the number of perſons living at the age of $N + a + b + 3$ years; and R^{IV}, R^{V}, R^{VI}, R^{VII}, R^{VIII}, R^{IX}, R^{X}, &c. the numbers living at the ſeveral following ages of $N + a + b + 4$ years, $N + a + b + 5$ years, $N + a + b + 6$ years, $N + a + b + 7$ years, $N + a + b + 8$ years, $N + a + b + 9$ years, $N + a + b + 10$ years, &c. reſpectively.

Theſe things being ſuppoſed, the preſent value of an annuity of one pound a year, to be enjoyed during the ſpace of n years, in caſe all the ſaid three lives, of the ages of N years, $N + a$ years, and $N + a + b$ years, ſhall ſo long continue, will be equal to the expreſſion, $\mathcal{L}1 \times$ the ſeries

$$\frac{P^{I} \times Q^{I} \times R^{I}}{P \times Q \times R \times r} + \frac{P^{II} \times Q^{II} \times R^{II}}{P \times Q \times R \times r^{2}} + \frac{P^{III} \times Q^{III} \times R^{III}}{P \times Q \times R \times r^{3}} +$$
$$\frac{P^{IV} \times Q^{IV} \times R^{IV}}{P \times Q \times R \times r^{4}} + \frac{P^{V} \times Q^{V} \times R^{V}}{P \times Q \times R \times r^{5}} + \frac{P^{VI} \times Q^{VI} \times R^{VI}}{P \times Q \times R \times r^{6}} +$$
$$\frac{P^{VII} \times Q^{VII} \times R^{VII}}{P \times Q \times R \times r^{7}} + \text{&c. continued to } n \text{ terms, or to the term } \frac{P^{n} \times Q^{n} \times R^{n}}{P \times Q \times R \times r^{n}},$$

or equal to the expreſſion, $\frac{\mathcal{L}1}{P \times Q \times R} +$ the ſeries

$$\frac{P^{I} \times Q^{I} \times R^{I}}{r} + \frac{P^{II} \times Q^{II} \times R^{II}}{r^2} + \frac{P^{III} \times Q^{III} \times R^{III}}{r^3} + \frac{P^{IV} \times Q^{IV} \times R^{IV}}{r^4}$$

$$+ \frac{P^{V} \times Q^{V} \times R^{V}}{r^5} + \frac{P^{VI} \times Q^{VI} \times R^{VI}}{r^6} + \frac{P^{VII} \times Q^{VII} \times R^{VII}}{r^7} + \text{&c. con-}$$

tinued to n terms, or to the term $\frac{P^{n} \times Q^{n} \times R^{n}}{r^n}$, or (dropping, for the sake of brevity, the marks $\times$ of the several multiplications,) equal to the expression $\frac{\pounds 1}{P\,Q\,R} \times$ the series $\frac{P^{I}\,Q^{I}\,R^{I}}{r} + \frac{P^{II}\,Q^{II}\,R^{II}}{r^2} + \frac{P^{III}\,Q^{III}\,R^{III}}{r^3}$

$$+ \frac{P^{IV}\,Q^{IV}\,R^{IV}}{r^4} + \frac{P^{V}\,Q^{V}\,R^{V}}{r^5} + \frac{P^{VI}\,Q^{VI}\,R^{VI}}{r^6} + \frac{P^{VII}\,Q^{VII}\,R^{VII}}{r^7} +$$

&c. continued to n terms, or to the term $\frac{P^{n}\,Q^{n}\,R^{n}}{r^n}$. This is evident from Prob. III, and its 6th and 7th Corollaries, Art. LIII, LIV, LV, LVI, pages 53, 54, 55, 56, 57.

CCCCLXXII. If n years is the greatest number of years through which it is possible (according to the table of the probabilities of the duration of human life adopted in the calculation,) for the oldest of the three given lives, or the life of the age of $N + a + b$ years, to be extended; or, in other words, if n is equal to $E - N - a - b$, the said expression $\frac{\pounds 1}{P\,Q\,R} \times$ the series $\frac{P^{I}\,Q^{I}\,R^{I}}{r} + \frac{P^{II}\,Q^{II}\,R^{II}}{r^2} + \frac{P^{III}\,Q^{III}\,R^{III}}{r^3} +$

$$\frac{P^{IV}\,Q^{IV}\,R^{IV}}{r^4} + \frac{P^{V}\,Q^{V}\,R^{V}}{r^5} + \frac{P^{VI}\,Q^{VI}\,R^{VI}}{r^6} + \frac{P^{VII}\,Q^{VII}\,R^{VII}}{r^7} + \text{&c.}$$

continued to n terms, or to the term $\frac{P^{n}\,Q^{n}\,R^{n}}{r^n}$, (which term in this case will be $\frac{P^{E-N-a-b} \times Q^{E-N-a-b} \times R^{E-N-a-b}}{r^n}$) will be the value of an annuity of one pound *per annum* for the whole joint continuance of the three given lives of the ages of N years, $N + a$ years, and $N + a + b$ years. But, if n is less than $E - N - a - b$, or the complement of $N + a + b$ (the number of years in the age of the oldest life) to E, or to the utmost possible duration of human life, the said expression will be less than the value of an annuity of one pound a year for the whole joint continuance of the said three lives, of the ages of N years, $N + a$ years, and $N + a + b$ years, and will be the value of an immediate, but

imperfect, life-annuity of one pound *per annum* during n years of the joint continuance of the ſaid three lives

An example of the computation of the value of an immediate and compleat life-annuity of one pound per annum *for the whole joint continuance of the lives of three perſons of given ages, by means of the foregoing expreſſion.*

CCCCLXXIII. Let it be required to find the value of an annuity of one pound *per annum* for the whole joint continuance of the lives of three perſons of the ages of 75 years, 80 years, and 85 years, according to Monſieur de Parcieux's table of the probabilities of the duration of human life, and upon a ſuppoſition that the intereſt of money is 3 per cent.

Here n, or the number of years through which the annuity is to continue, in caſe all the three lives (of which the oldeſt is of the age of 85 years,) ſhall laſt ſo long, is the greateſt poſſible number of years through which, according to Monſieur de Parcieux's table, a life of the age of 85 years can be extended, that is (94 — 85 yeays, or) 9 years. Therefore the ſeries $\frac{P^{\mathrm{I}}\,Q^{\mathrm{I}}\,R^{\mathrm{I}}}{r} + \frac{P^{\mathrm{II}}\,Q^{\mathrm{II}}\,R^{\mathrm{II}}}{r^2} + \frac{P^{\mathrm{III}}\,Q^{\mathrm{III}}\,R^{\mathrm{III}}}{r^3} + \frac{P^{\mathrm{IV}}\,Q^{\mathrm{IV}}\,R^{\mathrm{IV}}}{r^4}$ + &c. in the foregoing expreſſion, muſt be continued to 9 terms; which terms may be computed as follows.

Here P is $= 211$, P^{I} is $= 192$, $P^{\mathrm{II}} = 173$, $P^{\mathrm{III}} = 154$, $P^{\mathrm{IV}} = 136$, $P^{\mathrm{V}} = 118$, $P^{\mathrm{VI}} = 101$, $P^{\mathrm{VII}} = 85$, $P^{\mathrm{VIII}} = 71$, and $P^{\mathrm{IX}} = 59$; and Q is $= 118$, $Q^{\mathrm{I}} = 101$, $Q^{\mathrm{II}} = 85$, $Q^{\mathrm{III}} = 71$, $Q^{\mathrm{IV}} = 59$, $Q^{\mathrm{V}} = 48$, $Q^{\mathrm{VI}} = 38$, $Q^{\mathrm{VII}} = 29$, $Q^{\mathrm{VIII}} = 22$, and $Q^{\mathrm{IX}} = 16$; and R is $= 48$, $R^{\mathrm{I}} = 38$, $R^{\mathrm{II}} = 29$, $R^{\mathrm{III}} = 22$, $R^{\mathrm{IV}} = 16$, $R^{\mathrm{V}} = 11$, $R^{\mathrm{VI}} = 7$, $R^{\mathrm{VII}} = 4$, $R^{\mathrm{VIII}} = 2$, and $R^{\mathrm{IX}} = 1$. And r is $= 1.03$, and $\frac{1}{r} = \frac{1}{1.03} = .9708$, and $\frac{1}{r^2} = .9425$, and $\frac{1}{r^3} = .9151$, $\frac{1}{r^4} = .8884$, $\frac{1}{r^5} = .8626$, $\frac{1}{r^6} = .8374$, $\frac{1}{r^7} = .8130$, $\frac{1}{r^8} = .7894$, and $\frac{1}{r^9} = .7664$. Therefore the expreſſion.

$\frac{£1}{P\,Q\,R}$ × the series $\frac{P^{I}\,Q^{I}\,R^{I}}{r} + \frac{P^{II}\,Q^{II}\,R^{II}}{r^2} + \frac{P^{III}\,Q^{III}\,R^{III}}{r^3} +$
$\frac{P^{IV}\,Q^{IV}\,R^{IV}}{r^4} + \frac{P^{V}\,Q^{V}\,R^{V}}{r^5} + \frac{P^{VI}\,Q^{VI}\,R^{VI}}{r^6} + \frac{P^{VII}\,Q^{VII}\,R^{VII}}{r^7} +$
$\frac{P^{VIII}\,Q^{VIII}\,R^{VIII}}{r^8} + \frac{P^{IX}\,Q^{IX}\,R^{IX}}{r^9}$ will be equal to $\frac{£1}{211 \times 118 \times 48}$ × the

feries

$$
\begin{aligned}
&192 \times 101 \times 38 \times .9708\\
+\ &173 \times 85 \times 29 \times .9425\\
+\ &154 \times 71 \times 22 \times .9151\\
+\ &136 \times 59 \times 16 \times .8884\\
+\ &118 \times 48 \times 11 \times .8626\\
+\ &101 \times 38 \times 7 \times .8374\\
+\ &85 \times 29 \times 4 \times .8130\\
+\ &71 \times 22 \times 2 \times .7894\\
+\ &59 \times 16 \times 1 \times .7664\\
=\ &\frac{£1}{24{,}898 \times 48} \times \text{the feries}\\
&19392 \times 38 \times .9708\\
+\ &14705 \times 29 \times .9425\\
+\ &10934 \times 22 \times .9151\\
+\ &8024 \times 16 \times .8884\\
+\ &5664 \times 11 \times .8626\\
+\ &3838 \times 7 \times .8374\\
+\ &2465 \times 4 \times .8130\\
+\ &1562 \times 2 \times .7894\\
+\ &944 \times 1 \times .7664\\
=\ &\frac{£1}{24{,}898 \times 48} \times \text{the feries}\\
&38 \times 18{,}825.7536\\
+\ &29 \times 13{,}859.4625\\
+\ &22 \times 10{,}005.7034\\
+\ &16 \times 7{,}288.4336\\
+\ &11 \times 4{,}835.7664\\
+\ &7 \times 3{,}213.9412
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 4 \times 2004.0450 \\
&+ \quad 2 \times 1233.0428 \\
&+ \quad 1 \times \ 723.4816 \\
&= \quad \frac{£1}{1,195,104} \times \text{the series} \\
&\qquad 715,378.6368 \\
&+ \quad 401,924.4125 \\
&+ \quad 220,125.4748 \\
&+ \quad 116,614.9376 \\
&+ \quad 53,743.4304 \\
&+ \quad 22,497.5884 \\
&+ \quad 8,016.1800 \\
&+ \quad 2,466.0856 \\
&+ \quad 723.4816
\end{aligned}
$$

$= \frac{£1}{1.195,104} \times 1,541,490.2277 = \frac{£1,541,490.2277}{1,195,104} = £1.2898.$ Therefore the value of an annuity of one pound a year for the whole joint continuance of three lives of the ages of 75 years, 80 years, and 85 years, when the intereſt of money is 3 per cent. is 1.2898, or £.1 5*s*. 9½*d*. Q E I.

S C H O L I U M.

CCCCLXXIV. When three, or more, lives are combined together, the differences that may be taken between their ſeveral ages are ſo immenſely numerous that it is totally impracticable to form tables that ſhall exhibit the values of all, or even any conſiderable part of, the ſeveral annuities that may be ſuppoſed to depend on them. And conſequently, whenever the values of ſuch annuities are wanted, it will be neceſſary to take the trouble of computing them.

And, if the accurate value of ſuch an annuity is wanted, I believe there is no other way of finding it but that which has been above ſet forth by computing the expreſſion, $\frac{£1}{P\,Q\,R} \times$ the ſeries $\frac{P^{I}\,Q^{I}\,R^{I}}{r} + \frac{P^{II}\,Q^{II}\,R^{II}}{r^2}$

$\frac{P_{III}\ Q_{III}\ R_{III}}{r^3} + \frac{P_{IV}\ Q_{IV}\ R_{IV}}{r^4} + \frac{P_V\ Q_V\ R_V}{r^5} + \frac{P_{VI}\ Q_{VI}\ R_{VI}}{r^6} +$
$\frac{P_{VII}\ Q_{VII}\ R_{VII}}{r^7} +$ &c. continued to the end of the table of probabilities of the duration of human life; as was done in the foregoing example. But this method of obtaining it is in all cases rather tedious, even when the oldest life is a very old one: but when the three lives are all young, the computation is so very long and troublesome that few persons will, probably, care to undertake it. It seems therefore to be highly expedient to endeavour to find out some less difficult method of obtaining the value of an annuity of this kind by a tolerable approximation. Now this may be done to a moderate degree of exactness by a method analogous to that explained in Art. CCCCXLVI, CCCCXLVII, CCCCXLVIII, CCCCXLIX, whereby the value of an annuity of one pound a year for two joint lives of given ages was derived from the value of the same annuity for the older of the two single lives by means of the expression $\overset{£}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$. For we shall find, upon examination, that the value of an annuity of one pound a year for three joint lives of given ages may be derived in the like manner from the value of the same annuity for the joint continuance of the two older lives, by means of an expression exactly similar to the aforesaid expression $\overset{£}{A} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$. For if $\overset{£}{B}$ is the value of an annuity of one pound a year for the joint continuance of the two older of the three lives, and g is $= \frac{Q_I\ R_I}{r}$, and $h = \frac{Q_{II}\ R_{II}}{r^2}$, the value of an annuity of one pound a year for the joint continuance of all the three lives will be, nearly, equal to $\overset{£}{B} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$; as may be shewn in the manner following.

A method

A method of deriving the value of an annuity of one pound a year for the joint continuance of three lives of any given ages, from the value of the same annuity for the joint continuance of the two older of the said lives, by approximation.

CCCCLXXV. Let S be put for the series $\frac{\mathfrak{Q}^{\text{I}} R^{\text{I}}}{r} + \frac{\mathfrak{Q}^{\text{II}} R^{\text{II}}}{r^2} + \frac{\mathfrak{Q}^{\text{III}} R^{\text{III}}}{r^3} + \frac{\mathfrak{Q}^{\text{IV}} R^{\text{IV}}}{r^4} + \frac{\mathfrak{Q}^{\text{V}} R^{\text{V}}}{r^5} + \frac{\mathfrak{Q}^{\text{VI}} R^{\text{VI}}}{r^6} + \frac{\mathfrak{Q}^{\text{VII}} R^{\text{VII}}}{r^7} +$ &c. continued to the end of the table of the probabilities of the duration of human life. And let the Greek capital letter Σ be put for the series $\frac{P^{\text{I}} \mathfrak{Q}^{\text{I}} R^{\text{I}}}{r} + \frac{P^{\text{II}} \mathfrak{Q}^{\text{II}} R^{\text{II}}}{r^2} + \frac{P^{\text{III}} \mathfrak{Q}^{\text{III}} R^{\text{III}}}{r^3} + \frac{P^{\text{IV}} \mathfrak{Q}^{\text{IV}} R^{\text{IV}}}{r^4} + \frac{P^{\text{V}} \mathfrak{Q}^{\text{V}} R^{\text{V}}}{r^5} + \frac{P^{\text{VI}} \mathfrak{Q}^{\text{VI}} R^{\text{VI}}}{r^6} + \frac{P^{\text{VII}} \mathfrak{Q}^{\text{VII}} R^{\text{VII}}}{r^7} +$ &c. continued likewise to the end of the same table of probabilities.

Then it is evident that $\overset{\text{£}}{B}$, or the value of an annuity of one pound a year for the joint continuance of the two older lives (which answer to the letters $\mathfrak{Q}$ and R,) will be $= \frac{\text{£}^{\text{I}}}{\mathfrak{Q} R} \times S$; and that $\overset{\text{£}}{C}$, or the value of the same annuity for the joint continuance of all the three lives, will be $= \frac{\text{£}^{\text{I}}}{P \mathfrak{Q} R} \times \Sigma$.

Let i be put $= \frac{P^{\text{I}} \mathfrak{Q}^{\text{I}} R^{\text{I}}}{r}$, and k be put $= \frac{P^{\text{II}} \mathfrak{Q}^{\text{II}} R^{\text{II}}}{r^2}$.

Then, since g was taken $= \frac{\mathfrak{Q}^{\text{I}} R^{\text{I}}}{r}$, and h was taken $= \frac{\mathfrak{Q}^{\text{II}} R^{\text{II}}}{r^2}$, we shall have $i = g \times P^{\text{I}}$, and $k = h \times P^{\text{II}}$.

Let G be put for the infinite geometrical progression $g + h + \frac{h\,h}{g} +$

$\frac{h^3}{gg} + \frac{h^4}{g^3} + \frac{h^5}{g^4} + \frac{h^6}{g^5}$ + &c. of which the two firſt terms, g and h, are reſpectively equal to, $\frac{Q^{I} R^{I}}{r}$ and $\frac{Q^{II} R^{II}}{r^2}$, the two firſt terms of the ſeries S. And let the Greek capital letter Γ be put for the infinite geometrical progreſſion $i + k + \frac{k\,k}{i} + \frac{k^3}{ii} + \frac{k^4}{i^3} + \frac{k^5}{i^4} + \frac{k^6}{i^5}$ + &c. of which the two firſt terms, i and k, are reſpectively equal to, $\frac{P^{I} Q^{I} R^{I}}{r}$ and $\frac{P^{II} Q^{II} R^{II}}{r^2}$, the two firſt terms of the ſeries Σ.

Then will G be $= \frac{g\,g}{g-h}$; and Γ will be $= \frac{ii}{i-k}$.

CCCCLXXVI. Theſe things being premiſed, it ſeems reaſonable to conjecture that the geometrical progreſſion $g + h + \frac{h\,h}{g} + \frac{h^3}{g\,g} + \frac{h^4}{g^3} + \frac{h^5}{g^4} + \frac{h^6}{g^5}$ + &c. *ad infinitum*, or G, will bear pretty nearly the ſame proportion to its kindred ſeries S, (of which the two firſt terms, $\frac{Q^{I} R^{I}}{r}$ and $\frac{Q^{II} R^{II}}{r^2}$, are reſpectively equal to its two firſt terms g and h,) as the geometrical progreſſion $i + k + \frac{k\,k}{i} + \frac{k^3}{ii} + \frac{k^4}{i^3} + \frac{k^5}{i^4} + \frac{k^6}{i^5}$ + &c. *ad infinitum* bears to its kindred ſeries Σ, of which the two firſt terms, $\frac{P^{I} Q^{I} R^{I}}{r}$ and $\frac{P^{II} Q^{II} R^{II}}{r^2}$, are reſpectively equal to its two firſt terms, i and k. And, if this conjecture is well-founded, we ſhall have Σ nearly $\frac{\Gamma \times S}{G}$, and conſequently nearly $= \frac{\frac{ii}{i-k} \times S}{\frac{g\,g}{g-h}} = \frac{ii}{i-k} \times S \times \frac{g-h}{g\,g} =$

A conjectural ſuppoſition upon which this method of approximation is founded.

$\frac{g \times P^{I} \times g \times P^{I}}{g \times P^{I} - h \times P^{II}} \times S \times \frac{g-h}{gg} = \frac{gg \times P^{I} \times P^{I}}{g \times P^{I} - h \times P^{II}} \times S \times \frac{g-h}{gg} = \frac{P^{I} \times P^{I}}{g \times P^{I} - h \times P^{II}} \times S \times \overline{g-h} = \frac{P^{I}}{g \times P^{I} - h \times P^{II}} \times S \times \overline{g-h} \times P^{I} = \frac{P^{I}}{g \times P^{I} - h \times P^{II}} \times S \times \overline{g \times P^{I} - h \times P^{I}} = P^{I} \times S \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$.

But $\overset{\pounds}{B}$ is $= \frac{\pounds^{I}}{\mathcal{Q}R} \times S$; and consequently S is $= \overset{\pounds}{B} \times \mathcal{Q}R$. Therefore $P^{I} \times S \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$ is $= P^{I} \times \overset{\pounds}{B} \times \mathcal{Q}R \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$; and consequently Σ is, nearly, $= P^{I} \times \overset{\pounds}{B} \times \mathcal{Q}R \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$.

Therefore $\overset{\pounds}{C}$, or $\frac{\pounds^{I}}{P\,\mathcal{Q}R} \times \Sigma$, is nearly $= \frac{\pounds^{I}}{P\,\mathcal{Q}R} \times P^{I} \times \overset{\pounds}{B} \times \mathcal{Q}R \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}} = \frac{I}{P} \times P^{I} \times \overset{\pounds}{B} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}} = \overset{\pounds}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$; that is, the value of an annuity of one pound a year for the joint continuance of all the three lives will be, nearly, equal to $\overset{\pounds}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, or may be derived from $\overset{\pounds}{B}$, the value of the same annuity for the joint continuance of the two older lives, by multiplying it into the fraction $\frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$. QEI.

CCCCLXXVII. This expreſſion $\overset{\pounds}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$ will, I believe, be almoſt always leſs than the true value of $\overset{\pounds}{C}$. But in what *degree*,

gree, or within what *limits*, it will differ from it, I do not know. But, that we may form ſome tolerable conjecture upon the ſubject, I will now proceed to apply this expreſſion to the computation of the values of a few annuities for three joint lives, of which the learned Mr. Morgan has given us the true values to five places of figures, computed ſtrictly by means of the above-mentioned expreſſion, $\frac{f.1}{P\,Q\,R} \times$ the ſeries $\frac{P^{I}\,Q^{I}\,R^{I}}{r} + \frac{P^{II}\,Q^{II}\,R^{II}}{r^2} + \frac{P^{III}\,Q^{III}\,R^{III}}{r^3} + \frac{P^{IV}\,Q^{IV}\,R^{IV}}{r^4} + \frac{P^{V}\,Q^{V}\,R^{V}}{r^5} + \frac{P^{VI}\,Q^{VI}\,R^{VI}}{r^6} + \frac{P^{VII}\,Q^{VII}\,R^{VII}}{r^7} +$ &c. continued to the end of the table of the probabilities of the duration of human life. Theſe true values are contained in the ſeventh table of Mr. Morgan's treatiſe on the doctrine of annuities, page 273. They are the values of an annuity of one pound a year for three joint lives of equal ages, from the age of 60 years to the age of 91 years, incluſive of both. And they are computed from the Northampton table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 4 per cent. This table is as follows.

TABLE

TABLE XLIX.

Containing the true values of an annuity of one pound a year for the joint continuance of three lives of equal ages, from the age of 60 years to the age of 91 years, both included; when the interest of money is 4 per cent.

Computed from the Northampton table of the probabilities of the duration of human life.

Years in the age of the first life.	*Years in the age of the second life.*	*Years in the age of the third life.*	*Values of an annuity of 1l. a year for the joint continuance of all the three lives.*	*Years in the age of the first life.*	*Years in the age of the second life.*	*Years in the age of the third life.*	*Values of an annuity of 1l. a year for the joint continuance of all the three lives.*
			£				£
60	60	60	4.7826	76	76	76	1.9089
61	61	61	4.6115	77	77	77	1.7846
62	62	62	4.4382	78	78	78	1.5843
63	63	63	4.2626	79	79	79	1.3906
64	64	64	4.0849	80	80	80	1.2121
65	65	65	3.9050	81	81	81	1.0685
66	66	66	3.7230	82	82	82	1.0117
67	67	67	3.5390	83	83	83	0.9617
68	68	68	3.3533	84	84	84	0.8981
69	69	69	3.1662	85	85	85	0.7906
70	70	70	2.9780	86	86	86	0.7690
71	71	71	2.7895	87	87	87	0.7568
72	72	72	2.6015	88	88	88	0.5368
73	73	73	2.4160	89	89	89	0.3233
74	74	74	2.2352	90	90	90	0.1346
75	75	75	2.0636	91	91	91	0.1202

CCCCLXXVIII. Mr. Morgan has alſo given us, in his valuable treatiſe on annuities before-mentioned, pages 74, 75, and 76, a compleat table of the true values of an annuity of one pound a year for the joint continuance of two lives of equal ages, when the intereſt of money is 4 per cent. computed ſtrictly from the aforeſaid Northampton table of the probabilities of the duration of human life, for every age of life from the age of one year to that of 91 years, incluſively. This table is as follows.

TABLE L.

Containing the true values of an annuity of one pound a year for the joint continuance of two lives of equal ages, from the age of one year to the age of 91 years, incluſively; when the intereſt of money is 4 per cent.

Computed from the Northampton table of the probabilities of the duration of human life.

Years in the age of the firſt life.	*Years in the age of the ſecond life.*	*Values of an annuity of 1l. a year for the joint continuance of both lives.*	*Years in the age of the firſt life.*	*Years in the age of the ſecond life.*	*Values of an annuity of 1l. a year for the joint continuance of both lives.*
		£			£
1	1	10.5432	14	14	13.5638
2	2	11.3227	15	15	13.3673
3	3	12.5931	16	16	13.1616
4	4	13.1723	17	17	12.9461
5	5	13.6352	18	18	12.7201
6	6	13.9355	19	19	12.5346
7	7	14.1320	20	20	12.3928
8	8	14.2031	21	21	12.2985
9	9	14.2356	22	22	12.2038
10	10	14.1709	23	23	12.1089
11	11	14.0530	24	24	12.0139
12	12	13.9311	25	25	11.9191
13	13	13.7516	26	26	11.8242

Years in the age of the first life.	*Years in the age of the second life.*	*Values of an annuity of 1l. a year for the joint continuance of both lives.*	*Years in the age of the first life.*	*Years in the age of the second life.*	*Values of an annuity of 1l. a year for the joint continuance of both lives.*
		£			£
27	27	11.7292	60	60	6.2468
28	28	11.6351	61	61	6.0407
29	29	11.5422	62	62	5.8313
30	30	11.4500	63	63	5.6188
31	31	11.3007	64	64	5.4031
32	32	11.1462	65	65	5.1843
33	33	10.9880	66	66	4.9626
34	34	10.8240	67	67	4.7381
35	35	10.6538	68	68	4.5111
36	36	10.4782	69	69	4.2821
37	37	10.2962	70	70	4.0516
38	38	10.1073	71	71	3.8203
39	39	9.9110	72	72	3.5893
40	40	9.7065	73	73	3.3605
41	41	9.5532	74	74	3.1364
42	42	9.3960	75	75	2.9192
43	43	9.2358	76	76	2.7182
44	44	9.0723	77	77	2.5427
45	45	8.9054	78	78	2.2976
46	46	8.7350	79	79	2.0694
47	47	8.5608	80	80	1.8570
48	48	8.3829	81	81	1.6868
49	49	8.2008	82	82	1.5998
50	50	8.0779	83	83	1.5144
51	51	7.9599	84	84	1.4143
52	52	7.7818	85	85	1.244,62
53	53	7.6006	86	86	1.213,51
54	54	7.4165	87	87	1.132,87
55	55	7.2294	88	88	0.840,92
56	56	7.0392	89	89	0.554,77
57	57	6.8459	90	90	0.298,169
58	58	6.6494	91	91	0.240,383
59	59	6.4497			

CCCCLXXIX. And

CCCCLXXIX. And the Northampton table of the probabilities of the duration of human life, from which the two foregoing tables of the values of an annuity of one pound for two and three joint lives were computed, is given us likewise in Mr. Morgan's aforesaid treatise on annuities, page 267, and is as follows.

TABLE LI.

Representing the probabilities of the duration of human life at the several ages therein mentioned, from the age of one year to the age of 92 years, inclusively; as deduced by Dr. Price from observations on the bills of mortality at Northampton.

Age.	*Persons living.*	*Age.*	*Persons living.*	*Age.*	*Persons living.*	*Age.*	*Persons living.*
1 year.	849	25	475	49	293	73	99
2 years.	722	26	467	50	284	74	91
3	672	27	459	51	275	75	83
4	646	28	451	52	267	76	75
5	625	29	443	53	259	77	67
6	609	30	435	54	251	78	60
7	596	31	428	55	243	79	53
8	586	32	421	56	235	80	46
9	577	33	414	57	227	81	39
10	570	34	407	58	219	82	32
11	564	35	400	59	211	83	26
12	558	36	393	60	203	84	21
13	553	37	386	61	195	85	17
14	548	38	379	62	187	86	13
15	543	39	372	63	179	87	10
16	538	40	365	64	171	88	8
17	533	41	357	65	163	89	6
18	528	42	349	66	155	90	4
19	522	43	341	67	147	91	2
20	515	44	333	68	139	92	1
21	507	45	325	69	131	93	0
22	499	46	317	70	123		
23	491	47	309	71	115		
24	483	48	301	72	107		

CCCCLXXX. By the help of theſe two laſt tables we may apply the expreſſion $B^{\pounds} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$ to the diſcovery of near values of the ſeveral annuities for three joint lives of equal ages, of which we have ſeen the true values above in Table XLIX, Art. CCCCLXXVII. This may be done in the manner following.

CCCCLXXXI. When the three lives are all of the ſame age, $\mathcal{Q}$ and R will be equal to P, and $\mathcal{Q}^{I}$ and R^{I} will be equal to P^{I}, and $\mathcal{Q}^{II}$ and R^{II} will be equal to P^{II}. Therefore in this caſe g, or $\frac{\mathcal{Q}^{I} R^{I}}{r}$, will be $= \frac{P^{I} P^{I}}{r}$, and h, or $\frac{\mathcal{Q}^{II} R^{II}}{r^2}$, will be $= \frac{P^{II} P^{II}}{r^2}$; and conſequently $g \times P^{I}$ will be $= \frac{P^{I} P^{I}}{r} \times P^{I} = \frac{P^{I} \times P^{I} \times P^{I}}{r}$; and $h \times P^{I}$ will be $= \frac{P^{II} P^{II}}{r^2} \times P^{I} = \frac{P^{II} \times P^{II} \times P^{I}}{r^2}$; and $h \times P^{II}$ will be $= \frac{P^{II} P^{II}}{r^2} \times P^{II} = \frac{P^{II} \times P^{II} \times P^{II}}{r^2}$. Therefore $g \times P^{I} - h \times P^{I}$ will be $= \frac{P^{I} \times P^{I} \times P^{I}}{r} - \frac{P^{II} \times P^{II} \times P^{I}}{r^2} = \frac{P^{I} \times P^{I} \times P^{I} \times r}{r^2} - \frac{P^{II} \times P^{II} \times P^{I}}{r^2} = \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{r^2}$; and $g \times P^{I} - h \times P^{II}$ will be $= \frac{P^{I} \times P^{I} \times P^{I}}{r} - \frac{P^{II} \times P^{II} \times P^{II}}{r^2} = \frac{P^{I} \times P^{I} \times P^{I} \times r}{r^2} - \frac{P^{II} \times P^{II} \times P^{II}}{r^2} = \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}{r^2}$; and conſequently $\frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$ will be $= \frac{\frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{r^2}}{\frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}{r^2}}$

$= \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$. Therefore $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$, or the near value of an annuity of one pound a year for the three joint lives, will, in this caſe of an equality of the three ages, be $= \overset{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$; which I take to be the moſt convenient form to which this expreſſion can be reduced.

An example of the computation of the near value of an annuity of one pound a year for the joint continuance of three lives of equal ages, by means of the foregoing expreſſion.

CCCCLXXXII. Let it now be required, by means of this expreſſion, $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$, to find a near value of an annuity of one pound a year for three joint lives all of the age of 60 years, when the intereſt of money is 4 per cent. according to the foregoing Northampton table of the probabilities of the duration of human life; of which annuity we have ſeen above, in Table XLIX, that the true value is £4.7826.

Now it appears from Table L, Art. CCCCLXXVIII, that $\overset{£}{B}$, or the value of an annuity of one pound a year for the joint continuance of two lives that are both of the ſame age of 60 years, when the intereſt of money is 4 per cent. and according to the Northampton table of the probabilities of the duration of human life, is £6.2468. And it appears by the ſaid table of probabilities, that the number of perſons therein ſuppoſed to be living at the ſeveral ages of 60 years, 61 years, and 62 years, are 203, 195, and 187. Therefore P is $= 203$, $P^{I} = 195$, and P^{II} is $= 187$.

Alſo, ſince the intereſt of money is ſuppoſed to be 4 per cent. r will be $= 1.04$.

Therefore $P^{I} \times P^{I} \times P^{I} \times r$ will be $(= 195 \times 195 \times 195 \times 1.04 = 7{,}414{,}875 \times 1.04) = 7{,}711{,}470.00$; and $P^{II} \times P^{II} \times P^{I}$ will be $(= 187 \times 187 \times 195 = 34{,}969 \times 195) = 6{,}818{,}955$; and $P^{II} \times P^{II} \times P^{II}$ will be $(= 187 \times 187 \times 187 = 34{,}969 \times 187) = 6{,}539{,}203$. Therefore

$P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}$ will be $(= 7,711,470 - 6,818,955) = 892,515$; and $P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}$ will be $(= 7,711,470 - 6,539,203) = 1,172,267$; and consequently $\frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$ will be $= \frac{892,515}{1,172,267} = 0.761,35$. Therefore $\frac{P^{I}}{P}$ × the fraction $\frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$ will be $\left(= \frac{P^{I}}{P} \times 0.761,35 = \frac{195}{203} \times 0.761,35 = \frac{148.46325}{203}\right) = 0.731,34$; and $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$ will be $(= B \times 0.731,34 = 6.2468 \times 0.731,34) = 4.5685$. Therefore 4.5685 is a near value of the propoſed annuity of one pound a year for the joint continuance of three lives, all of the age of 60 years, when the intereſt of money is 4 per cent. according to the Northampton table of the probabilities of the duration of human life. Q E I.

This value of the propoſed annuity is leſs than its true value £4.7826, (given above in Table XLIX,) by the difference £0.2141, which is about a 22d part of the ſaid true value, £4.7826.

Near values of an annuity of one pound a year for three equal joint lives of the ages of 61 years, 62 years, 63 years, 64 years, &c. to the age of 91 years, inclufively, computed from the ſame expreſſion.

CCCCLXXXIII. If the values of an annuity of one pound a year for the joint continuance of three equal lives of the ages of 61 years, 62 years, 63 years, 64 years, &c. to the age of 91 years, inclufively, are computed in the ſame manner by means of the ſaid expreſſion, $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$, or $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$ they will be found to be as follows; to wit.

for three equal joint lives of the age of 61 years, = 4.3992,
of the age of 62 years, = 4.2281,
of the age of 63 years, = 4.0555,
of the age of 64 years, = 3.8814,
of the age of 65 years, = 3.7059,

of the

of the age of 66 years, = [illegible],
of the age of 67 years, = [illegible],
of the age of 68 years, = [illegible],
of the age of 69 years, = 2.9943,
of the age of 70 years, = 2.8153,
of the age of 71 years, = 2.6370,
of the age of 72 years, = 2.4600,
of the age of 73 years, = 2.2857,
of the age of 74 years, = 2.1156,
of the age of 75 years, = 1.9513,
of the age of 76 years, = 1.7961,
of the age of 77 years, = 1.6861,
of the age of 78 years, = 1.5067,
of the age of 79 years, = 1.3398,
of the age of 80 years, = 1.1842,
of the age of 81 years, = 1.0439,
of the age of 82 years, = 0.98201,
of the age of 83 years, = 0.92350,
of the age of 84 years, = 0.87917,
of the age of 85 years, = 0.72954,
of the age of 86 years, = 0.72759,
of the age of 87 years, = 0.69995,
of the age of 88 years, = 0.50505,
of the age of 89 years, = 0.31932,
of the age of 90 years, = 0.12872,
and of the age of 91 years, = 0.12019.

These near values, we may obſerve, are all leſs than the true values of the ſame annuities, given above in Table XLIX. And this will be found to be the caſe in moſt inſtances, except when ſome of the lives are very young; and then it ſometimes happens that the near values of an annuity for three joint lives, obtained by the foregoing expreſſion, is greater than its true value.

The differences of the foregoing near values of an annuity of one pound a year for three joint lives and the corresponding true values of the same annuity.

CCCCLXXXIV. The differences of these near values of the aforesaid annuities for three equal joint lives from their true values contained above in Table XLIX, are as follows; to wit,

£.		£.		£.
4.6115	—	4.3992	=	0.2123;
4.4382	—	4.2281	=	0.2101;
4.2626	—	4.0555	=	0.2071;
4.0849	—	3.8814	=	0.2035;
3.9050	—	3.7059	=	0.1991;
3.7230	—	3.5293	=	0.1937;
3.5390	—	3.3516	=	0.1874;
3.3533	—	3.1730	=	0.1803;
3.1662	—	2.9942	=	0.1720;
2.9780	—	2.8153	=	0.1627;
2.7895	—	2.6370	=	0.1525;
2.6015	—	2.4600	=	0.1415;
2.4160	—	2.2857	=	0.1303;
2.2352	—	2.1156	=	0.1196;
2.0636	—	1.9513	=	0.1123;
1.9089	—	1.7961	=	0.1128;
1.7846	—	1.6861	=	0.0985;
1.5843	—	1.5067	=	0.0776;
1.3906	—	1.3398	=	0.0508;
1.2121	—	1.1842	=	0.0279;
1.0685	—	1.0439	=	0.0246;
1.0117	—	0.982,01	=	0.029,69;
0.9617	—	0.923,50	=	0.038,20;
0.8981	—	0.879,17	=	0.078,93;
0.7606	—	0.729,54	=	0.031,06;
0.7690	—	0.727,59	=	0.041,41;
0.7568	—	0.699,95	=	0.056,85;
0.5368	—	0.505,05	=	0.031,75;

0.3233 — 0.319,32 = 0.003,98 ;
0.1346 — 0.128,72 = 0.005,88 ;
0.1202 — 0.120,19 = 0.000,01.

CCCCLXXXV. And the proportions of theſe differences to the ſaid true values themſelves are expreſſed by the following fractions, to wit,

The proportions of the foregoing differences to the ſaid ſeveral true values, reſpectively.

$\frac{1}{21}$, $\frac{1}{21}$, $\frac{1}{20}$, $\frac{1}{20}$, $\frac{1}{19}$, $\frac{1}{19}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$,

$\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{20}$, $\frac{1}{27}$, $\frac{1}{43}$, $\frac{1}{43}$, $\frac{1}{34}$, $\frac{1}{25}$, $\frac{1}{11}$, $\frac{1}{24}$, $\frac{1}{18}$,

$\frac{1}{13}$, $\frac{1}{16}$, $\frac{1}{81}$, $\frac{1}{22}$, and $\frac{1}{12020}$.

Theſe fractions (though they are not ſo ſmall as might have been wiſhed,) are ſufficiently ſmall to make the foregoing near values of an annuity of one pound a year for three equal joint lives of the ages of 60 years, 61 years, 62 years, &c. to the end of life, of very conſiderable uſe in practice. And therefore we may conclude from them that, in eſtimating annuities for three equal joint livesin the latter period of human life, after the age of 60 years, the expreſſion

A concluſion from the foregoing examples in favour of the uſefulneſs of the foregoing expreſſion, when the ages of the three lives are equal to each other and are not leſs than 60 years.

$$B \times \frac{P_I}{P} \times \frac{P_I \times P_I \times P_I \times r - P_{II} \times P_{II} \times P_I}{P_I \times P_I \times P_I \times r - P_{II} \times P_{II} \times P_{II}},$$ or $$B \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}},$$

may ſafely be adopted, as a means of determining the values of ſuch annuities to a tolerable degree of exactneſs.

CCCCLXXXVI. But it may be doubted, perhaps, whether this expreſſion will be equally uſeful in determining the values of annuities for three joint lives, when the ages of the lives are younger than 60 years, and when the three lives are not all of equal ages. Now, in order to form ſome judgement upon this matter, it will be neceſſary to try this approximation in ſome other inſtances, in which the three lives ſhall be much younger than 60 years of age, and likewiſe in ſome inſtances in which the three lives ſhall not be all of equal ages. And with this view I ſhall now preſent the reader

An inquiry into the uſefulneſs of the ſaid expreſſion in other caſes.

reader with four more tables of the value of annuities for three joint lives, of which the two former contain the accurate value of an annuity of one pound a year for three equal joint lives of the ages of 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35, 40, 45, 50, 55, 60, 65, and 70 years, and for three lives of unequal ages that differ from each other by 10 years and 20 years, and of which the youngeſt lives are of the ages of 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35, 40, 45, 50, 55, 60, 65, 70, and 75 years, and the two latter contain the near values of the annuities mentioned in the two former tables, which reſult from the expreſſion $\frac{\pounds}{B} \times \frac{P_1}{P} \times \frac{g \times P_1 - h \times P_1}{g \times P_1 - h \times P_{11}}$. The two firſt tables have been communicated to me by Dr. Price, and the two latter have been computed by an able arithmetician, who was employed for that purpoſe by Mr. Morgan, of the Society for equitable Aſſurances near Black-friars Bridge. In all the four tables the intereſt of money is ſuppoſed to be 4 per cent. and the table of the probabilities of the duration of human life, upon which the calculations are founded, is not that of Monſieur de Parcieux, (ſo often mentioned in the courſe of this work,) nor yet the Northampton table of thoſe probabilities given above in Art. ccccLxxix, but a new table derived from the ſaid Northampton table by Dr. Price, and called by him "*The new Northampton Table of thoſe probabilities*," being an improvement on the ſaid former Northampton table, and (as I am informed,) differing but little from it. Theſe four tables are as follows.

TABLE

TABLE LII.

Containing the true values of an annuity of one pound a year for the joint continuance of the lives of three perſons all of the ſame age, at the ſeveral ages of 5 *years,* 10 *years,* 15 *years,* 20 *years,* 25 *years,* 30 *years,* 35 *years,* 40 *years,* 45 *years,* 50 *years,* 55 *years,* 60 *years,* 65 *years, and* 70 *years; when the intereſt of money is* 4 *per cent.*

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life.

Years in the age of the firſt life.	*Years in the age of the ſecond life.*	*Years in the age of the third life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	5	5	11.1704
10	10	10	12.2006
15	15	15	11.2746
20	20	20	10.3429
25	25	25	9.796,42
30	30	30	9.221,10
35	35	35	8.585,22
40	40	40	7.865,05
45	45	45	7.126,40
50	50	50	6.317,17
55	55	55	5.550,60
60	60	60	4.755,03
65	65	65	3.914,00
70	70	70	2.995,84

TABLE LIII.

Containing the true values of an annuity of one pound a year for the joint continuance of the lives of three perſons of unequal ages, that differ from each other by 10 *years and* 20 *years, when the age of the youngeſt life is either* 5 *years, or* 10 *years, or* 15 *years, or* 20 *years, or* 25 *years, or* 30 *years, or* 35, *or* 40, 45 50, 55, 60, 65, 70, *or* 75 *years; upon a ſuppoſition that the intereſt of money is* 4 *per cent.*

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life.

Years in the age of the firſt, or youngeſt, life.	*Years in the age of the ſecond life.*	*years in the age of the third, or oldeſt, life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	15	25	10.6551
10	20	30	10.4379
15	25	35	9.738,56
20	30	40	8.986,72
25	35	45	8.313,10
30	40	50	7.570,83
35	45	55	6.816,07
40	50	60	5.994,15
45	55	65	5.145,62
50	60	70	4.219,37
55	65	75	3.297,98
60	70	80	2.408,48
65	75	85	1.623,48
70	80	90	1.122,51
75	85	95	0.169,378

TABLE LIV.

Containing approximations to the values of the annuities for three equal joint lives mentioned above in Table LII; when the intereſt of money is 4 *per cent.*

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by means of the expreſſion $\overset{£}{B} \times \frac{P^{1}}{P} \times \frac{g \times P^{1} - h \times P^{1}}{g \times P^{1} - h \times P^{11}}$, or $\overset{£}{B} \times \frac{P^{1}}{P} \times \frac{P^{1} \times P^{1} \times P^{1} \times r - P^{11} \times P^{11} \times P^{1}}{P^{1} \times P^{1} \times P^{1} \times r - P^{11} \times P^{11} \times P^{11}}$, *given above in Art.* CCCCLXXVI *and* CCCCLXXXI.

Years in the age of the firſt life.	*Years in the age of the ſecond life.*	*Years in the age of the third life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	5	5	10.4912
10	10	10	12.3942
15	15	15	11.4322
20	20	20	10.2377
25	25	25	9.667,62
30	30	30	9.073,29
35	35	35	8.426,20
40	40	40	7.691,08
45	45	45	6.941,47
50	50	50	6.118,36
55	55	55	5.342,53
60	60	60	4.543,77
65	65	65	3.718,59
70	70	70	2.839,99

TABLE LV.

Containing approximations to the values of the annuities for three joint lives of different ages, mentioned above in Table LIII; when the intereſt of money is 4 *per cent.*

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by means of the expreſſion $B^{£} \times \frac{P_1}{P} \times \frac{g \times P_1 - h \times P_1}{g \times P_1 - h \times P_{11}}$, *given above in Art.* CCCCLXXVI.

Years in the age of the firſt, or youngeſt, life.	*Years in the age of the ſecond life.*	*years in the age of the third, or oldeſt, life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	15	25	9.1325
10	20	30	10 5080
15	25	35	9.858,45
20	30	40	8.756,27
25	35	45	8.068,36
30	40	50	7.343,65
35	45	55	6.594,61
40	50	60	5.786,88
45	55	65	4.947,33
50	60	70	4.044,10
55	65	75	3.181,57
60	70	80	2.332,36
65	75	85	1.604,14
70	80	90	1.093,25
75	85	95	0.169,378

CCCCLXXXVII. The differences between the true values of the annuities for three equal joint lives, mentioned in Table LII, and the near values of the same annuities which are contained in Table LIV, are as follows; to wit,

The differences of the true values of the annuities for three equal joint lives in Table LII and the near values of the same annuities in Table LIV.

£.		£.		£.
11.1704	—	10.4912	=	0.6792;
— 12.2006	+	12.3940	=	0.1936;
— 11.2746	+	11.4322	=	0.1576;
10.3429	—	10.2377	=	0.1052;
9.796,42	—	9.667,62	=	0.128,80;
9.221,10	—	9.073,29	=	0.147,81;
8.585,22	—	8.426,20	=	0.159,02;
7.865,05	—	7.691,08	=	0.173,97;
7.126,40	—	6.941,47	=	0.184,93;
6.317,17	—	6.118,36	=	0.198,81;
5.550,60	—	5.342,53	=	0.208,07;
4.755,03	—	4.543,77	=	0.211,26;
3.914,00	—	3.718,59	=	0.195,41;
2.995,84	—	2.839,99	=	0.155,85.

CCCCLXXXVIII. It may be here observed that all the foregoing near values of annuities for three equal joint lives, contained in Table LIV, and which were obtained by means of the expression $\overset{£}{B} \times \frac{P_I}{P} \times \frac{P_I \times P_I \times P_I \times r - P_{II} \times P_{II} \times P_I}{P_I \times P_I \times P_I \times r - P_{II} \times P_{II} \times P_{II}}$, or $\overset{£}{B} \times \frac{P_I}{P} \times \frac{g \times P_I - h \times P_I}{g \times P_I - h \times P_{II}}$, are not less than the true values of the same annuities respectively, (as was the case with the near values of annuities for three equal joint lives of the ages of 60 years, 61 years, &c. to 91 years, obtained above in Art. CCCCLXXXII and CCCCLXXXIII by means of the same expression;) but two of the said near values, to wit, £12.3940 and £11.4322, (which relate to the ages of 10 years and 15 years,) are greater than the corresponding true values, £12.2006 and 11.2746.

A comparison of the said near values with the corresponding true values.

A remark on the magnitude of the ſaid differences with reſpect to one year's annuity.

CCCCLXXXIX. It may alſo be obſerved that all the foregoing differences between the true values and the near values of thoſe annuities, except the firſt difference £0.6792, (which relates to the age of 5 years, and is much greater than any of the others,) are leſs than £0.25, or a quarter of a year's annuity; which, with a view to practical purpoſes, can hardly be conſidered as a very important error.

The proportions of the ſaid differences to the ſeveral correſponding true values of the ſaid annuities.

CCCCXC. The proportions of the foregoing differences to their ſeveral correſponding true values are expreſſed by the following fractions; to wit, $\frac{1}{16}$, $\frac{1}{63}$, $\frac{1}{71}$, $\frac{1}{98}$, $\frac{1}{76}$, $\frac{1}{62}$, $\frac{1}{53}$, $\frac{1}{45}$, $\frac{1}{38}$, $\frac{1}{31}$, $\frac{1}{26}$, $\frac{1}{22}$, $\frac{1}{20}$, and $\frac{1}{19}$. Theſe fractions are, for the moſt part, conſiderably leſs than the fractions in Art. CCCCLXXXV, which expreſs the proportions of the differences of the near values and true values of the former ſet of annuities for three equal joint lives, (where the ages were 60 years and upwards,) to their correſponding true values. We may therefore conclude that the expreſſion $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{III}}$, or $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$, will be as uſeful, or rather more ſo, in computing the values of annuities for three equal joint lives, when the lives are under the age of 60 years as when they go beyond it.

The differences between the true values of the annuities for three joint lives of unequal ages, mentioned in Table LIII, and the near values of the ſame annuities in Table LV.

CCCCXCI. The differences between the true values of the annuities for three joint lives of unequal ages which differ from each other by 10 years and 20 years, given above in Table LIII, and the near values of the ſame annuities which are contained in Table LV, are as follows; to wit,

£		£		£
10.6551	—	9.1325	=	1.5226;
— 10.4379	+	10.5080	=	0.0701;
— 9.738,56	+	9.858,45	=	0.119,89;
				8.986,72

£.		£.		£.
8.986,72	—	8.756,27	=	0.230,45 ;
8.313,10	—	8.068,36	=	0.244,74 ;
7.570,83	—	7.343,65	=	0.227,18 ;
6.816,07	—	6.594,61	=	0.221,46 ;
5.994,15	—	5.786,88	=	0.207,27 ;
5.145,62	—	4.947,33	=	0.198,29 ;
4.219,37	—	4.044,10	=	0.175,27 ;
3.297,98	—	3.181,57	=	0.116,41 ;
2.408,48	—	2.332,36	=	0.076,12 ;
1.623,48	—	1.604,14	=	0.019,34 ;
1.122,51	—	1.093,25	=	0.029,26 ;
0.169,378	—	0.169,378	=	0.000,000.

CCCCXCII. In looking over the near values and true values contained in the foregoing article we may obſerve, that the near values are, for the moſt part, leſs than the correſponding true values; but not conſtantly ſo: for the near values £10.5080 and £9.858,45, (the former of which relates to three lives of the ages of 10, 20, and 30 years, and the latter to three lives of the ages of 15, 25, and 35 years,) are greater than the correſponding true values, £10.4379 and £9.738,56: which agrees pretty much with what was obſerved in Art. CCCCLXXXVIII concerning the near values and the true values contained in Art. CCCCLXXXVII.

A compariſon of the ſaid near values with the correſponding true values.

CCCCXCIII. It may alſo be obſerved that all the foregoing differences between the near values in Table LV and the correſponding true values in Table LIII, contained in Art. CCCCXCI, except the firſt difference, £1.5226, (which relates to three lives of the ages of 5 years, 15 years, and 25 years, and which is vaſtly greater than any of the others,) are leſs than £0.25, or a quarter of a year's annuity; which (as was before obſerved) is no very important variation from the truth.

A remark on the magnitude of the ſaid differences with reſpect to one year's annuity.

CCCCXCIV. And the proportions of the foregoing differences (contained in Art. CCCCXCI) to their ſeveral correſponding true values are expreſſed

The proportions of the ſaid differences to the ſeveral correſponding true values of the ſaid annuities.

preſſed by the following fractions; to wit, $\frac{1}{7}$, $\frac{1}{148}$, $\frac{1}{81}$, $\frac{1}{38}$, $\frac{1}{33}$, $\frac{1}{33}$, $\frac{1}{30}$, $\frac{1}{28}$, $\frac{1}{25}$, $\frac{1}{24}$, $\frac{1}{28}$, $\frac{1}{31}$, $\frac{1}{83}$, and $\frac{1}{38}$.

Theſe fractions are for the moſt part ſomewhat greater than the fractions in Art. CCCCXC, but ſomewhat leſs than thoſe in Art. CCCCLXXXV, which expreſs the proportions of the differences of the near values and true values of the former ſet of annuities for three equal joint lives (where the ages were 60 years and upwards,) to their correſponding true values. We may therefore conclude that the expreſſion $\overset{\text{£}}{B} \times \frac{P^{\text{I}}}{P} \times \frac{g \times P^{\text{I}} - h \times P^{\text{I}}}{g \times P^{\text{I}} - h \times P^{\text{II}}}$ will be as uſeful, or rather more ſo, in computing the values of annuities for three joint lives whoſe ages differ from each other by 10 years and 20 years, throughout all the periods of life, or, at leaſt, when the youngeſt life is not younger than 10 years, as in computing the values of annuities for three equal joint lives of the age of 60 years, or upwards.

A general concluſion, from all the foregoing trials, in favour of the uſefulneſs of the expreſſion $\overset{\text{£}}{B} \times \frac{P^{\text{I}}}{P} \times \frac{g \times P^{\text{I}} - h \times P^{\text{I}}}{g \times P^{\text{I}} - h \times P^{\text{II}}}$.

CCCCXCV. And from all theſe trials, taken together, it ſeems reaſonable to conjecture, that the expreſſion $\overset{\text{£}}{B} \times \frac{P^{\text{I}}}{P} \times \frac{g \times P^{\text{I}} - h \times P^{\text{I}}}{g \times P^{\text{I}} - h \times P^{\text{II}}}$ will always give us a tolerable approximation to the true value of an annuity for three joint lives of any ages whatſoever.

Another method of approximating to the value of an annuity for the joint continuance of three lives of given ages.

Of Mr. Thomas Simpſon's method of finding a near value of an annuity for three joint lives.

CCCCXCVI. The late very learned mathematician, Mr. Thomas Simpſon, of Woolwich Academy, in his book entitled, *Select Exerciſes in the Mathematicks*, page 279, has given us another method of finding a near value of an annuity for the joint continuance of three lives of any given ages, which is exceedingly ſhort and ſimple, and which alſo (as he informs us in the ſame book, page 312,) gives the quantity ſought to a very conſiderable degree of exactneſs, ſo as ſeldom to differ from the true value of the annuity by

by more than an eighth part of a years annuity. This method may be described as follows.

CCCCXCVII. Let the propoſed annuity, of which we are to find the value, be an annuity of one pound a year, as in all the foregoing inſtances. And let it be ſuppoſed that we have tables of the values of an annuity of one pound a year for ſingle lives of all ages, already computed to our hands; and likewiſe that we have tables of the values of an annuity of one pound a year for the joint continuance of two lives, computed to our hands, which contain a ſufficient variety of different ages of the two lives to enable us to find, (by the help of the method of Interpolation above explained in Art. ccccxi, &c. —— ccccxxxi,) the value of ſuch an annuity for any two propoſed lives of any given ages whatſoever to a conſiderable degree of exactneſs. A deſcription of the ſaid method.

Theſe things being ſuppoſed, we muſt, in the firſt place, find (by the tables, or by the rules given for finding the values of annuities for two joint lives,) the value of the propoſed annuity of one pound a year for the joint continuance of the two oldeſt of the three given lives. This value, it is evident, will be equal to the value of the ſame annuity of one pound a year for a ſingle life that is ſtill older than either of the ſaid two oldeſt of the three given lives. Let this fourth life be found by means of a table of the values of an annuity of one pound a year for ſingle lives, by looking along the column containing the values of ſuch an annuity till we meet with one that is equal, or nearly equal, to the ſaid value of the ſame annuity of one pound a year for the joint continuance of the ſaid two oldeſt of the three given lives. For the age correſponding to this value will be the age of this fourth, or imaginary, life.

And, laſtly, find the value of a like annuity of one pound a year for the joint continuance of this fourth, or imaginary, life, and the firſt, or youngeſt of the three given lives.

This laſt value will be nearly equal (as Mr. Simpſon aſſures us) to the value of the propoſed annuity of one pound a year during the joint continuance of the three given lives. Q E I.

CCCCXCVIII. This may be expreſſed more conciſely in the following manner. Call the youngeſt life *A*, the next *B*, and the third, or oldeſt, *C*. And let *D* be the fourth, or imaginary, life, an annuity for which is equal A more conciſe manner of expreſſing the foregoing deſcription.

equal in value to the ſame annuity for the joint continuance of the lives *B* and *C*.

Then, in the firſt place, we muſt find the value of an annuity of one pound a year for the joint continuance of the lives *B* and *C*. And, ſecondly, we muſt find the ſingle life *D*, an annuity for which is equal in value to the ſame annuity for the two joint lives *B* and *C*. And, thirdly, we muſt find the value of an annuity of one pound a year for the joint continuance of the two lives *D* and *A*. This laſt value will be nearly equal to the value ſought, or the value of an annuity of one pound a year for the joint continuance of all the three given lives. Q E I.

An example of the foregoing method of approximating to the value of an annuity for three joint lives.

CCCCXCIX Let us ſuppoſe the three given lives to be all of the ſame age of 60 years, as in the example given above in Art. CCCCLXXXII; and let the annuity be, as before, an annuity of one pound a year. And let it be required to find the value of this annuity for the joint continuance of theſe three lives of the age of 60 years, according to the Northampton table of the probabilities of the duration of human life given above in Art. CCCCLXXIX, Table XLI, and upon a ſuppoſition that the intereſt money is 4 per cent. To find this value by the foregoing method, we muſt proceed as follows.

The value of an annuity of one pound a year during the joint continuance of two of theſe lives, upon the ſuppoſitions here made, appears from Table L, Art. CCCCLXXVIII, to be £6.2468. We muſt therefore look into Mr. Morgan's table of the values of an annuity of one pound a year for ſingle lives in his treatiſe on the doctrine of annuities, pages 64, 65, 66, in order to find a value equal, or nearly equal, to the ſaid value £6.2468, that table of Mr. Morgan having been formed from the aforeſaid Northampton table of the probabilities of the duration of human life, and upon a ſuppoſition that the intereſt of money is 4 per cent. And we ſhall there find that the value that comes neareſt to £.6.2468 is £.6.263,15, which anſwers to the age of 70 years, or is the value of an annuity of one pound a year for a ſingle life of the age of 70 years. We muſt therefore, in the laſt place, ſeek for the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 60 years. Now the value of ſuch an annuity appears by Mr. Morgan's ſixth table (in the Appendix to his

his aforesaid Treatise on Annuities, pages 271, 272) to be, nearly, £.4.858. Therefore £.4.858 will (according to Mr. Simpson's assertion,) be nearly equal to the value of an annuity of one pound a year for the joint continuance of the three given lives, all of the age of 60 years. Q. E. I.

CCCCC. This near value, £4.858, of the said annuity of one pound a year for the said three equal joint lives of the age of 60 years, is somewhat greater than its true value, which we have seen above, in Table XLIX, to be £.4.7826. But the difference between them is but trifling, being only £.0.0754, which is less than the 63d part of the said true value £.4.7826, and less also than £.0.0833, or than the 12th part of a pound, or than the 12th part of a year's annuity, or than a month's annuity.

The difference between the foregoing near value of an annuity for three equal joint lives (obtained by Mr. Simpson's method,) and the true value of the same annuity.

CCCCCI. The former near value of this annuity for three equal joint lives of the age of 60 years, which was obtained in Art. CCCCLXXXII, by means of the expression $\overset{£}{B} \times \frac{P^I}{P} \times \frac{P^I \times P^I \times P^I \times r - P^{II} \times P^{II} \times P^I}{P^I \times P^I \times P^I \times r - P^{II} \times P^{II} \times P^{II}}$, or $\overset{£}{B} \times \frac{P^I}{P} \times \frac{g \times P^I - h \times P^I}{g \times P^I - h \times P^{II}}$, was £4.585; which is less than the true value £4.7826, and differs from it by the quantity £0.1976, which is greater than the difference £0.0754. Therefore in the present instance Mr. Simpson's method of approximating to the values of these annuities comes nearer to the truth than the other method. But whether or no it does so in general, can only be known by trying it in a variety of instances, and comparing the values resulting from it with the true values of the same annuities. With this view I shall present the reader with the two following tables of near values of the two sets of annuities, whose true values have been set down in Tables LII and LIII, computed according to the foregoing method of Mr. Simpson; which we may afterwards compare with the said true values. These tables of near values are as follows.

This near value (obtained by Mr. Simpson's method,) is more exact than the former near value of the same annuity, obtained in Art. CCCCLXXXII.

Two tables of near values of annuities for three joint lives, computed by Mr. Simpson's method of approximation.

TABLE LVI.

Containing approximations to the values of the annuities for three equal joint lives mentioned above in Table LII; when the intereſt of money is 4 *per cent.*

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by the foregoing method of Mr. Thomas Simpſon, of Woolwich.

Years in the age of the firſt life.	*Years in the age of the ſecond life.*	*Years in the age of the third life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	5	5	11.2119
10	10	10	12.2447
15	15	15	11.3543
20	20	20	10.4653
25	25	25	9.902,20
30	30	30	9.321,74
35	35	35	8.718,71
40	40	40	7.939,79
45	45	45	7.274,62
50	50	50	6.403,16
55	55	55	5.656,00
60	60	60	4.900,62
65	65	65	3.969,56
70	70	70	3.099,22

TABLE LVII.

Containing approximations to the values of the annuities for three joint lives of different ages, mentioned above in Table LIII; when the intereſt of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by the foregoing method of Mr. Thomas Simpſon, of Woolwich.

Years in the age of the firſt, or youngeſt, life.	*Years in the age of the ſecond life.*	*years in the age of the third, or oldeſt, life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	15	25	10.5974
10	20	30	10.4853
15	25	35	9.872,64
20	30	40	9.037,79
25	35	45	8.426,81
30	40	50	7.622,29
35	45	55	6.944,67
40	50	60	6.003,39
45	55	65	5.243,74
50	60	70	4.285,36
55	65	75	3.397,35
60	70	80	2.362,16
65	75	85	1.575,23
70	80	90	1.043,73

The differences of the true values of the annuities for three equal joint lives, in Table LII, and the near values of the same annuities (obtained by Mr. Simpson's method,) in Table LVI.

CCCCCII. The differences between the near values of annuities for three equal joint lives in Table LVI, and the true values of the same annuities in Table LII, are as follows; to wit,

—	11.1704	+	11,2119	=	0.0415;
—	12.2006	+	12.2447	=	0.0441;
—	11.2746	+	11.3543	=	0.0797;
—	10.3429	+	10.4653	=	0.1224;
—	9.796,42	+	9.902,20	=	0.105,78;
—	9.221,10	+	9.321,74	=	0.100,64;
—	8.585,22	+	8.718,71	=	0.133,49;
—	7.865,05	+	7.939,79	=	0.074,74;
—	7.126,40	+	7.274,62	=	0.148,22;
—	6.317,17	+	6.403,16	=	0.085,99;
—	5.550,60	+	5.656,00	=	0.105,40;
—	4.755,03	+	4.900,62	=	0.145,59;
—	3.914,00	+	3.969,56	=	0.055,56;
—	2.995,84	+	3.099,22	=	0.103,38.

A comparison between the said near values and the corresponding true values.

CCCCCIII. It is remarkable that all the near values in the foregoing article (which were obtained by Mr. Simpson's method,) are greater than the true values of the same annuities respectively; whereas the near values of them obtained above in Art. CCCCLXXXVI, Table LIV, by means of the expression $\overset{\pounds}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$, or $\overset{\pounds}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, were, for the most part, less than the said true values.

The proportions of the foregoing differences of the true and near values of the said annuities to the said true values, respectively.

CCCCCIV. The proportions of the foregoing differences in Art. CCCCCII to their corresponding true values are expressed by the following fractions;

fractions; to wit, $\frac{1}{269}$, $\frac{1}{290}$, $\frac{1}{141}$, $\frac{1}{84}$, $\frac{1}{92}$, $\frac{1}{91}$, $\frac{1}{64}$, $\frac{1}{105}$, $\frac{1}{48}$, $\frac{1}{73}$, $\frac{1}{52}$, $\frac{1}{32}$, $\frac{1}{70}$, and $\frac{1}{28}$.

CCCCCV. All these fractions, except the fourth, $\frac{1}{84}$, are considerably less than the fractions in Art. cccccxc, which express the proportions of the differences of the former near values of the same annuities (which were obtained by means of the expression $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$, or $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$,) from their several true values respectively, to the said true values. Therefore in these instances, as well as in the example given in Art. CCCCXCIX, Mr. Simpson's method of approximation seems to be preferable to the former method by means of the said expression.

All the foregoing near values of annuities for three equal joint lives (obtained by Mr. Simpson's method,) except one, are more exact than the near values obtained above in Art. CCCCLXXXVI, Table LIV.

CCCCCVI. The differences between the near values of annuities for three joint lives of different ages, contained above in Table LVII, and the true values of the same annuities in Table LIII, are as follows; to wit,

	£.		£.		£.
	10.6551	—	10.5974	=	0.0577;
—	10.4379	+	10.4853	=	0.0474;
—	9.738,56	+	9.872,64	=	0.134,08;
—	8.986,72	+	9.037,79	=	0.051,05;
—	8.313,10	+	8.426,81	=	0.113,71;
—	7.570,83	+	7.622,29	=	0.051,46;
—	6.816,07	+	6.994,67	=	0.128,60;
—	5.994,15	+	6.003,39	=	0.009,24;
—	5.145,62	+	5.243,74	=	0.098,12;
—	4.219,37	+	4.285,36	=	0.065,99;
—	3.297,98	+	3.397,35	=	0.099,37;
				+	2.408,48

The differences between the true values of annuities for three joint lives of different ages, in Table LIII, and the near values of the same annuities (obtained by Mr. Simpson's method,) in Table LVII.

$$+ \quad 2.408,48 \quad - \quad 2.362,16 \quad = \quad 0.046,32;$$
$$+ \quad 1.623,48 \quad - \quad 1.575,23 \quad = \quad 0.048,25;$$
$$+ \quad 1.122,51 \quad - \quad 1.043,73 \quad = \quad 0.078,78.$$

All the foregoing near values of annuities for three joint lives, of different ages, (which were obtained by Mr. Simpſon's method,) except the firſt and the three laſt, are greater than the true values of the ſame annuities, reſpectively.

CCCCCVII. It is remarkable that all the near values ſet down in Table LVII and in the foregoing article, (and which were obtained by Mr. Simpſon's method of approximation,) except the firſt and the three laſt, are greater than the ſeveral true values of the ſame annuities, reſpectively; as was obſerved in Art. cccccIII concerning all the near values, without any exception, that are ſet down in Table LVI and in Art. cccccII, and which were likewiſe obtained by Mr. Simpſon's method of approximation; whereas the near values of theſe annuities obtained above by means of the expreſſion $\overset{£}{B} \times \frac{P_1}{P} \times \frac{g \times P_1 - h \times P_1}{g \times P_1 - h \times P_{11}}$, and ſet down in Table LV and Art. ccccxcI, are, for the moſt part, leſs than the ſaid true values, reſpectively.

The proportions of the foregoing differences to the correſponding true values.

CCCCCVIII. The proportions of the foregoing differences in Art. cccccVI to their correſponding true values are expreſſed by the following fractions; to wit, $\frac{1}{184}$, $\frac{1}{220}$, $\frac{1}{72}$, $\frac{1}{176}$, $\frac{1}{73}$, $\frac{1}{147}$, $\frac{1}{53}$, $\frac{1}{648}$, $\frac{1}{52}$, $\frac{1}{63}$, $\frac{1}{33}$, $\frac{1}{51}$, $\frac{1}{33}$, and $\frac{1}{14}$.

All the foregoing near values of annuities for three joint lives of different ages, (obtained by Mr. Simpſon's method,) except the firſt and the two laſt, are more exact than the near values of the ſame annuities obtained above in Art. CCCCLXXXVI, Table LV.

CCCCCIX. All theſe fractions, except the third, to wit, $\frac{1}{72}$, and the two laſt, to wit, $\frac{1}{33}$, and $\frac{1}{14}$, are conſiderably leſs than the fractions in Art. cccexcIV, which expreſs the proportions of the differences of the former near values of the ſame annuities (which were obtained by means of the expreſſion $\overset{£}{B} \times \frac{P_1}{P} \times \frac{g \times P_1 - h \times P_1}{g \times P_1 - h \times P_{11}}$) from their ſeveral true values reſpectively to their ſaid true values. Therefore in the greater part of theſe inſtances, as well as in thoſe of the annuities for three equal joint lives contained

mined in Table LVI, and in the example given in Art. ccccxcix, this method of Mr. Simpſon ſeems to be preferable, in point of exactneſs, to the former method of approximation by means of the expreſſion $B^{\pounds} \times \frac{P^{1}}{P} \times \frac{g \times P^{1} - b \times P^{1}}{g \times P^{1} - b \times P^{11}}$. And it is certainly much ſhorter and eaſier in practice than that other method, which (as we have ſeen above in Art. ccccLxxxii) requires a good deal of calculation. And therefore I think it muſt be conſidered, upon the whole, as the better method of the two. Yet, as there are now and then ſome inſtances in which the other method comes nearer to the truth than this, I think it is convenient to be poſſeſſed of both methods, to the end that in doubtful caſes we may reſort to one of them as a kind of confirmation of the reſult obtained by the other to a moderate degree of exactneſs.

Mr. Simpſon's method of approximation ſeems, therefore, upon the whole, to be preferable to the former method of approximation by means of the expreſſion $B^{\pounds} \times \frac{P^{1}}{P} \times \frac{g \times P^{1} - b \times P^{1}}{g \times P^{1} - b \times P^{11}}$.

CCCCCX. The only thing that ſeems wanting to make Mr. Simpſon's method ſatisfactory, is a demonſtration of its truth, or an inveſtigation of it in ſome way or other. But this is what Mr. Simpſon has not given us. For he only ſays (in his *Select Exerciſes*, page 312,) "That the "reaſonableneſs of this method of proceeding is evident from the nature "of the ſubject, without calling in the aſſiſtance of any kind of computa"tion; and that, in a number of examples reſpecting lives of different "ages, he ſcarce ever found the error to exceed an eighth part of a year's "purchaſe." And this account of the degree of exactneſs to which this method of Mr. Simpſon gives the values of theſe annuities is confirmed by the foregoing trials; ſince of all the fourteen differences in Art. ccccci (which relate to annuities for three equal joint lives) only the ſeventh difference, to wit £0.133,49, the ninth, to wit, £.0.148,22, and the twelfth, to wit, £.0.145,59, are greater than £0.125,00, or an eighth part of a year's annuity; and of all the fourteen differences in Art. cccccvi, (which relate to annuities for three joint lives of different ages, which differ from each other by 10 years and 20 years,) only the third difference, to wit, £.0.134,08, and the ſeventh difference, to wit, £.0.128,60, are greater than £.0.125,00, or an eighth part of a year's annuity. It ſeems probable, therefore, that the differences of the near values of annuities for three joint lives, that would be obtained by this method of Mr. Simpſon in any other inſtances, (in which the ages of the lives were different from thoſe above-ſuppoſed,) from the true values of the ſame annuities, reſpectively, would ſeldom be greater than £.0.125,00, or one eighth part of a year's annuity; and conſequently, that this method of Mr. Simpſon will, in all thoſe

Of the degree of exactneſs to which it may reaſonably be conjectured that Mr. Simpſon's method of approximation will give, in moſt caſes, the value of an annuity for three joint lives.

thoſe caſes be a very uſeful method of approximating to the values of ſuch annuities. And with this I ſhall conclude what I had to offer concerning the valuation of annuities for the joint continuance of three lives of given ages.

[*End of the doctrine of the valuation of annuities for the joint continuance of three lives.*]

Of the values of annuities that depend on the continuance of the longeſt of two, or more, lives of given ages.

CCCCCXI. Having now gone through the doctrine of the valuation of annuities depending on the *joint* continuance of two, or more, lives of given ages, it remains that I ſhew how to eſtimate the values of annuities that depend on the continuance of *any one* out of two, or more, lives of given ages.

The principles upon which the determination of theſe values is founded, are explained above in Prob. IV. and its corollaries, Art. LVIII, LIX, LX, &c. —— —— LXXVI, pages 58, 59, 60, &c. —— —— 90. It is there ſhewn in Coroll. IV, pages 63, 64, 65, 66, "That the value of "an annuity of one pound a year for the lives of two perſons of given ages "and the life of the longer liver of them, is equal to the exceſs of the ſum "of two ſeparate annuities of one pound a year each, for the ſingle lives "of the ſame perſons, above the value of a like annuity of one pound a "year for their joint lives." And it is ſhewn in Coroll. XI. Art. LXXVI, pages 83, 84, 85, &c. —— —— 90, "That the value of an annuity of "one pound a year for the lives of three perſons of given ages, and the "life of the longeſt liver of them, is equal to the exceſs of the ſum of the "four following values, to wit, 1ſt. the value of an annuity of one pound "a year for the life of the firſt of the ſaid three perſons; 2dly, the value "of a like annuity for the life of the ſecond of them; 3dly, the value of a "like annuity for the life of the third of them; and, 4thly, the value of a "like annuity for the joint lives of all the three, above the ſum of the three "following values, to wit, 1ſt. the value of a like annuity of one pound a "year during the joint lives of the firſt and ſecond perſons; 2dly, the va-

"lue

" lue of the like annuity during the joint lives of the firſt and third perſons; " and, 3dly, the value of a like annuity during the joint lives of the ſe- " cond and third perſons." Therefore, if A be put for the value of an annuity of one pound a year for the life of the younger of two perſons of given ages, and B for the value of a like annuity of one pound a year for the life of the older of the ſaid two perſons, and AB be put for the value of a like annuity of one pound a year for the joint continuance of both lives; I ſay, if theſe are ſuppoſed to be the values of A, B, and AB, the value of a like annuity of one pound a year for the lives of both the ſaid perſons and the life of the longer liver of them, will be equal to $A + B - AB$. And, if C be put for the value of a like annuity of one pound a year for the life of a third perſon that is older than either of the two former perſons, and AC be put for the value of an annuity of one pound a year for the joint continuance of the firſt and third lives, and BC be put for the value of a like annuity of one pound a year for the joint continuance of the ſecond and third lives, and ABC be put for the value of a like annuity of one pound a year for the joint continuance of all the three lives; I ſay, if theſe are ſuppoſed to be the values of A, B, C, AB, AC, BC, and ABC, the value of an annuity of one pound a year for the life of the longer liver of the ſaid three perſons will be equal to $A + B + C + ABC - AB - AC - BC$, or $A + B + C - AB - AC - BC + ABC$. And, in like manner the value of an annuity for the longeſt of four, or more, lives may be deduced from the values of the like annuities for the ſame ſingle lives and for their joint continuance, by the principles laid down above in Prob. IV. and its corollaries, pages 58, 59, 60, &c.——90, whatever the number of lives may be. But it is ſeldom thought neceſſary, in treating of this ſubject of life-annuities, to ſuppoſe the lives to be more than three.

An example of the computation of the value of an annuity of one pound a year for the longeſt of two lives of given ages, by means of the expreſſion $A + B - AB$.

CCCCCXII. Let the younger life be ſuppoſed to be of the age of 20 years, and the older of the age of 30 years. And let the intereſt of money be ſuppoſed to be $3\frac{1}{2}$ per cent. and the probabilities of the duration of human life to be ſuch as they are repreſented to be in Monſieur de Parcieux's table.

Then will A, or the value of an annuity of one pound a year for the firſt, or younger life, be $= £19.440,616$; and B, or the value of a like annuity of one pound a year for the ſecond, or older, life, will be $= £18.068,798$; and AB, or the value of a like annuity of one pound a year for the joint continuance of both lives, will be $= £15.298,75$; as appears by Tables XV and XXXI, in pages 224 and 494. Therefore

$A + B - AB$, or the value of an annuity of one pound a year for the longer of the two given lives of the ages of 20 years and 30 years, will be $= £19.440,616 + £18.068,798 - £15.298,750$ ($= £37.509,414 - £15.298,750$) $= £22.210,664$. Q. E. I.

An example of the computation of the value of an annuity of one pound a year for the longeſt of three lives of given ages, by means of the expreſſion $A + B + C - AB - AC - BC + ABC$.

CCCCCXIII. Let the youngeſt life be ſuppoſed (as in the laſt example) to be of the age of 20 years, and the ſecond life to be of the age of 30 years; and let the third, or oldeſt, life be of the age of 40 years. And let the intereſt of money be ſuppoſed (as in the laſt example) to be $3\frac{1}{2}$ per cent. and the probabilities of the duration of human life to be ſuch as they are repreſented to be in Monſieur de Parcieux's table of them.

Then, in the firſt place, it appears from Table XV, page 224, that A, or the value of an annuity of one pound a year for the firſt, or youngeſt, life (which is of the age of 20 years) is $= £19.440,616$; and that B, or the value of an annuity of one pound a year for the ſecond life (which is of the age of 30 years) is $= £18.068,798$; and that C, or the value of an annuity of one pound a year for the third, or oldeſt, life (which is of the age of 40 years) is $= £16.104,542$. Therefore $A + B + C$ will be ($= £19.440,616 + £18.068,798 + £16.104,542$) $= £53.613,956$.

In the ſecond place, it appears from Table XXXI, page 494, that, when the intereſt of money is $3\frac{1}{2}$ per cent. the value of an annuity of one pound a year for two joint lives of the ages of 20 years and 30 years is $£15.298,75$; and the value of a like annuity for two joint lives of the ages of 30 years and 40 years is $£13.709,61$. And it appears from Table XXXII, page 495, that the value of an annuity of one pound a year for two joint lives of the ages of 20 years and 40 years (when the intereſt of money is at the ſame rate of $3\frac{1}{2}$ per cent.) is $£14.028,29$. Therefore AB is $= £15.298,75$, and BC is $= £13.709,61$, and AC is $= £14.028,29$; and conſequently $AB + BC + AC$ is ($= £15.298,75 + £13.709,61 + £14.028,29$) $= £43.036,65$. Therefore $A + B + C - AB - BC - AC$, or $A + B + C - AB - AC - BC$, is ($= A + B + C - £43.036,65 = £53.613,956 - 43.036,65$) $= £10.577,306$.

In the laſt place we muſt find the value of ABC, or of an annuity of one pound a year for the joint continuance of the three given lives of the ages of 20 years, 30 years, and 40 years. Now this may be done to a moderate degree of exactneſs by means of Mr. Simpſon's method of approximation, (above explained in Art. CCCCXCVII, CCCCXCVIII, pages 591, 592) in the manner following.

The

The value of an annuity of one pound a year for the joint continuance of the two older lives (which are of the ages of 30 years and 40 years) has been ſeen to be £13.709,61. We muſt therefore look into Table XV, page 224, in order to find the age of the ſingle life, for which an annuity of one pound a year will be worth the ſame ſum, or nearly the ſame ſum, as is the value of the ſame annuity of one pound a year for the joint continuance of the ſaid two older lives, (of the ages of 30 years and 40 years) to wit, the ſum of £13.709,61. Now it appears from Table XV, page 224, that the value of an annuity of one pound a year for a ſingle life of the age of 48 years (when the intereſt of money is 3 ½ per cent.) is £13.793,859, which is but little greater than £13.709,61. Therefore 48 years is, pretty nearly, the age ſought. We muſt therefore now ſeek the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 20 years and 48 years. This may be done in the manner following.

It appears from Table XXXII, page 495, that, when the intereſt of money is 3 ½ per cent. the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 20 years and 40 years is £14.028,29; and it appears from Table XXXIII, page 496, that, at the ſame rate of intereſt, the value of a like annuity of one pound a year for the joint continuance of two lives of the ages of 20 years and 50 years is £11.801,15. The difference of theſe two values, (or £14.028,29 — £11.801,15) is £2.227,14; and the fifth part of this difference is £0.445,428. Therefore, if we ſuppoſe the values of an annuity of one pound a year for the ſix following pairs of joint lives, to wit,

two joint lives of the ages of 20 years and 40 years,
two joint lives of the ages of 20 years and 42 years,
two joint lives of the ages of 20 years and 44 years,
two joint lives of the ages of 20 years and 46 years,
two joint lives of the ages of 20 years and 48 years,
and two joint lives of the ages of 20 years and 50 years,

to form, pretty nearly, an arithmetical progreſſion, or to decreaſe by, nearly, equal differences, the difference of the laſt value but one from the laſt value will be equal to one fifth part of the difference of the firſt value from the laſt, that is, to £0.445,428. Therefore by adding £0.445,428 to the laſt value, which is £11.801,15, we ſhall obtain, pretty nearly, the laſt value but one, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 20 years and 48 years; which value will therefore be, nearly = £12.246,578. This therefore is, pretty nearly, equal to the value of ABC, or of an annuity of one pound a year for the joint continuance of the three lives originally propoſed, which are of the ages of 20 years, 30 years, and 40 years. Therefore $A + B + C - AB - AC - BC + ABC$ will be nearly equal to $(A + B + C - AB - AC$

— *AC* — *BC* + £12.246,578, or to £10.577,306 + £12.246,578, or) £22.823,884; that is, the value of an annuity of one pound a year for the longeſt of three lives of the ages of 20 years, 30 years, and 40 years, when the intereſt of money is $3\frac{1}{2}$ per cent. will be, nearly, equal to £22.823,884.

CONCLUSION.

CCCCCXIV. I have now gone through every thing that ſeemed to me to be neceſſary to compleat the object of this treatiſe, which was to explain in a clear and familiar manner *the principles* of the doctrine of life-annuities. Much more, indeed, might be added to it concerning *the application* of this doctrine to a variety of uſeful queſtions concerning life-annuities, that may often occur in the courſe of men's dealings with each other; ſuch as the methods of finding the values of an annuity for one or more lives of given ages, that ſhall take place in caſe of the failure of one or more other lives of given ages, or in caſe of the failure of one, or more, other lives of given ages, before a third ſet of lives of given ages, or that ſhall depend on a variety of other contingencies. But for the ſolution of queſtions of this kind I ſhall refer my reader to Mr. Thomas Simpſon's *Doctrine of Life-Annuities*, and to his *Select Exerciſes in the Mathematicks*, and to Dr. Price's *Obſervations on Reverſionary Payments*, and Mr. Morgan's *Doctrine of Annuities and Aſſurances on Lives and Survivorſhips*, and to various other learned and uſeful tracts on this ſubject. I am contented with having explained the fundamental principles of the doctrine, and with having preſented the reader with a compleat ſet of tables of the values of life-annuities for ſingle lives at the ſeveral different rates of the intereſt of money at which Mr. Smart has given us tables of the values of annuities for terms of years, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. all fairly computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, without having recourſe to Mr. de Moivre's Hypotheſis, or any other inaccurate ſuppoſition, in order to facilitate the computation; and likewiſe with a conſiderable number of tables of the values of annuities for two joint lives at the two different rates of intereſt of $3\frac{1}{2}$ per cent and $4\frac{1}{2}$ per cent all fairly computed likewiſe from the ſaid table of Monſieur de Parcieux; and with having furniſhed the reader with rules for finding tolerably near values of other annuities, for two joint lives, and likewiſe of annuities for *three* joint lives, of any ages whatſoever, at the ſame, or any other, rates of intereſt; by all which, I flatter myſelf, the foregoing treatiſe will be juſtly intitled to be conſidered as a *uſeful appendix*, or *ſupplement*, to Mr. John Smart's very valuable *tables of intereſt*.

FINIS.

APPENDIX.

AS the tables of the values of remote life-annuities that were computed for the bill mentioned above in the ſcholium, page 34, (which was brought into the Houſe of Commons by the late Mr. Dowdeſwell in the year 1773,) have never been publiſhed, I preſume they will be thought to make no improper addition to thoſe which have been inſerted in the preceeding work. Nor can I ſuppoſe that a copy of that bill itſelf, to which the ſaid tables of the values of remote life-annuities were annexed, will be unwelcome to ſuch of my readers as ſhall approve the ſcope and view of it, which was "to encourage the poor to induſtry and frugality "by accommodating them with a ſafe and convenient method of "laying out what little money they could ſave out of the earnings "of their labour." I ſhall therefore now proceed to add to the foregoing ſheets an exact copy of this bill, in its laſt form, as it paſſed the Houſe of Commons, after a variety of amendments and improvements made in it, with great care and pains, by the gentlemen who were concerned for its ſucceſs, and alſo a copy of the tables of the values of remote life-annuities which were annexed to the ſaid bill, and were conſidered as a part of it. This bill was as follows.

A BILL,

A

BILL,

INTITULED

An ACT for the better Support of Poor Perſons in certain Circumſtances, by enabling Pariſhes to grant them Annuities for Life, upon Purchaſe, and under certain Reſtrictions.

WHEREAS it often happens that perſons engaged as journeymen in manufactures and handicraft trades, and likewiſe houſehold ſervants, labourers, and divers other perſons, get more money, as the wages of their labour and ſervice, than is ſufficient for their preſent maintenance, and might eaſily, if they were ſo minded, lay by, out of their ſaid gettings, a ſufficient ſum to provide for their ſupport in their old age : The preamble.

And whereas it would be highly uſeful, both to the ſaid perſons themſelves and to the nation in general, that they ſhould endeavour to make ſuch proviſion for their ſupport, in the latter periods of their lives, as they would thereby become more ſober and virtuous in their ordinary courſe of life, and more induſtrious in the proſecution of their ſeveral callings and employments,

employments, which would tend to the increaſe of the riches and manufactures of this kingdom:

And whereas it is probable that many of the ſaid perſons might be induced to lay up ſome part of their earnings in their youth and middle age, in order to make ſuch proviſion for their old age, if a convenient opportunity were offered them of employing the money they ſhould ſo lay up in a ſafe and advantageous manner:

And whereas the moſt ſafe and advantageous way, in which the ſaid frugal and induſtrious perſons can employ the ſeveral ſums of money, which they may be able to ſave out of their wages, for the ſupport of their old age, ſeems to be to purchaſe therewith annuities for their lives, which ſhould commence at ſome remote period, when their ſtrength and ability to work will be conſiderably impaired; and the poor's rates of the ſeveral pariſhes in England and Wales ſeem likely both to be, and to be thought by the ſaid induſtrious and frugal perſons, a ſufficient and convenient fund to ſecure, at all events, the payment of ſuch life-annuities as aforeſaid, to the ſeveral perſons who ſhall have purchaſed them, in caſe any deficiency ſhould happen in the fund created for the payment of the ſaid annuities, by the monies that ſhall have been paid for the purchaſe of them; be it therefore enacted by the king's moſt excellent majeſty, by and with the advice and conſent of the lords ſpiritual and temporal, and commons, in this preſent parliament aſſembled, and by the authority of the ſame, That, from and after the fifth day of July, one thouſand ſeven hundred and ſeventy-three, the plan and method herein-after preſcribed, for granting annuities to ſuch induſtrious and frugal perſons for their lives, ſhall take place and be carried into execution, in all pariſhes and townſhips which maintain their own poor, within England and Wales, and the town of Berwick upon Tweed, where there ſhall be two churchwardens, and two or more overſeers of the poor, or one churchwarden, and three or more overſeers of the poor, and where the ſame ſhall have been approved by a majority of the inhabitants of the ſaid pariſhes or townſhips, who are liable to be charged to the poor's rates of the ſame, aſſembled at two different pariſh meetings, held in the churches of the ſaid pariſhes or townſhips, after due notice given thereof, and at the diſtance of, at leaſt, twenty-one days one from the other, ſubject to the rules and regulations herein-after preſcribed; ſuch majority in number being alſo charged, in the laſt rate made for the relief of the poor of the ſaid pariſh or townſhip, in a ſum greater than that which was aſſeſſed in the ſaid rate upon the reſt of the inhabitants aſſembled at the ſaid meeting; and ſuch notice ſhall expreſs the times of both the ſaid meetings, and ſhall be publiſhed in the reſpective pariſh churches, on a Sunday immediately after divine ſervice, and affixed upon the door of the ſaid pariſh church,

After July 5, 1773, the following plan ſhall take place in all pariſhes and townſhips in England, in which the majority of the inhabitants, both in number and value, ſhall adopt it in two different pariſh-meetings held for the purpoſe after due publick notice.

church, by the clerk of the ſaid pariſh, or the perſon acting as ſuch, who is and are hereby required to publiſh and affix the ſame, upon the requeſt of any perſon deſirous of purchaſing any ſuch annuity; and the rector, vicar, or perpetual curate of any ſuch pariſh or townſhip, together with the churchwardens and overſeers of the poor in the ſame, ſhall be, and they are hereby, authoriſed and required to receive, on the behalf of all the ſaid inhabitants of ſuch pariſh or townſhip ſo liable, as is aforeſaid, to be charged to the poor's rate of the ſame, from any perſon to whom the ſaid inhabitants ſhall have agreed to grant a like annuity in manner herein-after mentioned, any ſum of money that ſuch perſon ſhall have agreed to pay them for the purchaſe of the ſame; and to grant to ſuch perſon, in the name and on the part of the ſaid rateable inhabitants of the ſame pariſh or townſhip, an annuity for the life of the grantee, equivalent to the purchaſe-money, to commence at ſuch future period of ſuch grantee's life, as ſhall have been ſettled between the ſaid inhabitants and the ſaid purchaſer, ſubject to the reſtrictions herein-after mentioned; and the poor's rate of ſuch pariſh or townſhip ſhall be, and they are hereby declared to be, ſubject by ſuch grant to the payment of the annuity thereby granted, from time to time, as the ſame ſhall become due and payable; and, if it ſhall happen that the rector, vicar, or perpetual curate of any ſuch pariſh or townſhip, ſhall be abſent from ſuch pariſh or townſhip, when the grant of any ſuch annuity ſhall be demanded, and ſhall continue to be abſent from the ſame for the ſpace of thirty days from the ſame time, it ſhall and may be lawful for the churchwardens and overſeers of the poor of the ſaid pariſh or townſhip, and they are hereby impowered and required, after the expiration of the ſaid thirty days from ſuch demand being made, notwithſtanding the abſence of ſuch rector, vicar, or perpetual curate, to make grants of ſuch annuities, without the concurrence of ſuch rector, vicar, or perpetual curate; and ſuch grants ſhall be, and are hereby declared to be, as good, valid, and effectual, as if ſuch rector, vicar, or perpetual curate, had been preſent, and concurred in the making thereof.

The rector, or vicar, or perpetual curate, of the pariſh, together with the church wardens and overſeers thereof, ſhall have power to make grants of life-annuities to any inhabitants thereof for certain reaſonable prices to be paid by the ſaid grantees, under certain reſtrictions.

And the poors rates of ſuch pariſhes ſhall become liable, in conſequence of theſe grants, to the payment of ſuch annuities, when they become due.

And in caſe of the abſence of the rector, vicar, or perpetual curate, for the ſpace of 30 days together, the ſaid grants may be made by the church wardens and overſeers of the poor without them.

And for obviating all doubts that may ariſe touching the value of ſuch life-annuities, and the money to be paid for the purchaſe thereof, be it further enacted by the authority aforeſaid, That in all pariſhes and townſhips within the cities of London and Weſtminſter, or of Southwark, the ſaid annuities ſhall be granted according to the reſpective rates herein-after mentioned and expreſſed in the ſet of tables hereunto annexed, No. I. and in all other pariſhes and townſhips within England, Wales, and the town of Berwick upon Tweed, the ſaid annuities ſhall be granted according to the reſpective prices herein-after mentioned and expreſſed in the ſet of tables hereunto annexed, No. II; and if any ſuch life-annuity ſhall be granted to any

The prices to be paid for ſuch annuities ſhall be thoſe which are contained in the tables hereunto annexed; of which the firſt ſet, called No. I, ſhall relate to annuities granted in the cities of London, Weſtminſter, and the borough of Southwark; and the ſecond ſet, called No. II, ſhall relate to thoſe granted in all other parts of England and Wales.

any perſon under the age of fifteen years, the ſame price ſhall be paid for ſuch annuity as if the grantee thereof was fifteen years of age.

No ſum leſs than 5l. ſhall be employed in the purchaſe of any of theſe annuities. And no annuity ſhall be greater than 20l. *per annum*.

Provided always, That no ſmaller ſum than five pounds of lawful money of Great Britain ſhall be laid out in the purchaſe of any of the ſaid annuities, nor ſhall any perſon be allowed to purchaſe any annuity or annuities from any one pariſh or townſhip, to a greater amount than twenty pounds per annum; nor ſhall any of the ſaid annuities commence after the grantee thereof ſhall have attained his age of ſeventy-five years; nor ſhall it commence for the life of any man, before he ſhall have attained the age of fifty years, nor for the life of any woman before ſhe ſhall have attained the age of thirty-five years; nor ſhall any ſuch annuity be granted to any perſon for any other life than that of the grantee.

Annuities granted to men ſhall not take place till they ſhall be 50 years old, nor to women till they ſhall be 35 years old.

And be it further enacted, That no ſuch annuity ſhall be granted, in any pariſh or townſhip, where the number of inhabitants aſſeſſed towards the relief of the poor ſhall exceed the number of eighteen, unleſs twelve, at the leaſt, of ſuch inhabitants ſhall be preſent in a veſtry, or other publick meeting to be held for that purpoſe; and in pariſhes or townſhips where the whole number of inhabitants, liable to be aſſeſſed as aforeſaid, ſhall be leſs than nineteen, no ſuch annuity ſhall be granted, unleſs at a veſtry or other publick meeting, where two-thirds of ſuch inhabitants ſhall be preſent; nor ſhall any ſuch annuity be granted, at any of the ſaid meetings, unleſs the major part of the inhabitants ſo aſſembled, being alſo charged in the laſt poor's rate, as aforeſaid, in a ſum greater than what is aſſeſſed in the ſame rate upon the reſt of the inhabitants there aſſembled, ſhall agree to grant the ſame; and no ſuch meeting ſhall be held for granting ſuch annuities, unleſs previous notice of holding a meeting for that purpoſe ſhall be publiſhed in the pariſh church upon two Sundays immediately after divine ſervice, and a written copy of ſuch notice affixed upon the door of ſuch church previous to the firſt publication thereof, as aforeſaid; and the clerk of the ſaid pariſh, or the perſon acting as ſuch, is and are hereby required to publiſh and affix the ſame, at the requeſt of any perſon deſirous of purchaſing ſuch annuity; and no ſuch annuity ſhall be granted to any perſon who ſhall not (in the opinion of the majority of ſuch inhabitants ſo aſſembled) appear to have a legal ſettlement within the pariſh or townſhip upon which the ſame ſhall be charged.

Theſe annuities ſhall not be granted without the conſent of ſuch a majority of the pariſhioners preſent at the meetings in which they are granted, as pays more to the poor's rate than the other pariſhioners there preſent.

Nor ſhall they be granted to any perſons but ſuch as are ſuppoſed to have a legal ſettlement in the pariſh.

Provided nevertheleſs, That if any annuity ſhall have been granted in the manner directed by this act, ſuch grant ſhall be valid and binding upon the pariſh, or townſhip, granting the ſame; and the annuity thereby granted ſhall be regularly paid, according to the purport and true meaning of ſuch grant,

Yet, if, through miſtake, they are granted in the manner directed in this act, to inhabitants of the pariſh who have not a legal ſettlement in it, they ſhall nevertheleſs be valid and binding.

grant, notwithſtanding it ſhall afterwards be diſcovered that, at the time of making ſuch grant, the grantee thereof was not legally ſettled in ſuch pariſh or townſhip.

The deeds of grant ſhall be made out on parchment.

And be it further enacted, That the ſaid annuities ſhall be granted by deeds of grant, fairly written or printed on parchment, and ſigned and ſealed by the churchwardens and overſeers of the poor, and by the rector, vicar, or perpetual curate (unleſs he ſhall be abſent as aforeſaid) of the pariſh, or townſhip, in behalf of which ſuch annuities ſhall be granted, and may be made in the form, or to the effect, following:

The form of the deeds by which theſe annuities ſhall be granted.

" AT a public meeting of the inhabitants of the pariſh or townſhip " of in the county of " holden in the veſtry of the church of the ſaid pariſh or townſhip, on the " day of in the year of the reign " of our ſovereign lord King George the Third, and in the year of our " Lord Chriſt after due " notice of the ſaid meeting being firſt given.

" *A. B.* rector (or vicar, or perpetual curate, as it may happen to be) " of the pariſh or townſhip of aforeſaid, in the county " aforeſaid, *C. D.* and *E. F.* churchwardens of the ſaid pariſh or townſhip, " *G. H. I. K.* and *L. M.* overſeers of the poor of the ſaid pariſh or town- " ſhip; To all to whom this preſent writing ſhall come, ſend greeting: " Whereas *N. O.* of the ſaid pariſh or townſhip aforeſaid, Bricklayer, (or " houſhold ſervant to P. Q. Eſquire, of the ſaid pariſh or townſhip, or day- " labourer or otherwiſe, *according to his proper addition or employment)* " appearing to us to be a perſon lawfully ſettled in the ſaid pariſh or town- " ſhip, and intitled to be relieved by the poors rate raiſed in the ſame, in " caſe he ſhould become poor and helpleſs, and now of the the age of " years, hath paid unto the hands of us the rector, or vicar, or " perpetual curate, churchwardens, and overſeers of the poor of the pa iſh " or townſhip, aforeſaid, the ſum of pounds of lawful " money of Great Britain, as the price of an annuity for his life, that ſhall " begin when he is years of age, that is, at the end " of years from this preſent time, and in the year of our " our Lord Chriſt to be paid " to him, or his certain attorney, by the rector, churchwardens, and over- " ſeers of the poor of the ſaid pariſh or townſhip for the time being, by " equal quarterly payments to be made at the four following feaſt days; " to wit, the feaſt day of the annunciation of the Bleſſed Virgin Mary, the " feaſt day of Saint John the Baptiſt, the feaſt day of Saint Michael the

" Archangel, and the feaſt day of the Nativity of our Lord Chriſt, or " within ſeven days after each of the ſaid feaſt days, according to the ſtatute " of the thirteenth year of the reign of King George the Third, in that " behalf made and provided: NOW know ye, that we the rector, (vicar, or " perpetual curate,) churchwardens, and overſeers of the poor of the ſaid " pariſh or townſhip, in conſideration of the ſaid ſum of pounds, " to us in hand paid, (the receipt whereof is hereby acknowledged,) and " in purſuance of the ſtatute aforeſaid, do by this preſent writing, in the " behalf of the inhabitants of the ſaid pariſh or townſhip of " give and grant unto the ſaid *N. O.* an annuity or yearly penſion of " of of lawful money of Great Britain, which ſhall " commence on the feaſt day of that ſhall be in " the year of our Lord Chriſt " and continue to be paid to the ſaid *N. O.* from thence forwards during " his life, by equal quarterly payments, at the four feaſt days above men- " tioned, or within ſeven days after each of them reſpectively: and fur- " ther, by virtue of the ſtatute aforeſaid, we do bind and engage the rates " of the ſaid pariſh or townſhip, that ſhall hereafter be raiſed therein for " the relief of the poor thereof, for the full and due payment of the ſaid " annuity of to the ſaid *N. O.* by equal quarterly " payments, on the four feaſt days above-mentioned, or within ſeven days " after each of them reſpectively, from the ſaid feaſt day of " to the feaſt day or day of payment that ſhall happen " next before the death of the ſaid *N. O.* including both the ſaid days. " In witneſs whereof, we have hereunto ſet our hands, and have fixed our " common ſeal, this day of in the " year of the reign of our ſovereign lord George the Third, and in the " year of our Lord Chriſt.

A. B. *Rector, or Vicar, or perpetual Curate, of the ſaid Pariſh or Townſhip.*

The Common Seal of the Pariſh of in the County of

C. D. } *Churchwardens.*
E. F. }

G. H. }
I. K. } *Overſeers of the Poor.*
L. M. }

And in caſe of the abſence of the rector, vicar, or perpetual curate as aforeſaid, ſuch grant ſhall be made in the names of the churchwardens and overſeers of the poor only; and ſuch abſence of the rector, vicar, or perpetual curate, ſhall be ſpecified in the ſame, and the charges and expences attending

ing the making out every ſuch grant ſhall be defrayed by the perſon to whom ſuch annuity ſhall be granted, who ſhall pay to the ſaid pariſh officers the ſum of two ſhillings and ſixpence for the ſame, and no more.

Such deed of grant ſhall be made out by the ſaid grantors for the ſum of 2s. 6d. to be paid them by the grantee.

And be it further enacted, That, in every ſuch deed of grant, the pounds, ſhillings, and pence, thereby granted, and the date of the year of our Lord Chriſt in which ſuch annuity is to commence, ſhall be written in words at length, and not in figures; and ſuch deeds of grant, immediately after the ſame ſhall be ſigned and ſealed as aforeſaid, ſhall be delivered to the grantee of the annuity thereby granted, to be kept by him or her, as the proof of his or her right to ſuch annuity.

The amount of theſe annuities, and the dates of their commencement, ſhall be written in theſe deeds in words at length.

And be it further enacted by the authority aforeſaid, That the rector, vicar, or perpetual curate, and the churchwardens and overſeers of the poor of every ſuch pariſh or townſhip, ſhall, and they are hereby required to cauſe a copy of every ſuch deed of grant to be entered in a book, to be kept in the pariſh cheſt for that purpoſe, and to ſeal and ſign the ſaid copy in the ſame manner as the original deed is hereby directed to be ſigned and ſealed, (both which ſhall be executed at a public meeting of the inhabitants of the pariſh or townſhip, to be holden for that purpoſe;) and the grantee of the ſaid annuity ſhall, at the ſame time ſign and ſeal an acknowledgement in writing, which ſhall be put at the bottom of the ſaid copy, in the words or to the effect following:

A copy of every ſuch deed of grant ſhall be entered by the grantors of it in a book to be kept in the pariſh for that purpoſe; and an acknowledgement of its being a true copy, ſhall be ſigned and ſealed by the grantee of the annuity.

I N. O. *of the pariſh or townſhip of* *in the county of* *do acknowledge, that the above is a true copy of the deed of grant of a certain life-annuity, which has been this day granted to me by the rector, vicar, or perpetual curate, churchwardens, and overſeers of the poor of the ſaid pariſh or townſhip, or by the churchwardens and overſeers of the poor*, in caſe of the abſence of the rector, vicar, or perpetual curate, as aforeſaid.

N. O.

Which ſaid copy ſhall be made at the expence of the inhabitants of the ſaid pariſh or townſhip, and be paid for out of the money received as the price of the ſaid annuity, from the ſaid grantee thereof, without any new expence to the ſaid grantee: and if it ſhall at any time happen, that any original deed of grant, delivered as aforeſaid, ſhall be loſt or deſtroyed, the copy thereof ſo entered as aforeſaid ſhall be deemed ſufficient evidence that ſuch grant had been made, and ſhall intitle ſuch grantee to receive his annuity according to the purport of ſuch copy. And, if ſuch grantee ſhall deſire to have a copy of ſuch deed of grant made from the ſaid pariſh copy, inſtead of the original deed ſo loſt or deſtroyed, the rector, vicar, or perpetual curate,

And, if the grantee of the annuity ſhall loſe his deed of grant of it, ſuch copy of the ſaid deed in the pariſh book ſhall be ſufficient evidence of it.

And the pariſh-officers ſhall give the grantee a new copy of the loſt deed of grant of his annuity from ſuch pariſh-copy of it, for the ſum of 2s. 6d.

curate, churchwardens, and overſeers of the poor of the ſaid pariſh or townſhip for the time being, ſhall cauſe ſuch copy to be made out upon parchment, and ſhall ſign the ſame, in atteſtation of its being a true copy from the ſaid copy in the pariſh book, and ſhall deliver ſuch atteſted copy to ſuch grantee, without requiring any new conſideration-money for the ſame, or any fee or reward, except the neceſſary expence of making out ſuch new copy; for which they ſhall take the ſum of two ſhillings and ſixpence, and no more: and the ſaid new copy, ſo atteſted, ſhall be ſufficient evidence that ſuch grant had been made, and ſhall intitle the grantee to receive his annuity from the ſaid pariſh or townſhip, according to the purport of ſuch atteſted copy, notwithſtanding the copy in the pariſh book ſhould afterwards happen to be loſt or deſtroyed.

And the ſaid new copy of the ſaid deed of grant ſhall be ſufficient evidence of its having been made, although the copy in the pariſh-book ſhould afterwards be loſt.

And be it further enacted by the authority aforeſaid, That, previous to the holding of any meeting in purſuance of this act, the overſeers of the poor for the pariſh or townſhip where ſuch meeting ſhall be held, ſhall cauſe proper books to be provided and kept in the public cheſt, belonging to ſuch pariſh or townſhip; in which books all orders, proceedings, and accounts, relating to the execution of this act, ſhall be fairly entered at length, by ſome perſon to be appointed by the reſpective churchwardens and overſeers of the poor; and the names of all the perſons who ſhall give a vote for, or againſt, the making of any order, or the coming to any reſolution relating to the execution of this act, ſhall be entered in one of the ſaid books, and the perſons agreeing to, or concurring in, ſuch order or reſolution ſhall ſign the ſame; and an account of all monies received or payed in purſuance of this act ſhall alſo be entered in one of the ſaid books, expreſſing particularly the times of all ſuch receipts and payments, the perſon to, or from, whom the ſame were paid or received, and for what purpoſes; which accounts ſhall be balanced and cloſed once in every year, as near as conveniently may be to the time for holding the petty ſeſſions for appointing overſeers of the poor for ſuch reſpective pariſh or townſhip, and (being ſigned by the rector, vicar, or perpetual curate, and the churchwardens and overſeers of the poor, for each ſuch pariſh or townſhip,) ſhall be laid before the juſtices of the peace at ſuch petty ſeſſions, who ſhall examine ſuch accounts with the vouchers thereto; and, if the ſame ſhall appear to ſuch juſtices to be juſt and true accounts, they ſhall certify the ſame in writing upon ſuch accounts; and a duplicate of every ſuch account ſhall be tranſmitted by the reſpective overſeers of the poor to the clerk of the peace for the county, riding, or diviſion, wherein ſuch pariſh or townſhip is ſituated, (whether the ſame be a town-corporate, having excluſive juriſdiction or not,) before the next general quarter ſeſſions of the peace; and each and every clerk of the peace is hereby required to cauſe every ſuch account to be filed amongſt the records of his office.

The overſeers of the poor ſhall enter all proceedings relating to the execution of this act, in proper books to be provided for that purpoſe.

An account of all the monies received and paid by virtue of this act ſhall be entered in one of theſe books. And the accounts ſhall be balanced once in every year, and laid before the juſtices of the peace at the petty ſeſſions, to be examined by them; and ſhall be certified by the ſaid juſtices to be juſt and true, if found to be ſo.

And a duplicate of every ſuch account ſhall be tranſmitted to the clerk of the peace for the county, before the next general quarter ſeſſions of the peace.

And

No ſtamp-duty ſhall be paid for the ſaid deeds of grant, or the copies of them.

And be it further enacted, That no deed of grant, or any of the copies thereof herein before directed to be made, nor any power of attorney for accepting and transferring ſtock, and for the receiving the dividends due thereon, ſhall be charged or chargeable with any ſtamp-duty whatſoever, but ſhall be good, valid, and effectual, to all intents and purpoſes, without any ſtamp being impreſſed thereon.

The ſaid annuities may, with the conſent of the grantees thereof expreſſed in the deeds of grant, be made unalienable.

And be it further enacted, That if any grantee of any ſuch life-annuity ſhall conſent that the ſame may be made unalienable, and thereupon a clauſe for that purpoſe, expreſſing his conſent that it ſhould be made ſo, ſhall be inſerted in the deed of grant delivered to him, and in the copy thereof kept in the pariſh regiſter of the ſaid grants of life-annuities, and which he ſhall have acknowledged to be a true copy of his ſaid grant in the manner above directed, in ſuch caſe the ſaid annuity ſhall be payable to the ſaid grantee alone, or his certain attorney, during his life, without any power in him to alienate, or aſſign, it to any perſon, or in any manner, whatſoever; and every aſſignment of ſuch annuity that ſhall afterwards be made by the ſaid grantee to any other perſon, ſhall be totally void in law and equity, and of no effect or operation whatſoever.

And, when they are not made unalienable, they ſhall not be alienated before an offer has been made of them to the pariſh by which they have been granted, at the values ſet down in the tables hereunto annexed.

But, if the pariſh refuſes to buy them in, the grantees ſhall be at liberty during the ſpace of ſix months to ſell them to any other perſons.

And be it further enacted, That if any grantee of any of the ſaid annuities that ſhall not have conſented as above to make his ſaid annuity unalienable, ſhall be deſirous of ſelling or diſpoſing of any ſuch annuity to any other perſon, he ſhall, in the firſt place, make an offer to ſell the ſame to the rector, vicar, or perpetual curate, churchwardens and overſeers of the poor, and other inhabitants chargeable to the poors rates of the ſaid pariſh or townſhip where the ſame was bought, at the price which ſuch annuity ſhall be then worth, according to the rates herein-after mentioned, or at any lower price; which offer ſhall be made at a veſtry or other publick meeting of the inhabitants of the ſaid pariſh or townſhip, notice of which meeting ſhall be given on two different Sundays, in the ſame manner as herein-before directed, concerning meetings for granting the ſaid annuities; and, if, upon ſuch offer being made, the major part of the rateable inhabitants ſo aſſembled, being alſo charged in the laſt poors rate as aforeſaid, in a ſum greater than what is aſſeſſed in the ſame rate upon the reſt of the inhabitants there aſſembled, ſhall think proper to purchaſe the ſame, the rector, vicar, or perpetual curate, and the churchwardens and overſeers of the poor of ſuch pariſh or townſhip, or the churchwardens and overſeers of the poor only, in the abſence of ſuch rector, vicar, or perpetual curate as aforeſaid, ſhall be, and are hereby, authorized and required to buy up the ſaid annuity, at the ſaid price, or at any lower price as ſhall be agreed for, and to defray the expence of ſuch purchaſe out of the fund created by the monies received from the grantees of the ſaid life-annuities; and in caſe the major part of the inhabitants

inhabitants, ſo aſſembled as aforeſaid, ſhall not agree with the ſaid grantee for the purchaſe of the ſaid annuity, the grantee thereof ſhall, for the ſpace of ſix months from the time of ſuch refuſal or non-agreement, be at liberty to ſell and aſſign his ſaid annuity to any perſon whatſoever, by a deed duly ſealed and delivered in the preſence of two credible witneſſes, and ſigned by the ſaid witneſſes, in atteſtation of their having ſeen the ſame ſo ſigned and delivered: and after the expiration of the ſaid ſix months, the ſaid grantee ſhall not have power to ſell or aſſign ſuch annuity to any perſon whatſoever, unleſs he ſhall again make an offer thereof, in the manner before preſcribed, to the rector, vicar, or perpetual curate, and churchwardens and overſeers of the poor, and other rateable inhabitants of the ſaid pariſh or townſhip; but in caſe of their refuſal or non-agreement to purchaſe the ſame, ſuch grantee ſhall again be at liberty, during the ſpace of ſix months from ſuch refuſal or non-agreement, to aſſign ſuch annuity to any perſon whatſoever: and ſo, in like manner, as often as he ſhall be deſirous of ſelling the ſaid annuity, he ſhall acquire a right ſo to do, for the ſpace of ſix months after making ſuch offer, and the refuſal of the ſame, or non-agreement, as aforeſaid. And all aſſignments made of ſuch annuities by the grantees thereof, otherwiſe than according to the rules herein preſcribed, ſhall be utterly void and of no effect. But, when any of the ſaid annuities ſhall have been aſſigned by the grantee thereof to any other perſon, the ſaid perſon, to whom it ſhall have been ſo aſſigned, ſhall be at liberty to aſſign it to whom he ſhall think fit, and in whatever time and manner he ſhall think fit, agreeable to the uſual rules of law, without being obliged to make a previous offer of it to the pariſh officers and rateable inhabitants of the pariſh, or townſhip, in which it was granted, as is required of the grantee thereof; and every ſubſequent aſſignee thereof ſhall have the ſame liberty.

After any ſuch annuity ſhall have been ſold, or aſſigned once, it may be aſſigned again without making any previous offer of it to the pariſh.

And be it further enacted, That, if any perſon ſhall at any time make, or cauſe to be made, any alteration in any deed of grant of any ſuch annuity, or any copy thereof which ſhall be made out as aforeſaid, ſo as to make the annual ſum thereby granted appear to be greater than it really was, or the time of the commencement of ſuch annuity appear to be earlier than that which was appointed by the ſaid deed, or ſhall know that any ſuch alteration has been made in ſuch deed or copy, and having made, or cauſed to be made, ſuch alteration, or knowing the ſame to have been made therein, ſhall demand and receive of the pariſh officers of the pariſh or townſhip in which ſuch annuity was granted, any part of ſuch annuity, according to the purport of ſuch fraudulent alteration, every perſon ſo offending, and being convicted thereof, ſhall be deemed and adjudged guilty of felony, and ſhall ſuffer death as a felon, without benefit of clergy; and, upon ſuch conviction, ſuch annuity ſhall ceaſe and determine.

If any perſon ſhall make any fraudulent alteration in any of the ſaid pariſh deeds of grant, or, knowing any ſuch fraudulent alteration to have been made, ſhall receive any money upon the ſame, he ſhall thereby become guilty of felony without benefit of clergy. And, upon his conviction the annuity ſhall ceaſe.

And

And be it further enacted, That if any perſon ſhall forge or frame, or cauſe to be forged or framed, any deed of grant of any ſuch pariſh annuity as is above mentioned, with an intent to defraud the inhabitants of the pariſh or townſhip, in which ſuch deed ſhall purport ſuch annuity to have been granted, and ſhall for that purpoſe counterfeit, or cauſe to be counterfeited, the names of the pariſh officers which are herein required to be ſubſcribed to every ſuch deed of grant, and, by means of ſuch forged deed, ſhall obtain from the ſaid officers of the pariſh, or townſhip, in which ſuch forged deed ſhall purport the ſaid annuity to have been granted, any ſum of money whatſoever, as part of ſuch pretended annuity, every perſon ſo offending, and being convicted thereof, ſhall be deemed and adjudged guilty of felony, and ſhall ſuffer death as a felon, without benefit of clergy.

If any perſon ſhall forge, or cauſe to be forged, any ſuch deed of grant, and ſhall, by means of ſuch forged deed, obtain from the officers of any pariſh any ſum of money, he ſhall thereby become guilty of felony without benefit of clergy.

And be it further enacted, That if any perſon ſhall, by any means whatſoever, get poſſeſſion of any real deed of grant of any ſuch annuity, or any copy thereof, given by the pariſh, or townſhip, wherein ſuch annuity is granted, as is above mentioned, without having a right to the annuity thereby granted, either as the grantee thereof, or the lawful aſſignee thereof, by one or more aſſignment or aſſignments thereof, in the manner above preſcribed, and ſhall falſely pretend to be the perſon lawfully intitled to ſuch annuity, and under ſuch pretence ſhall produce any ſuch deed of grant to the pariſh officers of the pariſh or townſhip in which ſuch annuity was granted, and in conſequence thereof ſhall demand and obtain, from the ſaid officers, any ſum of money whatſoever, as a part of ſuch annuity, every perſon ſo offending, and being convicted thereof, ſhall be deemed and adjudged to be guilty of felony.

And, if any perſon ſhall, by any means, get poſſeſſion of any *real* deed of grant of ſuch an annuity, belonging to another perſon, and ſhall, by falſely pretending that he is the perſon to whom the ſaid annuity belongs, obtain payment of any of the money due upon it, he ſhall thereby become guilty of felony.

And be it further enacted, That the ſeveral perſons intitled to the ſaid annuities ſhall apply to the ſaid officers of the pariſhes, or townſhips, in which the ſame have been reſpectively granted, for the payment thereof, as ſoon as they conveniently can, after the ſeveral quarterly feaſt days in which they are made payable; and if any ſuch annuitant ſhall neglect to apply (either by himſelf or his lawful attorney) for the payment of his annuity, to the ſaid officers of the pariſh or townſhip in which the ſame was granted, for more than the ſpace of one year and a quarter, ſo that five or more quarterly payments thereof ſhall be in arrear, he ſhall not be entitled to receive more than the four laſt quarterly payments of ſuch annuity, and ſhall forfeit his right to all the former payments ſo in arrear.

Any grantee of any one of theſe annuities who ſhall neglect to apply for payment of it for more than five quarters of a year, ſhall forfeit all the arrears due to him, except thoſe of the laſt four quarters of a year.

Provided always, That nothing in this clauſe ſhall extend, or be conſtrued to extend, to any grantee, who, from being beyond the ſeas, or any unavoidable incapacity, ſhall have been diſabled from making his demand within

A proviſoe in favour of perſons beyond the ſeas, or who ſhall have been under any unavoidable incapacity of making application for the payment of their ſaid annuities in due time.

the time ſo limited, and ſhall make ſuch demand within ſix months after his return from abroad, or the removal of ſuch incapacity.

The money paid for the purchaſe of theſe pariſh-annuities ſhall be the property of the inhabitants of the pariſh that are chargeable to the poors rates; and ſhall be inveſted in the three per cent. bank annuities in their name.

And be it further enacted, That the money to be received by the rector, vicar, or perpetual curate, and the churchwardens and overſeers of the poor of every pariſh or townſhip, in which any ſuch annuities ſhall be granted, ſhall be the property of all the inhabitants of ſuch pariſh or townſhip, chargeable to the poors rates. And the reſpective rectors, vicars, or perpetual curates, churchwardens, and overſeers of the poor ſhall, and they are hereby required as ſoon as conveniently may be, after the receipt of any ſuch purchaſe-money, to lay out the ſame in the purchaſe of ſome of the public annuities, eſtabliſhed by authority of parliament, and payable at the bank of England, after the rate of three *per centum per annum:* which money ſo laid out, ſhall ſtand in the books of the governor and company of the ſaid bank, in the name of the rector, vicar, or perpetual curate, churchwardens, overſeers of the poor, and inhabitants of ſuch pariſh or townſhip, chargeable to the poors rates of the ſame; who ſhall be, and are hereby declared to be, a body politick and corporate, having perpetual ſucceſſion for the ſeveral purpoſes of this act, and ſhall be known by the name of *The Rector, Vicar, or perpetual Curate, Churchwardens, Overſeers of the Poor, and Inhabitants of ſuch Pariſh or Townſhip, chargeable to the Poors Rates of the ſame*, and ſhall have and uſe a common ſeal, on which ſeal ſhall be engraven the name of ſuch pariſh or townſhip, and the county, city, or town in which the ſame is ſituated, and ſuch ſeal ſhall be carefully kept in the cheſt of ſuch pariſh or townſhip. And the rector, vicar, or perpetual curate of ſuch pariſh or townſhip, and the churchwardens and overſeers of the poor thereof, or a majority of them, ſhall be, and are hereby conſtituted and declared to be, *the acting members* of ſuch body corporate, and ſhall have power to purchaſe the ſaid bank annuities, in the name of the whole body, and to receive the dividends of intereſt that ſhall become due thereupon, and to ſell and transfer the ſaid annuities whenſoever they ſhall think fit.

And the ſaid inhabitants ſhall be a body politick and corporate for the ſeveral purpoſes of this act, and ſhall have a corporate name and a common ſeal.

And the rector, vicar, or perpetual curate, of the pariſh, and the churchwardens and overſeers of the poor, or a majority of them, ſhall be *the acting members* of every ſuch body corporate.

The ſaid corporations may appoint agents, or attornies, to tranſact their buſineſs at the Bank of England.

And, for the more eaſy tranſacting the buſineſs relating to the ſaid bank annuities, be it further enacted, That the rector, vicar, or perpetual curate, and the churchwardens and overſeers of the poor of every ſuch pariſh or townſhip, or the majority of them, ſhall, and they are hereby impowered, with the concurrence of a majority of the rateable inhabitants aſſembled at a pariſh meeting, (ſuch majority in number being alſo charged in the laſt rate made for the relief of the poor of the ſaid pariſh or townſhip, in a ſum greater than that which was aſſeſſed in the ſame rate upon the reſt of the inhabitants aſſembled at the ſaid meeting,) upon notice given as aforeſaid, to conſtitute and appoint any perſon or perſons that they ſhall think proper, to be

be their agent or attorney, agents or attornies, to purchaſe, ſell, or transfer, in their ſtead and place, and in the behalf of the ſaid body politick and corporate, the ſaid bank annuities, or to receive the dividends of intereſt due thereupon; which appointment ſhall be by a letter, or power, of attorney for that purpoſe, in writing, ſealed and delivered by a majority of them, the ſaid rector, vicar, or perpetual curate, churchwardens and overſeers of the poor in manner herein-after directed, and likewiſe ſigned by them in preſence of two credible witneſſes, who ſhall likewiſe ſign their names thereto, in atteſtation of their having ſeen the ſaid power ſo ſigned, ſealed, and delivered; and ſhall continue in force until it ſhall be expreſsly revoked by another inſtrument in writing, made and executed in the ſame manner, by ſuch rector, vicar, or perpetual curate, churchwardens and overſeers of the poor, or the major part of them, as aforeſaid, and atteſted by the ſame number of ſubſcribing witneſſes, as the ſaid letter, or power, of attorney is hereby directed to be atteſted by. And every ſuch letter, or power, of attorney ſhall (during the time it ſhall continue in force) be ſufficient to impower the perſon to whom it was given, (if it ſhall purport ſo to impower him) to purchaſe and accept any ſtock in the ſaid bank-annuities, for, and in behalf of, the ſaid body politick and corporate; and likewiſe to receive, in the ſame behalf, all the dividends of intereſt that ſhall become due on the ſtock of ſuch body; and likewiſe to ſell and transfer all, or any part of, the ſtock which ſhall belong to ſuch body corporate, either at the time of executing ſuch letter of attorney, or at any time after, while ſuch letter of attorney ſhall continue in force.

The aforeſaid money ſo inveſted in the three per cent. bank-annuities ſhall be kept and uſed as a fund for the payment of the ſaid pariſh life-annuities, and ſhall be applied to no other uſe or purpoſe whatſoever.

And be it further enacted, That all the money which ſhall be paid for the purchaſe of any of the ſaid life-annuities, and which is herein before directed to be inveſted in the three *per centum* bank annuities, ſhall be kept and uſed as a fund, for the payment of the ſaid life-annuities, as they ſhall become due, and ſhall not be applied to any other uſe, or purpoſe, whatſoever; and the reſpective rectors, vicars, or perpetual curates, and churchwardens and overſeers of the poor of every pariſh or townſhip, in which ſuch life-annuities ſhall be granted, ſhall, and they are hereby required, either by themſelves or their lawful attorney, impowered in the manner before-mentioned, to receive the dividends of intereſt that ſhall become due upon the ſaid fund or ſtock of the ſaid bodies politick and corporate reſpectively, in the ſaid bank annuities, as often and as ſoon as they ſhall become due and payable, and to inveſt the money, ariſing by ſuch dividends, immediately in the purchaſe of new ſtock in the ſaid bank-annuities, ſo as thereby to encreaſe ſuch pariſh fund continually, and enable it to furniſh the payments of the ſeveral life-annuities charged thereupon, and to pay out of the ſaid dividends ſuch annuities as ſhall be then due to the perſons entitled to receive the ſame, by equal quarterly payments in every year, at the four feaſt-days before-mentioned,

or within ſeven days after each of the ſaid feaſt days reſpectively; and in caſe ſuch dividends ſhall not be ſufficient for that purpoſe, to pay the ſame out of the fund ſo veſted in bank-annuities; and for that purpoſe to ſell and transfer, from time to time, whenever it ſhall be neceſſary, ſo much of the ſaid fund or ſtock of the ſaid pariſhes or townſhips reſpectively, in the ſaid bank annuities, as ſhall be ſufficient to enable them to make the ſaid payments.

Excepting only the neceſſary charges of managing this fund; which may be defrayed out of it.

Provided always, and be it hereby further enacted, That it ſhall and may be lawful for the ſaid rector, vicar, or perpetual curate, and churchwardens and overſeers of the poor in every ſuch pariſh or townſhip, to defray the neceſſary charges and expences of inveſting the money received from the purchaſers of the ſaid life-annuities in the ſaid bank-annuities, and of receiving the dividends thereof, and inveſting them in the purchaſe of freſh ſtock in the ſame, and of ſelling and transferring the ſame, and of preparing the ſaid letters of attorney for the tranſaction of this buſineſs, and of procuring the aforeſaid books for entering and regiſtering copies of the grants of the ſaid annuities, or any other books neceſſary for carrying this act into execution, and of every other thing neceſſary to be done by any other perſon than themſelves in the execution of this act, out of the monies received by them from time to time from the purchaſers of ſuch life-annuities, or out of the dividends payable out of the ſaid ſtock or fund, they keeping an exact account of the particulars of the ſaid charges and expences; any thing to the contrary hereof above-mentioned in any-wiſe notwithſtanding.

Sums of money given by charitable perſons in aid of the funds to be eſtabliſhed by this act ſhall be employed in the purchaſe of three per cent. bank-annuities in the ſame manner as the money paid for the purchaſe of the ſaid life-annuities.

And be it further enacted, That if any ſum or ſums of money ſhall be given by charitable perſons, or otherwiſe, for the purpoſe of enlarging the ſaid original fund of any pariſh or townſhip, and enabling it with more certainty to furniſh the ſeveral payments of the life-annuities charged thereupon, without any more particular directions from the donor or donors, as to the manner of applying the ſame, the rector, vicar, or perpetual curate, churchwardens, and overſeers of the poor of ſuch pariſh or townſhip, ſhall, and they are hereby authoriſed and required to, inveſt the money ariſing by ſuch gifts, or otherwiſe, in the ſaid bank-annuities, in the ſame manner as the money contributed by the purchaſers of the ſaid life-annuities is herein-before directed to be inveſted, and to employ and uſe ſuch money as an additional fund for the payment of the ſaid life-annuities, in the ſame manner as they are herein-before authoriſed and required to employ the aforeſaid original fund, contributed by the purchaſers of the ſaid life-annuities.

If theſe funds ſhould prove inſufficient for the payment of the life-annuities that ſhall have been granted by virtue of this act, the deficiencies ſhall be made good out of the poors rate.

And, for ſecuring the payment of the ſaid life-annuities, **be it further enacted,** That if it ſhall at any time happen in any pariſh or townſhip, where ſuch

ſuch life-annuities ſhall have been granted, that the aforeſaid original fund ariſing from the contributions of the purchaſers of the ſaid life-annuities, together with the ſaid additional fund, ſhall not be ſufficient to furniſh the quarterly payments of the ſaid life-annuities, the churchwardens and overſeers of the poor ſhall, and they are hereby required to, make the ſaid payments out of any other money in their hands, raiſed for the relief of the poor of ſuch pariſh or townſhip; and if ſuch money in their hands be not ſufficient for that purpoſe, they ſhall, and are hereby required to, make ſuch an addition to the rates, which they are impowered to make, by a ſtatute made in the forty-third year of the reign of Queen Elizabeth, intituled, *An Act for the Relief of the Poor*, as ſhall be ſufficient to ſupply the deficiencies of the ſaid funds, and to enable them to compleat the ſaid quarterly payments of the ſaid life-annuities, and to defray the neceſſary charges and expences of executing this act; which ſaid additional rate ſhall be made, levied, and recovered, under the ſame regulations and reſtrictions, and by the ſame ways and means, as the rates made for the relief of the poor are, by the ſaid laſt-mentioned ſtatute, or any ſubſequent act or acts of parliament, directed to be made, levied, and recovered.

And be it further enacted by the authority aforeſaid, That, if any of the capital ſtock of any pariſh or townſhip, in the ſaid bank annuities, ſhall remain, after all the annuities ſhall become extinct, the intereſt of ſuch ſtock ſhall be applied in aid of the rate to be raiſed and levied for the relief of the poor of the ſaid pariſh or townſhip, from time to time, as ſuch intereſt ſhall become due; and it ſhall and may be lawful for the rector, vicar, or perpetual curate, churchwardens, and overſeers of the poor of ſuch pariſh or townſhip, with the advice and conſent of the inhabitants of ſuch pariſh or townſhip, rateable towards the relief of the poor, or the major part of them, aſſembled in veſtry, or other publick meeting for that purpoſe, (due notice of the ſaid meeting being given in manner herein-before directed,) to ſell and diſpoſe of ſuch capital ſtock, and to apply the money ariſing by ſuch ſale, either in repairing, amending, or new-building any alms-houſes, in ſuch pariſh or townſhip, or furniſhing them in a more convenient manner, or in building or furniſhing new alms-houſes, or in any other manner for the benefit of the poor of ſuch pariſh or townſhip, as the rector, vicar, or perpetual curate, churchwardens, and overſeers of the poor, and inhabitants ſo aſſembled, ſhall order and direct.

And, on the other hand, if after the extinction of all the life-annuities that have been granted in any pariſh, there ſhall remain any part of the ſaid pariſh-fund in the ſaid three per cent. bank-annuities, ſuch fund ſhall be employed in aid of the poors rate in the ſaid pariſh.

And be it further enacted by the authority aforeſaid, That every ſuch rector, vicar, or perpetual curate, concerned in the execution of this act, who ſhall remove from any ſuch pariſh, and every churchwarden, or overſeer of the poor, at the time of going out of his office, and the executors or adminiſtrators

The acting members of theſe corporations who ſhall have any of the money belonging to the funds in their hands at the time of their removal from the pariſh or the expiration of their offices in it, ſhall pay over the ſame to their ſucceſſors within one month after ſuch removal or expiration of their offices. And their executors or adminiſtrators ſhall do the ſame after their deaths.

miniſtrators of every ſuch rector, vicar, or perpetual curate, churchwarden, or overſeer of the poor, who ſhall happen to die, during the time he ſhall be concerned in the execution of this act, ſhall deliver in a true and perfect account of, and ſhall pay, or cauſe to be paid, all monies remaining in his or their hands, for the purpoſes of this act, to the rector, vicar, or perpetual curate, churchwardens, or overſeers of the poor, for ſuch pariſh or townſhip, within one month after any ſuch removal, going out of office, or death; and in caſe of neglect or refuſal ſo to do, it ſhall and may be lawful for any two juſtices of the peace for the county or place where the offender ſhall reſide, to make inquiry concerning the ſame, in a ſummary way, either by confeſſion of the party, or by the teſtimony of any credible witneſs or witneſſes upon oath, (which oath ſuch juſtices are hereby impowered to adminiſter;) and to cauſe the money remaining in the hands of ſuch rector, vicar, or perpetual curate, churchwarden, overſeer of the poor, executor, or adminiſtrator reſpectively, to be recovered by diſtreſs and ſale of his goods and chattels, rendering the overplus to the owner of ſuch goods and chattels, after deducting the charges attending ſuch diſtreſs and ſale; and ſuch juſtices are hereby impowered to cauſe the books and accounts beforementioned to be brought, and ſuch witneſſes to be ſummoned to appear before them as they ſhall think neceſſary for their information: and if any perſon ſhall refuſe to appear, or to give evidence, or to produce ſuch books and accounts, as aforeſaid, to ſuch juſtices, it ſhall and may be lawful for ſuch juſtices to impoſe any fine or fines upon ſuch perſon and perſons ſo offending as they ſhall think fit, (ſo as no ſuch fine ſhall exceed the ſum of ten pounds upon any one perſon for one offence,) and to levy ſuch fines by diſtreſs and ſale of the offenders goods and chattels; and, if no ſuch diſtreſs can be found, it ſhall and may be lawful for ſuch juſtices to commit the offender to the common gaol of the county or place, for any time not exceeding ſix months, unleſs ſuch fine ſhall be ſooner paid.

And, in caſe of neglect or refuſal ſo to do, any two juſtices of the peace for the county in which the offender ſhall reſide, may inquire into the matter in a ſummary way, and compel him to do juſtice.

And be it further enacted by the authority aforeſaid, That, if any ſuch annuity ſhall be behind and unpaid for the ſpace of ſeven days after the ſame ſhall become due, and have been demanded, it ſhall and may be lawful for any one juſtice of the peace, for the county or place where the pariſh or townſhip from which ſuch annuity ſhall be ſo due and unpaid ſhall be ſituate, to make enquiry concerning the ſame, in a ſummary way, either by confeſſion of the party, or by the teſtimony of any credible witneſs or witneſſes upon oath, (which oath ſuch juſtice is hereby impowered to adminiſter,) and to cauſe the money ſo due to be levied by warrant under his hand and ſeal, by diſtreſs and ſale of the goods and chattels of any one of the churchwardens or overſeers of the poor of ſuch pariſh, rendering the overplus to the owner of ſuch goods and chattels, after deducting the charges attending ſuch diſtreſs and ſale.

And, in caſe the churchwardens and overſeers of the poor ſhall neglect, or refuſe, to pay any annuity for the ſpace of ſeven days after it ſhall have become due and been demanded, any one juſtice of the peace of the county in which the pariſh is ſituated, may inquire into the matter in a ſummary way, and compel them to do juſtice.

And

And whereas, by ſundry acts of parliament, ſeveral pariſhes and townſhips have been united into ſeveral corporations, and the management, maintenance, and regulation of the poor of ſuch pariſhes and townſhips is thereby veſted in the governors, guardians, and acting members of ſuch corporations reſpectively; be it enacted by the authority aforeſaid, That in all places where ſuch corporations exiſt, and where the plan and method herein-before preſcribed for granting annuities to induſtrious and frugal perſons, ſhall have been approved by the majority of a general court, to be called and held for that purpoſe, the power of granting life-annuities in purſuance of this act to perſons appearing to be legally ſettled in any pariſh or townſhip, ſo incorporated as aforeſaid, ſhall be veſted in the governors, guardians, and acting members of the corporation, to which ſuch pariſh or townſhip does reſpectively belong, and not in the rector, vicar, or perpetual curate, churchwardens, and overſeers of the poor, and rateable inhabitants, of ſuch pariſh or townſhip; and the governors, guardians, and acting members of ſuch corporation, aſſembled in a general court, or the major part of them ſo aſſembled, are hereby impowered to agree, according to the reſpective prices expreſſed in the ſet of tables hereunto annexed, No. 2, for the ſale of any annuity or annuities for life, to any perſon or perſons, appearing to them to have a lawful ſettlement in any pariſh or townſhip, within the limits of the juriſdiction of the ſaid corporation, the purchaſe-money not being leſs, nor the annuity or annuities greater, than the ſums before limited, nor the commencement of any ſuch annuity at any other age than as before limited for other pariſhes and townſhips; and upon ſuch agreement made, the ſaid governors, guardians, and acting members, are hereby impowered and required to receive the purchaſe-money for, and to do every act for granting and ſecuring ſuch life-annuity or annuities by deeds, to be executed by them in the ſame form and manner as other deeds are executed by them, for veſting the purchaſe-money received by them in the purchaſe of three *per cent.* bank annuities, in the name of the ſaid corporation, for receiving the dividends, and transferring the ſaid ſtock, ſo purchaſed by them, for paying, or cauſing to be paid, the life-annuities ſo granted by them, and for re-purchaſing the ſame in the manner before directed, which the ſeveral rectors, vicars, or perpetual curates, churchwardens, and overſeers of the poor of other pariſhes and townſhips, are by this act impowered or required to do; for all and every the purpoſes before mentioned, and in the ſame form and manner, as near as the inſtitution of ſuch corporation, and the ſeveral regulations touching its manner of acting and proceeding, will admit. And in caſe at any time the fund or funds for ſecuring ſuch life-annuities ſhall prove deficient, the governors, guardians, and acting members of every ſuch corporation, are hereby reſpectively impowered and required to pay the ſame out of any money in their hands, applicable to the maintenance

The foregoing regulations ſhall extend to ſuch pariſhes and townſhips as have been united together, by acts of parliament, into ſeveral corporations for the purpoſes of maintaining and managing their poor.

And they ſhall be carried into execution by the governors, guardians, and acting members of ſuch corporations.

nance of the poor of the parish or townſhip in which ſuch annuitant appeared to have been lawfully ſettled at the time of granting ſuch annuity, or to make, or cauſe to be made, a ſufficient rate upon ſuch inhabitants within the limits of the juriſdiction of the ſaid corporation, as would have been liable to contribute to the relief of the poor of ſuch pariſh or townſhip; and the ſaid governors, guardians, and acting members of the ſaid corporations, are hereby further impowered and required to compel the ſeveral perſons concerned in the receipt and payment of money, upon account of ſuch life-annuities, to produce and paſs their accounts of all monies received and paid by them, in ſuch manner, and by ſuch means, as the ſeveral accountants are compellable to account for other monies received and paid by them, by the ſaid ſeveral acts of parliament; and no conſent of the rateable inhabitants of the ſaid ſeveral pariſhes and townſhips ſhall be requiſite to make valid, or confirm any of the ſaid proceedings of the governors, guardians, and acting members of ſuch corportions.

And the aforeſaid penal clauſes againſt perſons guilty of frauds with reſpect to annuities granted by ſingle pariſhes ſhall extend to perſons guilty of the like frauds with reſpect to annuities granted by the ſaid corporations conſiſting of ſeveral pariſhes united together.

And be it further enacted by the authority aforeſaid, That every clauſe, proviſion, and regulation herein contained, as well with reſpect to perſons who ſhall alter, forge, frame, in manner before mentioned, or, not being the owner thereof, ſhall get poſſeſſion of, and demand and obtain money for, any grant, or copy of any grant, of any annuity, to be granted by virtue of this act, by rectors, vicars, or perpetual curates, churchwardens, and overſeers of the poor, as with reſpect to every other matter or thing directed to be done by them in the execution hereof, ſhall, in every caſe where, and as far as it can be made applicable, extend, and take place, alſo with reſpect to ſuch grants, or copies of grants, of annuities, which, in purſuance hereof, ſhall be granted by the ſaid corporations.

And be it further enacted by the authority aforeſaid, That in all pariſhes and townſhips, which have not been united into ſeveral corporations, for the management, maintenance, and regulation of their poor, the following form ſhall be obſerved in all powers of attorney to be executed by the rector, vicar, or perpetual curate, churchwardens, and overſeers of the poor of ſuch pariſhes or townſhips, for the acceptance of ſtock, and the receipt of dividends due thereupon; that is to ſay,

The Pariſh of *in the County of*

The form of the powers of attorney which ſhall be granted by ſingle pariſhes for the acceptance of ſtock in the three per cent. bank-annuities and the receipt of dividends due thereupon.

"KNOW all men by theſe preſents, That we, the rector, *(vicar, or perpetual curate, as the caſe may happen,)* churchwardens and overſeers of the poor, and rateable inhabitants of the pariſh of in the county of in veſtry aſſembled, by virtue of

" of an act of parliament of the thirteenth year of the reign of his Majesty " King George the Third, (intituled, *An Act for the better Support of* " *Poor Persons in certain Circumstances, by enabling Parishes to grant them* " *Annuities for Life upon Purchase, and under certain Restrictions,*) do make, " constitute, and appoint, *A. B.* of the parish of " in the county of Gentleman, our true and " lawful attorney, for us, in our names, and on our behalf, to accept all " transfers that are, or may, at any time or times, be made unto us, the " said rector, *(vicar, or perpetual curate, as the case may happen,)* church- " wardens, overseers of the poor, and rateable inhabitants of the parish " of in the county of and " our successors, of any interest or shares in the capital, or joint, stock of " three *per cent.* reduced, *(or consolidated,)* bank annuities, *(as the case may* " *happen)*; and also, on our behalf, to receive, and give receipts for, all divi- " dends and interest that shall grow due and payable, on our interest or " share in the above-said capital, or joint-stock, or on any part thereof, for " the time being, and to do all lawful acts requisite for effecting the pre- " mises, hereby ratifying and confirming all that our said attorney shall do " therein by virtue hereof. In witness whereof, we, the acting members " of the said body corporate, have hereunto set our hands, and affixed " our common seal, the day of " in the year of our Lord

Signed, sealed, and delivered in the presence of *N. O.* of Gentleman, *R. S.* of Yeoman.

D. E. Rector of the Parish of in the County of

J. R., *S. M.* Churchwardens of the said Parish.

T. N., *E. A.*, *F. M.* Overseers of the Poor of the said Parish.

The Common Seal of the Parish of in the County of

And in all such parishes or townships the following form shall be observed in all powers of attorney, for the transfer of all such stock as shall have been purchased by them; that is to say,

The Pariſh of *in the County of*

The form of the powers of attorney which ſhall be granted by ſingle pariſhes for the ſelling and transferring of ſtock in the ſaid 3 per cent. bank-annuities.

" KNOW all men by theſe preſents, That we, the rector, *(vicar, or perpetual curate, as the caſe may happen,)* churchwardens, overſeers of the poor, and rateable inhabitants of the pariſh of in the county of, in veſtry aſſembled, by virtue of an act of parliament, of the thirteenth year of the reign of his Majeſty King George the Third, (intituled, *An Act for the better Support of Poor Perſons in certain Circumſtances, by enabling Pariſhes to grant them Annuities for Life, upon Purchaſe, and under certain Reſtrictions,)* do make, conſtitute, and appoint *A. B.* of the pariſh of in the county of Gentleman, our true and lawful attorney, for us, in our names, and on our behalf, to ſell, aſſign, and transfer or any part thereof, being part of [*or* all] the intereſt or ſhare in the capital, or joint ſtock, of three *per cent.* reduced *(or conſolidated)* bank-annuities, *(as the caſe may happen)* ſtanding in the names of us the rector, *(vicar, or perpetual curate, as the caſe may happen,)* churchwardens, overſeers of the poor, and rateable inhabitants of the ſaid pariſh of in the county of and to receive the conſideration-money, and to give diſcharges and receipts for the ſame, and to do all lawful acts requiſite for effecting the premiſes, hereby ratifying and confirming all that our ſaid attorney ſhall do herein by virtue hereof. In witneſs whereof, we, the acting members of the ſaid body corporate, have hereunto ſet our hands, and affixed our common ſeal, the day of in the year of our Lord

Signed, ſealed, and delivered in the preſence of *N. O.* of Gentleman, *E. S.* of Yeoman. } *D. E.* Rector of the Pariſh of in the County of

J. R. } Churchwardens of
S. M. } the ſaid Pariſh.

T. N. } Overſeers of the
E. A. } Poor of the ſaid
F. M. } Pariſh.

The Common Seal of the Pariſh of in the County of

TABLES.

TABLES

SHEWING THE VALUES,

IN A SINGLE PRESENT PAYMENT,

OF AN

ANNUITY OF ONE POUND,

PAYABLE QUARTERLY,

For the Lives of Perſons of all Ages, from 15 to 73;

Such Annuity being ſuppoſed to commence at any Age, not younger than 35 nor older than 75 Years.

DIRECTIONS

FOR USING THE FOLLOWING

TABLES.

Drawn up by the Rev. Dr. Richard Price, of Newington-Green, near Iſlington, in the County of Middleſex.

THESE tables give the preſent payments that ought to be made (reckoning the intereſt at three per cent.) by perſons of all ages, from 15 to 73, for a life-annuity of one pound, payable quarterly, (that is, five ſhillings each quarter,) to commence at any given age not leſs than 35, nor exceeding 75.

For inſtance. Table the firſt, in the ſecond ſet of tables, ſhews, that the payment due from a perſon whoſe age is between $14\frac{3}{4}$ and $15\frac{1}{4}$ is £4 0*s.* 1*d.* for a life-annuity of one pound, payable quarterly, to commence when he attains to 43 years of age, or (more exactly) at the end of 28 years and a quarter from the day on which the payment is made. In like manner the ſame table ſhews, that the preſent payment due from a perſon of the ſame age, for the ſame annuity, is *one pound*, ſuppoſing the firſt quarterly payment of the annuity to be made at the end of forty-five years and a quarter, or not till he attains to 60 years of age; and *ſix ſhillings*, if he chuſes to wait for the firſt quarterly payment 55 years and a quarter, or till he attains to 70 years of age.

A life-annuity therefore of £2. to commence at theſe different periods, muſt be worth double theſe ſums; and a life-annuity of £10. muſt be worth ten times theſe ſums.

Univerſally

Univerſally therefore. In order to find from the following tables the preſent money that ought to be given for any annuity payable quarterly to purchaſers of given ages, after attaining any other given ages----*Multiply the value in the table for the given age, by the number of pounds in the propoſed annuity, and the product will be the anſwer.*

Example. From table 7th, in the ſecond ſet of tables, (calculated for the country,) it will appear by inſpection, that the value to a purchaſer aged 21, of an annuity of £1. for life, payable quarterly, to commence at 50 years of age, is £3. 1*s.* 2*d.* If therefore the annuity is £5. its value will be this ſum multiplyed by 5, or £15. 5*s.* 10*d.* If the annuity is £20. its value will be the ſame multiplyed by 20, or £61. 3*s.* 4*d.*

Second Example. Suppoſe a perſon whoſe age is 25, to apply for an annuity (payable quarterly) of £10. for his life, after attaining his 55th year. From table 11, in the ſecond ſet of tables, it appears, that, if the annuity had been one pound, its preſent value would have been £2. 9*s.* therefore the annuity being £10. its value is £2. 9*s.* multiplied by 10, or £24. 10*s.*

The value of the ſame annuity, according to table 11, in the firſt ſet of tables, calculated for London, is £1. 14*s.* 3*d.* multiplied by 10, or £17. 2*s.* 6*d.*

Again. Suppoſe a perſon of a given age to deſire to be informed "what annuity, as a proviſion for old age, he can purchaſe with a given ſum of money, of which he is in poſſeſſion."— In order to diſcover the anſwer, look over the table for the given age, and find there the given ſum; or, if it be not found exactly, find the ſum neareſt to it; and the correſpondent age will ſhew, that with the given ſum he may purchaſe an annuity of one pound for life, to commence at that age.——In like manner, the ages correſponding to half, a third, a fourth, &c. of the ſum will ſhew, that it will purchaſe for him an annuity of £2. £3. £4. &c. to commence at thoſe ages reſpectively.

Example. A poor perſon, aged 22, has ſaved by his induſtry the the ſum of £5. and with this in his hands, he applies for ſuch an annuity, to commence in ſome future year of his life, as it can purchaſe for him.

By looking over the table for the age of 22, or table 8 in the ſecond ſet of tables, it will appear, that £4. 19*s.* 11*d.* or, very nearly the ſum he offers, will purchaſe for him an annuity of one pound for his life,

to

to commence at the age of 44 years; and it appears alſo from the ſame table, that £2. 10s. 3d. (or about half his money,) will purchaſe the ſame annuity, to commence when he is 53; and that £1. 4s. 2d. (or about a quarter of his money,) will purchaſe the ſame annuity, to commence when he is 61; and that 19s. 7d. (or about a fifth part of his money,) will purchaſe the ſame annuity, to commence when he is 63. If, therefore, he thinks an annuity of one pound for his life, to commence at 44, too little, he may be offered for £5. 0s. 6d. £4. 16s. 8d. £4 17s. 11d. (that is for ſums nearly equal to £5.) an annuity of £2. for life, to commence at 53, or of £4. for life, to commence at 61, or of £5. for life, to commence at 63. From table 8, in the firſt ſet of tables, it will in the ſame way appear, that in London £5. would intitle ſuch a perſon to a life-annuity of either £2. to commence at 49, or of £4. to commence at 57, or of £5. to commence at 59, or four years earlier than by the ſecond ſet of tables, which are calculated for the country.

Obſervation 1ſt. The payments of perſons who happen to die before the age agreed on for the commencement of their annuities, are in theſe tables ſuppoſed to be an advantage ſhared amongſt ſurvivors, without which the money advanced would be inſufficient to bear the expence of the annuities.

Theſe tables alſo ſuppoſe, that annuitants will be intitled to nothing for any part of that quarter of the year in which they ſhall happen to die.

Obſervation 2d. It ſhould be remembered, that the firſt of the three columns in theſe tables is intended to be an explanation of the column next to it, and to expreſs with more preciſion the time at which the calculations ſuppoſe the annuity to commence. — Thus, if it were only expreſſed in table 1ſt, that a perſon, whoſe age is exactly $14\frac{3}{4}$, would be entitled, for a preſent payment of £2. 7s. 11d. to an annuity, payable quarterly, of £1. for life, to commence at the age of 50, it would only apper that the annuity was to commence ſome time or other after he had attained to that age, or entered his 51ſt year. But the firſt column removes this uncertainty, by ſpecifying, that the firſt quarterly payment is to be made at the end of 35 years and a quarter after purchaſing, or exactly upon his attaining the age of 50. In like manner; ſuppoſing his age 15, 15 and a quarter, or 15 and a half, the ſame column, by ſpecifying that the annuity was to commence at the end of 35 years and $\frac{1}{4}$, would ſhew, that the firſt payment was to be made when he came to be a quarter of a year, half a year, or three quarters of a year, turned of 50.

Obſervation

Obſervation 3d. When there is any uncertainty with reſpect to the preciſe age of a purchaſer, a younger age ſhould always be taken, rather than an older, in order to guard againſt the loſſes to pariſhes, that would ariſe from intitling perſons to higher annuities than are adequate to their payments. Much, however, will not depend on determining the age of any purchaſer to greater exactneſs than half a year, or a year.

Obſervation 4th. A conſiderable difference will be found at all ages under 45 or 50 years between the values in the following tables for London, and for the country. The reaſon is, that the inhabitants of London, and of great towns in general, are much more ſhort-lived than the inhabitants of ſmall towns, and country pariſhes and villages. This appears from undeniable obſervations, and has created the neceſſity of calculating diſtinct tables for London and the country.

It may be proper to add, for the ſatisfaction of thoſe who may wiſh to examine the following tables, that they have been calculated in the method explained and demonſtrated, by Mr. Maſeres, in a pamphlet intitled, *A Propoſal for eſtabliſhing Life-Annuities in Pariſhes, for the Benefit of the induſtrious Poor*, or by a rule in Dr. Price's Treatiſe on Annuities, Queſt. 6th, page 17th.———In calculating the ſecond ſet of tables, the probabilities of life at Northampton, as given in Table 4th, page 323, of the ſaid Treatiſe of Dr. Price, have been combined with the values of lives in Table 6th, page 325. And in calculating the firſt ſet of tables, the probabilities and values of lives have been taken from Mr. Simpſon's tables given in pages 332, and 334, of the ſame Treatiſe, or in page 254, and 260, of Mr. Simpſon's Select Exerciſes, with no other than the following difference.——The tables which have been mentioned give the values of lives in yearly payments only. An annuity, payable quarterly, is worth three eighths of a years purchaſe, or 7 s. 6d. more than an annuity payable yearly. Three eighths, therefore, or, in decimals, .375, have been always added to the values of lives taken from theſe tables, in order to obtain from thence the values in the following tables.

FIRST

FIRST SET OF TABLES.

Intended for the Uſe of LONDON.

TABLE I.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $14\frac{3}{4}$ to $15\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$20\frac{1}{4}$, or at Age 35	5	10	8
$21\frac{1}{4}$, — — 36	5	3	5
$22\frac{1}{4}$, — — 37	4	16	6
$23\frac{1}{4}$, — — 38	4	9	8
$24\frac{1}{4}$, — — 39	4	3	5
$25\frac{1}{4}$, — — 40	3	17	3
$26\frac{1}{4}$, — — 41	3	11	7
$27\frac{1}{4}$, — — 42	3	6	0
$28\frac{1}{4}$, — — 43	3	1	2
$29\frac{1}{4}$, — — 44	2	16	7
$30\frac{1}{4}$, — — 45	2	12	3
$31\frac{1}{4}$, — — 46	2	8	0
$32\frac{1}{4}$, — — 47	2	4	5
$33\frac{1}{4}$, — — 48	2	0	10
$34\frac{1}{4}$, — — 49	1	17	8
$35\frac{1}{4}$, — — 50	1	14	7
$36\frac{1}{4}$, — — 51	1	11	10
$37\frac{1}{4}$, — — 52	1	9	0
$38\frac{1}{4}$, — — 53	1	6	6
$39\frac{1}{4}$, — — 54	1	4	0
$40\frac{1}{4}$, — — 55	1	2	0

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d
$41\frac{1}{4}$, or at Age 56	1	0	0
$42\frac{1}{4}$, — — 57	0	18	2
$43\frac{1}{4}$, — — 58	0	16	4
$44\frac{1}{4}$, — — 59	0	14	11
$45\frac{1}{4}$, — — 60	0	13	5
$46\frac{1}{4}$, — — 61	0	12	1
$47\frac{1}{4}$, — — 62	0	10	10
$48\frac{1}{4}$, — — 63	0	9	9
$49\frac{1}{4}$, — — 64	0	8	8
$50\frac{1}{4}$, — — 65	0	7	9
$51\frac{1}{4}$, — — 66	0	6	9
$52\frac{1}{4}$, — — 67	0	6	0
$53\frac{1}{4}$, — — 68	0	5	2
$54\frac{1}{4}$, — — 69	0	4	7
$55\frac{1}{4}$, — — 70	0	4	0
$56\frac{1}{4}$, — — 71	0	3	6
$57\frac{1}{4}$, — — 72	0	3	0
$58\frac{1}{4}$, — — 73	0	2	7
$59\frac{1}{4}$, — — 74	0	2	2
$60\frac{1}{4}$, — — 75	0	1	11

TABLE II.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 15 ¾ to 16 ½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
19 ¼,	or at Age 35	5	15	3
20 ¼,	— — 36	5	7	11
21 ¼,	— — 37	5	0	6
22 ¼,	— — 38	4	13	8
23 ¼,	— — 39	4	6	11
24 ¼,	— — 40	4	0	9
25 ¼,	— — 41	3	14	7
26 ¼,	— — 42	3	9	2
27 ¼,	— — 43	3	3	10
28 ¼,	— — 44	2	19	1
29 ¼,	— — 45	2	14	5
30 ¼,	— — 46	2	10	4
31 ¼,	— — 47	2	6	3
32 ¼,	— — 48	2	2	9
33 ¼,	— — 49	1	19	3
34 ¼,	— — 50	1	16	2
35 ¼,	— — 51	1	13	2
36 ¼,	— — 52	1	10	5
37 ¼,	— — 53	1	7	8
38 ¼,	— — 54	1	5	3
39 ¼,	— — 55	1	2	11

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
40 ¼,	or at Age 56	1	0	11
41 ¼,	— — 57	0	18	11
42 ¼,	— — 58	0	17	2
43 ¼,	— — 59	0	15	6
44 ¼,	— — 60	0	14	0
45 ¼,	— — 61	0	12	7
46 ¼,	— — 62	0	11	4
47 ¼,	— — 63	0	10	2
48 ¼,	— — 64	0	9	1
49 ¼,	— — 65	0	8	0
50 ¼,	— — 66	0	7	2
51 ¼,	— — 67	0	6	3
52 ¼,	— — 68	0	5	6
53 ¼,	— — 69	0	4	9
54 ¼,	— — 70	0	4	2
55 ¼,	— — 71	0	3	8
56 ¼,	— — 72	0	3	2
57 ¼,	— — 73	0	2	8
58 ¼,	— — 74	0	2	3
59 ¼,	— — 75	0	1	11

TABLE III.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 16 ¾ to 17 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
18 ¼, or at Age 35	6	0	7	39 ¼, or at Age 56	1	1	9
19 ¼, — — 36	5	12	7	40 ¼, — — 57	0	19	9
20 ¼, — — 37	5	5	1	41 ¼, — — 58	0	17	9
21 ¼, — — 38	4	17	7	42 ¼, — — 59	0	16	2
22 ¼, — — 39	4	10	11	43 ¼, — — 60	0	14	7
23 ¼, — — 40	4	4	2	44 ¼, — — 61	0	13	2
24 ¼, — — 41	3	18	0	45 ¼, — — 62	0	11	10
25 ¼, — — 42	3	11	11	46 ¼, — — 63	0	10	8
26 ¼, — — 43	3	6	9	47 ¼, — — 64	0	9	6
27 ¼, — — 44	3	1	7	48 ¼, — — 65	0	8	5
28 ¼, — — 45	2	17	0	49 ¼, — — 66	0	7	4
29 ¼, — — 46	2	12	4	50 ¼, — — 67	0	6	6
30 ¼, — — 47	2	8	5	51 ¼, — — 68	0	5	8
31 ¼, — — 48	2	4	6	52 ¼, — — 69	0	5	0
32 ¼, — — 49	2	1	1	53 ¼, — — 70	0	4	4
33 ¼, — — 50	1	17	9	54 ¼, — — 71	0	3	10
34 ¼, — — 51	1	14	9	55 ¼, — — 72	0	3	4
35 ¼, — — 52	1	11	9	56 ¼, — — 73	0	2	11
36 ¼, — — 53	1	9	0	57 ¼, — — 74	0	2	5
37 ¼, — — 54	1	6	3	58 ¼, — — 75	0	2	0
38 ¼, — — 55	1	4	0				

TABLE IV.

For the Uſe of LONDON.

Shewing the Payment due, (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 17 ¾ to 18 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	.	d.
17 ¼, or at Age	35	6	5	8
18 ¼, — —	36	5	17	7
19 ¼, — —	37	5	9	6
20 ¼, — —	38	5	2	1
21 ¼, — —	39	4	14	8
22 ¼, — —	40	4	8	0
23 ¼, — —	41	4	1	3
24 ¼, — —	42	3	15	5
25 ¼, — —	43	3	9	7
26 ¼, — —	44	3	4	5
27 ¼, — —	45	2	19	3
28 ¼, — —	46	2	14	10
29 ¼, — —	47	2	10	5
30 ¼, — —	48	2	6	7
31 ¼, — —	49	2	2	10
32 ¼, — —	50	1	19	6
33 ¼, — —	51	1	16	2
34 ¼, — —	52	1	13	2
35 ¼, — —	53	1	10	2
36 ¼, — —	54	1	7	7
37 ¼, — —	55	1	5	0

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
38 ¼, or at Age	56	1	2	9
39 ¼, — —	57	1	0	6
40 ¼, — —	58	0	18	8
41 ¼, — —	59	0	16	10
42 ¼, — —	60	0	15	3
43 ¼, — —	61	0	13	9
44 ¼, — —	62	0	12	4
45 ¼, — —	63	0	11	1
46 ¼, — —	64	0	9	11
47 ¼, — —	65	0	8	9
48 ¼, — —	66	0	7	9
49 ¼, — —	67	0	6	9
50 ¼, — —	68	0	6	0
51 ¼, — —	69	0	5	3
52 ¼, — —	70	0	4	7
53 ¼, — —	71	0	4	0
54 ¼, — —	72	0	3	6
55 ¼, — —	73	0	3	0
56 ¼, — —	74	0	2	7
57 ¼, — —	75	0	2	2

TABLE V.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 18 ¾ to 19 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
16 ¼, or at Age 35	6	11	6
17 ¼, — — 36	6	2	9
18 ¼, — — 37	5	14	7
19 ¼, — — 38	5	6	5
20 ¼, — — 39	4	19	1
21 ¼, — — 40	4	11	9
22 ¼, — — 41	4	5	1
23 ¼, — — 42	3	18	5
24 ¼, — — 43	3	12	9
25 ¼, — — 44	3	7	2
26 ¼, — — 45	3	2	1
27 ¼, — — 46	2	17	0
28 ¼, — — 47	2	12	9
29 ¼, — — 48	2	8	6
30 ¼, — — 49	2	4	10
31 ¼, — — 50	2	1	1
32 ¼, — — 51	1	17	10
33 ¼, — — 52	1	14	7
34 ¼, — — 53	1	11	7
35 ¼, — — 54	1	8	7
36 ¼, — — 55	1	6	2

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
37 ¼, or at Age 56	1	3	8
38 ¼, — — 57	1	1	6
39 ¼, — — 58	0	19	4
40 ¼, — — 59	0	17	8
41 ¼, — — 60	0	16	0
42 ¼, — — 61	0	14	5
43 ¼, — — 62	0	12	11
44 ¼, — — 63	0	11	7
45 ¼, — — 64	0	10	4
46 ¼, — — 65	0	9	2
47 ¼, — — 66	0	8	1
48 ¼, — — 67	0	7	1
49 ¼, — — 68	0	6	2
50 ¼, — — 69	0	5	6
51 ¼, — — 70	0	4	9
52 ¼, — — 71	0	4	2
53 ¼, — — 72	0	3	7
54 ¼, — — 73	0	3	1
55 ¼, — — 74	0	2	7
56 ¼, — — 75	0	2	3

TABLE VI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $19\frac{3}{4}$ to $20\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$15\frac{1}{4}$, or at Age 35	6	17	1	$36\frac{1}{4}$, or at Age 56	1	4	10
$16\frac{1}{4}$, — — 36	6	8	3	$37\frac{1}{4}$, — — 57	1	2	5
$17\frac{1}{4}$, — — 37	5	19	5	$38\frac{1}{4}$, — — 58	1	0	5
$18\frac{1}{4}$, — — 38	5	11	4	$39\frac{1}{4}$, — — 59	0	18	5
$19\frac{1}{4}$, — — 39	5	3	3	$40\frac{1}{4}$, — — 60	0	16	8
$20\frac{1}{4}$, — — 40	4	15	11	$41\frac{1}{4}$, — — 61	0	15	0
$21\frac{1}{4}$, — — 41	4	8	8	$42\frac{1}{4}$, — — 62	0	13	6
$22\frac{1}{4}$, — — 42	4	2	3	$43\frac{1}{4}$, — — 63	0	12	1
$23\frac{1}{4}$, — — 43	3	15	10	$44\frac{1}{4}$, — — 64	0	10	10
$24\frac{1}{4}$, — — 44	3	10	3	$45\frac{1}{4}$, — — 65	0	9	7
$25\frac{1}{4}$, — — 45	3	4	8	$46\frac{1}{4}$, — — 66	0	8	6
$26\frac{1}{4}$, — — 46	2	19	10	$47\frac{1}{4}$, — — 67	0	7	5
$27\frac{1}{4}$, — — 47	2	14	11	$48\frac{1}{4}$, — — 68	0	6	7
$28\frac{1}{4}$, — — 48	2	10	10	$49\frac{1}{4}$, — — 69	0	5	8
$29\frac{1}{4}$, — — 49	2	6	8	$50\frac{1}{4}$, — — 70	0	5	0
$30\frac{1}{4}$, — — 50	2	3	0	$51\frac{1}{4}$, — — 71	0	4	4
$31\frac{1}{4}$, — — 51	1	19	5	$52\frac{1}{4}$, — — 72	0	3	10
$32\frac{1}{4}$, — — 52	1	16	2	$53\frac{1}{4}$, — — 73	0	3	3
$33\frac{1}{4}$, — — 53	1	12	11	$54\frac{1}{4}$, — — 74	0	2	10
$34\frac{1}{4}$, — — 54	1	10	0	$55\frac{1}{4}$, — — 75	0	2	5
$35\frac{1}{4}$, — — 55	1	7	2				

TABLE VII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 20 $\frac{3}{4}$ to 21 $\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
14 $\frac{1}{4}$, or at Age 35	7	3	5
15 $\frac{1}{4}$, — — 36	6	13	11
16 $\frac{1}{4}$, — — 37	6	5	0
17 $\frac{1}{4}$, — — 38	5	16	1
18 $\frac{1}{4}$, — — 39	5	8	1
19 $\frac{1}{4}$, — — 40	5	0	1
20 $\frac{1}{4}$, — — 41	4	12	10
21 $\frac{1}{4}$, — — 42	4	5	7
22 $\frac{1}{4}$, — — 43	3	19	5
23 $\frac{1}{4}$, — — 44	3	13	3
24 $\frac{1}{4}$, — — 45	3	7	9
25 $\frac{1}{4}$, — — 46	3	2	2
26 $\frac{1}{4}$, — — 47	2	17	7
27 $\frac{1}{4}$, — — 48	2	12	11
28 $\frac{1}{4}$, — — 49	2	8	10
29 $\frac{1}{4}$, — — 50	2	4	10
30 $\frac{1}{4}$, — — 51	2	1	3
31 $\frac{1}{4}$, — — 52	1	17	9
32 $\frac{1}{4}$, — — 53	1	14	6
33 $\frac{1}{4}$, — — 54	1	11	2
34 $\frac{1}{4}$, — — 55	1	8	6

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
35 $\frac{1}{4}$, or at Age 56	1	5	10
36 $\frac{1}{4}$, — — 57	1	3	6
37 $\frac{1}{4}$, — — 58	1	1	1
38 $\frac{1}{4}$, — — 59	0	19	3
39 $\frac{1}{4}$, — — 60	0	17	5
40 $\frac{1}{4}$, — — 61	0	15	8
41 $\frac{1}{4}$, — — 62	0	14	0
42 $\frac{1}{4}$, — — 63	0	12	7
43 $\frac{1}{4}$, — — 64	0	11	3
44 $\frac{1}{4}$, — — 65	0	10	0
45 $\frac{1}{4}$, — — 66	0	8	10
46 $\frac{1}{4}$, — — 67	0	7	10
47 $\frac{1}{4}$, — — 68	0	6	10
48 $\frac{1}{4}$, — — 69	0	6	0
49 $\frac{1}{4}$, — — 70	0	5	2
50 $\frac{1}{4}$, — — 71	0	4	7
51 $\frac{1}{4}$, — — 72	0	3	11
52 $\frac{1}{4}$, — — 73	0	3	5
53 $\frac{1}{4}$, — — 74	0	2	11
54 $\frac{1}{4}$, — — 75	0	2	7

TABLE VIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 21 $\frac{3}{4}$ to 22 $\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
13 $\frac{1}{4}$, or at Age 35	7	9	6	34 $\frac{1}{4}$, or at Age 56	1	7	1
14 $\frac{1}{4}$, — — 36	6	19	11	35 $\frac{1}{4}$, — — 57	1	4	5
15 $\frac{1}{4}$, — — 37	6	10	4	36 $\frac{1}{4}$, — — 58	1	2	3
16 $\frac{1}{4}$, — — 38	6	1	6	37 $\frac{1}{4}$, — — 59	1	0	0
17 $\frac{1}{4}$, — — 39	5	12	7	38 $\frac{1}{4}$, — — 60	0	18	2
18 $\frac{1}{4}$, — — 40	5	4	8	39 $\frac{1}{4}$, — — 61	0	16	4
19 $\frac{1}{4}$, — — 41	4	16	8	40 $\frac{1}{4}$, — — 62	0	14	9
20 $\frac{1}{4}$, — — 42	4	9	9	41 $\frac{1}{4}$, — — 63	0	13	1
21 $\frac{1}{4}$, — — 43	4	2	9	42 $\frac{1}{4}$, — — 64	0	11	9
22 $\frac{1}{4}$, — — 44	3	16	8	43 $\frac{1}{4}$, — — 65	0	10	5
23 $\frac{1}{4}$, — — 45	3	10	7	44 $\frac{1}{4}$, — — 66	0	9	3
24 $\frac{1}{4}$, — — 46	3	5	3	45 $\frac{1}{4}$, — — 67	0	8	1
25 $\frac{1}{4}$, — — 47	3	0	0	46 $\frac{1}{4}$, — — 68	0	7	2
26 $\frac{1}{4}$, — — 48	2	15	5	47 $\frac{1}{4}$, — — 69	0	6	3
27 $\frac{1}{4}$, — — 49	2	11	0	48 $\frac{1}{4}$, — — 70	0	5	6
28 $\frac{1}{4}$, — — 50	2	7	0	49 $\frac{1}{4}$, — — 71	0	4	9
29 $\frac{1}{4}$, — — 51	2	3	0	50 $\frac{1}{4}$, — — 72	0	4	2
30 $\frac{1}{4}$, — — 52	1	19	5	51 $\frac{1}{4}$, — — 73	0	3	6
31 $\frac{1}{4}$, — — 53	1	15	11	52 $\frac{1}{4}$, — — 74	0	3	0
32 $\frac{1}{4}$, — — 54	1	12	10	53 $\frac{1}{4}$, — — 75	0	2	7
33 $\frac{1}{4}$, — — 55	1	9	8				

TABLE

TABLE IX.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 22 $\frac{3}{4}$ to 23 $\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
12 $\frac{1}{4}$, or at Age 35	7	16	6
13 $\frac{1}{4}$, — — 36	7	6	2
14 $\frac{1}{4}$, — — 37	6	16	5
15 $\frac{1}{4}$, — — 38	6	6	9
16 $\frac{1}{4}$, — — 39	5	18	0
17 $\frac{1}{4}$, — — 40	5	9	2
18 $\frac{1}{4}$, — — 41	5	1	3
19 $\frac{1}{4}$, — — 42	4	13	4
20 $\frac{1}{4}$, — — 43	4	6	8
21 $\frac{1}{4}$, — — 44	4	0	0
22 $\frac{1}{4}$, — — 45	3	13	11
23 $\frac{1}{4}$, — — 46	3	7	11
24 $\frac{1}{4}$, — — 47	3	2	10
25 $\frac{1}{4}$, — — 48	2	17	9
26 $\frac{1}{4}$, — — 49	2	13	4
27 $\frac{1}{4}$, — — 50	2	9	0
28 $\frac{1}{4}$, — — 51	2	5	1
29 $\frac{1}{4}$, — — 52	2	1	2
30 $\frac{1}{4}$, — — 53	1	17	8
31 $\frac{1}{4}$, — — 54	1	14	1
32 $\frac{1}{4}$, — — 55	1	11	2

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d
33 $\frac{1}{4}$, or at Age 56	1	8	2
34 $\frac{1}{4}$, — — 57	1	5	8
35 $\frac{1}{4}$, — — 58	1	3	1
36 $\frac{1}{4}$, — — 59	1	1	0
37 $\frac{1}{4}$, — — 60	0	19	0
38 $\frac{1}{4}$, — — 61	0	17	2
39 $\frac{1}{4}$, — — 62	0	15	4
40 $\frac{1}{4}$, — — 63	0	13	10
41 $\frac{1}{4}$, — — 64	0	12	3
42 $\frac{1}{4}$, — — 65	0	10	11
43 $\frac{1}{4}$, — — 66	0	9	7
44 $\frac{1}{4}$, — — 67	0	8	6
45 $\frac{1}{4}$, — — 68	0	7	5
46 $\frac{1}{4}$, — — 69	0	6	6
47 $\frac{1}{4}$, — — 70	0	5	8
48 $\frac{1}{4}$, — — 71	0	5	0
49 $\frac{1}{4}$, — — 72	0	4	4
50 $\frac{1}{4}$, — — 73	0	3	9
51 $\frac{1}{4}$, — — 74	0	3	2
52 $\frac{1}{4}$, — — 75	0	2	8

TABLE X.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $23\frac{3}{4}$ to $24\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$11\frac{1}{4}$, or at Age 35	8	3	9	$32\frac{1}{4}$, or at Age 56	1	9	8
$12\frac{1}{4}$, — — 36	7	13	3	$33\frac{1}{4}$, — — 57	1	6	9
$13\frac{1}{4}$, — — 37	7	2	8	$34\frac{1}{4}$, — — 58	1	4	4
$14\frac{1}{4}$, — — 38	6	13	0	$35\frac{1}{4}$, — — 59	1	2	0
$15\frac{1}{4}$, — — 39	6	3	5	$36\frac{1}{4}$, — — 60	0	19	11
$16\frac{1}{4}$, — — 40	5	14	8	$37\frac{1}{4}$, — — 61	0	17	11
$17\frac{1}{4}$, — — 41	5	5	11	$38\frac{1}{4}$, — — 62	0	16	2
$18\frac{1}{4}$, — — 42	4	18	3	$39\frac{1}{4}$, — — 63	0	14	5
$19\frac{1}{4}$, — — 43	4	10	7	$40\frac{1}{4}$, — — 64	0	12	11
$20\frac{1}{4}$, — — 44	4	4	0	$41\frac{1}{4}$, — — 65	0	11	4
$21\frac{1}{4}$, — — 45	3	17	4	$42\frac{1}{4}$, — — 66	0	10	1
$22\frac{1}{4}$, — — 46	3	11	6	$43\frac{1}{4}$, — — 67	0	8	10
$23\frac{1}{4}$, — — 47	3	5	8	$44\frac{1}{4}$, — — 68	0	7	10
$24\frac{1}{4}$, — — 48	3	0	9	$45\frac{1}{4}$, — — 69	0	6	10
$25\frac{1}{4}$, — — 49	2	15	9	$46\frac{1}{4}$, — — 70	0	6	0
$26\frac{1}{4}$, — — 50	2	11	5	$47\frac{1}{4}$, — — 71	0	5	2
$27\frac{1}{4}$, — — 51	2	7	1	$48\frac{1}{4}$, — — 72	0	4	6
$28\frac{1}{4}$, — — 52	2	3	2	$49\frac{1}{4}$, — — 73	0	3	11
$29\frac{1}{4}$, — — 53	1	19	4	$50\frac{1}{4}$, — — 74	0	3	4
$30\frac{1}{4}$, — — 54	1	16	0	$51\frac{1}{4}$, — — 75	0	2	10
$31\frac{1}{4}$, — — 55	1	12	6				

TABLE XI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 24 ¼ to 25 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
10 ¼, or at Age 35	8	12	1	31 ¼, or at Age 56	1	11	0
11 ¼, — — 36	8	0	8	32 ¼, — — 57	1	8	2
12 ¼, — — 37	7	10	0	33 ¼, — — 58	1	5	4
13 ¼, — — 38	6	19	3	34 ¼, — — 59	1	3	1
14 ¼, — — 39	6	9	8	35 ¼, — — 60	1	0	11
15 ¼, — — 40	6	0	1	36 ¼, — — 61	0	18	10
16 ¼, — — 41	5	11	4	37 ¼, — — 62	0	16	10
17 ¼, — — 42	5	2	8	38 ¼, — — 63	0	15	2
18 ¼, — — 43	4	15	3	39 ¼, — — 64	0	13	6
19 ¼, — — 44	4	7	11	40 ¼, — — 65	0	12	0
20 ¼, — — 45	4	1	3	41 ¼, — — 66	0	10	6
21 ¼, — — 46	3	14	8	42 ¼, — — 67	0	9	4
22 ¼, — — 47	3	9	1	43 ¼, — — 68	0	8	2
23 ¼, — — 48	3	3	6	44 ¼, — — 69	0	7	2
24 ¼, — — 49	2	18	8	45 ¼, — — 70	0	6	3
25 ¼, — — 50	2	13	10	46 ¼, — — 71	0	5	6
26 ¼, — — 51	2	9	6	47 ¼, — — 72	0	4	9
27 ¼, — — 52	2	5	3	48 ¼, — — 73	0	4	1
28 ¼, — — 53	2	1	4	49 ¼, — — 74	0	3	6
29 ¼, — — 54	1	17	6	50 ¼, — — 75	0	3	0
30 ¼, — — 55	1	14	3				

TABLE XII.

For the Uſe of LONDON.

Shewing the Payment due, (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $25\frac{3}{4}$ to $26\frac{1}{2}$.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment.		
		£.	s.	d.
$9\frac{1}{4}$, or at Age	35	9	0	2
$10\frac{1}{4}$, — —	36	8	8	7
$11\frac{1}{4}$, — —	37	7	17	0
$12\frac{1}{4}$, — —	38	7	6	4
$13\frac{1}{4}$, — —	39	6	15	8
$14\frac{1}{4}$, — —	40	6	6	1
$15\frac{1}{4}$, — —	41	5	16	6
$16\frac{1}{4}$, — —	42	5	8	1
$17\frac{1}{4}$, — —	43	4	19	8
$18\frac{1}{4}$, — —	44	4	12	4
$19\frac{1}{4}$, — —	45	4	5	0
$20\frac{1}{4}$, — —	46	3	18	7
$21\frac{1}{4}$, — —	47	3	12	3
$22\frac{1}{4}$, — —	48	3	6	9
$23\frac{1}{4}$, — —	49	3	1	4
$24\frac{1}{4}$, — —	50	2	16	7
$25\frac{1}{4}$, — —	51	2	11	10
$26\frac{1}{4}$, — —	52	2	7	6
$27\frac{1}{4}$, — —	53	2	3	3
$28\frac{1}{4}$, — —	54	1	19	6
$29\frac{1}{4}$, — —	55	1	15	10

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment.		
		£.	s.	d.
$30\frac{1}{4}$, or at Age	56	1	12	7
$31\frac{1}{4}$, — —	57	1	9	5
$32\frac{1}{4}$, — —	58	1	6	9
$33\frac{1}{4}$, — —	59	1	4	2
$34\frac{1}{4}$, — —	60	1	1	11
$35\frac{1}{4}$, — —	61	0	19	8
$36\frac{1}{4}$, — —	62	0	17	9
$37\frac{1}{4}$, — —	63	0	15	10
$38\frac{1}{4}$, — —	64	0	14	2
$39\frac{1}{4}$, — —	65	0	12	6
$40\frac{1}{4}$, — —	66	0	11	1
$41\frac{1}{4}$, — —	67	0	9	9
$42\frac{1}{4}$, — —	68	0	8	7
$43\frac{1}{4}$, — —	69	0	7	6
$44\frac{1}{4}$, — —	70	0	6	7
$45\frac{1}{4}$, — —	71	0	5	8
$46\frac{1}{4}$, — —	72	0	5	0
$47\frac{1}{4}$, — —	73	0	4	3
$48\frac{1}{4}$, — —	74	0	3	8
$49\frac{1}{4}$, — —	75	0	3	1

TABLE XIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 26 ¾ to 27 ½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
8 ¼,	or at Age	35	9	9	4
9 ¼,	— —	36	8	16	10
10 ¼,	— —	37	8	5	1
11 ¼,	— —	38	7	13	3
12 ¼,	— —	39	7	2	8
13 ¼,	— —	40	6	12	1
14 ¼,	— —	41	6	2	7
15 ¼,	— —	42	5	13	0
16 ¼,	— —	43	5	4	11
17 ¼,	— —	44	4	16	9
18 ¼,	— —	45	4	9	6
19 ¼,	— —	46	4	2	2
20 ¼,	— —	47	3	16	0
21 ¼,	— —	48	3	9	11
22 ¼,	— —	49	3	4	6
23 ¼,	— —	50	2	19	2
24 ¼,	— —	51	2	14	6
25 ¼,	— —	52	2	9	11
26 ¼,	— —	53	2	5	7
27 ¼,	— —	54	2	1	3
28 ¼,	— —	55	1	17	8

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
29 ¼,	or at Age	56	1	14	2
30 ¼,	— —	57	1	11	0
31 ¼,	— —	58	1	7	11
32 ¼,	— —	59	1	5	5
33 ¼,	— —	60	1	3	0
34 ¼,	— —	61	1	0	9
35 ¼,	— —	62	0	18	6
36 ¼,	— —	63	0	16	8
37 ¼,	— —	64	0	14	11
38 ¼,	— —	65	0	13	3
39 ¼,	— —	66	0	11	7
40 ¼,	— —	67	0	10	3
41 ¼,	— —	68	0	9	0
42 ¼,	— —	69	0	7	11
43 ¼,	— —	70	0	6	11
44 ¼,	— —	71	0	6	0
45 ¼,	— —	72	0	5	2
46 ¼,	— —	73	0	4	6
47 ¼,	— —	74	0	3	10
48 ¼,	— —	75	0	3	3

TABLE XIV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 27 ¾ to 28 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
7 ¼, or at Age 35	9	18	9	28 ¼, or at Age 56	1	16	0
8 ¼, — — 36	9	5	11	29 ¼, — — 57	1	12	6
9 ¼, — — 37	8	13	2	30 ¼, — — 58	1	9	7
10 ¼, — — 38	8	1	5	31 ¼, — — 59	1	6	8
11 ¼, — — 39	7	9	8	32 ¼, — — 60	1	4	2
12 ¼, — — 40	6	19	1	33 ¼, — — 61	1	1	9
13 ¼, — — 41	6	8	6	34 ¼, — — 62	0	19	7
14 ¼, — — 42	5	19	3	35 ¼, — — 63	0	17	6
15 ¼, — — 43	5	10	0	36 ¼, — — 64	0	15	8
16 ¼, — — 44	5	1	11	37 ¼, — — 65	0	13	10
17 ¼, — — 45	4	13	9	38 ¼, — — 66	0	12	3
18 ¼, — — 46	4	6	9	39 ¼, — — 67	0	10	9
19 ¼, — — 47	3	19	8	40 ¼, — — 68	0	9	6
20 ¼, — — 48	3	13	8	41 ¼, — — 69	0	8	3
21 ¼, — — 49	3	7	8	42 ¼, — — 70	0	7	3
22 ¼, — — 50	3	2	5	43 ¼, — — 71	0	6	4
23 ¼, — — 51	2	17	2	44 ¼, — — 72	0	5	6
24 ¼, — — 52	2	12	5	45 ¼, — — 73	0	4	8
25 ¼, — — 53	2	7	9	46 ¼, — — 74	0	4	0
26 ¼, — — 54	2	3	7	47 ¼, — — 75	0	3	5
27 ¼, — — 55	1	19	6				

TABLE XV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 28 ¼ to 29 ½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
6 ¼,	or at Age	35	10	9	5	27 ¼,	or at Age	56	1	17	9
7 ¼,	— —	36	9	15	7	28 ¼,	— —	57	1	14	3
8 ¼,	— —	37	9	2	6	29 ¼,	— —	58	1	10	11
9 ¼,	— —	38	8	9	6	30 ¼,	— —	59	1	8	2
10 ¼,	— —	39	7	17	9	31 ¼,	— —	60	1	5	5
11 ¼,	— —	40	7	6	1	32 ¼,	— —	61	1	2	11
12 ¼,	— —	41	6	15	6	33 ¼,	— —	62	1	0	6
13 ¼,	— —	42	6	4	11	34 ¼,	— —	63	0	18	6
14 ¼,	— —	43	5	15	11	35 ¼,	— —	64	0	16	6
15 ¼,	— —	44	5	7	0	36 ¼,	— —	65	0	14	8
16 ¼,	— —	45	4	18	11	37 ¼,	— —	66	0	12	10
17 ¼,	— —	46	4	10	10	38 ¼,	— —	67	0	11	4
18 ¼,	— —	47	4	4	0	39 ¼,	— —	68	0	9	11
19 ¼,	— —	48	3	17	3	40 ¼,	— —	69	0	8	9
20 ¼,	— —	49	3	11	4	41 ¼,	— —	70	0	7	7
21 ¼,	— —	50	3	5	6	42 ¼,	— —	71	0	6	8
22 ¼,	— —	51	3	0	4	43 ¼,	— —	72	0	5	10
23 ¼,	— —	52	2	15	2	44 ¼,	— —	73	0	5	0
24 ¼,	— —	53	2	10	4	45 ¼,	— —	74	0	4	2
25 ¼,	— —	54	2	5	7	46 ¼,	— —	75	0	3	7
26 ¼,	— —	55	2	1	8						

TABLE XVI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $29\frac{3}{4}$ to $30\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$5\frac{1}{4}$, or at Age 35	11	0	3	$26\frac{1}{4}$, or at Age 56	1	19	11
$6\frac{1}{4}$, — — 36	10	6	1	$27\frac{1}{4}$, — — 57	1	16	0
$7\frac{1}{4}$, — — 37	9	11	11	$28\frac{1}{4}$, — — 58	1	12	9
$8\frac{1}{4}$, — — 38	8	18	11	$29\frac{1}{4}$, — — 59	1	9	7
$9\frac{1}{4}$, — — 39	8	5	11	$30\frac{1}{4}$, — — 60	1	6	10
$10\frac{1}{4}$, — — 40	7	14	2	$31\frac{1}{4}$, — — 61	1	4	1
$11\frac{1}{4}$, — — 41	7	2	5	$32\frac{1}{4}$, — — 62	1	1	9
$12\frac{1}{4}$, — — 42	6	12	2	$33\frac{1}{4}$, — — 63	0	19	4
$13\frac{1}{4}$, — — 43	6	1	11	$34\frac{1}{4}$, — — 64	0	17	4
$14\frac{1}{4}$, — — 44	5	12	11	$35\frac{1}{4}$, — — 65	0	15	4
$15\frac{1}{4}$, — — 45	5	4	0	$36\frac{1}{4}$, — — 66	0	13	8
$16\frac{1}{4}$, — — 46	4	16	2	$37\frac{1}{4}$, — — 67	0	11	11
$17\frac{1}{4}$, — — 47	4	8	4	$38\frac{1}{4}$, — — 68	0	10	7
$18\frac{1}{4}$, — — 48	4	1	8	$39\frac{1}{4}$, — — 69	0	9	2
$19\frac{1}{4}$, — — 49	3	15	0	$40\frac{1}{4}$, — — 70	0	8	1
$20\frac{1}{4}$, — — 50	3	9	2	$41\frac{1}{4}$, — — 71	0	7	0
$21\frac{1}{4}$, — — 51	3	3	4	$42\frac{1}{4}$, — — 72	0	6	1
$22\frac{1}{4}$, — — 52	2	18	2	$43\frac{1}{4}$, — — 73	0	5	3
$23\frac{1}{4}$, — — 53	2	12	11	$44\frac{1}{4}$, — — 74	0	4	6
$24\frac{1}{4}$, — — 54	2	8	4	$45\frac{1}{4}$, — — 75	0	3	9
$25\frac{1}{4}$, — — 55	2	3	10				

TABLE XVII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $30\frac{1}{4}$ to $31\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$4\frac{1}{4}$, or at Age 35	11	12	9
$5\frac{1}{4}$, — — 36	10	17	4
$6\frac{1}{4}$, — — 37	10	2	10
$7\frac{1}{4}$, — — 38	9	8	4
$8\frac{1}{4}$, — — 39	8	15	4
$9\frac{1}{4}$, — — 40	8	2	4
$10\frac{1}{4}$, — — 41	7	10	7
$11\frac{1}{4}$, — — 42	6	18	10
$12\frac{1}{4}$, — — 43	6	8	10
$13\frac{1}{4}$, — — 44	5	18	11
$14\frac{1}{4}$, — — 45	5	9	11
$15\frac{1}{4}$, — — 46	5	1	0
$16\frac{1}{4}$, — — 47	4	13	5
$17\frac{1}{4}$, — — 48	4	5	11
$18\frac{1}{4}$, — — 49	3	19	4
$19\frac{1}{4}$, — — 50	3	12	10
$20\frac{1}{4}$, — — 51	3	7	0
$21\frac{1}{4}$, — — 52	3	1	3
$22\frac{1}{4}$, — — 53	2	16	0
$23\frac{1}{4}$, — — 54	2	10	8
$24\frac{1}{4}$, — — 55	2	6	4

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$25\frac{1}{4}$, or at Age 56	2	2	0
$26\frac{1}{4}$, — — 57	1	18	1
$27\frac{1}{4}$, — — 58	1	14	3
$28\frac{1}{4}$, — — 59	1	11	3
$29\frac{1}{4}$, — — 60	1	8	3
$30\frac{1}{4}$, — — 61	1	5	6
$31\frac{1}{4}$, — — 62	1	2	10
$32\frac{1}{4}$, — — 63	1	0	7
$33\frac{1}{4}$, — — 64	0	18	3
$34\frac{1}{4}$, — — 65	0	16	3
$35\frac{1}{4}$, — — 66	0	14	3
$36\frac{1}{4}$, — — 67	0	12	8
$37\frac{1}{4}$, — — 68	0	11	0
$38\frac{1}{4}$, — — 69	0	9	9
$39\frac{1}{4}$, — — 70	0	8	6
$40\frac{1}{4}$, — — 71	0	7	5
$41\frac{1}{4}$, — — 72	0	6	5
$42\frac{1}{4}$, — — 73	0	5	7
$43\frac{1}{4}$, — — 74	0	4	8
$44\frac{1}{4}$, — — 75	0	4	0

TABLE XVIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 31 ¾ to 32 ½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
3 ¼,	or at Age 35	12	5	1
4 ¼,	— — 36	11	9	4
5 ¼,	— — 37	10	13	7
6 ¼,	— — 38	9	19	1
7 ¼,	— — 39	9	4	7
8 ¼,	— — 40	8	11	7
9 ¼,	— — 41	7	18	6
10 ¼,	— — 42	7	7	1
11 ¼,	— — 43	6	15	8
12 ¼,	— — 44	6	5	8
13 ¼,	— — 45	5	15	8
14 ¼,	— — 46	5	7	0
15 ¼,	— — 47	4	18	4
16 ¼,	— — 48	4	10	11
17 ¼,	— — 49	4	3	6
18 ¼,	— — 50	3	17	0
19 ¼,	— — 51	3	10	6
20 ¼,	— — 52	3	4	8
21 ¼,	— — 53	2	18	10
22 ¼,	— — 54	2	13	9
23 ¼,	— — 55	2	8	8

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
24 ¼,	or at Age 56	2	4	5
25 ¼,	— — 57	2	0	1
26 ¼,	— — 58	1	16	6
27 ¼,	— — 59	1	12	11
28 ¼,	— — 60	1	9	10
29 ¼,	— — 61	1	6	10
30 ¼,	— — 62	1	4	2
31 ¼,	— — 63	1	1	7
32 ¼,	— — 64	0	19	4
33 ¼,	— — 65	0	17	1
34 ¼,	— — 66	0	15	2
35 ¼,	— — 67	0	13	3
36 ¼,	— — 68	0	11	9
37 ¼,	— — 69	0	10	2
38 ¼,	— — 70	0	9	0
39 ¼,	— — 71	0	7	10
40 ¼,	— — 72	0	6	10
41 ¼,	— — 73	0	5	10
42 ¼,	— — 74	0	5	0
43 ¼,	— — 75	0	4	3

TABLE XIX.

For the Use of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 32 ¼ to 33 ½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
2 ¼,	or at Age	35	12	19	3
3 ¼,	— —	36	12	2	1
4 ¼,	— —	37	11	5	11
5 ¼,	— —	38	10	9	10
6 ¼,	— —	39	9	15	4
7 ¼,	— —	40	9	0	11
8 ¼,	— —	41	8	7	9
9 ¼,	— —	42	7	14	8
10 ¼,	— —	43	7	3	7
11 ¼,	— —	44	6	12	6
12 ¼,	— —	45	6	2	6
13 ¼,	— —	46	5	12	6
14 ¼,	— —	47	5	4	1
15 ¼,	— —	48	4	15	8
16 ¼,	— —	49	4	8	4
17 ¼,	— —	50	4	1	1
18 ¼,	— —	51	3	14	8
19 ¼,	— —	52	3	8	3
20 ¼,	— —	53	3	2	4
21 ¼,	— —	54	2	16	6
22 ¼,	— —	55	2	11	7

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
23 ¼,	or at Age	56	2	6	9
24 ¼,	— —	57	2	2	6
25 ¼,	— —	58	1	18	2
26 ¼,	— —	59	1	14	10
27 ¼,	— —	60	1	11	6
28 ¼,	— —	61	1	8	5
29 ¼,	— —	62	1	5	4
30 ¼,	— —	63	1	2	10
31 ¼,	— —	64	1	0	4
32 ¼,	— —	65	0	18	2
33 ¼,	— —	66	0	15	11
34 ¼,	— —	67	0	14	1
35 ¼,	— —	68	0	12	3
36 ¼,	— —	69	0	10	10
37 ¼,	— —	70	0	9	5
38 ¼,	— —	71	0	8	3
39 ¼,	— —	72	0	7	2
40 ¼,	— —	73	0	6	2
41 ¼,	— —	74	0	5	2
42 ¼,	— —	75	0	4	6

TABLE XX.

For the Uſe of LONDON.

Shewing the Payment due, (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $33\frac{3}{4}$ to $34\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$1\frac{1}{4}$, or at Age 35	13	13	4	$22\frac{1}{4}$, or at Age 56	2	9	6
$2\frac{1}{4}$, — — 36	12	15	10	$23\frac{1}{4}$, — — 57	2	4	8
$3\frac{1}{4}$, — — 37	11	18	2	$24\frac{1}{4}$, — — 58	2	0	8
$4\frac{1}{4}$, — — 38	11	2	0	$25\frac{1}{4}$, — — 59	1	16	8
$5\frac{1}{4}$, — — 39	10	5	11	$26\frac{1}{4}$, — — 60	1	13	3
$6\frac{1}{4}$, — — 40	9	11	4	$27\frac{1}{4}$, — — 61	1	9	11
$7\frac{1}{4}$, — — 41	8	16	9	$28\frac{1}{4}$, — — 62	1	7	0
$8\frac{1}{4}$, — — 42	8	4	0	$29\frac{1}{4}$, — — 63	1	4	0
$9\frac{1}{4}$, — — 43	7	11	3	$30\frac{1}{4}$, — — 64	1	1	6
$10\frac{1}{4}$, — — 44	7	0	2	$31\frac{1}{4}$, — — 65	0	19	0
$11\frac{1}{4}$, — — 45	6	9	0	$32\frac{1}{4}$, — — 66	0	17	0
$12\frac{1}{4}$, — — 46	5	19	4	$33\frac{1}{4}$, — — 67	0	14	10
$13\frac{1}{4}$, — — 47	5	9	8	$34\frac{1}{4}$, — — 68	0	13	1
$14\frac{1}{4}$, — — 48	5	1	4	$35\frac{1}{4}$, — — 69	0	11	4
$15\frac{1}{4}$, — — 49	4	13	1	$36\frac{1}{4}$, — — 70	0	10	0
$16\frac{1}{4}$, — — 50	4	5	10	$37\frac{1}{4}$, — — 71	0	8	8
$17\frac{1}{4}$, — — 51	3	18	7	$38\frac{1}{4}$, — — 72	0	7	7
$18\frac{1}{4}$, — — 52	3	12	2	$39\frac{1}{4}$, — — 73	0	6	6
$19\frac{1}{4}$, — — 53	3	5	8	$40\frac{1}{4}$, — — 74	0	5	7
$20\frac{1}{4}$, — — 54	3	0	0	$41\frac{1}{4}$, — — 75	0	4	9
$21\frac{1}{4}$, — — 55	2	14	4				

TABLE XXI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $34\frac{3}{4}$ to $35\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 35	14	9	6
$1\frac{1}{4}$, — — 36	13	10	4
$2\frac{1}{4}$, — — 37	12	12	4
$3\frac{1}{4}$, — — 38	11	14	3
$4\frac{1}{4}$, — — 39	10	18	2
$5\frac{1}{4}$, — — 40	10	2	0
$6\frac{1}{4}$, — — 41	9	7	4
$7\frac{1}{4}$, — — 42	8	12	8
$8\frac{1}{4}$, — — 43	8	0	3
$9\frac{1}{4}$, — — 44	7	7	11
$10\frac{1}{4}$, — — 45	6	16	9
$11\frac{1}{4}$, — — 46	6	5	7
$12\frac{1}{4}$, — — 47	5	16	2
$13\frac{1}{4}$, — — 48	5	6	10
$14\frac{1}{4}$, — — 49	4	18	8
$15\frac{1}{4}$, — — 50	4	10	7
$16\frac{1}{4}$, — — 51	4	3	4
$17\frac{1}{4}$, — — 52	3	16	2
$18\frac{1}{4}$, — — 53	3	9	8
$19\frac{1}{4}$, — — 54	3	3	1
$20\frac{1}{4}$, — — 55	2	17	8

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$21\frac{1}{4}$, or at Age 56	2	12	2
$22\frac{1}{4}$, — — 57	2	7	5
$23\frac{1}{4}$, — — 58	2	2	8
$24\frac{1}{4}$, — — 59	1	18	11
$25\frac{1}{4}$, — — 60	1	15	2
$26\frac{1}{4}$, — — 61	1	11	9
$27\frac{1}{4}$, — — 62	1	8	4
$28\frac{1}{4}$, — — 63	1	5	7
$29\frac{1}{4}$, — — 64	1	2	9
$30\frac{1}{4}$, — — 65	1	0	3
$31\frac{1}{4}$, — — 66	0	17	9
$32\frac{1}{4}$, — — 67	0	15	9
$33\frac{1}{4}$, — — 68	0	13	8
$34\frac{1}{4}$, — — 69	0	12	1
$35\frac{1}{4}$, — — 70	0	10	6
$36\frac{1}{4}$, — — 71	0	9	3
$37\frac{1}{4}$, — — 72	0	8	0
$38\frac{1}{4}$, — — 73	0	6	11
$39\frac{1}{4}$, — — 74	0	5	10
$40\frac{1}{4}$, — — 75	0	5	0

TABLE XXII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $35\frac{1}{4}$ to $36\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 36	14	5	6	$20\frac{1}{4}$, or at Age 56	2	15	4
$1\frac{1}{4}$, — — 37	13	6	4	$21\frac{1}{4}$, — — 57	2	10	0
$2\frac{1}{4}$, — — 38	12	8	3	$22\frac{1}{4}$, — — 58	2	5	6
$3\frac{1}{4}$, — — 39	11	10	2	$23\frac{1}{4}$, — — 59	2	1	0
$4\frac{1}{4}$, — — 40	10	14	0	$24\frac{1}{4}$, — — 60	1	17	2
$5\frac{1}{4}$, — — 41	9	17	8	$25\frac{1}{4}$, — — 61	1	13	5
$6\frac{1}{4}$, — — 42	9	3	5	$26\frac{1}{4}$, — — 62	1	10	2
$7\frac{1}{4}$, — — 43	8	9	2	$27\frac{1}{4}$, — — 63	1	6	11
$8\frac{1}{4}$, — — 44	7	16	8	$28\frac{1}{4}$, — — 64	1	4	1
$9\frac{1}{4}$, — — 45	7	4	3	$29\frac{1}{4}$, — — 65	1	1	4
$10\frac{1}{4}$, — — 46	6	13	5	$30\frac{1}{4}$, — — 66	0	18	11
$11\frac{1}{4}$, — — 47	6	2	7	$31\frac{1}{4}$, — — 67	0	16	6
$12\frac{1}{4}$, — — 48	5	13	4	$32\frac{1}{4}$, — — 68	0	14	8
$13\frac{1}{4}$, — — 49	5	4	1	$33\frac{1}{4}$, — — 69	0	12	9
$14\frac{1}{4}$, — — 50	4	16	0	$34\frac{1}{4}$, — — 70	0	11	2
$15\frac{1}{4}$, — — 51	4	8	0	$35\frac{1}{4}$, — — 71	0	9	8
$16\frac{1}{4}$, — — 52	4	0	8	$36\frac{1}{4}$, — — 72	0	8	6
$17\frac{1}{4}$, — — 53	3	13	5	$37\frac{1}{4}$, — — 73	0	7	3
$18\frac{1}{4}$, — — 54	3	7	1	$38\frac{1}{4}$, — — 74	0	6	3
$19\frac{1}{4}$, — — 55	3	0	9	$39\frac{1}{4}$, — — 75	0	5	3

TABLE XXIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 36 ¾ to 37 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age	37 14	1	6
1 ¼, — —	38 13	2	5
2 ¼, — —	39 12	4	3
3 ¼, — —	40 11	6	2
4 ¼, — —	41 10	9	9
5 ¼, — —	42 9	13	4
6 ¼, — —	43 8	19	6
7 ¼, — —	44 8	5	8
8 ¼, — —	45 7	13	2
9 ¼, — —	46 7	0	8
10 ¼, — —	47 6	10	1
11 ¼, — —	48 5	19	7
12 ¼, — —	49 5	10	6
13 ¼, — —	50 5	1	4
14 ¼, — —	51 4	13	4
15 ¼, — —	52 4	5	4
16 ¼, — —	53 3	18	0
17 ¼, — —	54 3	10	8
18 ¼, — —	55 3	4	6
19 ¼, — —	56 2	18	5

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
20 ¼, or at Age	57 2	13	1
21 ¼, — —	58 2	7	9
22 ¼, — —	59 2	3	7
23 ¼, — —	60 1	19	4
24 ¼, — —	61 1	15	6
25 ¼, — —	62 1	11	8
26 ¼, — —	63 1	8	7
27 ¼, — —	64 1	5	6
28 ¼, — —	65 1	2	8
29 ¼, — —	66 0	19	11
30 ¼, — —	67 0	17	7
31 ¼, — —	68 0	15	4
32 ¼, — —	69 0	13	7
33 ¼, — —	70 0	11	10
34 ¼, — —	71 0	10	4
35 ¼, — —	72 0	8	11
36 ¼, — —	73 0	7	9
37 ¼, — —	74 0	6	6
38 ¼, — —	75 0	5	7

TABLE XXIV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 37 ¼ to 38 ½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 38	13	17	6	19 ¼,	or at Age 57	2	16	2
1 ¼,	— — 39	12	18	9	20 ¼,	— — 58	2	11	1
2 ¼,	— — 40	12	0	6	21 ¼,	— — 59	2	6	1
3 ¼,	— — 41	11	2	2	22 ¼,	— — 60	2	1	10
4 ¼,	— — 42	10	6	1	23 ¼,	— — 61	1	17	7
5 ¼,	— — 43	9	10	1	24 ¼,	— — 62	1	13	10
6 ¼,	— — 44	8	16	1	25 ¼,	— — 63	1	10	2
7 ¼,	— — 45	8	2	2	26 ¼,	— — 64	1	7	1
8 ¼,	— — 46	7	10	0	27 ¼,	— — 65	1	4	0
9 ¼,	— — 47	6	17	9	28 ¼,	— — 66	1	1	3
10 ¼,	— — 48	6	7	4	29 ¼,	— — 67	0	18	7
11 ¼,	— — 49	5	17	0	30 ¼,	— — 68	0	16	5
12 ¼,	— — 50	5	7	11	31 ¼,	— — 69	0	14	3
13 ¼,	— — 51	4	18	9	32 ¼,	— — 70	0	12	7
14 ¼,	— — 52	4	10	8	33 ¼,	— — 71	0	11	0
15 ¼,	— — 53	4	2	6	34 ¼,	— — 72	0	9	6
16 ¼,	— — 54	3	15	5	35 ¼,	— — 73	0	8	1
17 ¼,	— — 55	3	8	3	36 ¼,	— — 74	0	7	0
18 ¼,	— — 56	3	2	2	37 ¼,	— — 75	0	5	11

TABLE XXV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 38 ¾ to 39 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age 39	13	13	6	19 ¼, or at Age 58	2	13	11
1 ¼, — — 40	12	15	2	20 ¼, — — 59	2	9	2
2 ¼, — — 41	11	16	8	21 ¼, — — 60	2	4	5
3 ¼, — — 42	10	18	2	22 ¼, — — 61	2	0	1
4 ¼, — — 43	10	2	6	23 ¼, — — 62	1	15	10
5 ¼, — — 44	9	6	11	24 ¼, — — 63	1	12	3
6 ¼, — — 45	8	12	9	25 ¼, — — 64	1	8	9
7 ¼, — — 46	7	18	8	26 ¼, — — 65	1	5	7
8 ¼, — — 47	7	6	10	27 ¼, — — 66	1	2	5
9 ¼, — — 48	6	14	11	28 ¼, — — 67	0	19	10
10 ¼, — — 49	6	4	8	29 ¼, — — 68	0	17	3
11 ¼, — — 50	5	14	4	30 ¼, — — 69	0	15	3
12 ¼, — — 51	5	5	4	31 ¼, — — 70	0	13	3
13 ¼, — — 52	4	16	3	32 ¼, — — 71	0	11	8
14 ¼, — — 53	4	8	0	33 ¼, — — 72	0	10	1
15 ¼, — — 54	3	19	8	34 ¼, — — 73	0	8	8
16 ¼, — — 55	3	12	10	35 ¼, — — 74	0	7	4
17 ¼, — — 56	3	5	11	36 ¼, — — 75	0	6	3
18 ¼, — — 57	2	19	11				

TABLE XXVI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $39\frac{3}{4}$ to $40\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 40	13	11	6	18 $\frac{1}{4}$, or at Age 58	2	17	9
1 $\frac{1}{4}$, — — 41	12	11	1	19 $\frac{1}{4}$, — — 59	2	12	1
2 $\frac{1}{4}$, — — 42	11	13	0	20 $\frac{1}{4}$, — — 60	2	7	3
3 $\frac{1}{4}$, — — 43	10	14	10	21 $\frac{1}{4}$, — — 61	2	2	6
4 $\frac{1}{4}$, — — 44	9	19	1	22 $\frac{1}{4}$, — — 62	1	18	4
5 $\frac{1}{4}$, — — 45	9	3	3	23 $\frac{1}{4}$, — — 63	1	14	2
6 $\frac{1}{4}$, — — 46	8	9	6	24 $\frac{1}{4}$, — — 64	1	10	7
7 $\frac{1}{4}$, — — 47	7	15	9	25 $\frac{1}{4}$, — — 65	1	7	1
8 $\frac{1}{4}$, — — 48	7	4	0	26 $\frac{1}{4}$, — — 66	1	4	0
9 $\frac{1}{4}$, — — 49	6	12	3	27 $\frac{1}{4}$, — — 67	1	1	0
10 $\frac{1}{4}$, — — 50	6	2	0	28 $\frac{1}{4}$, — — 68	0	18	7
11 $\frac{1}{4}$, — — 51	5	11	8	29 $\frac{1}{4}$, — — 69	0	16	2
12 $\frac{1}{4}$, — — 52	5	2	6	30 $\frac{1}{4}$, — — 70	0	14	3
13 $\frac{1}{4}$, — — 53	4	13	3	31 $\frac{1}{4}$, — — 71	0	12	4
14 $\frac{1}{4}$, — — 54	4	5	3	32 $\frac{1}{4}$, — — 72	0	10	9
15 $\frac{1}{4}$, — — 55	3	17	2	33 $\frac{1}{4}$, — — 73	0	9	2
16 $\frac{1}{4}$, — — 56	3	10	4	34 $\frac{1}{4}$, — — 74	0	7	10
17 $\frac{1}{4}$, — — 57	3	3	5	35 $\frac{1}{4}$, — — 75	0	6	8

TABLE XXVII.

For the Use of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 40 ¾ to 41 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age 41	13	7	6
1 ¼, — — 42	12	7	1
2 ¼, — — 43	11	9	4
3 ¼, — — 44	10	11	7
4 ¼, — — 45	9	15	8
5 ¼, — — 46	8	19	8
6 ¼, — — 47	8	6	3
7 ¼, — — 48	7	12	10
8 ¼, — — 49	7	1	2
9 ¼, — — 50	6	9	7
10 ¼, — — 51	5	19	4
11 ¼, — — 52	5	9	1
12 ¼, — — 53	4	19	8
13 ¼, — — 54	4	10	3
14 ¼, — — 55	4	2	6
15 ¼, — — 56	3	14	8
16 ¼, — — 57	3	7	10
17 ¼, — — 58	3	1	0

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
18 ¼, or at Age 59	2	15	8
19 ¼, — — 60	2	10	3
20 ¼, — — 61	2	5	5
21 ¼, — — 62	2	0	7
22 ¼, — — 63	1	16	7
23 ¼, — — 64	1	12	7
24 ¼, — — 65	1	9	0
25 ¼, — — 66	1	5	5
26 ¼, — — 67	1	2	6
27 ¼, — — 68	0	19	7
28 ¼, — — 69	0	17	4
29 ¼, — — 70	0	15	1
30 ¼, — — 71	0	13	3
31 ¼, — — 72	0	11	5
32 ¼, — — 73	0	9	11
33 ¼, — — 74	0	8	4
34 ¼, — — 75	0	7	1

TABLE XXVIII.

For the Uſe of LONDON.

Shewing the Payment due, (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 41 $\frac{3}{4}$ to 42 $\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment.		
			£.	s.	d.
$\frac{1}{4}$,	or at Age	42	13	3	6
1 $\frac{1}{4}$,	— —	43	12	4	6
2 $\frac{1}{4}$,	— —	44	11	6	6
3 $\frac{1}{4}$,	— —	45	10	8	6
4 $\frac{1}{4}$,	— —	46	9	12	10
5 $\frac{1}{4}$,	— —	47	8	17	2
6 $\frac{1}{4}$,	— —	48	8	3	10
7 $\frac{1}{4}$,	— —	49	7	10	5
8 $\frac{1}{4}$,	— —	50	6	18	9
9 $\frac{1}{4}$,	— —	51	6	7	1
10 $\frac{1}{4}$,	— —	52	5	16	7
11 $\frac{1}{4}$,	— —	53	5	6	1
12 $\frac{1}{4}$,	— —	54	4	17	0
13 $\frac{1}{4}$,	— —	55	4	7	9
14 $\frac{1}{4}$,	— —	56	4	0	0
15 $\frac{1}{4}$,	— —	57	3	12	3
16 $\frac{1}{4}$,	— —	58	3	5	9
17 $\frac{1}{4}$,	or at Age	59	2	19	3
18 $\frac{1}{4}$,	— —	60	2	13	9
19 $\frac{1}{4}$,	— —	61	2	8	3
20 $\frac{1}{4}$,	— —	62	2	3	7
21 $\frac{1}{4}$,	— —	63	1	18	10
22 $\frac{1}{4}$,	— —	64	1	14	10
23 $\frac{1}{4}$,	— —	65	1	10	9
24 $\frac{1}{4}$,	— —	66	1	7	4
25 $\frac{1}{4}$,	— —	67	1	3	11
26 $\frac{1}{4}$,	— —	68	1	1	2
27 $\frac{1}{4}$,	— —	69	0	18	4
28 $\frac{1}{4}$,	— —	70	0	16	3
29 $\frac{1}{4}$,	— —	71	0	14	1
30 $\frac{1}{4}$,	— —	72	0	12	3
31 $\frac{1}{4}$,	— —	73	0	10	5
32 $\frac{1}{4}$,	— —	74	0	9	0
33 $\frac{1}{4}$,	— —	75	0	7	7

TABLE XXIX.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 42 ¾ to 43 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment.		
		£.	s.	d.
¼,	or at Age 43	12	19	6
1 ¼,	— — 44	12	2	0
2 ¼,	— — 45	11	3	8
3 ¼,	— — 46	10	5	5
4 ¼,	— — 47	9	10	1
5 ¼,	— — 48	8	14	8
6 ¼,	— — 49	8	1	5
7 ¼,	— — 50	7	8	1
8 ¼,	— — 51	6	16	4
9 ¼,	— — 52	6	4	8
10 ¼,	— — 53	5	13	11
11 ¼,	— — 54	5	3	2
12 ¼,	— — 55	4	14	3
13 ¼,	— — 56	4	5	4
14 ¼,	— — 57	3	17	7
15 ¼,	— — 58	3	9	10
16 ¼,	— — 59	3	3	8
17 ¼,	or at Age 60	2	17	6
18 ¼,	— — 61	2	11	11
19 ¼,	— — 62	2	6	4
20 ¼,	— — 63	2	1	9
21 ¼,	— — 64	1	17	2
22 ¼,	— — 65	1	13	1
23 ¼,	— — 66	1	9	0
24 ¼,	— — 67	1	5	9
25 ¼,	— — 68	1	2	5
26 ¼,	— — 69	0	19	10
27 ¼,	— — 70	0	17	2
28 ¼,	— — 71	0	15	2
29 ¼,	— — 72	0	13	1
30 ¼,	— — 73	0	11	3
31 ¼,	— — 74	0	9	6
32 ¼,	— — 75	0	8	2

TABLE XXX.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 43 ¾ to 44 ½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 44	12	17	0	16 ¼,	or at Age 60	3	1	4
1 ¼,	— — 45	11	18	0	17 ¼,	— — 61	2	15	1
2 ¼,	— — 46	11	0	0	18 ¼,	— — 62	2	9	9
3 ¼,	— — 47	10	2	2	19 ¼,	— — 63	2	4	4
4 ¼,	— — 48	9	6	11	20 ¼,	— — 64	1	19	8
5 ¼,	— — 49	8	11	7	21 ¼,	— — 65	1	15	1
6 ¼,	— — 50	7	18	4	22 ¼,	— — 66	1	11	2
7 ¼,	— — 51	7	5	0	23 ¼,	— — 67	1	7	3
8 ¼,	— — 52	6	13	0	24 ¼,	— — 68	1	4	2
9 ¼,	— — 53	6	1	1	25 ¼,	— — 69	1	1	0
10 ¼,	— — 54	5	10	7	26 ¼,	— — 70	0	18	6
11 ¼,	— — 55	5	0	2	27 ¼,	— — 71	0	16	0
12 ¼,	— — 56	4	11	3	28 ¼,	— — 72	0	14	0
13 ¼,	— — 57	4	2	4	29 ¼,	— — 73	0	12	0
14 ¼,	— — 58	3	15	0	30 ¼,	— — 74	0	10	3
15 ¼,	— — 59	3	7	8	31 ¼,	— — 75	0	8	8

TABLE XXXI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 44 ¾ to 45 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age	45 12	13	6
1 ¼, — —	46 11	13	11
2 ¼, — —	47 10	16	5
3 ¼, — —	48 9	18	11
4 ¼, — —	49 9	3	9
5 ¼, — —	50 8	8	7
6 ¼, — —	51 7	15	3
7 ¼, — —	52 7	1	11
8 ¼, — —	53 6	9	8
9 ¼, — —	54 5	17	6
10 ¼, — —	55 5	7	4
11 ¼, — —	56 4	17	2
12 ¼, — —	57 4	8	3
13 ¼, — —	58 3	19	5
14 ¼, — —	59 3	12	5
15 ¼, — —	60 3	5	6

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
16 ¼, or at Age	61 2	19	1
17 ¼, — —	62 2	12	9
18 ¼, — —	63 2	7	7
19 ¼, — —	64 2	2	4
20 ¼, — —	65 1	17	8
21 ¼, — —	66 1	13	0
22 ¼, — —	67 1	9	3
23 ¼, — —	68 1	5	6
24 ¼, — —	69 1	2	7
25 ¼, — —	70 0	19	7
26 ¼, — —	71 0	17	3
27 ¼, — —	72 0	14	11
28 ¼, — —	73 0	12	11
29 ¼, — —	74 0	10	11
30 ¼, — —	75 0	9	3

TABLE

TABLE XXXII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 45 ¼ to 46 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age 46	12	9	6	15 ¼, or at Age 61	3	2	11
1 ¼, — — 47	11	10	9	16 ¼, — — 62	2	16	9
2 ¼, — — 48	10	13	4	17 ¼, — — 63	2	10	7
3 ¼, — — 49	9	15	11	18 ¼, — — 64	2	5	4
4 ¼, — — 50	9	0	8	19 ¼, — — 65	2	0	1
5 ¼, — — 51	8	5	6	20 ¼, — — 66	1	15	7
6 ¼, — — 52	7	11	10	21 ¼, — — 67	1	11	1
7 ¼, — — 53	6	18	2	22 ¼, — — 68	1	7	6
8 ¼, — — 54	6	6	3	23 ¼, — — 69	1	4	0
9 ¼, — — 55	5	14	4	24 ¼, — — 70	1	1	2
10 ¼, — — 56	5	4	2	25 ¼, — — 71	0	18	3
11 ¼, — — 57	4	14	0	26 ¼, — — 72	0	16	0
12 ¼, — — 58	4	5	7	27 ¼, — — 73	0	13	8
13 ¼, — — 59	3	17	2	28 ¼, — — 74	0	11	9
14 ¼, — — 60	3	10	0	29 ¼, — — 75	0	9	11

TABLE XXXIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $46\frac{3}{4}$ to $47\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment.		
	£.	s.	d.
$\frac{1}{4}$, or at Age 47	12	5	6
$1\frac{1}{4}$, — — 48	11	7	7
$2\frac{1}{4}$, — — 49	10	10	3
$3\frac{1}{4}$, — — 50	9	12	11
$4\frac{1}{4}$, — — 51	8	17	8
$5\frac{1}{4}$, — — 52	8	2	5
$6\frac{1}{4}$, — — 53	7	8	5
$7\frac{1}{4}$, — — 54	6	14	5
$8\frac{1}{4}$, — — 55	6	2	10
$9\frac{1}{4}$, — — 56	5	11	2
$10\frac{1}{4}$, — — 57	5	1	0
$11\frac{1}{4}$, — — 58	4	10	11
$12\frac{1}{4}$, — — 59	4	2	11
$13\frac{1}{4}$, — — 60	3	14	11
$14\frac{1}{4}$, — — 61	3	7	8

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment.		
	£.	s.	d.
$15\frac{1}{4}$, or at Age 62	3	0	4
$16\frac{1}{4}$, — — 63	2	14	5
$17\frac{1}{4}$, — — 64	2	8	6
$18\frac{1}{4}$, — — 65	2	3	2
$19\frac{1}{4}$, — — 66	1	17	10
$20\frac{1}{4}$, — — 67	1	13	6
$21\frac{1}{4}$, — — 68	1	9	2
$22\frac{1}{4}$, — — 69	1	5	10
$23\frac{1}{4}$, — — 70	1	2	5
$24\frac{1}{4}$, — — 71	0	19	9
$25\frac{1}{4}$, — — 72	0	17	0
$26\frac{1}{4}$, — — 73	0	14	8
$27\frac{1}{4}$, — — 74	0	12	5
$28\frac{1}{4}$, — — 75	0	10	7

TABLE XXXIV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $47\frac{3}{4}$ to $48\frac{1}{4}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	48	12	3	6
$1\frac{1}{4}$,	——	49	11	4	2
$2\frac{1}{4}$,	——	50	10	6	9
$3\frac{1}{4}$,	——	51	9	9	5
$4\frac{1}{4}$,	——	52	8	13	9
$5\frac{1}{4}$,	——	53	7	18	1
$6\frac{1}{4}$,	——	54	7	4	6
$7\frac{1}{4}$,	——	55	6	10	10
$8\frac{1}{4}$,	——	56	5	19	3
$9\frac{1}{4}$,	——	57	5	7	7
$10\frac{1}{4}$,	——	58	4	18	0
$11\frac{1}{4}$,	——	59	4	8	4
$12\frac{1}{4}$,	——	60	4	0	2
$13\frac{1}{4}$,	——	61	3	12	0
$14\frac{1}{4}$,	——	62	3	5	0
$15\frac{1}{4}$,	——	63	2	17	11
$16\frac{1}{4}$,	——	64	2	11	10
$17\frac{1}{4}$,	——	65	2	5	10
$18\frac{1}{4}$,	——	66	2	0	9
$19\frac{1}{4}$,	——	67	1	15	7
$20\frac{1}{4}$,	——	68	1	11	6
$21\frac{1}{4}$,	——	69	1	7	5
$22\frac{1}{4}$,	——	70	1	4	2
$23\frac{1}{4}$,	——	71	1	1	0
$24\frac{1}{4}$,	——	72	0	18	3
$25\frac{1}{4}$,	——	73	0	15	7
$26\frac{1}{4}$,	——	74	0	13	5
$27\frac{1}{4}$,	——	75	0	11	4

TABLE XXXV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any age from 35 to 75.

Age of the Purchaſer from $48\frac{1}{4}$ to $49\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	49	11	19	6
$1\frac{1}{4}$,	——	50	11	0	10
$2\frac{1}{4}$,	——	51	10	3	4
$3\frac{1}{4}$,	——	52	9	5	11
$4\frac{1}{4}$,	——	53	8	9	10
$5\frac{1}{4}$,	——	54	7	13	10
$6\frac{1}{4}$,	——	55	7	0	7
$7\frac{1}{4}$,	——	56	6	7	3
$8\frac{1}{4}$,	——	57	5	15	8
$9\frac{1}{4}$,	——	58	5	4	0
$10\frac{1}{4}$,	——	59	4	14	11
$11\frac{1}{4}$,	——	60	4	5	9
$12\frac{1}{4}$,	——	61	3	17	5
$13\frac{1}{4}$,	——	62	3	9	1
$14\frac{1}{4}$,	——	63	3	2	3
$15\frac{1}{4}$,	——	64	2	15	6
$16\frac{1}{4}$,	——	65	2	9	4
$17\frac{1}{4}$,	——	66	2	3	3
$18\frac{1}{4}$,	——	67	1	18	4
$19\frac{1}{4}$,	——	68	1	13	5
$20\frac{1}{4}$,	——	69	1	9	6
$21\frac{1}{4}$,	——	70	1	5	8
$22\frac{1}{4}$,	——	71	1	2	7
$23\frac{1}{4}$,	——	72	0	19	6
$24\frac{1}{4}$,	——	73	0	16	10
$25\frac{1}{4}$,	——	74	0	14	2
$26\frac{1}{4}$,	——	75	0	12	2

TABLE XXXVI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $49\frac{3}{4}$ to $50\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 50	11	15	6
$1\frac{1}{4}$, —— 51	10	16	9
$2\frac{1}{4}$, —— 52	9	18	10
$3\frac{1}{4}$, —— 53	9	1	0
$4\frac{1}{4}$, —— 54	8	5	4
$5\frac{1}{4}$, —— 55	7	9	8
$6\frac{1}{4}$, —— 56	6	16	5
$7\frac{1}{4}$, —— 57	6	3	1
$8\frac{1}{4}$, —— 58	5	12	1
$9\frac{1}{4}$, —— 59	5	1	0
$10\frac{1}{4}$, —— 60	4	11	9
$11\frac{1}{4}$, —— 61	4	2	4
$12\frac{1}{4}$, —— 62	3	14	4
$13\frac{1}{4}$, —— 63	3	6	3
$14\frac{1}{4}$, —— 64	2	19	4
$15\frac{1}{4}$, —— 65	2	12	6
$16\frac{1}{4}$, —— 66	2	6	7
$17\frac{1}{4}$, —— 67	2	0	9
$18\frac{1}{4}$, —— 68	1	16	1
$19\frac{1}{4}$, —— 69	1	11	4
$20\frac{1}{4}$, —— 70	1	7	8
$21\frac{1}{4}$, —— 71	1	4	0
$22\frac{1}{4}$, —— 72	1	0	11
$23\frac{1}{4}$, —— 73	0	17	10
$24\frac{1}{4}$, —— 74	0	15	4
$25\frac{1}{4}$, —— 75	0	13	0

TABLE XXXVII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer fron: $50\frac{1}{4}$ to $51\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 51	11	11	6
$1\frac{1}{4}$, —— 52	10	12	8
$2\frac{1}{4}$, —— 53	9	14	4
$3\frac{1}{4}$, —— 54	8	16	0
$4\frac{1}{4}$, —— 55	8	0	10
$5\frac{1}{4}$, —— 56	7	5	7
$6\frac{1}{4}$, —— 57	6	12	4
$7\frac{1}{4}$, —— 58	5	19	0
$8\frac{1}{4}$, —— 59	5	8	7
$9\frac{1}{4}$, —— 60	4	18	1
$10\frac{1}{4}$, —— 61	4	8	7
$11\frac{1}{4}$, —— 62	3	19	0
$12\frac{1}{4}$, —— 63	3	11	3
$13\frac{1}{4}$, —— 64	3	3	6
$14\frac{1}{4}$, —— 65	2	16	6
$15\frac{1}{4}$, —— 66	2	9	6
$16\frac{1}{4}$, —— 67	2	3	11
$17\frac{1}{4}$, —— 68	1	18	3
$18\frac{1}{4}$, —— 69	1	13	10
$19\frac{1}{4}$, —— 70	1	9	4
$20\frac{1}{4}$, —— 71	1	5	10
$21\frac{1}{4}$, —— 72	1	2	3
$20\frac{1}{4}$, —— 73	0	19	3
$23\frac{1}{4}$, —— 74	0	16	3
$24\frac{1}{4}$, —— 75	0	13	11

TABLE XXXVIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $51\frac{3}{4}$ to $52\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	52	11.	7	6
$1\frac{1}{4}$,	——	53	10	7	8
$2\frac{1}{4}$,	——	54	9	9	9
$3\frac{1}{4}$,	——	55	8	11	9
$4\frac{1}{4}$,	——	56	7	16	7
$5\frac{1}{4}$,	——	57	7	1	3
$6\frac{1}{4}$,	——	58	6	8	8
$7\frac{1}{4}$,	——	59	5	16	0
$8\frac{1}{4}$,	——	60	5	5	3
$9\frac{1}{4}$,	——	61	4	14	6
$10\frac{1}{4}$,	——	62	4	5	3
$11\frac{1}{4}$,	——	63	3	16	0
$12\frac{1}{4}$,	——	64	3	8	1
$13\frac{1}{4}$,	——	65	3	0	3
$14\frac{1}{4}$,	——	66	2	13	6
$15\frac{1}{4}$,	——	67	2	6	9
$16\frac{1}{4}$,	——	68	2	1	5
$17\frac{1}{4}$,	——	69	1	16	0
$18\frac{1}{4}$,	——	70	1	11	9
$19\frac{1}{4}$,	——	71	1	7	6
$20\frac{1}{4}$,	——	72	1	4	0
$21\frac{1}{4}$,	——	73	1	0	6
$22\frac{1}{4}$,	——	74	0	17	7
$23\frac{1}{4}$,	——	75	0	14	5

TABLE XXXIX.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $52\frac{3}{4}$ to $53\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	53	11	1	6
$1\frac{1}{4}$,	——	54	10	2	8
$2\frac{1}{4}$,	——	55	9	5	2
$3\frac{1}{4}$,	——	56	8	7	7
$4\frac{1}{4}$,	——	57	7	12	4
$5\frac{1}{4}$,	——	58	6	17	0
$6\frac{1}{4}$,	——	59	6	5	0
$7\frac{1}{4}$,	——	60	5	12	11
$8\frac{1}{4}$,	——	61	5	2	0
$9\frac{1}{4}$,	——	62	4	11	0
$10\frac{1}{4}$,	——	63	4	2	0
$11\frac{1}{4}$,	——	64	3	13	1
$12\frac{1}{4}$,	——	65	3	5	0
$13\frac{1}{4}$,	——	66	2	17	0
$14\frac{1}{4}$,	——	67	2	10	6
$15\frac{1}{4}$,	——	68	2	4	0
$16\frac{1}{4}$,	——	69	1	18	11
$17\frac{1}{4}$,	——	70	1	13	10
$18\frac{1}{4}$,	——	71	1	9	9
$19\frac{1}{4}$,	——	72	1	5	8
$20\frac{1}{4}$,	——	73	1	2	2
$21\frac{1}{4}$,	——	74	0	18	9
$22\frac{1}{4}$,	——	75	0	16	0

TABLE XL.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $53\frac{3}{4}$ to $54\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	54	10	17	6
$1\frac{1}{4}$,	——	55	9	18	6
$2\frac{1}{4}$,	——	56	9	0	11
$3\frac{1}{4}$,	——	57	8	3	3
$4\frac{1}{4}$,	——	58	7	8	7
$5\frac{1}{4}$,	——	59	6	14	0
$6\frac{1}{4}$,	——	60	6	1	7
$7\frac{1}{4}$,	——	61	5	9	2
$8\frac{1}{4}$,	——	62	4	18	7
$9\frac{1}{4}$,	——	63	4	7	10
$10\frac{1}{4}$,	——	64	3	18	8
$11\frac{1}{4}$,	——	65	3	9	7
$12\frac{1}{4}$,	——	66	3	1	9
$13\frac{1}{4}$,	——	67	2	14	0
$14\frac{1}{4}$,	——	68	2	7	9
$15\frac{1}{4}$,	——	69	2	1	7
$16\frac{1}{4}$,	——	70	1	16	8
$17\frac{1}{4}$,	——	71	1	11	9
$18\frac{1}{4}$,	——	72	1	7	9
$19\frac{1}{4}$,	——	73	1	3	8
$20\frac{1}{4}$,	——	74	1	0	4
$21\frac{1}{4}$,	——	75	0	17	3

TABLE XLI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $54\frac{3}{4}$ to $55\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	55	10	13	6
$1\frac{1}{4}$,	——	56	9	14	5
$2\frac{1}{4}$,	——	57	8	16	8
$3\frac{1}{4}$,	——	58	7	18	11
$4\frac{1}{4}$,	——	59	7	4	11
$5\frac{1}{4}$,	——	60	6	10	11
$6\frac{1}{4}$,	——	61	5	18	2
$7\frac{1}{4}$,	——	62	5	5	6
$8\frac{1}{4}$,	——	63	4	15	2
$9\frac{1}{4}$,	——	64	4	4	9
$10\frac{1}{4}$,	——	65	3	15	5
$11\frac{1}{4}$,	——	66	3	6	1
$12\frac{1}{4}$,	——	67	2	18	7
$13\frac{1}{4}$,	——	68	2	11	0
$14\frac{1}{4}$,	——	69	2	5	1
$15\frac{1}{4}$,	——	70	1	19	2
$16\frac{1}{4}$,	——	71	1	14	6
$17\frac{1}{4}$,	——	72	1	9	9
$18\frac{1}{4}$,	——	73	1	5	9
$19\frac{1}{4}$,	——	74	1	1	8
$20\frac{1}{4}$,	——	75	0	18	7

TABLE

TABLE XLII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $55\frac{3}{4}$ to $57\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	56	10	9	6
$1\frac{1}{4}$,	——	57	9	10	2
$2\frac{1}{4}$,	——	58	8	13	1
$3\frac{1}{4}$,	——	59	7	16	0
$4\frac{1}{4}$,	——	60	7	1	7
$5\frac{1}{4}$,	——	61	6	7	2
$6\frac{1}{4}$,	——	62	5	14	8
$7\frac{1}{4}$,	——	63	5	2	4
$8\frac{1}{4}$,	——	64	4	11	8
$9\frac{1}{4}$,	——	65	4	1	0
$10\frac{1}{4}$,	——	66	3	12	0
$11\frac{1}{4}$,	——	67	3	2	10
$12\frac{1}{4}$,	——	68	2	15	8
$13\frac{1}{4}$,	——	69	2	8	5
$14\frac{1}{4}$,	——	70	2	2	8
$15\frac{1}{4}$,	——	71	1	17	0
$16\frac{1}{4}$,	——	72	1	12	3
$17\frac{1}{4}$,	——	73	1	7	7
$18\frac{1}{4}$,	——	74	1	3	8
$19\frac{1}{4}$,	——	75	1	0	1

TABLE XLIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $56\frac{3}{4}$ to $57\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1 in one preſent payment. £.	s.	d
$\frac{1}{4}$,	or at Age	57	10	5	6
$1\frac{1}{4}$,	——	58	9	5	11
$2\frac{1}{4}$,	——	59	8	9	6
$3\frac{1}{4}$,	——	60	7	13	2
$4\frac{1}{4}$,	——	61	6	18	3
$5\frac{1}{4}$,	——	62	6	3	5
$6\frac{1}{4}$,	——	63	5	11	3
$7\frac{1}{4}$,	——	64	4	19	2
$8\frac{1}{4}$,	——	65	4	8	3
$9\frac{1}{4}$,	——	66	3	17	4
$10\frac{1}{4}$,	——	67	3	8	6
$11\frac{1}{4}$,	——	68	2	19	8
$12\frac{1}{4}$,	——	69	2	12	10
$13\frac{1}{4}$,	——	70	2	5	11
$14\frac{1}{4}$,	——	71	2	0	4
$15\frac{1}{4}$,	——	72	1	14	10
$16\frac{1}{4}$,	——	73	1	10	1
$17\frac{1}{4}$,	——	74	1	5	5
$18\frac{1}{4}$,	——	75	1	1	8

TABLE XLIV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $57\frac{3}{4}$ to $58\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	58	9	19	6
$1\frac{1}{4}$,	——	59	9	1	9
$2\frac{1}{4}$,	——	60	8	5	0
$3\frac{1}{4}$,	——	61	7	8	2
$4\frac{1}{4}$,	——	62	6	13	8
$5\frac{1}{4}$,	——	63	5	19	2
$6\frac{1}{4}$,	——	64	5	6	9
$7\frac{1}{4}$,	——	65	4	14	5
$8\frac{1}{4}$,	——	66	4	3	10
$9\frac{1}{4}$,	——	67	3	13	3
$10\frac{1}{4}$,	——	68	3	4	10
$11\frac{1}{4}$,	——	69	2	16	5
$12\frac{1}{4}$,	——	70	2	9	9
$13\frac{1}{4}$,	——	71	2	3	1
$14\frac{1}{4}$,	——	72	1	17	7
$15\frac{1}{4}$,	——	73	1	12	2
$16\frac{1}{4}$,	——	74	1	7	7
$17\frac{1}{4}$,	——	75	1	3	5

TABLE XLV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $58\frac{3}{4}$ to $59\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	59	9	15	6
$1\frac{1}{4}$,	——	60	8	17	8
$2\frac{1}{4}$,	——	61	8	0	5
$3\frac{1}{4}$,	——	62	7	3	2
$4\frac{1}{4}$,	——	63	6	9	1
$5\frac{1}{4}$,	——	64	5	15	0
$6\frac{1}{4}$,	——	65	5	2	4
$7\frac{1}{4}$,	——	66	4	9	8
$8\frac{1}{4}$,	——	67	3	19	6
$9\frac{1}{4}$,	——	68	3	9	3
$10\frac{1}{4}$,	——	69	3	1	2
$11\frac{1}{4}$,	——	70	2	13	2
$12\frac{1}{4}$,	——	71	2	6	9
$13\frac{1}{4}$,	——	72	2	0	4
$14\frac{1}{4}$,	——	73	1	14	11
$15\frac{1}{4}$,	——	74	1	9	6
$16\frac{1}{4}$,	——	75	1	5	2

TABLE XLVI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 59¾ to 60¾.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	60	9	11	6
1 ¼,	——	61	8	12	7
2 ¼,	——	62	7	15	9
3 ¼,	——	63	6	18	10
4 ¼,	——	64	6	4	5
5 ¼,	——	65	5	10	0
6 ¼,	——	66	4	17	8
7 ¼,	——	67	4	5	4
8 ¼,	——	68	3	15	7
9 ¼,	——	69	3	5	9
10 ¼,	——	70	2	18	0
11 ¼,	——	71	2	10	2
12 ¼,	——	72	2	3	10
13 ¼,	——	73	1	17	5
14 ¼,	——	74	1	12	1
15 ¼,	——	75	1	7	0

TABLE XLVII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 60¾ to 61½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	61	9	5	6
1 ¼,	——	62	8	7	7
2 ¼,	——	63	7	11	1
3 ¼,	——	64	6	14	7
4 ¼,	——	65	5	19	10
5 ¼,	——	66	5	5	0
6 ¼,	——	67	4	13	0
7 ¼,	——	68	4	1	1
8 ¼,	——	69	3	11	8
9 ¼,	——	70	3	2	3
10 ¼,	——	71	2	14	9
11 ¼,	——	72	2	7	3
12 ¼,	——	73	2	0	11
13 ¼,	——	74	1	14	6
14 ¼,	——	75	1	9	3

TABLE

TABLE XLVIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 61½ to 62½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	62	9	1	6
1 ¼,	——	63	8	3	4
2 ¼,	——	64	7	6	4
3 ¼,	——	65	6	9	4
4 ¼,	——	66	5	14	11
5 ¼,	——	67	5	0	5
6 ¼,	——	68	4	8	11
7 ¼,	——	69	3	17	4
8 ¼,	——	70	3	8	3
9 ¼,	——	71	2	19	1
10 ¼,	——	72	2	11	6
11 ¼,	——	73	2	4	0
12 ¼,	——	74	1	17	10
13 ¼,	——	75	1	12	0

TABLE XLIX.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 62¼ to 63¼.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	63	8	17	6
1 ¼,	——	64	7	19	2
2 ¼,	——	65	7	1	8
3 ¼,	——	66	6	4	2
4 ¼,	——	67	5	10	0
5 ¼,	——	68	4	15	11
6 ¼,	——	69	4	4	10
7 ¼,	——	70	3	13	8
8 ¼,	——	71	3	4	10
9 ¼,	——	72	2	15	11
10 ¼,	——	73	2	8	4
11 ¼,	——	74	2	0	10
12 ¼,	——	75	1	14	10

TABLE L.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $63\frac{3}{4}$ to $64\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	64	8	13	6
$1\frac{1}{4}$,	——	65	7	14	0
$2\frac{1}{4}$,	——	66	6	16	9
$3\frac{1}{4}$,	——	67	5	19	6
$4\frac{1}{4}$,	——	68	5	5	9
$5\frac{1}{4}$,	——	69	4	12	1
$6\frac{1}{4}$,	——	70	4	1	3
$7\frac{1}{4}$,	——	71	3	10	4
$8\frac{1}{4}$,	——	72	3	1	4
$9\frac{1}{4}$,	——	73	2	12	5
$10\frac{1}{4}$,	——	74	2	5	0
$11\frac{1}{4}$,	——	75	1	18	0

TABLE LI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $64\frac{3}{4}$ to $65\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	65	8	7	6
$1\frac{1}{4}$,	——	66	7	8	11
$2\frac{1}{4}$,	——	67	6	11	11
$3\frac{1}{4}$,	——	68	5	14	11
$4\frac{1}{4}$,	——	69	5	1	7
$5\frac{1}{4}$,	——	70	4	8	4
$6\frac{1}{4}$,	——	71	3	17	8
$7\frac{1}{4}$,	——	72	3	7	0
$8\frac{1}{4}$,	——	73	2	17	11
$9\frac{1}{4}$,	——	74	2	8	11
$10\frac{1}{4}$,	——	75	2	1	9

TABLE

TABLE LII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 65¾ to 66½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	66	8	3	6
1 ¼,	——	67	7	4	6
2 ¼,	——	68	6	7	11
3 ¼,	——	69	5	11	3
4 ¼,	——	70	4	18	1
5 ¼,	——	71	4	5	0
6 ¼,	——	72	3	14	2
7 ¼,	——	73	3	3	3
8 ¼,	——	74	2	14	5
9 ¼,	——	75	2	6	9

TABLE LIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 66¾ to 67½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	67	7	19	6
1 ¼,	——	68	7	0	2
2 ¼,	——	69	6	3	11
3 ¼,	——	70	5	7	8
4 ¼,	——	71	4	14	8
5 ¼,	——	72	4	1	8
6 ¼,	——	73	3	10	8
7 ¼,	——	74	2	19	7
8 ¼,	——	75	2	10	11

TABLE LIV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $67\frac{3}{4}$ to $68\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	68	7	15	6
$1\frac{1}{4}$,	——	69	6	16	1
$2\frac{1}{4}$,	——	70	6	0	0
$3\frac{1}{4}$,	——	71	5	3	11
$4\frac{1}{4}$,	——	72	4	10	7
$5\frac{1}{4}$,	——	73	3	17	4
$6\frac{1}{4}$,	——	74	3	6	6
$7\frac{1}{4}$,	——	75	2	16	0

TABLE LV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $68\frac{3}{4}$ to $69\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	69	7	9	6
$1\frac{1}{4}$,	——	70	6	12	0
$2\frac{1}{4}$,	——	71	5	16	0
$3\frac{1}{4}$,	——	72	5	0	2
$4\frac{1}{4}$,	——	73	4	6	7
$5\frac{1}{4}$,	——	74	3	13	1
$6\frac{1}{4}$,	——	75	3	2	4

TABLE

TABLE LVI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $69\frac{3}{4}$ to $70\frac{1}{4}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	70	7	5	6
$1\frac{1}{4}$,	——	71	6	7	7
$2\frac{1}{4}$,	——	72	5	11	3
$3\frac{1}{4}$,	——	73	4	15	0
$4\frac{1}{4}$,	——	74	4	1	8
$5\frac{1}{4}$,	——	75	3	9	0

TABLE LVII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $70\frac{3}{4}$ to $71\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	71	7	1	6
$1\frac{1}{4}$,	——	72	6	3	3
$2\frac{1}{4}$,	——	73	5	6	7
$3\frac{1}{4}$,	——	74	4	9	11
$4\frac{1}{4}$,	——	75	3	16	9

TABLE

TABLE LVIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $71\frac{1}{4}$ to $72\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1 in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 72	6	17	3
$1\frac{1}{4}$, —— 73	5	18	1
$2\frac{1}{4}$, —— 74	5	1	9
$3\frac{1}{4}$, —— 75	4	6	6

TABLE LIX.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $72\frac{1}{4}$ to $73\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 73	6	11	6
$1\frac{1}{4}$, —— 74	5	13	7
$2\frac{1}{4}$, —— 75	4	16	11

SECOND

SECOND SET OF TABLES.

Intended for the Uſe of COUNTRY PARISHES.

TABLE I.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $14\frac{3}{4}$ to $15\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$20\frac{1}{4}$, or at Age 35	6	12	8
$21\frac{1}{4}$, — — 36	6	4	10
$22\frac{1}{4}$, — — 37	5	17	8
$23\frac{1}{4}$, — — 38	5	10	7
$24\frac{1}{4}$, — — 39	5	4	1
$25\frac{1}{4}$, — — 40	4	17	7
$26\frac{1}{4}$, — — 41	4	11	7
$27\frac{1}{4}$, — — 42	4	5	7
$28\frac{1}{4}$, — — 43	4	0	1
$29\frac{1}{4}$, — — 44	3	14	7
$30\frac{1}{4}$, — — 45	3	9	8
$31\frac{1}{4}$, — — 46	3	4	9
$32\frac{1}{4}$, — — 47	3	0	4
$33\frac{1}{4}$, — — 48	2	15	11
$34\frac{1}{4}$, — — 49	2	11	11
$35\frac{1}{4}$, — — 50	2	7	11
$36\frac{1}{4}$, — — 51	2	4	4
$37\frac{1}{4}$, — — 52	2	0	8
$38\frac{1}{4}$, — — 53	1	17	7
$39\frac{1}{4}$, — — 54	1	14	6
$40\frac{1}{4}$, — — 55	1	11	9

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$41\frac{1}{4}$, or at Age 56	1	9	0
$42\frac{1}{4}$, — — 57	1	6	7
$43\frac{1}{4}$, — — 58	1	4	2
$44\frac{1}{4}$, — — 59	1	2	1
$45\frac{1}{4}$, — — 60	1	0	0
$46\frac{1}{4}$, — — 61	0	18	1
$47\frac{1}{4}$, — — 62	0	16	2
$48\frac{1}{4}$, — — 63	0	14	7
$49\frac{1}{4}$, — — 64	0	13	0
$50\frac{1}{4}$, — — 65	0	11	8
$51\frac{1}{4}$, — — 66	0	10	3
$52\frac{1}{4}$, — — 67	0	9	2
$53\frac{1}{4}$, — — 68	0	8	0
$54\frac{1}{4}$, — — 69	0	7	0
$55\frac{1}{4}$, — — 70	0	6	0
$56\frac{1}{4}$, — — 71	0	5	2
$57\frac{1}{4}$, — — 72	0	4	5
$58\frac{1}{4}$, — — 73	0	3	9
$59\frac{1}{4}$, — — 74	0	3	1
$60\frac{1}{4}$, — — 75	0	2	6

TABLE II.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 15 ¾ to 16 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
19 ¼, or at Age 35	6	17	5	40 ¼, or at Age 56	1	10	3
20 ¼, — — 36	6	9	10	41 ¼, — — 57	1	7	7
21 ¼, — — 37	6	2	2	42 ¼, — — 58	1	5	3
22 ¼, — — 38	5	15	1	43 ¼, — — 59	1	2	11
23 ¼, — — 39	5	8	1	44 ¼, — — 60	1	0	9
24 ¼, — — 40	5	1	6	45 ¼, — — 61	0	18	9
25 ¼, — — 41	4	15	0	46 ¼, — — 62	0	16	11
26 ¼, — — 42	4	9	0	47 ¼, — — 63	0	15	2
27 ¼, — — 43	4	3	1	48 ¼, — — 64	0	13	8
28 ¼, — — 44	3	17	8	49 ¼, — — 65	0	12	1
29 ¼, — — 45	3	12	3	50 ¼, — — 66	0	10	10
30 ¼, — — 46	3	7	6	51 ¼, — — 67	0	9	4
31 ¼, — — 47	3	2	8	52 ¼, — — 68	0	8	4
32 ¼, — — 48	2	18	3	53 ¼, — — 69	0	7	3
33 ¼, — — 49	2	13	10	54 ¼, — — 70	0	6	3
34 ¼, — — 50	2	9	11	55 ¼, — — 71	0	5	5
35 ¼, — — 51	2	6	0	56 ¼, — — 72	0	4	7
36 ¼, — — 52	2	2	5	57 ¼, — — 73	0	3	10
37 ¼, — — 53	1	19	0	58 ¼, — — 74	0	3	3
38 ¼, — — 54	1	16	0	59 ¼, — — 75	0	2	8
39 ¼, — — 55	1	12	11				

TABLE

TABLE III.

For the Uſe of Country Parithes.

Shewing the Payment due, (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 16 ¼ to 17 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
18 ¼, or at Age 35	7	3	4	39 ¼, or at Age 56	1	11	4
19 ¼, — — 36	6	14	10	40 ¼, — — 57	1	8	9
20 ¼, — — 37	6	7	1	41 ¼, — — 58	1	6	2
21 ¼, — — 38	5	19	5	42 ¼, — — 59	1	3	11
22 ¼, — — 39	5	12	6	43 ¼, — — 60	1	1	7
23 ¼, — — 40	5	5	7	44 ¼, — — 61	0	19	7
24 ¼, — — 41	4	19	0	45 ¼, — — 62	0	17	7
25 ¼, — — 42	4	12	5	46 ¼, — — 63	0	15	10
26 ¼, — — 43	4	6	6	47 ¼, — — 64	0	14	1
27 ¼, — — 44	4	0	7	48 ¼, — — 65	0	12	8
28 ¼, — — 45	3	15	4	49 ¼, — — 66	0	11	2
29 ¼, — — 46	3	10	0	50 ¼, — — 67	0	9	11
30 ¼, — — 47	3	5	3	51 ¼, — — 68	0	8	7
31 ¼, — — 48	3	0	6	52 ¼, — — 69	0	7	7
32 ¼, — — 49	2	16	1	53 ¼, — — 70	0	6	6
33 ¼, — — 50	2	11	9	54 ¼, — — 71	0	5	7
34 ¼, — — 51	2	7	11	55 ¼, — — 72	0	4	9
35 ¼, — — 52	2	4	0	56 ¼, — — 73	0	4	0
36 ¼, — — 53	2	0	8	57 ¼, — — 74	0	3	3
37 ¼, — — 54	1	17	4	58 ¼, — — 75	0	2	8
38 ¼, — — 55	1	14	4				

TABLE IV.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 17 ¾ to 18 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
17 ¼, or at Age 35	7	8	11	38 ¼, or at Age 56	1	12	9
18 ¼, — — 36	7	0	6	39 ¼, — — 57	1	9	10
19 ¼, — — 37	6	12	1	40 ¼, — — 58	1	7	4
20 ¼, — — 38	6	4	6	41 ¼, — — 59	1	4	9
21 ¼, — — 39	5	16	10	42 ¼, — — 60	1	2	7
22 ¼, — — 40	5	9	10	43 ¼, — — 61	1	0	3
23 ¼, — — 41	5	2	9	44 ¼, — — 62	0	18	4
24 ¼, — — 42	4	16	4	45 ¼, — — 63	0	16	4
25 ¼, — — 43	4	9	10	46 ¼, — — 64	0	14	8
26 ¼, — — 44	4	4	1	47 ¼, — — 65	0	13	0
27 ¼, — — 45	3	18	2	48 ¼, — — 66	0	11	8
28 ¼, — — 46	3	12	11	49 ¼, — — 67	0	10	3
29 ¼, — — 47	3	7	8	50 ¼, — — 68	0	9	0
30 ¼, — — 48	3	3	0	51 ¼, — — 69	0	7	9
31 ¼, — — 49	2	18	3	52 ¼, — — 70	0	6	10
32 ¼, — — 50	2	13	11	53 ¼, — — 71	0	5	9
33 ¼, — — 51	2	9	8	54 ¼, — — 72	0	4	11
34 ¼, — — 52	2	6	0	55 ¼, — — 73	0	4	2
35 ¼, — — 53	2	2	2	56 ¼, — — 74	0	3	5
36 ¼, — — 54	1	18	11	57 ¼, — — 75	0	2	9
37 ¼, — — 55	1	15	8				

TABLE V.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 18 $\frac{3}{4}$ to 19 $\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
16 $\frac{1}{4}$, or at Age 35	7	15	3	37 $\frac{1}{4}$, or at Age 56	1	14	0
17 $\frac{1}{4}$, — — 36	7	6	1	38 $\frac{1}{4}$, — — 57	1	11	2
18 $\frac{1}{4}$, — — 37	6	17	9	39 $\frac{1}{4}$, — — 58	1	8	4
19 $\frac{1}{4}$, — — 38	6	9	5	40 $\frac{1}{4}$, — — 59	1	5	10
20 $\frac{1}{4}$, — — 39	6	1	11	41 $\frac{1}{4}$, — — 60	1	3	4
21 $\frac{1}{4}$, — — 40	5	14	4	42 $\frac{1}{4}$, — — 61	1	1	2
22 $\frac{1}{4}$, — — 41	5	7	2	43 $\frac{1}{4}$, — — 62	0	19	0
23 $\frac{1}{4}$, — — 42	5	0	0	44 $\frac{1}{4}$, — — 63	0	17	1
24 $\frac{1}{4}$, — — 43	4	13	8	45 $\frac{1}{4}$, — — 64	0	15	2
25 $\frac{1}{4}$, — — 44	4	7	4	46 $\frac{1}{4}$, — — 65	0	13	7
26 $\frac{1}{4}$, — — 45	4	1	7	47 $\frac{1}{4}$, — — 66	0	12	0
27 $\frac{1}{4}$, — — 46	3	15	10	48 $\frac{1}{4}$, — — 67	0	10	8
28 $\frac{1}{4}$, — — 47	3	10	7	49 $\frac{1}{4}$, — — 68	0	9	4
29 $\frac{1}{4}$, — — 48	3	5	5	50 $\frac{1}{4}$, — — 69	0	8	2
30 $\frac{1}{4}$, — — 49	3	0	9	51 $\frac{1}{4}$, — — 70	0	7	0
31 $\frac{1}{4}$, — — 50	2	16	1	52 $\frac{1}{4}$, — — 71	0	6	1
32 $\frac{1}{4}$, — — 51	2	11	10	53 $\frac{1}{4}$, — — 72	0	5	1
33 $\frac{1}{4}$, — — 52	2	7	7	54 $\frac{1}{4}$, — — 73	0	4	4
34 $\frac{1}{4}$, — — 53	2	4	0	55 $\frac{1}{4}$, — — 74	0	3	7
35 $\frac{1}{4}$, — — 54	2	0	5	56 $\frac{1}{4}$, — — 75	0	2	11
36 $\frac{1}{4}$, — — 55	1	17	2				

TABLE VI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 19 ¾ to 20 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
15 ¼, or at Age 35	8	2	0.
16 ¼, — — 36	7	12	10
17 ¼, — — 37	7	3	9.
18 ¼, — — 38	6	15	5
19 ¼, — — 39	6	7	2
20 ¼, — — 40	5	19	6
21 ¼, — — 41	5	11	10
22 ¼, — — 42	5	4	9
23 ¼, — — 43	4	17	8
24 ¼, — — 44	4	11	4
25 ¼, — — 45	4	5	1
26 ¼, — — 46	3	19	4
27 ¼, — — 47	3	13	8
28 ¼, — — 48	3	8	5
29 ¼, — — 49	3	3	3
30 ¼, — — 50	2	18	8
31 ¼, — — 51	2	14	0
32 ¼, — — 52	2	9	11
33 ¼, — — 53	2	5	10
34 ¼, — — 54	2	2	4
35 ¼, — — 55	1	18	9

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
36 ¼, or at Age 56	1	15	7
37 ¼, — — 57	1	12	6
38 ¼, — — 58	1	9	9
39 ¼, — — 59	1	6	11
40 ¼, — — 60	1	4	6
41 ¼, — — 61	1	2	1
42 ¼, — — 62	0	19	11
43 ¼, — — 63	0	17	10
44 ¼, — — 64	0	16	0
45 ¼, — — 65	0	14	2
46 ¼, — — 66	0	12	7
47 ¼, — — 67	0	11	1
48 ¼, — — 68	0	9	9
49 ¼, — — 69	0	8	5
50 ¼, — — 70	0	7	4
51 ¼, — — 71	0	6	3
52 ¼, — — 72	0	5	4
53 ¼, — — 73	0	4	6
54 ¼, — — 74	0	3	9
55 ¼, — — 75	0	3	0

TABLE VII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 20 ¾ to 21 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
14 ¼, or at Age 35	8	9	7
15 ¼, — — 36	7	19	7
16 ¼, — — 37	7	10	6
17 ¼, — — 38	7	1	5
18 ¼, — — 39	6	13	2
19 ¼, — — 40	6	4	11
20 ¼, — — 41	5	17	2
21 ¼, — — 42	5	9	4
22 ¼, — — 43	5	2	4
23 ¼, — — 44	4	15	5
24 ¼, — — 45	4	9	1
25 ¼, — — 46	4	2	10
26 ¼, — — 47	3	17	2
27 ¼, — — 48	3	11	7
28 ¼, — — 49	3	6	4
29 ¼, — — 50	3	1	2
30 ¼, — — 51	2	16	7
31 ¼, — — 52	2	12	0
32 ¼, — — 53	2	8	1
33 ¼, — — 54	2	4	2
34 ¼, — — 55	2	0	8

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
35 ¼, or at Age 56	1	17	2
36 ¼, — — 57	1	14	0
37 ¼, — — 58	1	11	0
38 ¼, — — 59	1	8	3
39 ¼, — — 60	1	5	7
40 ¼, — — 61	1	3	2
41 ¼, — — 62	1	0	10
42 ¼, — — 63	0	18	9
43 ¼, — — 64	0	16	9
44 ¼, — — 65	0	15	0
45 ¼, — — 66	0	13	2
46 ¼, — — 67	0	11	8
47 ¼, — — 68	0	10	2
48 ¼, — — 69	0	8	11
49 ¼, — — 70	0	7	7
50 ¼, — — 71	0	6	7
51 ¼, — — 72	0	5	7
52 ¼, — — 73	0	4	9
53 ¼, — — 74	0	3	11
54 ¼, — — 75	0	3	2

TABLE VIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 21 ¾ to 22 ½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
13 ¼,	or at Age	35	8	17	2
14 ¼,	— —	36	8	7	2
15 ¼,	— —	37	7	17	2
16 ¼,	— —	38	7	8	2
17 ¼,	— —	39	6	19	1
18 ¼,	— —	40	6	10	9
19 ¼,	— —	41	6	2	4
20 ¼,	— —	42	5	14	7
21 ¼,	— —	43	5	6	10
22 ¼,	— —	44	4	19	11
23 ¼,	— —	45	4	13	0
24 ¼,	— —	46	4	6	10
25 ¼,	— —	47	4	0	7
26 ¼,	— —	48	3	14	11
27 ¼,	— —	49	3	9	4
28 ¼,	— —	50	3	4	2
29 ¼,	— —	51	2	19	1
30 ¼,	— —	52	2	14	8
31 ¼,	— —	53	2	10	3
32 ¼,	— —	54	2	6	3
33 ¼,	— —	55	2	2	5

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
34 ¼,	or at Age	56	1	18	11
35 ¼,	— —	57	1	15	6
36 ¼,	— —	58	1	12	6
37 ¼,	— —	59	1	9	6
38 ¼,	— —	60	1	6	9
39 ¼,	— —	61	1	4	2
40 ¼,	— —	62	1	1	10
41 ¾,	— —	63	0	19	7
42 ¼,	— —	64	0	17	7
43 ¼,	— —	65	0	15	7
44 ¼,	— —	66	0	13	11
45 ¾,	— —	67	0	12	2
46 ¼,	— —	68	0	10	9
47 ¼,	— —	69	0	9	3
48 ¾,	— —	70	0	8	1
49 ¼,	— —	71	0	6	11
50 ¼,	— —	72	0	5	10
51 ¼,	— —	73	0	4	11
52 ¼,	— —	74	0	4	1
53 ¼,	— —	75	0	3	4

TABLE IX.

For the Uſe of Country Pariſhes

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 22 ¾ to 23 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
12 ¼, or at Age 35	9	5	10
13 ¼, — — 36	8	14	10
14 ¼, — — 37	8	4	10
15 ¼, — — 38	7	14	10
16 ¼, — — 39	7	5	10
17 ¼, — — 40	6	16	10
18 ¼, — — 41	6	8	4
19 ¼, — — 42	5	19	10
20 ¼, — — 43	5	12	1
21 ¼, — — 44	5	4	5
22 ¼, — — 45	4	17	7
23 ¼, — — 46	4	10	8
24 ¼, — — 47	4	4	7
25 ¼, — — 48	3	18	5
26 ¼, — — 49	3	12	9
27 ¼, — — 50	3	7	1
28 ¼, — — 51	3	2	0
29 ¼, — — 52	2	17	0
30 ¼, — — 53	2	12	8
31 ¼, — — 54	2	8	5
32 ¼, — — 55	2	4	6

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
33 ¼, or at Age 56	2	0	8
34 ¼, — — 57	1	17	3
35 ¼, — — 58	1	13	11
36 ¼, — — 59	1	11	0
37 ¼, — — 60	1	8	0
38 ¼, — — 61	1	5	5
39 ¼, — — 62	1	2	10
40 ¼, — — 63	1	0	7
41 ¼, — — 64	0	18	4
42 ¼, — — 65	0	16	5
43 ¼, — — 66	0	14	5
44 ¼, — — 67	0	12	10
45 ¼, — — 68	0	11	2
46 ¼, — — 69	0	9	10
47 ¼, — — 70	0	8	5
48 ¼, — — 71	0	7	3
49 ¼, — — 72	0	6	2
50 ¼, — — 73	0	5	2
51 ¼, — — 74	0	4	3
52 ¼, — — 75	0	3	6

TABLE X.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 23 ¾ to 24 ½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
11 ¼,	or at Age	35	9	14	3
12 ¼,	— —	36	9	3	3
13 ¼,	— —	37	8	12	4
14 ¼,	— —	38	8	2	4
15 ¼,	— —	39	7	12	5
16 ¼,	— —	40	7	3	3
17 ¼,	— —	41	6	14	1
18 ¼,	— —	42	6	5	8
19 ¼,	— —	43	5	17	2
20 ¼,	— —	44	5	9	6
21 ¼,	— —	45	5	1	11
22 ¼,	— —	46	4	15	3
23 ¼,	— —	47	4	8	7
24 ¼,	— —	48	4	2	3
25 ¼,	— —	49	3	16	0
26 ¼,	— —	50	3	10	7
27 ¼,	— —	51	3	5	2
28 ¼,	— —	52	3	0	1
29 ¼,	— —	53	2	15	0
30 ¼,	— —	54	2	10	10
31 ¼,	— —	55	2	6	9

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
32 ¼,	or at Age	56	2	2	10
33 ¼,	— —	57	1	18	11
34 ¼,	— —	58	1	15	8
35 ¼,	— —	59	1	12	6
36 ¼,	— —	60	1	9	6
37 ¼,	— —	61	1	6	6
38 ¼,	— —	62	1	4	0
39 ¼,	— —	63	1	1	7
40 ¼,	— —	64	0	19	4
41 ¼,	— —	65	0	17	1
42 ¼,	— —	66	0	15	3
43 ¼,	— —	67	0	13	6
44 ¼,	— —	68	0	11	10
45 ¼,	— —	69	0	10	2
46 ¼,	— —	70	0	8	11
47 ¼,	— —	71	0	7	8
48 ¼,	— —	72	0	6	8
49 ¼,	— —	73	0	5	7
50 ¼,	— —	74	0	4	6
51 ¼,	— —	75	0	3	8

TABLE XI.

For the Uſe of Country Pariſhes.

Shewing the Payment due, (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 24 ¾ to 25 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
10 ¼, or at Age 35	10	3	9
11 ¼, — — 36	9	11	9
12 ¼, — — 37	9	0	9
13 ¼, — — 38	8	9	10
14 ¼, — — 39	7	19	11
15 ¼, — — 40	7	10	0
16 ¼, — — 41	7	0	8
17 ¼, — — 42	6	11	5
18 ¼, — — 43	6	3	0
19 ¼, — — 44	5	14	7
20 ¼, — — 45	5	7	0
21 ¼, — — 46	4	19	5
22 ¼, — — 47	4	12	11
23 ¼, — — 48	4	6	6
24 ¼, — — 49	4	0	0
25 ¼, — — 50	3	13	7
26 ¼, — — 51	3	8	5
27 ¼, — — 52	3	3	3
28 ¼, — — 53	2	18	2
29 ¼, — — 54	2	13	0
30 ¼, — — 55	2	9	0
31 ¼, or at Age 56	2	5	1
32 ¼, — — 57	2	1	2
33 ¼, — — 58	1	17	2
34 ¼, — — 59	1	14	1
35 ¼, — — 60	1	11	1
36 ¼, — — 61	1	8	0
37 ¼, — — 62	1	5	0
38 ¼, — — 63	1	2	8
39 ¼, — — 64	1	0	5
40 ¼, — — 65	0	18	2
41 ¼, — — 66	0	15	11
42 ¼, — — 67	0	14	2
43 ¼, — — 68	0	12	6
44 ¼, — — 69	0	10	10
45 ¼, — — 70	0	9	2
46 ¼, — — 71	0	8	1
47 ¼, — — 72	0	7	0
48 ¼, — — 73	0	6	0
49 ¼, — — 74	0	4	11
50 ¼, — — 75	0	3	10

TABLE XII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $25\frac{3}{4}$ to $26\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$9\frac{1}{4}$, or at Age 35	10	13	1
$10\frac{1}{4}$, — — 36	10	1	1
$11\frac{1}{4}$, — — 37	9	9	1
$12\frac{1}{4}$, — — 38	8	18	2
$13\frac{1}{4}$, — — 39	8	7	3
$14\frac{1}{4}$, — — 40	7	17	2
$15\frac{1}{4}$, — — 41	7	7	1
$16\frac{1}{4}$, — — 42	6	17	10
$17\frac{1}{4}$, — — 43	6	8	7
$18\frac{1}{4}$, — — 44	6	0	3
$19\frac{1}{4}$, — — 45	5	11	10
$20\frac{1}{4}$, — — 46	5	4	4
$21\frac{1}{4}$, — — 47	4	16	10
$22\frac{1}{4}$, — — 48	4	10	5
$23\frac{1}{4}$, — — 49	4	4	0
$24\frac{1}{4}$, — — 50	3	17	7
$25\frac{1}{4}$, — — 51	3	11	2
$26\frac{1}{4}$, — — 52	3	6	1
$27\frac{1}{4}$, — — 53	3	1	0
$28\frac{1}{4}$, — — 54	2	16	0
$29\frac{1}{4}$, — — 55	2	11	0

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$30\frac{1}{4}$, or at Age 56	2	7	1
$31\frac{1}{4}$, — — 57	2	3	2
$32\frac{1}{4}$, — — 58	1	19	3
$33\frac{1}{4}$, — — 59	1	15	4
$34\frac{1}{4}$, — — 60	1	12	4
$35\frac{1}{4}$, — — 61	1	9	5
$36\frac{1}{4}$, — — 62	1	6	5
$37\frac{1}{4}$, — — 63	1	3	6
$38\frac{1}{4}$, — — 64	1	1	3
$39\frac{1}{4}$, — — 65	0	19	0
$40\frac{1}{4}$, — — 66	0	16	10
$41\frac{1}{4}$, — — 67	0	14	8
$42\frac{1}{4}$, — — 68	0	13	0
$43\frac{1}{4}$, — — 69	0	11	5
$44\frac{1}{4}$, — — 70	0	9	10
$45\frac{1}{4}$, — — 71	0	8	3
$46\frac{1}{4}$, — — 72	0	7	2
$47\frac{1}{4}$, — — 73	0	6	1
$48\frac{1}{4}$, — — 74	0	5	0
$49\frac{1}{4}$, — — 75	0	4	0

TABLE XIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 26 ¼ to 27 ½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
8 ¼,	or at Age	35	11	3	7
9 ¼,	— —	36	10	10	5
10 ¼,	— —	37	9	18	6
11 ¼,	— —	38	9	6	6
12 ¼,	— —	39	8	15	8
13 ¼,	— —	40	8	4	9
14 ¼,	— —	41	7	14	6
15 ¼,	— —	42	7	4	3
16 ¼,	— —	43	6	15	0
17 ¼,	— —	44	6	5	10
18 ¼,	— —	45	5	17	6
19 ¼,	— —	46	5	9	2
20 ¼,	— —	47	5	1	9
21 ¼,	— —	48	4	14	4
22 ¼,	— —	49	4	7	11
23 ¼,	— —	50	4	1	6
24 ¼,	— —	51	3	15	1
25 ¼,	— —	52	3	8	8
26 ¼,	— —	53	3	3	9
27 ¼,	— —	54	2	18	10
28 ¼,	— —	55	2	13	11

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
29 ¼,	or at Age	56	2	9	0
30 ¼,	— —	57	2	5	2
31 ¼,	— —	58	2	1	4
32 ¼,	— —	59	1	17	5
33 ¼,	— —	60	1	13	7
34 ¼,	— —	61	1	10	8
35 ¼,	— —	62	1	7	10
36 ¼,	— —	63	1	4	11
37 ¼,	— —	64	1	2	0
38 ¼,	— —	65	0	19	10
39 ¼,	— —	66	0	17	8
40 ¼,	— —	67	0	15	7
41 ¼,	— —	68	0	13	5
42 ¼,	— —	69	0	11	11
43 ¼,	— —	70	0	10	5
44 ¼,	— —	71	0	8	11
45 ¼,	— —	72	0	7	5
46 ¼,	— —	73	0	6	4
47 ¼,	— —	74	0	5	3
48 ¼,	— —	75	0	4	2

TABLE XIV.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 27 ¾ to 28 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
7 ¼, or at Age 35	11	14	1	28 ¼, or at Age 56	2	11	9
8 ¼, — — 36	11	0	10	29 ¼, — — 57	2	7	2
9 ¼, — — 37	10	7	8	30 ¼, — — 58	2	3	2
10 ¼, — — 38	9	15	9	31 ¼, — — 59	1	19	2
11 ¼, — — 39	9	3	10	32 ¼, — — 60	1	15	7
12 ¼, — — 40	8	12	9	33 ¼, — — 61	1	12	1
13 ¼, — — 41	8	1	8	34 ¼, — — 62	1	9	0
14 ¼, — — 42	7	11	5	35 ¼, — — 63	1	6	0
15 ¼, — — 43	7	1	2	36 ¼, — — 64	1	3	4
16 ¼, — — 44	6	12	0	37 ¼, — — 65	1	0	9
17 ¼, — — 45	6	2	11	38 ¼, — — 66	0	18	6
18 ¼, — — 46	5	14	8	39 ¼, — — 67	0	16	3
19 ¼, — — 47	5	6	5	40 ¼, — — 68	0	14	4
20 ¼, — — 48	4	19	0	41 ¼, — — 69	0	12	7
21 ¼, — — 49	4	11	7	42 ¼, — — 70	0	10	10
22 ¼, — — 50	4	5	0	43 ¼, — — 71	0	9	3
23 ¼, — — 51	3	18	6	44 ¼, — — 72	0	7	11
24 ¼, — — 52	3	12	5	45 ¼, — — 73	0	6	8
25 ¼, — — 53	3	6	4	46 ¼, — — 74	0	5	6
26 ¼, — — 54	3	1	4	47 ¼, — — 75	0	4	5
27 ¼, — — 55	2	16	4				

TABLE XV.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 28 ¾ to 29 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
6 ¼, or at Age 35	12	5	11
7 ¼, — — 36	11	11	5
8 ¼, — — 37	10	18	2
9 ¼, — — 38	10	5	0
10 ¼, — — 39	9	13	1
11 ¼, — — 40	9	1	2
12 ¼, — — 41	8	9	10
13 ¼, — — 42	7	18	7
14 ¼, — — 43	7	8	4
15 ¼, — — 44	6	18	2
16 ¼, — — 45	6	9	1
17 ¼, — — 46	6	0	0
18 ¼, — — 47	5	11	11
19 ¼, — — 48	5	3	9
20 ¼, — — 49	4	16	3
21 ¼, — — 50	4	8	10
22 ¼, — — 51	4	2	2
23 ¼, — — 52	3	15	6
24 ¼, — — 53	3	9	9
25 ¼, — — 54	3	4	0
26 ¼, — — 55	2	18	11

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
27 ¼, or at Age 56	2	13	10
28 ¼, — — 57	2	9	7
29 ¼, — — 58	2	5	5
30 ¼, — — 59	2	1	2
31 ¼, — — 60	1	17	0
32 ¼, — — 61	1	13	10
33 ¼, — — 62	1	10	7
34 ¼, — — 63	1	7	5
35 ¼, — — 64	1	4	2
36 ¼, — — 65	1	1	10
37 ¼, — — 66	0	19	6
38 ¼, — — 67	0	17	2
39 ¼, — — 68	0	14	10
40 ¼, — — 69	0	13	1
41 ¼, — — 70	0	11	6
42 ¼, — — 71	0	9	9
43 ¼, — — 72	0	8	1
44 ¼, — — 73	0	6	11
45 ¼, — — 74	0	5	10
46 ¼, — — 75	0	4	8

TABLE XVI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 29 ¾ to 30 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
5 ¼, or at Age 35	12	17	2
6 ¼, — — 36	12	2	9
7 ¼, — — 37	11	8	3
8 ¼, — — 38	10	15	1
9 ¼, — — 39	10	1	11
10 ¼, — — 40	9	9	9
11 ¼, — — 41	8	17	7
12 ¼, — — 42	8	6	4
13 ¼, — — 43	7	15	2
14 ¼, — — 44	7	5	1
15 ¼, — — 45	6	15	0
16 ¼, — — 46	6	5	11
17 ¼, — — 47	5	16	11
18 ¼, — — 48	5	8	9
19 ¼, — — 49	5	0	7
20 ¼, — — 50	4	13	3
21 ¼, — — 51	4	5	10
22 ¼, — — 52	3	19	4
23 ¼, — — 53	3	12	9
24 ¼, — — 54	3	7	2
25 ¼, — — 55	3	1	7

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
26 ¼, or at Age 56	2	16	6
27 ¼, — — 57	2	11	6
28 ¼, — — 58	2	7	3
29 ¼, — — 59	2	3	0
30 ¼, — — 60	1	19	1
31 ¼, — — 61	1	15	3
32 ¼, — — 62	1	11	11
33 ¼, — — 63	1	8	7
34 ¼, — — 64	1	5	8
35 ¼, — — 65	1	2	9
36 ¼, — — 66	1	0	4
37 ¼, — — 67	0	17	10
38 ¼, — — 68	0	15	9
39 ¼, — — 69	0	13	8
40 ¼, — — 70	0	11	10
41 ¼, — — 71	0	10	2
42 ¼, — — 72	0	8	8
43 ¼, — — 73	0	7	2
44 ¼, — — 74	0	5	11
45 ¼, — — 75	0	4	10

TABLE XVII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 30 ¾ to 31 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
4 ¼, or at Age 35	13	10	0
5 ¼, — — 36	12	14	0
6 ¼, — — 37	11	19	7
7 ¼, — — 38	11	5	2
8 ¼, — — 39	10	12	0
9 ¼, — — 40	9	18	10
10 ¼, — — 41	9	6	5
11 ¼, — — 42	8	14	0
12 ¼, — — 43	8	2	11
13 ¼, — — 44	7	11	10
14 ¼, — — 45	7	1	10
15 ¼, — — 46	6	11	10
16 ¼, — — 47	6	2	10
17 ¼, — — 48	5	13	10
18 ¼, — — 49	5	5	8
19 ¼, — — 50	4	17	6
20 ¼, — — 51	4	10	3
21 ¼, — — 52	4	2	11
22 ¼, — — 53	3	16	7
23 ¼, — — 54	3	10	2
24 ¼, — — 55	3	4	8

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
25 ¼, or at Age 56	2	19	2
26 ¼, — — 57	2	14	2
27 ¼, — — 58	2	9	2
28 ¼, — — 59	2	4	11
29 ¼, — — 60	2	0	7
30 ¼, — — 61	1	17	1
31 ¼, — — 62	1	13	7
32 ¼, — — 63	1	10	1
33 ¼, — — 64	1	6	7
34 ¼, — — 65	1	4	0
35 ¼, — — 66	1	1	5
36 ¼, — — 67	0	18	10
37 ¼, — — 68	0	16	2
38 ¼, — — 69	0	14	4
39 ¼, — — 70	0	12	6
40 ¼, — — 71	0	10	8
41 ¼, — — 72	0	8	10
42 ¼, — — 73	0	7	7
43 ¼, — — 74	0	6	3
44 ¼, — — 75	0	5	0

TABLE XVIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $31\frac{3}{4}$ to $32\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$3\frac{1}{4}$, or at Age 35	14	2	3
$4\frac{1}{4}$, — — 36	13	6	4
$5\frac{1}{4}$, — — 37	12	10	5
$6\frac{1}{4}$, — — 38	11	16	0
$7\frac{1}{4}$, — — 39	11	1	7
$8\frac{1}{4}$, — — 40	10	8	3
$9\frac{1}{4}$, — — 41	9	14	11
$10\frac{1}{4}$, — — 42	9	2	7
$11\frac{1}{4}$, — — 43	8	10	3
$12\frac{1}{4}$, — — 44	7	19	3
$13\frac{1}{4}$, — — 45	7	8	2
$14\frac{1}{4}$, — — 46	6	18	3
$15\frac{1}{4}$, — — 47	6	8	4
$16\frac{1}{4}$, — — 48	5	19	4
$17\frac{1}{4}$, — — 49	5	10	5
$18\frac{1}{4}$, — — 50	5	2	3
$19\frac{1}{4}$, — — 51	4	14	2
$20\frac{1}{4}$, — — 52	4	7	1
$21\frac{1}{4}$, — — 53	3	19	11
$22\frac{1}{4}$, — — 54	3	13	9
$23\frac{1}{4}$, — — 55	3	7	6

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$24\frac{1}{4}$, or at Age 56	3	2	0
$25\frac{1}{4}$, — — 57	2	16	7
$26\frac{1}{4}$, — — 58	2	11	8
$27\frac{1}{4}$, — — 59	2	6	10
$28\frac{1}{4}$, — — 60	2	2	8
$29\frac{1}{4}$, — — 61	1	18	5
$30\frac{1}{4}$, — — 62	1	14	11
$31\frac{1}{4}$, — — 63	1	11	4
$32\frac{1}{4}$, — — 64	1	8	2
$33\frac{1}{4}$, — — 65	1	5	0
$34\frac{1}{4}$, — — 66	1	2	4
$35\frac{1}{4}$, — — 67	0	19	7
$36\frac{1}{4}$, — — 68	0	17	4
$37\frac{1}{4}$, — — 69	0	15	0
$38\frac{1}{4}$, — — 70	0	13	1
$39\frac{1}{4}$, — — 71	0	11	2
$40\frac{1}{4}$, — — 72	0	9	6
$41\frac{1}{4}$, — — 73	0	7	11
$42\frac{1}{4}$, — — 74	0	6	7
$43\frac{1}{4}$, — — 75	0	5	3

TABLE XIX.

For the Uſe of Country Pariſhes.

Shewing the Payment due, (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 32 ¼ to 33 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
2 ¼, or at Age 35	14	16	1	23 ¼, or at Age 56	3	4	10
3 ¼, — — 36	13	18	7	24 ¼, — — 57	2	19	5
4 ¼, — — 37	13	2	8	25 ¼, — — 58	2	14	0
5 ¼, — — 38	12	6	10	26 ¼, — — 59	2	9	3
6 ¼, — — 39	11	12	5	27 ¼, — — 60	2	4	7
7 ¼, — — 40	10	18	0	28 ¼, — — 61	2	0	6
8 ¼, — — 41	10	4	6	29 ¼, — — 62	1	16	4
9 ¼, — — 42	9	11	0	30 ¼, — — 63	1	12	9
10 ¼, — — 43	8	18	10	31 ¼, — — 64	1	9	2
11 ¼, — — 44	8	6	7	32 ¼, — — 65	1	6	4
12 ¼, — — 45	7	15	7	33 ¼, — — 66	1	3	6
13 ¼, — — 46	7	4	7	34 ¼, — — 67	1	0	8
14 ¼, — — 47	6	14	9	35 ¼, — — 68	0	17	10
15 ¼, — — 48	6	4	10	36 ¼, — — 69	0	15	10
16 ¼, — — 49	5	15	11	37 ¼, — — 70	0	13	10
17 ¼, — — 50	5	7	0	38 ¼, — — 71	0	11	10
18 ¼, — — 51	4	18	11	39 ¼, — — 72	0	9	10
19 ¼, — — 52	4	10	10	40 ¼, — — 73	0	8	5
20 ¼, — — 53	4	3	11	41 ¼, — — 74	0	7	0
21 ¼, — — 54	3	17	0	42 ¼, — — 75	0	5	7
22 ¼, — — 55	3	10	11				

TABLE XX.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $33\frac{3}{4}$ to $34\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$1\frac{1}{4}$, or at Age 35	15	9	9
$2\frac{1}{4}$, — — 36	14	12	3
$3\frac{1}{4}$, — — 37	13	14	9
$4\frac{1}{4}$, — — 38	12	18	11
$5\frac{1}{4}$, — — 39	12	3	1
$6\frac{1}{4}$, — — 40	11	8	5
$7\frac{1}{4}$, — — 41	10	13	9
$8\frac{1}{4}$, — — 42	10	0	4
$9\frac{1}{4}$, — — 43	9	6	11
$10\frac{1}{4}$, — — 44	8	14	10
$11\frac{1}{4}$, — — 45	8	2	8
$12\frac{1}{4}$, — — 46	7	11	9
$13\frac{1}{4}$, — — 47	7	0	10
$14\frac{1}{4}$, — — 48	6	11	0
$15\frac{1}{4}$, — — 49	6	1	1
$16\frac{1}{4}$, — — 50	5	12	3
$17\frac{1}{4}$, — — 51	5	3	5
$18\frac{1}{4}$, — — 52	4	15	7
$19\frac{1}{4}$, — — 53	4	7	9
$20\frac{1}{4}$, — — 54	4	0	11
$21\frac{1}{4}$, — — 55	3	14	2

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$22\frac{1}{4}$, or at Age 56	3	8	1
$23\frac{1}{4}$, — — 57	3	2	1
$24\frac{1}{4}$, — — 58	2	16	9
$25\frac{1}{4}$, — — 59	2	11	6
$26\frac{1}{4}$, — — 60	2	6	10
$27\frac{1}{4}$, — — 61	2	2	2
$28\frac{1}{4}$, — — 62	1	18	2
$29\frac{1}{4}$, — — 63	1	14	2
$30\frac{1}{4}$, — — 64	1	10	8
$31\frac{1}{4}$, — — 65	1	7	3
$32\frac{1}{4}$, — — 66	1	4	4
$33\frac{1}{4}$, — — 67	1	1	6
$34\frac{1}{4}$, — — 68	0	19	0
$35\frac{1}{4}$, — — 69	0	16	6
$36\frac{1}{4}$, — — 70	0	14	5
$37\frac{1}{4}$, — — 71	0	12	3
$38\frac{1}{4}$, — — 72	0	10	6
$39\frac{1}{4}$, — — 73	0	8	9
$40\frac{1}{4}$, — — 74	0	7	3
$41\frac{1}{4}$, — — 75	0	5	10

TABLE XXI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 34 $\frac{3}{4}$ to 35 $\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 35	16	5	0
1 $\frac{1}{4}$, — — 36	15	6	0
2 $\frac{1}{4}$, — — 37	14	8	6
3 $\frac{1}{4}$, — — 38	13	11	0
4 $\frac{1}{4}$, — — 39	12	15	2
5 $\frac{1}{4}$, — — 40	11	19	5
6 $\frac{1}{4}$, — — 41	11	4	6
7 $\frac{1}{4}$, — — 42	10	9	7
8 $\frac{1}{4}$, — — 43	9	16	2
9 $\frac{1}{4}$, — — 44	9	2	10
10 $\frac{1}{4}$, — — 45	8	10	10
11 $\frac{1}{4}$, — — 46	7	18	9
12 $\frac{1}{4}$, — — 47	7	8	0
13 $\frac{1}{4}$, — — 48	6	17	2
14 $\frac{1}{4}$, — — 49	6	7	4
15 $\frac{1}{4}$, — — 50	5	17	5
16 $\frac{1}{4}$, — — 51	5	8	7
17 $\frac{1}{4}$, — — 52	4	19	10
18 $\frac{1}{4}$, — — 53	4	12	3
19 $\frac{1}{4}$, — — 54	4	4	8
20 $\frac{1}{4}$, — — 55	3	18	0

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
21 $\frac{1}{4}$, or at Age 56	3	11	4
22 $\frac{1}{4}$, — — 57	3	5	4
23 $\frac{1}{4}$, — — 58	2	19	4
24 $\frac{1}{4}$, — — 59	2	14	2
25 $\frac{1}{4}$, — — 60	2	9	0
26 $\frac{1}{4}$, — — 61	2	4	5
27 $\frac{1}{4}$, — — 62	1	19	10
28 $\frac{1}{4}$, — — 63	1	15	11
29 $\frac{1}{4}$, — — 64	1	12	0
30 $\frac{1}{4}$, — — 65	1	8	8
31 $\frac{1}{4}$, — — 66	1	5	4
32 $\frac{1}{4}$, — — 67	1	2	5
33 $\frac{1}{4}$, — — 68	0	19	7
34 $\frac{1}{4}$, — — 69	0	17	5
35 $\frac{1}{4}$, — — 70	0	15	2
36 $\frac{1}{4}$, — — 71	0	13	0
37 $\frac{1}{4}$, — — 72	0	10	9
38 $\frac{1}{4}$, — — 73	0	9	2
39 $\frac{1}{4}$, — — 74	0	7	8
40 $\frac{1}{4}$, — — 75	0	6	2

TABLE XXII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $35\frac{3}{4}$ to $36\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 36	16	0	10	$20\frac{1}{4}$, or at Age 56	3	14	10
$1\frac{1}{4}$, — — 37	15	2	0	$21\frac{1}{4}$, — — 57	3	8	3
$2\frac{1}{4}$, — — 38	14	4	6	$22\frac{1}{4}$, — — 58	3	2	5
$3\frac{1}{4}$, — — 39	13	7	2	$23\frac{1}{4}$, — — 59	2	16	7
$4\frac{1}{4}$, — — 40	12	11	0	$24\frac{1}{4}$, — — 60	2	11	6
$5\frac{1}{4}$, — — 41	11	15	0	$25\frac{1}{4}$, — — 61	2	6	5
$6\frac{1}{4}$, — — 42	11	0	2	$26\frac{1}{4}$, — — 62	2	1	11
$7\frac{1}{4}$, — — 43	10	5	3	$27\frac{1}{4}$, — — 63	1	17	6
$8\frac{1}{4}$, — — 44	9	12	0	$28\frac{1}{4}$, — — 64	1	13	8
$9\frac{1}{4}$, — — 45	8	18	8	$29\frac{1}{4}$, — — 65	1	9	11
$10\frac{1}{4}$, — — 46	8	6	9	$30\frac{1}{4}$, — — 66	1	6	7
$11\frac{1}{4}$, — — 47	7	14	9	$31\frac{1}{4}$, — — 67	1	3	4
$12\frac{1}{4}$, — — 48	7	3	11	$32\frac{1}{4}$, — — 68	1	0	7
$13\frac{1}{4}$, — — 49	6	13	1	$33\frac{1}{4}$, — — 69	0	17	10
$14\frac{1}{4}$, — — 50	6	3	4	$34\frac{1}{4}$, — — 70	0	15	8
$15\frac{1}{4}$, — — 51	5	13	7	$35\frac{1}{4}$, — — 71	0	13	5
$16\frac{1}{4}$, — — 52	5	5	0	$36\frac{1}{4}$, — — 72	0	11	6
$17\frac{1}{4}$, — — 53	4	16	5	$37\frac{1}{4}$, — — 73	0	9	7
$18\frac{1}{4}$, — — 54	4	8	11	$38\frac{1}{4}$, — — 74	0	7	11
$19\frac{1}{4}$, — — 55	4	1	5	$39\frac{1}{4}$, — — 75	0	6	6

TABLE XXIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 36 ¾ to 37 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age	37 15	16	9
1 ¼, — —	38 14	18	0
2 ¼, — —	39 14	0	6
3 ¼, — —	40 13	3	4
4 ¼, — —	41 12	6	11
5 ¼, — —	42 11	10	7
6 ¼, — —	43 10	15	10
7 ¼, — —	44 10	1	0
8 ¼, — —	45 9	7	10
9 ¼, — —	46 8	14	7
10 ¼, — —	47 8	2	8
11 ¼, — —	48 7	10	10
12 ¼, — —	49 6	19	11
13 ¼, — —	50 6	9	0
14 ¼, — —	51 5	19	5
15 ¼, — —	52 5	9	10
16 ¼, — —	53 5	1	5
17 ¼, — —	54 4	13	0
18 ¼, — —	55 4	5	7
19 ¼, — —	56 3	18	2

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
20 ¼, or at Age	57 3	11	8
21 ¼, — —	58 3	5	2
22 ¼, — —	59 2	19	6
23 ¼, — —	60 2	13	10
24 ¼, — —	61 2	8	10
25 ¼, — —	62 2	3	10
26 ¼, — —	63 1	19	6
27 ¼, — —	64 1	15	2
28 ¼, — —	65 1	11	6
29 ¼, — —	66 1	7	10
30 ¼, — —	67 1	4	7
31 ¼, — —	68 1	1	5
32 ¼, — —	69 0	18	10
33 ¼, — —	70 0	16	2
34 ¼, — —	71 0	14	0
35 ¼, — —	72 0	11	10
36 ¼, — —	73 0	10	1
37 ¼, — —	74 0	8	5
38 ¼, — —	75 0	6	9

TABLE XXIV.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $37\frac{3}{4}$ to $38\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment.		
			£.	s.	d.
$\frac{1}{4}$,	or at Age	38	15	12	6
$1\frac{1}{4}$,	— —	39	14	13	11
$2\frac{1}{4}$,	— —	40	13	16	0
$3\frac{1}{4}$,	— —	41	12	18	7
$4\frac{1}{4}$,	— —	42	12	2	3
$5\frac{1}{4}$,	— —	43	11	5	11
$6\frac{1}{4}$,	— —	44	10	11	3
$7\frac{1}{4}$,	— —	45	9	16	6
$8\frac{1}{4}$,	— —	46	9	3	5
$9\frac{1}{4}$,	— —	47	8	10	3
$10\frac{1}{4}$,	— —	48	7	18	4
$11\frac{1}{4}$,	— —	49	7	6	5
$12\frac{1}{4}$,	— —	50	6	15	8
$13\frac{1}{4}$,	— —	51	6	4	11
$14\frac{1}{4}$,	— —	52	5	15	6
$15\frac{1}{4}$,	— —	53	5	6	1
$16\frac{1}{4}$,	— —	54	4	17	10
$17\frac{1}{4}$,	— —	55	4	9	7
$18\frac{1}{4}$,	— —	56	4	2	3

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment.		
			£.	s.	d.
$19\frac{1}{4}$,	or at Age	57	3	15	0
$20\frac{1}{4}$,	— —	58	3	8	7
$21\frac{1}{4}$,	— —	59	3	2	0
$22\frac{1}{4}$,	— —	60	2	16	7
$23\frac{1}{4}$,	— —	61	2	11	0
$24\frac{1}{4}$,	— —	62	2	6	2
$25\frac{1}{4}$,	— —	63	2	1	4
$26\frac{1}{4}$,	— —	64	1	17	1
$27\frac{1}{4}$,	— —	65	1	12	10
$28\frac{1}{4}$,	— —	66	1	9	3
$29\frac{1}{4}$,	— —	67	1	5	8
$30\frac{1}{4}$,	— —	68	1	2	7
$31\frac{1}{4}$,	— —	69	0	19	7
$32\frac{1}{4}$,	— —	70	0	17	1
$33\frac{1}{4}$,	— —	71	0	14	7
$34\frac{1}{4}$,	— —	72	0	12	6
$35\frac{1}{4}$,	— —	73	0	10	5
$36\frac{1}{4}$,	— —	74	0	8	9
$37\frac{1}{4}$,	— —	75	0	7	1

TABLE XXV.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 38 ¾ to 39 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age 39	15	8	6
1 ¼, — — 40	14	9	10
2 ¼, — — 41	13	11	10
3 ¼, — — 42	12	13	10
4 ¼, — — 43	11	17	7
5 ¼, — — 44	11	1	4
6 ¼, — — 45	10	6	8
7 ¼, — — 46	9	12	0
8 ¼, — — 47	8	19	0
9 ¼, — — 48	8	6	0
10 ¼, — — 49	7	14	0
11 ¼, — — 50	7	2	1
12 ¼, — — 51	6	11	6
13 ¼, — — 52	6	0	10
14 ¼, — — 53	5	11	7
15 ¼, — — 54	5	2	5
16 ¼, — — 55	4	14	3
17 ¼, — — 56	4	6	2
18 ¼, — — 57	3	19	0

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
19 ¼, or at Age 58	3	11	10
20 ¼, — — 59	3	5	6
21 ¼, — — 60	2	19	2
22 ¼, — — 61	2	13	8
23 ¼, — — 62	2	8	2
24 ¼, — — 63	2	3	6
25 ¼, — — 64	1	18	10
26 ¼, — — 65	1	14	8
27 ¼, — — 66	1	10	7
28 ¼, — — 67	1	7	1
29 ¼, — — 68	1	3	7
30 ¼, — — 69	1	0	8
31 ¼, — — 70	0	17	10
32 ¼, — — 71	0	15	5
33 ¼, — — 72	0	13	0
34 ¼, — — 73	0	11	0
35 ¼, — — 74	0	9	0
36 ¼, — — 75	0	7	5

TABLE XXVI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $39\frac{3}{4}$ to $40\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 40	15	4	4	$18\frac{1}{4}$, or at Age 58	3	15	8
$1\frac{1}{4}$, — — 41	14	5	2	$19\frac{1}{4}$, — — 59	3	8	7
$2\frac{1}{4}$, — — 42	13	7	2	$20\frac{1}{4}$, — — 60	3	2	5
$3\frac{1}{4}$, — — 43	12	9	2	$21\frac{1}{4}$, — — 61	2	16	3
$4\frac{1}{4}$, — — 44	11	13	0	$22\frac{1}{4}$, — — 62	2	10	10
$5\frac{1}{4}$, — — 45	10	16	10	$23\frac{1}{4}$, — — 63	2	5	6
$6\frac{1}{4}$, — — 46	10	2	4	$24\frac{1}{4}$, — — 64	2	0	11
$7\frac{1}{4}$, — — 47	9	7	9	$25\frac{1}{4}$, — — 65	1	16	4
$8\frac{1}{4}$, — — 48	8	14	8	$26\frac{1}{4}$, — — 66	1	12	4
$9\frac{1}{4}$, — — 49	8	1	6	$27\frac{1}{4}$, — — 67	1	8	4
$10\frac{1}{4}$, — — 50	7	9	8	$28\frac{1}{4}$, — — 68	1	5	0
$11\frac{1}{4}$, — — 51	6	17	10	$29\frac{1}{4}$, — — 69	1	1	8
$12\frac{1}{4}$, — — 52	6	7	5	$30\frac{1}{4}$, — — 70	0	18	10
$13\frac{1}{4}$, — — 53	5	17	0	$31\frac{1}{4}$, — — 71	0	16	1
$14\frac{1}{4}$, — — 54	5	7	10	$32\frac{1}{4}$, — — 72	0	13	9
$15\frac{1}{4}$, — — 55	4	18	9	$33\frac{1}{4}$, — — 73	0	11	6
$16\frac{1}{4}$, — — 56	4	10	9	$34\frac{1}{4}$, — — 74	0	9	7
$17\frac{1}{4}$, — — 57	4	2	9	$35\frac{1}{4}$, — — 75	0	7	9

TABLE XXVII.

For the Uſe of Country Pariſhes.

Shewing the Payment due, (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 40 ¾ to 41 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age 41	15	0	0	18 ¼, or at Age 59	3	12	5
1 ¼, — — 42	14	0	7	19 ¼, — — 60	3	5	5
2 ¼, — — 43	13	2	7	20 ¼, — — 61	2	19	4
3 ¼, — — 44	12	4	7	21 ¼, — — 62	2	13	4
4 ¼, — — 45	11	8	6	22 ¼, — — 63	2	8	1
5 ¼, — — 46	10	12	5	23 ¼, — — 64	2	2	10
6 ¼, — — 47	9	18	0	24 ¼, — — 65	1	18	4
7 ¼, — — 48	9	3	7	25 ¼, — — 66	1	13	10
8 ¼, — — 49	8	10	4	26 ¼, — — 67	1	10	0
9 ¼, — — 50	7	17	1	27 ¼, — — 68	1	6	2
10 ¼, — — 51	7	5	4	28 ¼, — — 69	1	2	11
11 ¼, — — 52	6	13	7	29 ¼, — — 70	0	19	9
12 ¼, — — 53	6	3	5	30 ¼, — — 71	0	17	1
13 ¼, — — 54	5	13	2	31 ¼, — — 72	0	14	5
14 ¼, — — 55	5	4	2	32 ¼, — — 73	0	12	2
15 ¼, — — 56	4	15	2	33 ¼, — — 74	0	10	0
16 ¼, — — 57	4	7	3	34 ¼, — — 75	0	8	2
17 ¼, — — 58	3	19	5				

TABLE XXVIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 41 ¾ to 42 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age 42	14	15	6	17 ¼, or at Age 59	3	16	1
1 ¼, — — 43	13	16	2	18 ¼, — — 60	3	9	2
2 ¼, — — 44	12	18	2	19 ¼, — — 61	3	2	4
3 ¼, — — 45	12	0	3	20 ¼, — — 62	2	16	4
4 ¼, — — 46	11	4	2	21 ¼, — — 63	2	10	5
5 ¼, — — 47	10	8	1	22 ¼, — — 64	2	5	4
6 ¼, — — 48	9	13	7	23 ¼, — — 65	2	0	2
7 ¼, — — 49	8	19	0	24 ¼, — — 66	1	15	10
8 ¼, — — 50	8	5	11	25 ¼, — — 67	1	11	5
9 ¼, — — 51	7	12	9	26 ¼, — — 68	1	7	9
10 ¼, — — 52	7	1	2	27 ¼, — — 69	1	4	0
11 ¼, — — 53	6	9	8	28 ¼, — — 70	1	0	11
12 ¼, — — 54	5	19	7	29 ¼, — — 71	0	17	10
13 ¼, — — 55	5	9	6	30 ¼, — — 72	0	15	4
14 ¼, — — 56	5	0	7	31 ¼, — — 73	0	12	9
15 ¼, — — 57	4	11	8	32 ¼, — — 74	0	10	7
16 ¼, — — 58	4	3	10	33 ¼, — — 75	0	8	7

TABLE XXIX.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 42 $\frac{3}{4}$ to 43 $\frac{1}{2}$.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment.		
		£.	s.	d.
$\frac{1}{4}$,	or at Age 43	14	11	0
1 $\frac{1}{4}$,	— — 44	13	11	9
2 $\frac{1}{4}$,	— — 45	12	13	10
3 $\frac{1}{4}$,	— — 46	11	15	11
4 $\frac{1}{4}$,	— — 47	10	19	10
5 $\frac{1}{4}$,	— — 48	10	3	10
6 $\frac{1}{4}$,	— — 49	9	9	2
7 $\frac{1}{4}$,	— — 50	8	14	6
8 $\frac{1}{4}$,	— — 51	8	1	6
9 $\frac{1}{4}$,	— — 52	7	8	5
10 $\frac{1}{4}$,	— — 53	6	17	1
11 $\frac{1}{4}$,	— — 54	6	5	10
12 $\frac{1}{4}$,	— — 55	5	15	10
13 $\frac{1}{4}$,	— — 56	5	5	10
14 $\frac{1}{4}$,	— — 57	4	17	0
15 $\frac{1}{4}$,	— — 58	4	8	2
16 $\frac{1}{4}$,	— — 59	4	0	6

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment.		
		£.	s.	d.
17 $\frac{1}{4}$,	or at Age 60	3	12	9
18 $\frac{1}{4}$,	— — 61	3	6	0
19 $\frac{1}{4}$,	— — 62	2	19	3
20 $\frac{1}{4}$,	— — 63	2	13	5
21 $\frac{1}{4}$,	— — 64	2	7	7
22 $\frac{1}{4}$,	— — 65	2	2	7
23 $\frac{1}{4}$,	— — 66	1	17	7
24 $\frac{1}{4}$,	— — 67	1	13	4
25 $\frac{1}{4}$,	— — 68	1	9	1
26 $\frac{1}{4}$,	— — 69	1	5	6
27 $\frac{1}{4}$,	— — 70	1	1	11
28 $\frac{1}{4}$,	— — 71	0	18	11
29 $\frac{1}{4}$,	— — 72	0	16	0
30 $\frac{1}{4}$,	— — 73	0	13	7
31 $\frac{1}{4}$,	— — 74	0	11	2
32 $\frac{1}{4}$,	— — 75	0	9	1

TABLE XXX.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 43 ¾ to 44 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age 44	14	6	9	16 ¼, or at Age 60	3	17	0
1 ¼, — — 45	13	7	2	17 ¼, — — 61	3	9	4
2 ¼, — — 46	12	9	3	18 ¼, — — 62	3	2	9
3 ¼, — — 47	11	11	4	19 ¼, — — 63	2	16	1
4 ¼, — — 48	10	15	2	20 ¼, — — 64	2	10	5
5 ¼, — — 49	9	19	0	21 ¼, — — 65	2	4	9
6 ¼, — — 50	9	4	5	22 ¼, — — 66	1	19	10
7 ¼, — — 51	8	9	10	23 ¼, — — 67	1	14	11
8 ¼, — — 52	7	17	0	24 ¼, — — 68	1	10	10
9 ¼, — — 53	7	4	2	25 ¼, — — 69	1	6	9
10 ¼, — — 54	6	13	0	26 ¼, — — 70	1	3	3
11 ¼, — — 55	6	1	10	27 ¼, — — 71	0	19	10
12 ¼, — — 56	5	11	11	28 ¼, — — 72	0	17	0
13 ¼, — — 57	5	2	0	29 ¼, — — 73	0	14	2
14 ¼, — — 58	4	13	3	30 ¼, — — 74	0	11	10
15 ¼, — — 59	4	4	7	31 ¼, — — 75	0	9	7

TABLE XXXI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 44 ¾ to 45 ½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment.		
		£.	s.	d.
¼,	or at Age 45	14	2	0
1 ¼,	— — 46	13	2	7
2 ¼,	— — 47	12	4	8
3 ¼,	— — 48	11	6	10
4 ¼,	— — 49	10	10	6
5 ¼,	— — 50	9	14	2
6 ¼,	— — 51	8	19	8
7 ¼,	— — 52	8	5	2
8 ¼,	— — 53	7	12	7
9 ¼,	— — 54	7	0	0
10 ¼,	— — 55	6	8	11
11 ¼,	— — 56	5	17	10
12 ¼,	— — 57	5	8	0
13 ¼,	— — 58	4	18	2
14 ¼,	— — 59	4	9	7
15 ¼,	— — 60	4	1	0

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment.		
		£.	s.	d.
16 ¼,	or at Age 61	3	13	6
17 ¼,	— — 62	3	6	0
18 ¼,	— — 63	2	19	6
19 ¼,	— — 64	2	13	0
20 ¼,	— — 65	2	7	5
21 ¼,	— — 66	2	1	11
22 ¼,	— — 67	1	17	1
23 ¼,	— — 68	1	12	4
24 ¼,	— — 69	1	8	5
25 ¼,	— — 70	1	4	5
26 ¼,	— — 71	1	1	1
27 ¼,	— — 72	0	17	10
28 ¼,	— — 73	0	15	1
29 ¼,	— — 74	0	12	5
30 ¼,	— — 75	0	10	2

TABLE XXXII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $45\frac{3}{4}$ to $46\frac{1}{2}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age 46	13	17	6	$15\frac{1}{4}$, or at Age 61	3	17	3
$1\frac{1}{4}$, — — 47	12	17	10	$16\frac{1}{4}$, — — 62	3	9	11
$2\frac{1}{4}$, — — 48	11	19	9	$17\frac{1}{4}$, — — 63	3	2	7
$3\frac{1}{4}$, — — 49	11	1	8	$18\frac{1}{4}$, — — 64	2	16	2
$4\frac{1}{4}$, — — 50	10	5	6	$19\frac{1}{4}$, — — 65	2	9	10
$5\frac{1}{4}$, — — 51	9	9	3	$20\frac{1}{4}$, — — 66	2	4	5
$6\frac{1}{4}$, — — 52	8	15	0	$21\frac{1}{4}$, — — 67	1	19	0
$7\frac{1}{4}$, — — 53	8	0	8	$22\frac{1}{4}$, — — 68	1	14	4
$8\frac{1}{4}$, — — 54	7	8	2	$23\frac{1}{4}$, — — 69	1	9	9
$9\frac{1}{4}$, — — 55	6	15	8	$24\frac{1}{4}$, — — 70	1	5	11
$10\frac{1}{4}$, — — 56	6	4	8	$25\frac{1}{4}$, — — 71	1	2	2
$11\frac{1}{4}$, — — 57	5	13	8	$26\frac{1}{4}$, — — 72	0	18	11
$12\frac{1}{4}$, — — 58	5	4	0	$27\frac{1}{4}$, — — 73	0	15	10
$13\frac{1}{4}$, — — 59	4	14	3	$28\frac{1}{4}$, — — 74	0	13	2
$14\frac{1}{4}$, — — 60	4	5	9	$29\frac{1}{4}$, — — 75	0	10	9

TABLE XXXIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $46\frac{3}{4}$ to $47\frac{1}{2}$.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment.		
		£.	s.	d.
$\frac{1}{4}$,	or at Age 47	13	12	6
$1\frac{1}{4}$,	— — 48	12	13	2
$2\frac{1}{4}$,	— — 49	11	14	11
$3\frac{1}{4}$,	— — 50	10	16	7
$4\frac{1}{4}$,	— — 51	10	0	6
$5\frac{1}{4}$,	— — 52	9	4	4
$6\frac{1}{4}$,	— — 53	8	10	4
$7\frac{1}{4}$,	— — 54	7	16	3
$8\frac{1}{4}$,	— — 55	7	3	10
$9\frac{1}{4}$,	— — 56	6	11	5
$10\frac{1}{4}$,	— — 57	6	0	6
$11\frac{1}{4}$,	— — 58	5	9	7
$12\frac{1}{4}$,	— — 59	5	0	0
$13\frac{1}{4}$,	— — 60	4	10	4
$14\frac{1}{4}$,	— — 61	4	2	0

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment.		
		£.	s.	d.
$15\frac{1}{4}$,	or at Age 62	3	13	7
$16\frac{1}{4}$,	— — 63	3	6	5
$17\frac{1}{4}$,	— — 64	2	19	2
$18\frac{1}{4}$,	— — 65	2	12	11
$19\frac{1}{4}$,	— — 66	2	6	9
$20\frac{1}{4}$,	— — 67	2	1	5
$21\frac{1}{4}$,	— — 68	1	16	2
$22\frac{1}{4}$,	— — 69	1	11	8
$23\frac{1}{4}$,	— — 70	1	7	2
$24\frac{1}{4}$,	— — 71	1	3	6
$25\frac{1}{4}$,	— — 72	0	19	10
$26\frac{1}{4}$,	— — 73	0	16	10
$27\frac{1}{4}$,	— — 74	0	13	10
$28\frac{1}{4}$,	— — 75	0	11	4

TABLE XXXIV.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 47 ¾ to 48 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¾, or at Age 48	13	7	9	14 ¾, or at Age 62	3	18	1
1 ¾, — — 49	12	7	10	15 ¾, — — 63	3	9	10
2 ¾, — — 50	11	9	7	16 ¾, — — 64	3	2	10
3 ¾, — — 51	10	11	5	17 ¾, — — 65	2	15	8
4 ¾, — — 52	9	15	6	18 ¾, — — 66	2	9	7
5 ¾, — — 53	8	19	7	19 ¾, — — 67	2	3	7
6 ¾, — — 54	8	5	7	20 ¾, — — 68	1	18	5
7 ¾, — — 55	7	11	8	21 ¾, — — 69	1	13	3
8 ¾, — — 56	6	19	4	22 ¾, — — 70	1	8	11
9 ¾, — — 57	6	7	0	23 ¾, — — 71	1	4	8
10 ¾, — — 58	5	16	2	24 ¾, — — 72	1	1	2
11 ¾, — — 59	5	5	4	25 ¾, — — 73	0	17	8
12 ¾, — — 60	4	15	10	26 ¾, — — 74	0	14	9
13 ¾, — — 61	4	6	4	27 ¾, — — 75	0	12	0

TABLE XXXV.

For the Uſe of Country Pariſhes.

Shewing the Payment due, (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 48 ¼ to 49 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age 49	13	2	6	14 ¼, or at Age 63	3	14	3
1 ¼, — — 50	12	2	6	15 ¼, — — 64	3	6	2
2 ¼, — — 51	11	4	5	16 ¼, — — 65	2	19	3
3 ¼, — — 52	10	6	3	17 ¼, — — 66	2	12	3
4 ¼, — — 53	9	10	6	18 ¼, — — 67	2	6	4
5 ¼, — — 54	8	14	10	19 ¼, — — 68	2	0	5
6 ¼, — — 55	8	0	11	20 ¼, — — 69	1	15	5
7 ¼, — — 56	7	7	1	21 ¼, — — 70	1	10	5
8 ¼, — — 57	6	14	10	22 ¼, — — 71	1	6	3
9 ¼, — — 58	6	2	7	23 ¼, — — 72	1	2	2
10 ¼, — — 59	5	11	10	24 ¼, — — 73	0	18	10
11 ¼, — — 60	5	1	1	25 ¼, — — 74	0	15	6
12 ¼, — — 61	4	11	9	26 ¼, — — 75	0	12	8
13 ¼, — — 62	4	2	5				

TABLE XXXVI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 49 ¾ to 50 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age 50	12	17	8	13 ¼, or at Age 63	3	18	7
1 ¼, — — 51	11	17	10	14 ¼, — — 64	3	10	7
2 ¼, — — 52	10	19	10	15 ¼, — — 65	3	2	7
3 ¼, — — 53	10	1	11	16 ¼, — — 66	2	15	10
4 ¼, — — 54	9	6	2	17 ¼, — — 67	2	8	11
5 ¼, — — 55	8	10	6	18 ¼, — — 68	2	3	2
6 ¼, — — 56	7	16	7	19 ¼, — — 69	1	17	5
7 ¼, — — 57	7	2	10	20 ¼, — — 70	1	12	7
8 ¼, — — 58	6	10	7	21 ¼, — — 71	1	7	9
9 ¼, — — 59	5	18	5	22 ¼, — — 72	1	3	9
10 ¼, — — 60	5	7	9	23 ¼, — — 73	0	19	10
11 ¼, — — 61	4	17	1	24 ¼, — — 74	0	16	7
12 ¼, — — 62	4	7	10	25 ¼, — — 75	0	13	6

TABLE XXXVII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 50 ¾ to 51 ½.

To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼, or at Age 51	12	12	6	13 ¼, or at Age 64	3	14	10
1 ¼, — — 52	11	13	2	14 ¼, — — 65	3	6	11
2 ¼, — — 53	10	15	4	15 ¼, — — 66	2	19	1
3 ¼, — — 54	9	17	7	16 ¼, — — 67	2	12	5
4 ¼, — — 55	9	1	11	17 ¼, — — 68	2	5	8
5 ¼, — — 56	8	6	2	18 ¼, — — 69	2	0	0
6 ¼, — — 57	7	12	4	19 ¼, — — 70	1	14	5
7 ¼, — — 58	6	18	7	20 ¼, — — 71	1	9	9
8 ¼, — — 59	6	6	5	21 ¼, — — 72	1	5	1
9 ¼, — — 60	5	14	3	22 ¼, — — 73	1	1	4
10 ¼, — — 61	5	3	9	23 ¼, — — 74	0	17	6
11 ¼, — — 62	4	13	2	24 ¼, — — 75	0	14	4
12 ¼, — — 63	4	4	0				

TABLE XXXVIII.

For the Uſe of Country Pariſhes

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 51 ¼ to 52 ½.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	52	12	7	6
1 ¼,	——	53	11	7	10
2 ¼,	——	54	10	10	1
3 ¼,	——	55	9	12	4
4 ¼,	——	56	8	16	9
5 ¼,	——	57	8	1	1
6 ¼,	——	58	7	7	4
7 ¼,	——	59	6	13	8
8 ¼,	——	60	6	1	7
9 ¼,	——	61	5	9	7
10 ¼,	——	62	4	19	2
11 ¼,	——	63	4	8	8
12 ¼,	——	64	3	19	8
13 ¼,	——	65	3	10	8
14 ¼,	——	66	3	2	11
15 ¼,	——	67	2	15	3
16 ¼,	——	68	2	8	9
17 ¼,	——	69	2	2	3
18 ¼,	——	70	1	16	9
19 ¼,	——	71	1	11	4
20 ¼,	——	72	1	6	10
21 ¼,	——	73	1	2	5
22 ¼,	——	74	0	18	9
23 ¼,	——	75	0	15	3

TABLE XXXIX.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any age from 35 to 75.

Age of the Purchaſer from 52 ¾ to 53 ¼.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	53	12	2	0
1 ¼,	——	54	11	2	7
2 ¼,	——	55	10	4	11
3 ¼,	——	56	9	7	2
4 ¼,	——	57	8	11	8
5 ¼,	——	58	7	16	1
6 ¼,	——	59	7	2	5
7 ¼,	——	60	6	8	9
8 ¼,	——	61	5	16	10
9 ¼,	——	62	5	4	11
10 ¼,	——	63	4	14	7
11 ¼,	——	64	4	4	3
12 ¼,	——	65	3	15	5
13 ¼,	——	66	3	6	7
14 ¼,	——	67	2	19	0
15 ¼,	——	68	2	11	6
16 ¼,	——	69	2	5	2
17 ¼,	——	70	1	18	10
18 ¼,	——	71	1	13	7
19 ¼,	——	72	1	8	3
20 ¼,	——	73	1	4	0
21 ¼,	——	74	0	19	9
22 ¼,	——	75	0	16	2

TABLE

TABLE XL.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $53\frac{3}{4}$ to $54\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	54	11	16	8
$1\frac{1}{4}$,	——	55	10	17	2
$2\frac{1}{4}$,	——	56	9	19	6
$3\frac{1}{4}$,	——	57	9	1	10
$4\frac{1}{4}$,	——	58	8	6	4
$5\frac{1}{4}$,	——	59	7	10	10
$6\frac{1}{4}$,	——	60	6	17	3
$7\frac{1}{4}$,	——	61	6	3	8
$8\frac{1}{4}$,	——	62	5	11	11
$9\frac{1}{4}$,	——	63	5	0	1
$10\frac{1}{4}$,	——	64	4	9	11
$11\frac{1}{4}$,	——	65	3	19	9
$12\frac{1}{4}$,	——	66	3	11	1
$13\frac{1}{4}$,	——	67	3	2	4
$14\frac{1}{4}$,	——	68	2	15	0
$15\frac{1}{4}$,	——	69	2	7	8
$16\frac{1}{4}$,	——	70	2	1	6
$17\frac{1}{4}$,	——	71	1	15	5
$18\frac{1}{4}$,	——	72	1	10	4
$19\frac{1}{4}$,	——	73	1	5	3
$20\frac{1}{4}$,	——	74	1	1	1
$21\frac{1}{4}$,	——	75	0	17	2

TABLE XLI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $54\frac{3}{4}$ to $55\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	55	11	11	3
$1\frac{1}{4}$,	——	56	10	11	9
$2\frac{1}{4}$,	——	57	9	14	1
$3\frac{1}{4}$,	——	58	8	16	6
$4\frac{1}{4}$,	——	59	8	1	0
$5\frac{1}{4}$,	——	60	7	5	7
$6\frac{1}{4}$,	——	61	6	12	1
$7\frac{1}{4}$,	——	62	5	18	7
$8\frac{1}{4}$,	——	63	5	7	0
$9\frac{1}{4}$,	——	64	4	15	4
$10\frac{1}{4}$,	——	65	4	5	4
$11\frac{1}{4}$,	——	66	3	15	3
$12\frac{1}{4}$,	——	67	3	6	9
$13\frac{1}{4}$,	——	68	2	18	2
$14\frac{1}{4}$,	——	69	2	11	0
$15\frac{1}{4}$,	——	70	2	3	11
$16\frac{1}{4}$,	——	71	1	17	11
$17\frac{1}{4}$,	——	72	1	12	0
$18\frac{1}{4}$,	——	73	1	7	2
$19\frac{1}{4}$,	——	74	1	2	4
$20\frac{1}{4}$,	——	75	0	18	2

TABLE XLII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $55\frac{3}{4}$ to $56\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	56	11	5	6
$1\frac{1}{4}$,	——	57	10	5	10
$2\frac{1}{4}$,	——	58	9	8	6
$3\frac{1}{4}$,	——	59	8	10	11
$4\frac{1}{4}$,	——	60	7	15	6
$5\frac{1}{4}$,	——	61	7	0	2
$6\frac{1}{4}$,	——	62	6	6	9
$7\frac{1}{4}$,	——	63	5	13	4
$8\frac{1}{4}$,	——	64	5	1	11
$9\frac{1}{4}$,	——	65	4	10	5
$10\frac{1}{4}$,	——	66	4	0	7
$11\frac{1}{4}$,	——	67	3	10	8
$12\frac{1}{4}$,	——	68	3	2	4
$13\frac{1}{4}$,	——	69	2	14	0
$14\frac{1}{4}$,	——	70	2	7	0
$15\frac{1}{4}$,	——	71	2	0	1
$16\frac{1}{4}$,	——	72	1	14	4
$17\frac{1}{4}$,	——	73	1	8	8
$18\frac{1}{4}$,	——	74	1	3	11
$19\frac{1}{4}$,	——	75	0	19	6

TABLE XLIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $56\frac{3}{4}$ to $57\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1 in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	57	11	0	0
$1\frac{1}{4}$,	——	58	10	0	5
$2\frac{1}{4}$,	——	59	9	2	11
$3\frac{1}{4}$,	——	60	8	5	4
$4\frac{1}{4}$,	——	61	7	10	0
$5\frac{1}{4}$,	——	62	6	14	9
$6\frac{1}{4}$,	——	63	6	1	6
$7\frac{1}{4}$,	——	64	5	8	2
$8\frac{1}{4}$,	——	65	4	16	10
$9\frac{1}{4}$,	——	66	4	5	6
$10\frac{1}{4}$,	——	67	3	15	10
$11\frac{1}{4}$,	——	68	3	6	1
$12\frac{1}{4}$,	——	69	2	18	0
$13\frac{1}{4}$,	——	70	2	9	10
$14\frac{1}{4}$,	——	71	2	3	1
$15\frac{1}{4}$,	——	72	1	16	4
$16\frac{1}{4}$,	——	73	1	10	10
$17\frac{1}{4}$,	——	74	1	5	4
$18\frac{1}{4}$,	——	75	1	0	9

TABLE XLIV.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $57\frac{3}{4}$ to $58\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	58	10	14	0
$1\frac{1}{4}$,	——	59	9	14	7
$2\frac{1}{4}$,	——	60	9	17	1
$3\frac{1}{4}$,	——	61	7	19	7
$4\frac{1}{4}$,	——	62	7	4	4
$5\frac{1}{4}$,	——	63	6	9	2
$6\frac{1}{4}$,	——	64	5	16	0
$7\frac{1}{4}$,	——	65	5	2	10
$8\frac{1}{4}$,	——	66	4	11	8
$9\frac{1}{4}$,	——	67	4	0	6
$10\frac{1}{4}$,	——	68	3	11	0
$11\frac{1}{4}$,	——	69	3	1	5
$12\frac{1}{4}$,	——	70	2	13	7
$13\frac{1}{4}$,	——	71	2	5	8
$14\frac{1}{4}$,	——	72	1	19	1
$15\frac{1}{4}$,	——	73	1	12	7
$16\frac{1}{4}$,	——	74	1	7	3
$17\frac{1}{4}$,	——	75	1	2	2

TABLE XLV.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $58\frac{3}{4}$ to $59\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	59	10	8	0
$1\frac{1}{4}$,	——	60	9	8	9
$2\frac{1}{4}$,	——	61	8	11	3
$3\frac{1}{4}$,	——	62	7	13	10
$4\frac{1}{4}$,	——	63	6	18	8
$5\frac{1}{4}$,	——	64	6	3	7
$6\frac{1}{4}$,	——	65	5	10	7
$7\frac{1}{4}$,	——	66	4	17	7
$8\frac{1}{4}$,	——	67	4	6	6
$9\frac{1}{4}$,	——	68	3	15	6
$10\frac{1}{4}$,	——	69	3	6	2
$11\frac{1}{4}$,	——	70	2	16	10
$12\frac{1}{4}$,	——	71	2	9	2
$13\frac{1}{4}$,	——	72	2	1	6
$14\frac{1}{4}$,	——	73	1	15	2
$15\frac{1}{4}$,	——	74	1	8	11
$16\frac{1}{4}$,	——	75	1	3	8

TABLE XLVI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 59¾ to 60½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment.		
		£.	s.	d.
¼,	or at Age 60	10	2	0
1 ¼,	— 61	9	2	8
2 ¼,	— 62	8	5	2
3 ¼,	— 63	7	7	10
4 ¼,	— 64	6	12	9
5 ¼,	— 65	5	17	9
6 ¼,	— 66	5	4	11
7 ¼,	— 67	4	12	1
8 ¼,	— 68	4	1	2
9 ¼,	— 69	3	10	4
10 ¼,	— 70	3	1	3
11 ¼,	— 71	2	12	2
12 ¼,	— 72	2	4	9
13 ¼,	— 73	1	17	5
14 ¼,	— 74	1	11	2
15 ¼,	— 75	1	5	5

TABLE XLVII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 60½ to 61½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment.		
		£.	s.	d.
¼,	or at Age 61	9	16	0
1 ¼,	— 62	8	16	7
2 ¼,	— 63	7	19	2
3 ¼,	— 64	7	1	10
4 ¼,	— 65	6	6	11
5 ¼,	— 66	5	12	0
6 ¼,	— 67	4	19	4
7 ¼,	— 68	4	6	7
8 ¼,	— 69	3	15	11
9 ¼,	— 70	3	5	3
10 ¼,	— 71	2	16	5
11 ¼,	— 72	2	7	7
12 ¼,	— 73	2	0	5
13 ¼,	— 74	1	13	4
14 ¼,	— 75	1	7	2

TABLE XLVIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $61\frac{3}{4}$ to $62\frac{3}{4}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	62	9	9	8
$1\frac{1}{4}$,	——	63	8	10	3
$2\frac{1}{4}$,	——	64	7	12	11
$3\frac{1}{4}$,	——	65	6	15	8
$4\frac{1}{4}$,	——	66	6	0	9
$5\frac{1}{4}$,	——	67	5	6	0
$6\frac{1}{4}$,	——	68	4	13	6
$7\frac{1}{4}$,	——	69	4	1	0
$8\frac{1}{4}$,	——	70	3	10	7
$9\frac{1}{4}$,	——	71	3	0	1
$10\frac{1}{4}$,	——	72	2	11	6
$11\frac{1}{4}$,	——	73	2	3	0
$12\frac{1}{4}$,	——	74	1	15	8
$13\frac{1}{4}$,	——	75	1	9	3

TABLE XLIX.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $62\frac{1}{4}$ to $63\frac{1}{4}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	63	9	3	3
$1\frac{1}{4}$,	——	64	8	3	11
$2\frac{1}{4}$,	——	65	7	6	8
$3\frac{1}{4}$,	——	66	6	9	6
$4\frac{1}{4}$,	——	67	5	14	9
$5\frac{1}{4}$,	——	68	5	0	0
$6\frac{1}{4}$,	——	69	4	7	9
$7\frac{1}{4}$,	——	70	3	15	6
$8\frac{1}{4}$,	——	71	3	5	3
$9\frac{1}{4}$,	——	72	2	15	0
$10\frac{1}{4}$,	——	73	2	6	8
$11\frac{1}{4}$,	——	74	1	18	5
$12\frac{1}{4}$,	——	75	1	11	5

TABLE L.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $63\frac{3}{4}$ to $64\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	64	8	16	8
$1\frac{1}{4}$,	——	65	7	17	4
$2\frac{1}{4}$,	——	66	7	0	2
$3\frac{1}{4}$,	——	67	6	3	0
$4\frac{1}{4}$,	——	68	5	8	6
$5\frac{1}{4}$,	——	69	4	13	11
$6\frac{1}{4}$,	——	70	4	1	10
$7\frac{1}{4}$,	——	71	3	9	9
$8\frac{1}{4}$,	——	72	2	19	10
$9\frac{1}{4}$,	——	73	2	9	10
$10\frac{1}{4}$,	——	74	2	1	7
$11\frac{1}{4}$,	——	75	1	14	0

TABLE LI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $64\frac{3}{4}$ to $65\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1 in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	65	8	10	0
$1\frac{1}{4}$,	——	66	7	10	10
$2\frac{1}{4}$,	——	67	6	13	8
$3\frac{1}{4}$,	——	68	5	16	7
$4\frac{1}{4}$,	——	69	5	2	3
$5\frac{1}{4}$,	——	70	4	7	11
$6\frac{1}{4}$,	——	71	3	16	0
$7\frac{1}{4}$,	——	72	3	4	1
$8\frac{1}{4}$,	——	73	2	14	5
$9\frac{1}{4}$,	——	74	2	4	9
$10\frac{1}{4}$,	——	75	1	16	7

TABLE

TABLE LII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 65¾ to 66½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 66	8	3	4
1 ¼,	—— 67	7	4	0
2 ¼,	—— 68	6	6	11
3 ¼,	—— 69	5	10	0
4 ¼,	—— 70	4	15	10
5 ¼,	—— 71	4	1	8
6 ¼,	—— 72	3	10	0
7 ¼,	—— 73	2	18	4
8 ¼,	—— 74	2	8	8
9 ¼,	—— 75	1	19	9

TABLE LIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 66¾ to 67½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 67	7	16	6
1 ¼,	—— 68	6	17	2
2 ¼,	—— 69	6	0	3
3 ¼,	—— 70	5	3	5
4 ¼,	—— 71	4	9	5
5 ¼,	—— 72	3	15	5
6 ¼,	—— 73	3	4	0
7 ¼,	—— 74	2	12	7
8 ¼,	—— 75	2	3	0

TABLE LIV.

For the Uſe of Country Pariſhes

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 67¾ to 68½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 68	7	9	6
1 ¼,	—— 69	6	10	1
2 ¼,	—— 70	5	13	4
3 ¼,	—— 71	4	16	7
4 ¼,	—— 72	4	2	10
5 ¼,	—— 73	3	9	0
6 ¼,	—— 74	2	17	6
7 ¼,	—— 75	2	7	1

TABLE LV.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any age from 35 to 75.

Age of the Purchaſer from 68¾ to 69½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 69	7	2	4
1 ¼,	—— 70	6	3	1
2 ¼,	—— 71	5	6	5
3 ¼,	—— 72	4	9	9
4 ¼,	—— 73	3	16	3
5 ¼,	—— 74	3	2	8
6 ¼,	—— 75	2	11	2

TABLE

TABLE LVI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $69\frac{3}{4}$ to $70\frac{3}{4}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	70	6	15	1
$1\frac{1}{4}$,	——	71	5	15	9
$2\frac{1}{4}$,	——	72	4	19	3
$3\frac{1}{4}$,	——	73	4	2	9
$4\frac{1}{4}$,	——	74	3	9	0
$5\frac{1}{4}$,	——	75	2	16	0

TABLE LVII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $70\frac{3}{4}$ to $71\frac{3}{4}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	71	6	7	9
$1\frac{1}{4}$,	——	72	5	8	6
$2\frac{1}{4}$,	——	73	4	12	1
$3\frac{1}{4}$,	——	74	3	15	9
$4\frac{1}{4}$,	——	75	3	1	11

TABLE

TABLE LVIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $71\frac{3}{4}$ to $72\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	72	6	0	0
$1\frac{1}{4}$,	——	73	5	0	11
$2\frac{1}{4}$,	——	74	4	4	2
$3\frac{1}{4}$,	——	75	3	8	6

TABLE LIX.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $72\frac{3}{4}$ to $73\frac{1}{2}$.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	73	5	12	6
$1\frac{1}{4}$,	——	74	4	13	4
$2\frac{1}{4}$,	——	75	3	16	3

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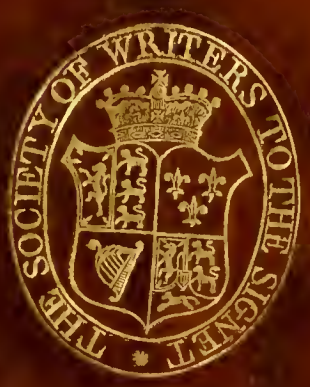
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